THE MINIMUM INVARIANT MASS - A TECHNIQUE FOR
HEAVY QUARK SEARCHES AT COLLIDER ENERGY

E.L. Berger^{*)}, D. DiBitonto, M. Jacob and W.J. Stirling

CERN - Geneva

A B S T R A C T

Simple scaling arguments based on ISR results suggest that the cross-section for the associated production of heavy quark mesons at the SPS $p\bar{p}$ collider may be quite sizeable. We consider the semi-leptonic decay of such a heavy meson, and emphasize a technique for determining the mass and longitudinal momentum distribution. The important features are illustrated using a simple theoretical model.

^{*)} Permanent address: High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, U.S.A.

In the standard three-family model of quarks and leptons a heavy charge 2/3 partner of the b quark is predicted. With no evidence so far from e^+e^- annihilation experiments, the search for this top quark has shifted to the $p\bar{p}$ collider where much higher energies are accessible. The main difficulty here is in establishing a clean experimental signature above the expected large background. In this letter we emphasize a technique for heavy quark searches at collider energy which should allow a rather precise determination of the mass and production distribution.

Although the method is potentially quite general, the particular process we have in mind is the associated production and semi-leptonic decay of a $t(\bar{t})$ meson - a low p_T hadronic process. Experience with the ISR shows that single large mass diffractive excitation is likely to give a particularly favourable signal to background ratio in such a case¹⁾:

$$\begin{aligned}\bar{p} + p &\rightarrow X + p \\ X &\rightarrow \bar{\Lambda}_t + T + X' \\ T &\rightarrow \mu^+(e^+) + \nu + \text{jet}(b\bar{q})\end{aligned}\quad (1)$$

where $T = (t\bar{q})$, $\bar{\Lambda}_t = (\bar{t}q\bar{q})$. We stress the quark content because the weak decay may not always occur from the lowest mass state. We include also the C mirror reaction where the $\bar{p}$ rather than the p is quasi-elastically scattered. It is difficult to predict the cross-section from first principles, but the observation of a large diffractive charm signal at the ISR¹⁾ suggests that the top quark rate may be sizeable. Assuming that such cross-sections decrease with quark mass as m_q^{-2} for m_q^2/s small²⁾, and noting that associated $\Lambda_c \bar{D}$ production at the ISR is reported to exceed 100 pb¹⁾, we may expect a cross-section of the order of 10-100 nb for the production of a 40-20 GeV top meson at the SPS $p\bar{p}$ collider. Insofar as the baryon is expected to keep some leading role, this mechanism suggests that the meson should be produced predominantly at lower x_F (the Feynman scaling variable) and with a rather small p_T . The T meson should therefore be more easily detected, and more often, since fragments of the $\bar{\Lambda}_t$ baryon are likely to be lost at very forward angles.

The method of analysis which we adopt is applicable only to clean events in which there is no significant contamination from large p_T jets not associated with the parent $T(\bar{T})$; otherwise, it is of course incorrect to

assign missing transverse energy to a single unobserved neutrino. As pointed out in Ref. 3), there is a special charge correlation effect in the semi-leptonic decay which can provide an additional signature. When the antiproton "flares" the $\bar{t}$ quark stays with the antibaryon whereas the t quark forms a meson with eventual μ^+ production. The opposite is true when the proton flares. We shall assume that in an experiment it is possible to isolate events in which only the decay products of a $T(\bar{T})$ meson - in the form of a single jet, a lepton of the appropriate charge, and missing transverse energy - are observed. Events with the wrong charge correlation and/or more than one jet would then correspond to a mixture of $\bar{\Lambda}_t$, T fragmentation.

To summarize, the situation we envisage is of a top meson produced with an a priori unknown (but assumed soft) x_F distribution and with small p_T , which subsequently decays into a lepton, a neutrino and a hadronic jet. We assume that the four-vectors of the muon and jet are measured. The missing transverse energy is ascribed to the neutrino. The minimum invariant mass method uses these measured parameters to determine (a) the mass and (b) the longitudinal momentum distribution of the top meson. We will start by defining the relevant quantities in a general context, and then illustrate the important features using a simple model.

Regarding the unknown neutrino longitudinal momentum as a free parameter z , the invariant mass of the jet- μ - ν system is

$$M^2(z) = \left(E_1 + E_2 + \sqrt{p_{T3}^2 + z^2} \right)^2 - (p_{L1} + p_{L2} + z)^2 - \left| \sum_{i=1}^3 \vec{p}_{Ti} \right|^2 \quad (2)$$

Here the four-momenta are labelled $(E_i, \vec{p}_{Ti}, p_{Li})$ with $i = 1, 2, 3$ for the jet muon, neutrino respectively. Now $M(z) \rightarrow \infty$ as $|z| \rightarrow \infty$ and in fact M has a unique positive minimum, i.e.,

$$\left. \frac{dM}{dz} \right|_{z=z_0} = 0$$

$$z_0 = (p_{L1} + p_{L2}) \left| \vec{p}_{T3} \right| \left[(E_1 + E_2)^2 - (p_{L1} + p_{L2})^2 \right]^{-\frac{1}{2}} \quad (3)$$

We define the minimum mass M^* to be the minimum invariant mass which can be

We define the minimum mass M^* to be the minimum invariant mass which can be constructed for the jet- μ - ν system, i.e., $M^* = M(z_0)$. Substituting for z_0 gives M^* as a function of the measured parameters:

$$\begin{aligned} M^{*2} &= \left[\left\{ (E_1 + E_2)^2 - (P_{L1} + P_{L2})^2 \right\}^{\frac{1}{2}} + |\vec{P}_{T3}| \right]^2 - \left| \sum_{i=1}^3 \vec{P}_{Ti} \right|^2 \\ &= \left[M_{T12} + |\vec{P}_{T3}| \right]^2 - \left| \sum_{i=1}^3 \vec{P}_{Ti} \right|^2 \end{aligned} \quad (4)$$

where M_{T12} is the transverse mass of the system composed of 1 and 2. The second expression for M^* in Eq. (4) shows that our "minimum invariant mass" variable is identical to the "cluster transverse mass" defined by Barger, Martin and Phillips in Ref. 4). We also define a corresponding "longitudinal momentum" for the system:

$$\begin{aligned} P_L^* &= P_{L1} + P_{L2} + z_0 \\ &= (P_{L1} + P_{L2}) \left[1 + \frac{|\vec{P}_{T3}|}{\left[(E_1 + E_2)^2 - (P_{L1} + P_{L2})^2 \right]^{\frac{1}{2}}} \right] \end{aligned} \quad (5)$$

The second expression for M^* in Eq. (4) shows that M^* is a quantity invariant under longitudinal boosts of the parent meson. As we shall now demonstrate, the distributions $d\sigma/dM^*$ and $d\sigma/dP_L^*$ are strongly peaked at the true invariant mass - the mass of the top meson M - and longitudinal momentum of the jet- μ - ν system respectively. They also make maximal use of the experimentally measured quantities (cf. the transverse mass, see below). With sufficient statistics, therefore, the top meson mass and x_F distribution can be readily determined.

To make these statements more precise, we consider a simple model for the process in question. Since $m_t, m_b \gg m_q$ it is reasonable to ignore the light quark spectator in the decay $T \rightarrow B\mu\nu$ and focus instead on $t \rightarrow b\mu\nu$. Consider, therefore, the three-body decay of a heavy quark Q of mass M produced at rest in the laboratory:

$$\begin{aligned} Q &\rightarrow p_1 + p_2 + p_3 \\ Q^\mu &= (M, \vec{0}, 0) \\ p_i^\mu &= (E_i, \vec{P}_{Ti}, P_{Li}) \end{aligned} \quad (6)$$

$$M^* = \left[(E_1 + E_2)^2 - (p_{L1} + p_{L2})^2 \right]^{\frac{1}{2}} + |\vec{p}_{T3}| \quad (7)$$

The minimum mass distribution is defined by

$$\frac{d\Gamma}{dM^*} = \frac{1}{2M} \frac{1}{(2\pi)^5} \int \prod_{i=1}^3 \frac{d^3 p_i}{2E_i} \delta^{(4)}(Q^M - \sum_{i=1}^3 p_i^M) |m|^2 \delta(M^* - \left[(E_1 + E_2)^2 - (p_{L1} + p_{L2})^2 \right]^{\frac{1}{2}} - |\vec{p}_{T3}|) \quad (8)$$

To begin with, set $|m|^2 = \text{constant}$. For this phase space model we obtain, after some algebra, a distribution differential in M^* and $p_T \equiv |\vec{p}_{T3}|$:

$$\frac{1}{\Gamma} \frac{d^2 \Gamma^{PS}}{dM^* dp_T^2} = \frac{8}{M^2 (M^2 - M^{*2})^{\frac{1}{2}}} \frac{(M^* - p_T)}{\{M^2 - (M^* - 2p_T)^2\}^{\frac{1}{2}}} \quad (9)$$

This analytic expression displays a Jacobian peak at $M^* = M$, for all p_T . Thus, selections may be considered which limit p_T , either from above or below, without biasing the M^* distribution. Integrating over p_T , we derive

$$\frac{1}{\Gamma} \frac{d\Gamma^{PS}}{d\xi} = \frac{2\xi^2 - 1}{\sqrt{1 - \xi^2}} \sin^{-1} \xi + \xi \quad (10)$$

with $\xi = M^*/M$. This shows a clear Jacobian peak at the upper limit, $\xi = 1$, i.e., $M^* = M$ (Fig. 1, dashed line). The limiting behaviour is

$$\frac{1}{\Gamma} \frac{d\Gamma^{PS}}{d\xi} \underset{\xi \rightarrow 1}{\sim} \frac{\pi}{2} (1 - \xi^2)^{-\frac{1}{2}} \quad (11)$$

With the exact weak interaction matrix element for $t(Q) \rightarrow b(p_1) + \mu^+(p_2) + \nu(p_3)$, the distribution becomes (Fig. 1, solid line)

$$\begin{aligned}
 \frac{1}{\Gamma} \frac{d\Gamma^{t \rightarrow b\mu\nu}}{d\xi} &= \frac{\sin^{-1}\xi}{\sqrt{1-\xi^2}} \left[-1 + \frac{7}{2}\xi^2 - 2\xi^4 \right] \\
 &+ \frac{2\xi}{\sqrt{1-\xi^2}} \left[\xi^2 - \frac{2}{3} \right] \\
 &+ \xi \left[\frac{7}{3} - \frac{17}{6}\xi^2 + \xi^4 \right]
 \end{aligned} \tag{12}$$

with limiting behaviour

$$\frac{1}{\Gamma} \frac{d\Gamma^{t \rightarrow b\mu\nu}}{d\xi} \underset{\xi \rightarrow 1}{\sim} \left(\frac{\pi}{4} + \frac{2}{3} \right) (1-\xi^2)^{-\frac{1}{2}} \tag{13}$$

The distributions are clearly very similar, and both show narrow peaks at $\xi = 1$.

From Eq. (4), we see that the minimum mass distribution is invariant under longitudinal boosts and therefore the same curves obtain for a heavy quark with an arbitrary x_F distribution. Furthermore, the distributions are insensitive to final state masses and transverse momentum smearing, provided $m_i, \langle Q_T \rangle \ll M$.

Now consider the distribution in the "longitudinal momentum" P_L^* . In our model, there is only one energy scale M and so, as before, it is convenient to define a dimensionless ratio $R^* = P_L^*/M$. We assume that the parent heavy quark Q has some longitudinal momentum $P_L = RM$ and calculate the corresponding distribution $1/\Gamma d\Gamma/dR^*$. Figure 2 shows the R^* distributions for $t \rightarrow b\mu\nu$ with $R = 0, 2, 4$. The distributions are different for different R (unlike M^* , P_L^* is not boost invariant) but in each case the peak is close to the "true" value of R . This peaking is more pronounced for small R - in fact the distribution vanishes outside the range $R - \sqrt{1+R^2} \leq R^* \leq R + \sqrt{1+R^2}$. So in practice, if the heavy quark is predominantly produced with a certain longitudinal momentum, this will be reflected in a peak at that value in the P_L^* distribution. This result may be used to unfold the experimental R^* spectrum to produce the true R spectrum.

To summarize, we have shown how the mass and longitudinal momentum of a top meson - produced via the mechanism in Eq. (1) - can be reconstructed from measurements of the four-vectors of its observed muon and jet decay products,

using the minimum mass technique. Of course, our model calculations are naïve in that we have not included the effects of detector geometry, experimental cuts, backgrounds, etc. Ultimately, the utility of the method depends on its ability to survive these effects and still yield useful information. Rather than try to anticipate the complicated cuts and thresholds of a realistic experimental situation, we will restrict our attention to two different but representative types of cuts. The first of these is straightforward and amounts to removing the contamination of leptons, neutrinos and jets from other (background) sources. Since these will be predominantly produced at small p_T , we can, for example, require that the jet muon and neutrino each have a transverse momentum greater than some threshold value, $p_T^{\min}$. Our analytic expression Eq. (9) suggests that p_T cuts will not affect the M^* distribution, and we have verified this numerically. Figure 3 shows the M^* distribution for $p_T^{\min} = M/8$. For, say, $M = 40 \text{ GeV}/c^2$ this corresponds to $p_T^{\min} = 5 \text{ GeV}/c$ which is well above all other intrinsic mass scales except that of the b quark. However, the analogous semi-leptonic B meson decay ($b \rightarrow c \mu \nu$) gives muons with the wrong charge correlation. For comparison, the distribution with no cuts from Fig. 1 is also shown. There is evidently very little difference in shape.

The second type of cut relates more to the geometry of the detector. If the muons are required to be central in the laboratory, then the minimum mass distribution is no longer invariant under longitudinal boosts. In fact for top mesons with $\langle x_F \rangle > 0$, such an experimental restriction introduces a bias in favour of events in which the muon is moving backwards in the top meson centre-of-mass frame. In effect, the transverse momentum of the muon in this frame is restricted. To investigate this in our model we take the longitudinal momentum of the heavy quark to be $2M$ and require that the (laboratory) pseudo-rapidity of the muon be between -1 and $+1$. The corresponding M^* distribution is shown in Fig. 4 (solid line). Although the normalization is reduced by a factor of about 4, the characteristic shape is the same, with a pronounced Jacobian peak at $\xi = 1$. We conclude therefore that the minimum mass is indeed a reliable indicator of the true invariant mass, even in the presence of quite severe cuts.

Finally we mention, as a possible alternative, the distribution in transverse mass M_T ⁴⁾, where

$$M_T^2 = \left[\sum_{i=1}^3 \{ |\vec{p}_{T_i}|^2 + m_i^2 \}^{\frac{1}{2}} \right]^2 - \left| \sum_{i=1}^3 \vec{p}_{T_i} \right|^2 \quad (14)$$

It can be shown that the distribution $1/\Gamma d\Gamma/dM_T$ also has a Jacobian peak at $M_T = M$. However, in the presence of the muon rapidity cut discussed above, the transverse mass no longer provides a good estimate of the true mass. This is illustrated in Fig. 4, where the dotted line shows the transverse mass distribution ($\xi = M_T/M$) for the same choice of parameters. Evidently the Jacobian peak has disappeared and the distribution has a maximum at $\xi = 0.85$. This can be understood physically as follows: the central rapidity cut biases the lepton and jet longitudinal momenta towards large values, in opposite directions, thus providing an important contribution to the invariant and minimum masses. On the other hand, the transverse mass variable is blind to this effect.

In conclusion, we believe the minimum mass technique should be a very useful method for estimating the true mass of a system for which one of the decay products is non-interacting. One must assume in the present context that the fragments of the top meson are isolated to a good approximation. However, there are cross checks (single large p_T jets, lepton charge asymmetry) which help in the selection of events. As we have shown, the location of the pronounced peak in the minimum mass distribution is insensitive to typical experimental cuts in other variables.

Although we have applied the method here to the associated production of top mesons, we stress its more general applicability to the analysis of many other systems. The method could also be applied to the analysis of leading particles in the fragmentation of heavy quark jets at large p_T . In a wider context, we note that states predicted in various extensions of the standard model, including supersymmetry, are often characterized by decays with missing transverse energy⁵⁾.

REFERENCES

- 1) For a review, consult: D. DiBitonto, in Proton-Antiproton Collider Physics, 1981 (Madison, WI), eds. V. Barger, D. Cline and F. Halzen (AIP, N.Y., 1982), p. 26.
- 2) M.G. Albrow et al. - Nucl.Phys. B108 (1976) 1;
M. Bozzo et al. - CERN/EP 83-189 (December 1983);
S.J. Brodsky, C. Peterson and N. Sakai - Phys.Rev. D23 (1981) 2745.
- 3) R. Horgan and M. Jacob - Phys.Lett. 107B (1981) 395.
- 4) V. Barger, A.D. Martin and R.J.N. Phillips - Phys.Lett. 125B (1983) 339, 343; Phys.Rev. D28 (1983) 145;
P. Darriulat - Private communication.
- 5) See, for example: J. Ellis - "Supersymmetry and Grand Unified Theories", CERN Preprint TH. 3718 (1983); "Supersymmetry Confronting Experiment", Physics Reports, to be published; eds. D. Nanopoulos and A. Savoy-Navarro.

FIGURE CAPTIONS

Figure 1 Minimum mass distribution for the decay $t \rightarrow b\mu\nu$. Also shown (dashed line) is the distribution for a constant matrix element. Both curves have unit area.

Figure 2 P_L^* ($= MR^*$) distributions for three different heavy quark longitudinal momenta, $P_L = 0, 2M, 4M$. (The distributions are calculated using Monte Carlo integration.)

Figure 3 Minimum mass distribution for the decay $t \rightarrow b\mu\nu$ where each final state particle is required to have transverse momentum $M/8$. Also shown for comparison is the no-cut distribution from Fig. 1 (dashed line).

Figure 4 Minimum mass distribution for $t \rightarrow b\mu\nu$ with $P_L = 2M$ and a cut on the muon pseudorapidity, $|\eta^\mu| \leq 1$. The corresponding transverse mass distribution is also shown (dotted line).

Fig. 1

Fig. 2

Fig. 3

Fig. 4