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A POTENTIAL PROBLEM FOR THE POTENTIAL MODEL

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A B S T R A C T

The difference between the b and c quark masses is determined by two methods which give conflicting results.

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For a decade the highly successful quark potential model¹⁾ has been based on the existence of a unique flavour independent interquark potential. This potential has been generally assumed to have a Coulomb-like singularity at short distance and to increase linearly at large distance.

In this paper we have two objectives. First we use the measured D, D* and B flavoured heavy meson masses to extract an accurate value for the mass difference between the b and c quarks. We then compare this mass difference to the corresponding result from heavy quarkonia analyses. Both from explicit fits and from rigorous inequalities like those of Bertlmann and Martin²⁾ (BM), we show that the heavy-light and heavy-heavy results are inconsistent.

(I) HEAVY-LIGHT ANALYSIS

The extraction of the $m_b - m_c$ quark mass difference using heavy-light states seems straightforward. For s wave quarks, meson masses are expressed as

$$m = m_Q + m_q + E(\mu_q) + \frac{A}{m_Q} \langle \vec{S}_1 \cdot \vec{S}_2 \rangle \quad (1)$$

where the meson is in a spin $\vec{S} = \vec{S}_1 + \vec{S}_2$ singlet or triplet state. The essential simplifying feature³⁾ in heavy-light dynamics is that the binding energy $E(\mu)$ is a function only of the reduced mass

$$\mu_q = \frac{m_q m_Q}{m_Q + m_q} \simeq m_q - \frac{m_q^2}{m_Q} \quad (2)$$

and hence will vary little as a function of the heavy quark mass m_Q . The binding energy will then nearly cancel when we consider differences in meson masses.

From the difference between the spin triplet D* and the spin singlet D meson masses, we find from Eq. (1)

$$m_{D^*} - m_D = \frac{A}{m_c} \quad (3)$$

From the difference between the spin singlet B and D mesons we obtain, from Eqs. (1) and (3),

$$m_B - m_D = m_b - m_c + \Delta E + \frac{3}{4} (m_{D^*} - m_D) \left(\frac{m_b - m_c}{m_b} \right) \quad (4)$$

where

$$\Delta E = E(\mu_b) - E(\mu_c) \quad (5)$$

It only remains to estimate the binding change ΔE . Even though potential models may not be strictly justified for heavy-light systems, we believe that the small correction ΔE can be reasonably estimated. Since the light, charm, and spin averaged (D, D^*) masses are roughly 0.3, 1.5, 2.0 GeV, respectively, we expect that the binding energy $E(\mu_c)$ is about 200 MeV.

By approximating the potential between the heavy and light quarks by a harmonic oscillator, the derivative of the binding energy with respect to the reduced mass is

$$\frac{\partial E}{\partial \mu} = -\frac{E}{2\mu} \quad (6)$$

and by Eq. (2) we have

$$\Delta E \approx \frac{\partial E}{\partial \mu} \Delta \mu \approx -\frac{E}{2m_c m_b} (m_b - m_c) \quad (7)$$

Combining Eqs. (4) and (7) and solving for $m_b - m_c$, we find

$$m_b - m_c = (m_B - m_D) \left[1 + \frac{m_q E}{2m_c m_b} - \frac{3}{4} \frac{(m_{D^*} - m_D)}{m_b} \right] \quad (8)$$

Numerically we use ^{4),5)} $m_{B_0} = 5274$ MeV, $m_{D_+} = 1869$ MeV, $m_{D^*} - m_D = 142$ MeV and we approximate $E \approx 200$ MeV, $m_q \approx 300$ MeV, $m_c \approx 1500$ MeV and $m_b \approx 4800$ MeV to obtain

$$m_b - m_c = 3405 [1 + 0.0042 - 0.0222] = 3342 \text{ MeV} \quad (9)$$

A conservative error estimate would be 20 MeV. The binding correction is probably the least reliable but since it only amounts to 14 MeV, a factor of two error is still unimportant.

(II) HEAVY-HEAVY ANALYSIS

Numerous potential models of heavy quarkonia have been proposed. One interesting aspect of these models, as pointed out by Martin⁶⁾, is that the difference between the b and c quark masses is much better determined than the quark masses individually. As examples, we mention two recent analyses: Buchmüller and Tye⁷⁾ and Hagiwara, Jacobs, Olsson and Miller⁸⁾, which agree in this respect and yield the value

$$m_b - m_c = 3410 \pm 20 \text{ MeV} \quad (10)$$

This exceeds the heavy-light result of Eq. (9) by 66 MeV.

An alternative approach exploits the inequalities established by BM where specific potentials are not used. By adopting reasonable qualitative criteria for the interquark potential, BM were able to show that

$$\begin{aligned} m^{1/3} T(m) & \text{ increases as } m \text{ increases} \\ T(m)/m & \text{ decreases as } m \text{ increases} \end{aligned} \quad (11)$$

where $T(m)$ is the expectation of the ground state kinetic energy for a quark of mass m . Integration of the Feynman-Hellman theorem⁹⁾ then yields the quark mass difference

$$m_b - m_c = \frac{1}{2} [m_{1s}(b\bar{b}) - m_{1s}(c\bar{c})] + \frac{1}{2} \int_{m_c}^{m_b} \frac{T(m) dm}{m} \quad (12)$$

where m_{1s} is the spin averaged ground state mass. The experimental values⁸⁾ of the lowest spin averaged p wave and the two lowest s wave masses in the $c\bar{c}$ and $b\bar{b}$ systems determine $T(m)$ at the c and b quark masses,

$$\begin{aligned} T(m_c) &= 431 \text{ MeV} \\ T(m_b) &= 420 \text{ MeV} \end{aligned} \quad (13)$$

The relations of Eq. (11) then can be expressed in terms of inequalities which are illustrated in Fig. 1. Upper and lower limits on $m_b - m_c$ are determined by

integrating Eq. (12) on the upper and lower curves of Fig. 1. The results are

$$\begin{aligned}
 m_b - m_c &< \frac{1}{2} [m_{1S}(b\bar{b}) - m_{1S}(c\bar{c})] - \frac{1}{2} [T(m_c) + 3 T(m_b)] + 2 \left(\frac{m_b}{m_c} \right)^{\frac{1}{4}} T(m_c)^{\frac{1}{4}} T(m_b)^{\frac{3}{4}} \\
 m_b - m_c &> \frac{1}{2} [m_{1S}(b\bar{b}) - m_{1S}(c\bar{c})] + \frac{1}{2} [3 T(m_c) + T(m_b)] - 2 \left(\frac{m_c}{m_b} \right)^{\frac{1}{4}} T(m_c)^{\frac{3}{4}} T(m_b)^{\frac{1}{4}}
 \end{aligned}
 \tag{14}$$

The upper limit is BM's but the lower limit is new.

Using $m_{1S}(b\bar{b}) = 9452$ MeV and $m_{1S}(c\bar{c}) = 3068$ MeV we can evaluate^{*)} the limits of given quark masses. The charm quark mass is not well known but no doubt lies between 1.2 GeV and 1.7 GeV with a preferred value⁸⁾ between 1.4 and 1.5 GeV. Since the quark mass difference is well determined, we shall assume that $m_b \approx 3.4 + m_c$ GeV in the calculation of the limits of Eq. (14). Variation of m_b by 100 MeV only changes the limits by 3 MeV.

In Fig. 2, we plot the upper and lower bounds as a function of the charm quark mass. We see again that the heavy-light value of $m_b - m_c$ lies about 70 MeV below the lower limit provided by heavy-heavy potential theory. For completeness, we show the result of a specific potential fit⁸⁾ to all available data.

(III) DISCUSSION

There are ways of resolving the above contradiction but they may involve giving up one of the usual assumptions of the potential model. Although the binding energy between heavy and light quarks should not depend strongly on the heavy quark mass, it is conceivable that we are making a large error in our estimate of ΔE in Eq. (8). On the other hand, the accurate prediction of the number of sharp up-silon states³⁾ was very impressive. The hyperfine correction of Eq. (1) works well in predicting the K^*-K splitting in the $(s\bar{s})$ system and will soon be checked when the B^*-B difference is measured.

*) Table 1 of Ref.8) has a misprint; $m_{1S}(b\bar{b}) = 9452$ not 9552 MeV. If vector masses are used $\frac{1}{2}(m_T - m_\psi) = 3182$ is only 9 MeV below $\frac{1}{2}[m_{1S}(b\bar{b}) - m_{1S}(c\bar{c})]$.

It is somewhat easier to enumerate changes in heavy quarkonia potentials which might imply a smaller $m_b - m_c$. A spin averaged potential with at least one oscillation can reduce the quark mass difference. Potentials of this sort are not compatible with the BM criteria and hence can violate the bounds on the mass difference. Another possibility is to give up strict flavour independence. Small flavour dependences are certainly introduced by relativistic corrections so perhaps some flavour dependence in the static potential is acceptable. An additional possible explanation¹⁰⁾ of at least part of the apparent shift of $m_b - m_c$ between the heavy-light and heavy-heavy systems might be a running mass or self-energy change associated with the change in scale.

A conflict between the two ways of computing $m_b - m_c$ definitely seems to be present, but it is not a gross effect. The flavour dependence introduced by the Bell-Bertlmann potential¹¹⁾ is of a different magnitude. In this case the leading power term at short distance, beyond the Coulomb singularity, is proportional to $m_Q r^4$. No reasonable amount of hiding this behaviour¹²⁾ will result in a successful potential.

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FIGURE CAPTIONS

Figure 1: Bounds on the expectation value of the quarkonium ground state kinetic energy as a function of quark mass. The boundary curves follow from the inequalities of Bertlmann and Martin, Eq. (11), and the calculated values of $T(m)$ for the $c\bar{c}$ and $b\bar{b}$ systems; Eq. (13).

Figure 2: Allowed region for $m_b - m_c$ from the bounds of Eq. (14). The bounds are plotted as a function of the charm quark mass m_c . Also shown are the values of $m_b - m_c$ from the heavy-light analysis, Eq. (9), and from a recent fit, Eq. (10), to $c\bar{c}$ and $b\bar{b}$ data.

FIG. 1

FIG. 2