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THE INSTANTON-INDUCED VACUUM STRUCTURE OF SQCD

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A B S T R A C T

The vacuum condensates of supersymmetric QCD with an arbitrary number of flavours (below the asymptotic freedom bound) are calculated by extending previous instanton methods. The outcome is in complete agreement with large-distance clustering and with the effective Lagrangian results.

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Vacuum condensates in supersymmetric quantum chromodynamics (SQCD) are strongly restricted¹⁻⁵ by unbroken supersymmetry and by ordinary and anomalous Ward identities. The fields of an $SU(N)$ -SQCD with M flavors are gauge fields $(\lambda^a, F_{\mu\nu}^a)$ contained in the superfield $W^a(x, \theta)$ ($a = 1, \dots, N^2-1$) and matter and antimatter fields $(\phi_\alpha^i, \psi_\alpha^i)$ and $(\tilde{\phi}_i^\alpha, \tilde{\psi}_i^\alpha)$, respectively, contained in chiral superfields $m_\alpha^i(x, \theta), \tilde{m}_i^\alpha(x, \theta)$. ($\alpha = 1, \dots, N; i = 1, \dots, M$)

If the flavor symmetry of the gauge interaction is softly broken by a supersymmetric mass term

$$\mathcal{L}_m = - \sum_{i=1}^M \int d^2\theta m_i m_\alpha^i \tilde{m}_i^\alpha \quad (1)$$

SUSY is known to be unbroken. It has been proposed^{4, 5} that in the SUSY vacuum the following condensates exist:

$$\langle \lambda \lambda \rangle \sim \frac{M}{\prod_{j=1}^M} (m_j)^{1/N} \quad (2)$$

$$\langle \phi^i \tilde{\phi}_i \rangle \sim m_i^{-1} \frac{M}{\prod_{j=1}^M} (m_j)^{1/N} \quad (3)$$

These relations were first indicated by a Wess-Zumino-type effective Lagrangian⁴ based on the axial anomaly. More general arguments using analyticity have been given in ref. 5; again the axial anomaly is crucial in this argument. The axial anomaly is related to topological structure, and very recently⁶ Rossi and Veneziano computed certain Green's

functions of the type

$$T < \frac{\alpha}{8\pi} \lambda\lambda(x_1) \dots \phi\tilde{\phi}(y_1) \dots >$$

in the $n = 1$ instanton field⁷ using supersymmetric instanton techniques developed in ref. 8. Since $\lambda\lambda$ and $\phi\tilde{\phi}$ are the lowest components of chiral superfields, such Green's functions should be constant for unbroken SUSY, and indeed some of them are calculated as constants unequal to zero in the instanton vacuum^{6, 8, 9}. Contrary to ref. 8 this result of an instanton calculation is taken seriously at small distances. A cluster decomposition at larger distances leads to condensates $\langle \lambda\lambda \rangle, \langle \phi\tilde{\phi} \rangle$. These of course cannot be explained by instantons, and one has to find other (but somewhere related) nonperturbative effects¹⁰. Using an identity based on an operator anomaly¹¹, the authors⁶ derive equations for the condensates $\langle \lambda\lambda \rangle, \langle \phi\tilde{\phi} \rangle$ in the cases where the number of flavors M is smaller or equal to the number of colors M . These are in complete agreement with (2) and (3).

In this letter I discuss a more general class of Green's functions in the instanton vacuum, and (2) and (3) are derived for all M if clustering is assumed. The constants of proportionality can be calculated.

Consider in SQCD with N colors and M flavors the set of N -point Green's functions

$$\overline{G}_N^{M, K} = T < \frac{\alpha}{8\pi} \lambda\lambda(x_1) \dots \frac{\alpha}{8\pi} \lambda\lambda(x_{N-K}) \phi^1\tilde{\phi}_1(x_{N-K+1}) \dots \phi^K\tilde{\phi}_K(x_N) > \quad (4)$$

(N, M)

with $0 \leq K \leq M$, $K \leq N$.

Through the perturbation (1) it can be connected to a Green's function which has a purely instanton-induced contribution. (has $2N$ λ -legs and $2M$ $\psi/\tilde{\psi}$ -legs, where ϕ counts as $\psi\lambda$, see fig. 1):

$$\begin{aligned} \overline{G}_N^{M,K} &= \prod_{i=1}^{M-K} \int d^4 x_{N+i} m_{i+K} \times \\ &\times T < \frac{\alpha}{8\pi} \lambda \lambda(x_1) \dots \frac{\alpha}{8\pi} \lambda \lambda(x_{N-K}) \phi^1 \tilde{\phi}_1(x_{N-K+1}) \dots \phi^K \tilde{\phi}_K(x_N) \times \\ &\times \psi^{K+1} \tilde{\psi}_{K+1}(x_{N+1}) \dots \psi^M \tilde{\psi}_M(x_{N+M-K}) >_{(N,M)} \end{aligned} \quad (5)$$

In the instanton contribution, the dependence on x of the fermionic zero modes¹² $\psi \tilde{\psi}$ is simple and can be integrated out explicitly:

$$\frac{2}{\pi^2} \int d^4 x \, g^2 [(x-x_0)^2 + g^2]^{-3} = 1 \quad (6)$$

The remaining zero modes are now just those necessary to give a constant $\overline{G}$ after inserting them in $\lambda \lambda(x_i)$ and $\phi \tilde{\phi}(x_j)$ and after integration over the size g and the position x_0 of the instanton. The calculation involved is precisely the one of ref. 6 for N colors and K flavors. One obtains

$$\begin{aligned} \overline{G}_N^{M,K} &= C_N^{M,K} \prod_{i=K+1}^M m_i \Lambda^{3N-M} \\ &= C_N^{M,K} \Lambda^{3N-K} \prod_{i=K+1}^M \left(\frac{m_i}{\Lambda} \right) \end{aligned} \quad (7)$$

where for $m_i \rightarrow 0$ the $C_N^{M,K}$ are dimensionless numerical constants and

$$\Lambda = \mu \exp \left(-8\pi^2 / (3N-1) g^2(\mu) \right) \quad (8)$$

is the one-loop renormalization group invariant scale parameter.

If we now use, as in ref. 6, clustering at large separations of the x_i , it is easy to see that the solution of the resulting huge set of equations for $\langle \lambda\lambda \rangle$ and $\langle \phi\tilde{\phi} \rangle$ indeed requires the m_i -dependence of eqs. (2) and (3).

Consistency also requires constraints among the constants $C_N^{M,K}$:

$$C_N^{M,K} = \alpha^K C_N^{M,0} \quad \left(\begin{array}{l} 0 \leq K \leq M \\ K \leq N \end{array} \right) \quad (9)$$

The validity of these equations could be checked and the constant α determined by explicit calculations, but such an exercise is complicated though straightforward. The confirmation of (9) appears to be guaranteed^x by use⁶ of the operator identity¹¹

$$\{ \bar{Q}_{\dot{\alpha}}, \bar{\Psi}^{\dot{\alpha}} \phi(x) \} / 2\sqrt{2} = -m \tilde{\phi} \phi(x) + \frac{\alpha}{8\pi} \lambda \lambda(x) \quad (10)$$

where the left-hand side is zero in unbroken SUSY Green's functions.

This leads to relations

^xThis applies even if the SUSY instanton vacuum is not the "real" one. A similar argument holds for the constancy of the functions $\bar{G}$ in the instanton approximation.

$$\bar{G}_N^{M,K} = m_{K+1} \bar{G}_N^{M,K+1} \quad (11)$$

and with eq. (7) to

$$C_N^{M,K} = C_N^{M,K+1} \quad (12)$$

i.e. to eq. (9) with $\alpha=1$.

Hence (2) and (3) can be written more precisely as

$$\langle \frac{\alpha}{8\pi} \lambda \lambda \rangle = (C_N^{M,0})^{1/N} \Lambda^3 \prod_{j=1}^M \left(\frac{m_j}{\Lambda} \right)^{1/N} \quad (2')$$

$$\langle \phi^i \tilde{\phi}_i \rangle = (C_N^{M,0})^{1/N} \frac{\Lambda^3}{m_i} \prod_{j=1}^M \left(\frac{m_j}{\Lambda} \right)^{1/N} \quad (3')$$

Note that for $M \leq N$ our method gives the results of ref. 6, which only discusses the functions $\bar{G}_N^{M,M}$ and makes systematic use of the identities¹¹ (10). However, for $M > N$ the approach of ref. 6 becomes inadequate because of the nonexistence of $\bar{G}_N^{M,M}$, while we consider the Green's functions $G_N^{M,K}$ ($K \leq N$) and arrive at a prediction even in that case.

Generalizing to gauge theories with an arbitrary (anomaly-free) set of chiral fields, we propose the following: Consider all Green's functions with lowest component chiral fields, which can be constructed by the pure instanton interaction and by perturbative inclusion of all terms of the

interaction Lagrangian. Then use the clustering argument of ref. 6 to extract maximal information about the irreducible chiral scalar and gaugino condensates.

In the effective Lagrangian of ref. 7 the case $N < M$ is excluded. Indeed the determinant $\det T_{iK}$ is zero because of Bose statistics, if the T_{iK} are local fields $m_{\alpha}^i(x) \tilde{m}_{\alpha}^{\kappa}(x)$ and if the effective Lagrangian is strictly local. However, the instanton calculation which leads to non-trivial results for $N < M$ in agreement with the effective Lagrangian of ref. 4 involves nonlocal effects and - even more important - is only a vehicle to calculate at small distances the constant Green's functions. Using the clustering argument, we completely lose the locality of the interaction.

An instanton-induced Green's function involving $2K$ lowest component chiral fields $\phi, \tilde{\phi}$ is easily constructed for $K \geq N$, if one takes "direct" $\langle \lambda \lambda \rangle$ condensates out of the vacuum. These have to exist because of clustering⁶. Indeed one obtains^x

$$T \langle \phi^1 \tilde{\phi}_1(x_1) \dots \phi^K \tilde{\phi}_K(x_K) \rangle \sim m_{K+1} \dots m_M \Lambda^{3N-M} \left(\frac{\alpha}{8\pi} \lambda \lambda \right)^{K-N} \quad (13)$$

Thus the gaugino condensate field plays an important role and should appear in the effective Lagrangian.

^xOne can also calculate the Green's functions $\langle \phi^{i_1} \tilde{\phi}_{j_1}(x_1) \dots \phi^{i_K} \tilde{\phi}_{j_K}(x_K) \rangle$ in the instanton field and obtains the r.h.s. of (13) multiplied with a tensor $\epsilon_{i_1 \dots i_K} \epsilon_{j_1 \dots j_K}$, i.e. with $SU(K) \times SU(K)$ symmetry. Clustering then seems to lead to strange results. But (comment of the authors of ref. 6) in this expression one still has $SU(K) \times SU(K)$ degenerate vacua! It reduces to (13) after symmetry breaking.

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Fig. 1