TRANSVERSE ENERGY-ENERGY CORRELATIONS: A TEST OF PERTURBATIVE QCD
FOR THE PROTON-ANTIPROTON COLLIDER

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A B S T R A C T

The energy-energy correlation function, and its associated asymmetry, has proved a powerful technique for quantitative tests of perturbative quantum chromodynamics in high energy e^+e^- annihilation. Here we present the natural analogue for the $\bar{p}p$ collider, constructed from the transverse energies and azimuthal angles of the final state hadrons. Leading order QCD predictions are calculated. We show how the correlation function provides a measure of three-jet production which depends only weakly on the parton structure functions, and should therefore allow a direct measurement of the QCD coupling constant α_s .

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The production of large p_T jets at the CERN $p\bar{p}$ collider is now firmly established as an important testing ground for perturbative QCD. The dominant two-jet structure seen in large E_T events at the expected rate¹⁾ is the most striking success of the QCD parton picture²⁾. The observed multi-jet events³⁾ also have a natural interpretation as evidence of hard parton bremsstrahlung^{4),5)}. Testing the theory at a more precise quantitative level is, however, rather difficult. The theoretical uncertainties associated with parton structure functions and higher order corrections (K-factors, higher twist contributions, etc.) translate directly into a large "systematic" uncertainty in the predicted two-jet cross-section. Since similar uncertainties presumably afflict the multi-jet cross-sections also, an alternative approach is to consider the ratio of, say, three-jet to two-jet production which should in principle be less sensitive to structure functions and more sensitive to the underlying QCD parton dynamics. Of course, such a ratio is also, in some sense, a measurement of α_s . The motivation for the present analysis is to find a quantity which makes this statement more precise.

In e^+e^- annihilation into hadrons, there is no structure function ambiguity and many tests have been developed for comparing data with the predictions of QCD⁶⁾. Paramount among these is the energy-energy correlation function measurement - the energy weighted angular distribution of produced hadron pairs^{7),8)}. A detailed discussion of the theory and phenomenology can be found for example in Refs. 9) and 10), but the principal advantages are that it is: (a) calculable directly in perturbation theory; (b) easily measured in experiments, and (c) stable with respect to higher order corrections.

The UA1/UA2 $p\bar{p}$ collider experiments are able to measure transverse hadronic energy flow over the complete azimuthal range in a restricted central rapidity region. We therefore consider events with large transverse energy E_T and define the transverse energy-energy correlation (EEC) function

$$\frac{\int_{E_T^{\min}}^{\sqrt{s}} dE_T \frac{d^2 \Sigma'}{dE_T d\phi}}{\int_{E_T^{\min}}^{\sqrt{s}} dE_T \frac{d\sigma}{dE_T}} = \frac{1}{N} \sum_{A=1}^N \frac{1}{\Delta\phi} \sum_{\text{pairs in } \Delta\phi} \frac{2 E_{Ta}^A E_{Tb}^A}{(E_T^A)^2} \quad (1)$$

The first sum on the right-hand side is a sum over events A with total transverse energy $E_T^A = \sum_a E_{T_a}^A \geq E_T^{\min}$. The second sum is over pairs of hadrons (a, b) whose transverse momenta have relative azimuthal angle ϕ to $\phi + \Delta\phi$. With this definition, the correlation function satisfies the usual sum rule $\int_0^\pi d\phi(\dots) = 1$ when self-correlations (a = b) at $\phi = 0$ are included⁷⁾.

In leading order QCD the transverse energy spectrum $d\sigma/dE_T$ is a convolution of parton structure functions with $2 \rightarrow 2$ hard scattering subprocesses ($O(\alpha_s)^2$). Likewise for $\phi \neq 0^0, 180^0$ the energy weighted cross-section $d^2\Sigma/dE_T d\phi$ is a convolution of structure functions with $2 \rightarrow 3$ subprocesses, e.g., $Q\bar{Q} \rightarrow Q\bar{Q}G$, etc., ($O(\alpha_s)^3$), i.e., schematically,

$$\frac{1}{\sigma'} \frac{d\Sigma'}{d\phi} = \frac{\sum_{a_i, b_i} G_{a_i/p} * G_{a_2/\bar{p}} * \hat{\sigma}^{a_1 a_2 \rightarrow b_1 b_2 b_3}}{\sum_{a_i, b_i} G_{a_i/p} * G_{a_2/\bar{p}} * \hat{\sigma}^{a_1 a_2 \rightarrow b_1 b_2}} \quad (2)$$

where we have introduced the notation

$$\sigma' = \int_{E_T^{\min}}^{\sqrt{s}} dE_T \frac{d\sigma}{dE_T}$$

etc. It is important to contrast this expression with the corresponding one for e^+e^- annihilation. There, the denominator is simply the total cross-section $\sigma_{\text{tot}}(e^+e^- \rightarrow q\bar{q}) \propto 1/W^2$, where W is the centre-of-mass energy. The numerator also contains a similar factor of W^{-2} and so the correlation function takes the form⁷⁾

$$\frac{1}{\sigma} \frac{d\Sigma^{e^+e^-}}{d\theta} = \frac{\alpha_s(W)}{\pi} F^{e^+e^-}(\theta) \quad (3)$$

The dimensionless function $F^{e^+e^-}(\theta)$ is calculable from the matrix element $e^+e^- \rightarrow q\bar{q}g$ and provides a direct test of QCD dynamics. We can therefore ask whether a similar function $F(\phi)$ exists in the present context, independent of the structure functions of the colliding partons.

The problem with the correlation function defined in Eq. (2) is that the $2 \rightarrow 3$ subprocess cross-sections depend not only on ϕ , but also on the ratio

$E_T^{\min}/\sqrt{s}$ (after removing the factor $1/s$) and the pseudorapidity variable η^*). The $2 \rightarrow 2$ subprocess cross-sections in the denominator also depend on $E_T^{\min}/\sqrt{s}$ and η but in general in a different way, so that the same structure functions are weighted differently in the numerator and denominator. In practice, therefore, what is measured is an "average" angular function $F^{a_1 a_2}$ weighted according to the different structure functions.

However, if one could show that certain normalized distributions for the various subprocesses contributing to the $2 \rightarrow 3$ hard scatterings are similar, and that the same combination of structure functions enters in the $2 \rightarrow 2$ and $2 \rightarrow 3$ cross-sections, at least for some domain of E_T and η values, then the relative differential cross-sections are approximately independent of the structure functions. In what follows, we show that this is precisely what happens for the transverse EEC cross-section, and to a very good approximation ($\sim 10\%$) one may, therefore, write

$$\frac{1}{\sigma'} \frac{d\Sigma'}{d\phi} \approx \frac{\alpha_s(\langle Q \rangle)}{\pi} F(\phi) \quad (4)$$

The numerical calculation of the correlation function is quite straightforward. We use the $2 \rightarrow 2$ and $2 \rightarrow 3$ subprocess matrix elements calculated, for example, in Ref. 4) and 5). All parton masses are set to zero. To remain close to the experimental conditions we have imposed a jet trigger and a cut in the pseudorapidity variable η . The procedure we adopt is to require that at least one parton (jet) has a certain trigger transverse energy, i.e., $\max(E_T^1, E_T^2, E_T^3) \geq E_T^{\text{trigger}}$. We also require that the detected partons lie in a central pseudorapidity range, $|\eta| < \eta_0^+$. (Note that this is not the same as requiring that all three final state partons satisfy this cut).

*) In e^+e^- annihilation, the η -dependence is completely integrable since the initial and final states can be collinear without being singular, in contrast to $p\bar{p}$ collisions, where the η -dependence cannot be completely integrated out.

+) All calculations reported here have assumed the pseudorapidity cut-off $\eta_0 = 2.5$.

In Fig. 1a we show the normalized transverse energy-energy correlation differential cross-section defined as

$$\frac{d\Sigma}{d\phi} \equiv \int_{E_T^{\text{trigger}}}^{\sqrt{s}} dE_T \frac{d^2\Sigma}{dE_T d\phi}$$

for the following $2 \rightarrow 3$ subprocesses

$$\begin{aligned} GG &\rightarrow 1 + 2 + 3 \\ GQ &\rightarrow 1 + 2 + 3 \\ Q\bar{Q} &\rightarrow 1 + 2 + 3 \end{aligned} \quad (5)$$

where 1, 2 and 3 represent all allowed Q , $\bar{Q}$ and G final states. In Figs. 1, we use the $GG \rightarrow 1+2+3$ process to fix the normalization, which is such that

$$\sum^{GG \rightarrow 1+2+3} (30^\circ < \phi < 150^\circ) \equiv \int_{30^\circ}^{150^\circ} d\phi \frac{d\Sigma^{GG \rightarrow 1+2+3}}{d\phi} = 1 \quad (6)$$

The cuts on the angle ϕ serve to remove the dominant contributions from the $2 \rightarrow 2$ subprocesses and subsequent fragmentation, and therefore define the domain in ϕ where fixed order perturbation theory can be used with justification^{*)}.

We remark that the EEC functions for the GG -, GQ - and $Q\bar{Q}$ -scatterings have very similar shapes. The relative contributions are consistent within 10% with the ratio of the colour factors $1:4/9:16/81$ for the GG , GQ ($= Q\bar{Q}$) and $Q\bar{Q}$ initial states over the entire range in ϕ shown in Fig. 1a. The relative cross-sections get closer to this ratio for the backward angles ($\phi \approx 180^\circ$) where the leading log approximation is expected to hold. The remarkable result is that even at $\phi \approx 90^\circ$, the sub-leading terms, which are not expected to show the colour factor ratio, do not substantially change this ratio.

^{*)} The cut on ϕ also removes the dominant effect of initial state bremsstrahlung and intrinsic k_T effects. This, for example, can be seen in the UA2 analysis (Ref. 3)) of the two-jet events.

It is known that the $2 \rightarrow 2$ subprocesses also approximately factorize and contribute in the ratio 1:4/9: 6/81 for the GG-, GQ- ($= G\bar{Q}$ -) and $Q\bar{Q}$ -scattering if, for example, one calculates the angular distribution in the centre-of-mass scattering angle. Thus, one may express the differential distribution in the form¹¹⁾

$$\frac{d^3\sigma}{dx_1 dx_2 d\cos\theta} = \left[\frac{1}{x_1} F(x_1) \right] \left[\frac{1}{x_2} F(x_2) \right] \frac{\pi \alpha_s^2}{2x_1 x_2 s} \frac{d\hat{\sigma}_{2 \rightarrow 2}}{d\cos\theta}$$

where the structure function is approximately

$$F(x) \simeq G(x) + \frac{4}{9} [Q(x) + \bar{Q}(x)] \quad (7)$$

and $G(x)$, $Q(x)$ and $\bar{Q}(x)$ are respectively the gluon, quark and antiquark structure functions of the proton.

It is evident from Fig. 1a that to a good approximation the $2 \rightarrow 3$ subprocesses also factorize like Eq. (7), leading to the result that the transverse EEC is approximately independent of the structure functions. The shape of the transverse EEC functions depends on E_T^{trigger} (or, equivalently, on $x_T^{\text{trigger}} \equiv 2 E_T^{\text{trigger}}/\sqrt{s}$) and this is shown for the process $GG \rightarrow 1+2+3$ in Fig. 1b. However, the approximate relation (7) still holds, becoming more accurate for low E_T^{trigger} . This is shown in Table I, which contains the relative transverse EEC differential cross-sections for two different choices of the x_T^{trigger} .

In Fig. 1c, we show the transverse EEC asymmetry for the subprocess $GG \rightarrow 1+2+3$ defined as

$$\frac{1}{\Sigma'} \frac{d\Sigma'^{\text{asy}}}{d\phi} \equiv \frac{1}{\Sigma'} \left. \frac{d\Sigma'}{d\phi} \right|_{\pi-\phi} - \frac{1}{\Sigma'} \left. \frac{d\Sigma'}{d\phi} \right|_{\phi} \quad (8)$$

The normalization is again such that $\Sigma'(30^\circ \leq \phi \leq 150^\circ) = 1$. We remark that the asymmetry changes sign in the region near $\phi = \pi/2$ as x_T^{trigger} is increased. There is no analogue of this behaviour in e^+e^- annihilation. Thus, it is clear that the transverse EEC and its asymmetry will depend on the

x_T^{trigger} conditions but this is an attribute of the underlying QCD dynamics, rather than of the choice of a particular structure function. Later we show that this qualitative behaviour is maintained if the structure functions are included.

To study the effect of the structure functions on the transverse EEC cross-section we use two different parametrizations of the parton structure functions - those due to Owens and Reya (OR)¹²⁾ with $\Lambda = 0.5$ GeV, and those due to Glück, Hoffman and Reya (GHR)¹³⁾ with $\Lambda = 0.4$ GeV. Roughly speaking the latter have a broader gluon distribution which gives a correspondingly harder E_T -spectrum. Both parametrizations are consistent with deep inelastic scattering data and both give a qualitatively good fit to the UA1/UA2 high- p_T data, though differing by a factor ~ 2 in rates⁵⁾. The corresponding transverse energy-energy correlation functions for the two structure functions are shown in Fig. 2a, assuming the cut-offs $E_T^{\text{trigger}} = 15$ and 30 GeV and $|\eta| \leq 2.5$. Figure 2b shows the asymmetries defined as usual by

$$\frac{1}{\sigma'} \frac{d\Sigma'}{d\phi}{}^{\text{asy}} = \frac{1}{\sigma'} \frac{d\Sigma'}{d\phi} \Big|_{\pi-\phi} - \frac{1}{\sigma'} \frac{d\Sigma'}{d\phi} \Big|_{\phi} \quad (9)$$

The asymmetry in Eq. (9) is an absolute prediction being proportional to α_s in the sense of Eq. (4), in contrast to Eq. (8). The correlation functions and their asymmetry have the characteristic shapes familiar from e^+e^- annihilation, with collinear parton and soft gluon singularities dominating in the limit $\phi \rightarrow 0^\circ, 180^\circ$, respectively. The structure function dependence is clearly weak, both in shape and normalization. More specifically, Table II shows the integrated correlation functions

$$\Sigma = \int_{20^\circ}^{160^\circ} d\phi \frac{1}{\sigma'} \frac{d\Sigma'}{d\phi} \quad (10a)$$

$$\Sigma^{\text{asy}} = \int_{20^\circ}^{90^\circ} d\phi \frac{1}{\sigma'} \frac{d\Sigma'}{d\phi}{}^{\text{asy}} \quad (10b)$$

for the two sets of structure functions. Also shown are the two-jet cross-sections, σ' . While the latter differ by a factor ~ 2 for the same E_T^{trigger} , the variation in the correlation function is reduced to $< 10\%^*$). We also remark that whereas the inclusive p_T cross-section is a rather sensitive function of E_T^{trigger} , the transverse EEC cross-section and its asymmetry depend mildly on the E_T trigger. This reduces considerably a source of large systematic errors in a quantitative comparison of the collider data with QCD predictions.

Figure 3 compares the shapes of the e^+e^- and $p\bar{p}$ correlation functions, with the GHR structure functions for the latter. We also show for comparison the results obtained by using a constant $2 \rightarrow 3$ matrix element. Here the integrated correlation functions defined in (10a) are the same. The effect of the QCD dynamics is very clear, especially at small ϕ where both the e^+e^- and $p\bar{p}$ functions show characteristic bremsstrahlung behaviour obtained in a gauge theory. On the other hand, the correlation function corresponding to pure phase space in $2 \rightarrow 3$ scattering goes to a constant as $\phi \rightarrow 0$. More precisely for the pure phase space model involving the $2 \rightarrow 3$ scattering one has the behaviour

$$\frac{d\Sigma}{d\cos\phi} \xrightarrow{\phi \rightarrow 0} \text{constant} \quad (11a)$$

$$\xrightarrow{\phi \rightarrow \pi} \ln \frac{1}{\omega}, \quad \omega = \cot^2 \phi/2 \quad (11b)$$

Note that QCD gives an extra power singularity at each end.

Finally, we study numerically the extent to which Eq. (4) holds, i.e., the extent to which the correlation function scales like α_s . This point can, in principle, be studied by varying Λ consistently in the calculation of the

*) The transverse EEC function is a specific example of a quantity where one measures the ratio of two cross-sections. Thus, the normalized transverse-thrust distribution and the normalized p_{out} distributions show similar insensitivity to the structure functions. However, these distributions are known to have large $O(\alpha_s)^2$ corrections in e^+e^- annihilation as well as larger contamination due to the non-perturbative effects in $2 \rightarrow 2$ sub-processes.

production cross-sections for the QCD subprocesses and scaling violations in the structure functions. Of course, a closely related point is the argument of $\alpha_s(Q)$. Clearly, this question can only be answered satisfactorily if one calculates higher order corrections to the $2 \rightarrow 2$ cross-section. However, it is reasonable to assume that higher order corrections are minimized by choosing the argument in $\alpha_s(Q)$ to be $Q = O(E_T)$. For our numerical calculations we choose $Q = \max(E_T^1, E_T^2, E_T^3)$, i.e., the largest parton transverse energy, which satisfies $1/3 \sum_i E_T^i \leq Q \leq \frac{1}{2} \sum_i E_T^i$. With this choice, the argument of α_s in Eq. (2) depends on the variables of the convolution integrals, and the powers of α_s in the numerator and the denominator in Eq. (2) do not cancel in principle. However, as we shall now show, the powers of α_s do tend to cancel in practice.

To study the Λ -dependence we use the GHR structure functions and vary Λ between 0.1 and 0.4. Now each different Λ value corresponds to a different set of structure functions with a different E_T -spectrum. To compensate for this we compute for each Λ the average value of

$$\langle Q \rangle = \frac{1}{\sigma'} \int_{E_T^{\min}}^{\sqrt{s}} dE_T Q \frac{d\sigma}{dE_T} \quad (12)$$

and use this to define an effective angular function for the correlation and its asymmetry

$$\begin{aligned} \frac{1}{\sigma'} \frac{d\Sigma'}{d\phi} &= \frac{\alpha_s(\langle Q \rangle)}{\pi} F(\phi) \\ \frac{1}{\sigma'} \frac{d\Sigma'^{\text{asy}}}{d\phi} &= \frac{\alpha_s(\langle Q \rangle)}{\pi} F^{\text{asy}}(\phi) \end{aligned} \quad (13)$$

If our conjecture about scaling like α_s is correct, then F 's should depend weakly on Λ . Table III shows the integrated correlation and asymmetry functions $F(\phi)$ and $F^{\text{asy}}(\phi)$ for different values of Λ , together with the corresponding values for $\langle Q \rangle$. The percentage variation of F 's over the range in Λ is small ($\lesssim 5\%$) and is a measure of the "systematic" error from the structure function uncertainty due to Λ ^{*)}. Note, however, that the effective value of α_s itself increases by 30% as Λ varies from 0.1 to 0.4.

^{*)} This suggests that the effect of scaling violation in the structure function is small in the transverse EEC function.

We remark that the scale-ambiguity in the absolute EEC cross-section is much smaller compared to the inclusive E_T or p_{out} cross-sections, since these are proportional to higher powers of $\alpha_s(Q)$, thereby exacerbating the scale ambiguity.

To summarize, we have shown how the transverse energy-energy correlation function provides a quantitative test of QCD at the $\bar{p}p$ collider. In particular, we have demonstrated the approximate factorization in this quantity for the $2 \rightarrow 3$ subprocesses, which is very similar to the functional form noted earlier for the angular distributions in $2 \rightarrow 2$ subprocesses¹⁰⁾. A corollary of this result is that the structure function dependence of the transverse EEC cross-section becomes very mild. We estimate it to be of the order of 10% which is probably comparable to the systematic errors in the data, and almost certainly less than the effect of higher order perturbative corrections. Of course, we cannot say anything quantitative about the latter but we note that for the e^+e^- correlation function the $O(\alpha_s)^2$ correction is $\sim 40\%$ and in the relevant angular range simply rescales the lowest order coupling constant^{9),10)}. For the asymmetry, the $O(\alpha_s)^2$ correction is very small. The technical difficulties in performing such an exact higher order calculation for the collider seem overwhelming at present. However, based on the $O(\alpha_s)^2$ e^+e^- -analysis of the EEC functions, we conjecture that the corresponding corrections for the collider correlation functions should have a similar behaviour.

Another important correction in e^+e^- annihilation is the non-perturbative contribution to the QCD parton distributions due to fragmentation. Such a contribution is also expected at the $\bar{p}p$ collider but at a smaller level due to the large energies of the jets. But more importantly, the inclusive jet-trigger available at the collider practically eliminates the fragmentation ambiguity, thus providing a clean quantitative test of the hard scattering picture in high energy hadron-hadron collisions.

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$\phi(\text{degrees})$	$x_T = 0.11$			$x_T = 0.33$		
	$\frac{1}{\Sigma} \frac{d\Sigma}{d\phi}$ (GG) $\times 10^{-3}$	GQ/GG	$Q\bar{Q}$ /GG	$\frac{1}{\Sigma} \frac{d\Sigma}{d\phi}$ (GG) $\times 10^{-3}$	GQ/GG	$Q\bar{Q}$ /GG
30°	2.75	0.38	0.15	6.35	0.38	0.13
45°	2.36	0.40	0.16	5.23	0.38	0.15
60°	2.44	0.40	0.16	4.74	0.39	0.16
75°	2.76	0.41	0.17	4.58	0.39	0.16
90°	3.61	0.41	0.17	4.57	0.39	0.16
105°	5.20	0.42	0.18	4.81	0.40	0.17
120°	8.24	0.42	0.18	6.29	0.40	0.17
135°	13.48	0.42	0.18	10.13	0.40	0.17
150°	25.80	0.42	0.18	19.90	0.40	0.17
Sum	1.00	0.42	0.17	1.00	0.39	0.16

TABLE I Normalized transverse energy-energy correlation cross-sections for $2 \rightarrow 3$ processes involving GG, GQ (= $G\bar{Q}$) and $Q\bar{Q}$ in the initial state at $\sqrt{s} = 540$ GeV and transverse energy trigger $x_T = 2E_T^{\text{trigger}}/\sqrt{s} = 0.11$ and 0.33 .

E_T^{trigger}	GHR structure functions			OR structure functions		
	$\frac{\text{corr}}{\Sigma} / \sigma_{2\text{jet}}$	$\frac{\text{asy}}{\Sigma} / \sigma_{2\text{jet}}$	$\sigma_{2\text{jet}} (\mu\text{b})$	$\frac{\text{corr}}{\Sigma} / \sigma_{2\text{jet}}$	$\frac{\text{asy}}{\Sigma} / \sigma_{2\text{jet}}$	$\sigma_{2\text{jet}} (\mu\text{b})$
15.0	0.41	0.10	34.80	0.38	0.09	24.0
30.0	0.27	0.06	1.00	0.26	0.05	0.46

TABLE II - Integrated $p\bar{p}$ cross-sections $\Sigma^{\text{corr}}(20^\circ \leq \phi \leq 160^\circ)$, $\Sigma^{\text{asy}}(\phi \geq 20^\circ)$ and $\sigma_{2\text{jet}}$ for two values of the transverse-energy trigger and the pseudorapidity cut-off $|\eta| \leq 2.5$.

$\Lambda(\text{GeV})$	$\langle p_T \rangle \text{ GeV}$	$\langle p_T^{\text{max}} \rangle \text{ GeV}$	$\alpha_s(\langle p_T^{\text{max}} \rangle)$	F	F ^{asy}
0.1	74.2	35.6	0.182	4.97 ± 0.05	1.1 ± 0.05
0.2	73.7	36.3	0.171	4.88 ± 0.05	1.06 ± 0.05
0.3	73.7	36.1	0.178	4.77 ± 0.05	1.02 ± 0.05
0.4	72.8	35.9	0.139	4.74 ± 0.05	1.0 ± 0.05

TABLE III - Dependence of the average transverse energy, $\langle p_T \rangle$, the average maximum transverse energy, $\langle p_T^{\text{max}} \rangle$, $\alpha_s(\langle p_T^{\text{max}} \rangle)$, the transverse-energy-energy correlation function F and its asymmetric part F^{asy} on the QCD scale-parameter, Λ for 2 → 3 processes in $p\bar{p}$ collisions. We have assumed $E_T^{\text{trigger}} = 30 \text{ GeV}$, $|\eta| \leq 2.5$ and $\sqrt{s} = 540 \text{ GeV}$. Errors shown are statistical due to the Monte Carlo integrations we use in the calculations.

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FIGURE CAPTIONS

- Figure 1 Transverse energy-energy correlation cross-sections and their asymmetry for the various $2 \rightarrow 3$ scattering processes involving partons at $\sqrt{s} = 540$ GeV, assuming no structure functions. The normalization is that of Eq. (6).
- (a) Relative contributions of the GG, GQ ($= G\bar{Q}$) and $Q\bar{Q}$ scattering cross-sections.
 - (b) Effect of x_T^{trigger} on the shape of the transverse energy-energy correlation, shown for the GG $\rightarrow$ three-parton subprocess.
 - (c) Effect of x_T^{trigger} on the shape of the asymmetry, shown for the GG $\rightarrow$ three-parton subprocess.
- Figure 2 (a) Dependence of the transverse energy-energy correlation cross-section on the structure functions and transverse energy trigger. Solid curves correspond to the GHR parametrization and the broken lines to the OR parametrization.
- (b) Dependence of the asymmetry in the transverse energy-energy correlations on the structure functions and transverse energy trigger.
- Figure 3 Comparison of the normalized energy-energy correlation function in e^+e^- annihilation and transverse energy-energy correlations in $p\bar{p}$ collisions with $|\eta| \leq 2.5$ and $E_T^{\text{trigger}} = 30$ GeV. The solid curve corresponds to the GHR parametrization of the structure functions and the dashed line to $2 \rightarrow 3$ scattering with constant matrix element. For normalization, see text.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 2a

Fig. 2b

Fig. 3