INSTANTON-ANTI-INSTANTON INDUCED VACUUM ENERGY
IN SUPERSYMMETRIC QCD

R. K. Kaul ^{*)}

CERN -- Geneva

and

Leah Mizrachi

Département de Physique Théorique
Université de Genève
CH-1211 Geneva 4

A B S T R A C T

We show that sufficiently separated instanton-anti-instanton configurations induce a small negative vacuum energy in supersymmetric QCD.

^{*)} On leave from Centre for Theoretical Studies,
Indian Institute of Science, Bangalore, India.

Supersymmetry ensures relative stability of the gauge hierarchy of various mass scales in a grand unified model^{1,2}. It may also provide an explanation as to why these mass scales are so widely separated. To that effect, it would be desirable if supersymmetry were broken dynamically². One possible mechanism could be provided by the non-perturbative quantum fluctuations of the instanton-type in the vacuum³. The role of instantons in supersymmetric gauge theories has been extensively studied in recent times^{4,5}.

How instantons affect the vacuum energy density was studied in the context of non-supersymmetric Yang-Mills theories, a long time ago^{6,7}. In a Yang-Mills gauge theory, there exists an infinity of degenerate classical ground states, characterized by the integer topological charge. Instantons provide a description of quantum mechanical tunnelling between ground states of different topological charge, thereby contributing non-trivially to the vacuum energy density. However, when massless fermions are added to the theory, the picture changes drastically. Due to the zero modes of the relevant Dirac operator in the topologically non-trivial background (with Pontryagin index non-zero), the quantum tunnelling between classical vacua of different topological charge is completely suppressed. Therefore, in supersymmetric Yang-Mills theories, where we have massless fermions, single instantons or anti-instantons (for that matter any configuration with non-zero Pontryagin index) do not contribute to the vacuum energy. These cannot break supersymmetry and vacuum energy stays at zero. However, topologically trivial (Pontryagin index = 0) but non-perturbative configurations, for which there are no exact fermionic zero modes,

may contribute to the vacuum energy. An instanton-anti-instanton configuration would be an example this type. In this paper we shall study these quantum fluctuations in the vacuum of a supersymmetric Yang-Mills theory. For definiteness we shall work with an $SU(2)$ gauge group. In the approximation of sufficiently widely separated instanton and anti-instanton, we show that the vacuum does receive small but non-zero and negative energy. Whether this result is altered when instanton and anti-instanton interactions are introduced or whether other non-perturbative effects wash it out is not clear. In case that does not happen it would suggest a breakdown of supersymmetry. Since the vacuum energy is negative, it cannot be a spontaneous breakdown but would have to be an explicit one.

In order to see the effect of instanton - anti-instanton configuration on the vacuum energy, let us consider a supersymmetric model based on an $SU(2)$ gauge group. Here we have a triplet of gauge fields, W_μ^a ($a = 1, 2, 3$) and a triplet of Majorana fermions, λ^a . The Euclidean action in the Wess-Zumino supergauge reads :

$$S_E = \int d^4x \left[F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\lambda}^a \bar{\Sigma}_\mu (i D_\mu \lambda)^a \right] \quad (1)$$

where $F_{\mu\nu}^a$ is the gluon field strength, $D_\mu^{ac} = \delta^{ac} \partial_\mu + g \epsilon^{abc} W_\mu^b$ is the covariant derivative. The fermions have been expressed in Euclideanised Weyl basis, with Dirac matrices as

$$\gamma_\mu = \begin{pmatrix} 0 & \Sigma_\mu \\ \bar{\Sigma}_\mu & 0 \end{pmatrix}, \quad \Sigma_\mu = (i\sigma_i, 1), \quad \bar{\Sigma}_\mu = (-i\sigma_i, 1)$$

and

$$\Sigma_\mu \bar{\Sigma}_\nu + \Sigma_\nu \bar{\Sigma}_\mu = 2\delta_{\mu\nu}, \quad \bar{\Sigma}_\mu \Sigma_\nu + \bar{\Sigma}_\nu \Sigma_\mu = 2\delta_{\mu\nu}$$

As mentioned earlier, there are infinitely many degenerate ground states. We represent them as $|n\rangle$, $n = 0, \pm 1, \pm 2, \dots$. To evaluate quantum mechanical tunnelling, $|n'\rangle \rightarrow |n\rangle$, we need to evaluate the functional integral

$$\langle n | e^{-H\tau} | n' \rangle \xrightarrow{\tau \rightarrow \infty} \int \mathcal{D}W_\mu \mathcal{D}\lambda e^{-S_E} \quad (2)$$

over all those configurations with Pontryagin index $n - n'$. In particular, for tunnellings between vacua differing by one unit of topological charge, the functional integral has to be evaluated over configurations such as single instanton or single anti-instanton³ :

$$W_\mu^{Ia} = \frac{2}{g} \frac{\eta_{a\mu\nu} (x - x_1)_\nu}{[(x - x_1)^2 + \rho_1^2]} \quad , \quad W_\mu^{\bar{I}a} = \frac{2}{g} \frac{\bar{\eta}_{a\mu\nu} (x - x_2)_\nu}{[(x - x_2)^2 + \rho_2^2]} \quad (3)$$

where, x_1, x_2 are the instanton (anti-instanton) locations, ρ_1, ρ_2 their sizes respectively and $\eta_{a\mu\nu}, \bar{\eta}_{a\mu\nu}$ are 't Hooft symbols.

In order to study the fluctuations around an instanton, we expand the Euclidean action in terms of $Q_\mu^a = W_\mu^a - W_\mu^{aI}$ in the functional integral. Integration over Q_μ^a yields the inverse of the square root of the bose determinant, which has eight zero modes. These are treated by the collective coordinate method, yielding an integration over the instanton location, x_1 , instanton size ρ_1 and the group orientation, Ω . Integration over the fermions results in the square root of the determinant of the Dirac operator $(i\gamma_\mu D_\mu)$. This has four left-handed zero modes,

$$(\lambda_{oss}^{(+)})_{\alpha}^a = \frac{\sqrt{2}}{\pi} \frac{\rho_1^{5/2}}{[(x-x_1)^2 + \rho_1^2]^2} (\sigma^a)_{\alpha}^{\beta} u_{\beta}^{(+)} \quad (4)$$

$$(\lambda_{osc}^{(+)})_{\alpha}^a = \frac{1}{\pi} \frac{\rho_1^{3/2}}{[(x-x_1)^2 + \rho_1^2]^2} (\sigma^a)_{\alpha}^{\beta} \left(\sum_{\mu} (x-x_1)_{\mu} \right)_{\beta} \delta^{\dot{\alpha}} \bar{v}^{(+)}_{\dot{\alpha}}$$

where $u^{(+)} = (1,0), (0,1)$ and $\bar{v}^{(+)} = (1,0), (0,1)$. These are respectively generated by the supersymmetric and superconformal transformation of the instanton⁸. For these fermionic zero modes, we introduce anticommuting collective coordinates, $\alpha_{1,2}^{(+)}$ and $\bar{\beta}_{1,2}^{(+)}$, respectively.

Unlike in a one-dimensional situation, the bose and Fermi determinants with zero modes factored out are equal and hence cancel out^{9,10}. Therefore, the contribution to the functional integral comes only from the zero modes of the Bose and Fermi determinants and can be written as⁶

$$\langle n+1 | e^{-H\tau} | n \rangle_{\pm} = \frac{1}{8\pi^4} \int d^4x \frac{d\rho_1}{\rho_1^5} d\Omega \left(\frac{8\pi^2}{g^2} \right)^4 e^{-8\pi^2/g^2(\rho_1)} \int d^2\alpha^{(+)} d^2\bar{\beta}^{(+)} \quad (5)$$

As is obvious, integration over the fermionic collective coordinates $\alpha_{1,2}^{(+)}$ and $\bar{\beta}_{1,2}^{(+)}$ renders this exactly zero.

A corresponding expression can be written for an anti-instanton. Here we have four right-handed fermionic zero modes :

$$(\bar{\lambda}_{oss}^{(+)})^{a\dot{\alpha}} = \frac{\sqrt{2}}{\pi} \frac{\rho_2^{5/2}}{[(x-x_2)^2 + \rho_2^2]^2} (\sigma^a)^{\dot{\alpha}}_{\dot{\beta}} \bar{u}^{(+)}_{\dot{\beta}} \quad (6)$$

$$(\bar{\lambda}_{osc}^{(+)})^{a\dot{\alpha}} = \frac{1}{\pi} \frac{\rho_2^{3/2}}{[(x-x_2)^2 + \rho_2^2]^2} \left(\sum_{\mu} (x-x_2)_{\mu} \right)^{\dot{\alpha}\beta} (\sigma^a)_{\beta}^{\dot{\gamma}} \bar{v}_{\dot{\gamma}}^{(+)}$$

where $\bar{u}^{(-)} = (1,0), (0,1)$ and $v^{(-)} = (1,0), (0,1)$. These again render single anti-instanton contribution to the functional integral zero.

Although direct tunnellings due to the single instanton or anti-instanton are suppressed, the vacuum still does contain quantum mechanical fluctuations due to non-perturbative configurations with topological charge zero⁷. A configuration which is instanton-like in some part (region I in the Figure) and anti-instanton-like in some other part (region II) and essentially pure vacuum everywhere else (region III) is this kind of fluctuation. The functional integral (2) may, therefore, be rewritten as

$$\langle \theta | e^{-H\tau} | \theta \rangle = \delta(\theta - \theta') \int (\mathcal{D}W_\mu)_0 \mathcal{D}\lambda e^{-S_E} \quad , \quad (7)$$

where the subscript 0 indicates that only configurations with Pontryagin index zero have to be considered. In order to ensure cluster decomposition, the quantum mechanical vacuum is still a linear superposition of the $|n\rangle$'s, $|\theta\rangle = \sum_n e^{in\theta} |n\rangle$.

The contribution to the functional integral (7) due to a pair of instanton and anti-instanton, sufficiently far away from each other, can be written as

$$\langle \theta | e^{-H_T} | \theta \rangle_{\mathbf{i}\bar{\mathbf{i}}} = \delta(\theta - \theta') \left(\frac{1}{8\pi^4} \right)^2 \int d^4x_1 \frac{d\rho_1}{\rho_1^5} d^4x_2 \frac{d\rho_2}{\rho_2^5} d\Omega d\Omega_R$$

$$\cdot \left(\frac{8\pi^2}{g^2(\rho_1)} \right)^4 \left(\frac{8\pi^2}{g^2(\rho_2)} \right)^4 \exp \left[-\frac{8\pi^2}{g^2(\rho_1)} - \frac{8\pi^2}{g^2(\rho_2)} \right] K_0(x_1, x_2, \rho_1, \rho_2, \Omega_R), \quad (8)$$

where, because of the assumption of "diluteness", we have ignored the instanton-anti-instanton interaction. In this approximation, the determinantal factor with the "quasi-zero modes" factored out is simply the product of the instanton and anti-instanton determinantal factors, and each of them is equal to one. $\int d\Omega_R$ is the integration over the relative orientation of the anti-instanton with respect to that of the instanton in the group space, $\int d\Omega$ is the integration over the initial instanton orientation. $K_0(x_1, x_2, \rho_1, \rho_2, \Omega_R)$ is the fermion determinant evaluated in the subspace of fermionic zero modes, listed in Eqs. (4) and (6), with σ^a in Eq. (6) replaced by $\sigma_R^a = R^a_b \sigma^b$, where the matrix R^a_b represents the anti-instanton relative group orientation :

$$K_0 = \det \begin{pmatrix} \lambda_{oss}^{(\psi)} i \Sigma_\mu D_\mu \bar{\lambda}_{oss}^{(\psi)} & \lambda_{oss}^{(\psi)} i \Sigma_\mu D_\mu \bar{\lambda}_{osc}^{(\psi)} \\ \lambda_{osc}^{(\psi)} i \Sigma_\mu D_\mu \bar{\lambda}_{oss}^{(\psi)} & \lambda_{osc}^{(\psi)} i \Sigma_\mu D_\mu \bar{\lambda}_{osc}^{(\psi)} \end{pmatrix} \quad (9)$$

In order to evaluate K_0 , we invoke the approximation that the gauge configuration W_μ^a is essentially pure vacuum everywhere except in the neighbourhoods of the instanton and anti-instanton,

so that the covariant derivative in (9) can be replaced by an ordinary derivative. With this, we may write K_0 as

$$K_0 = \det \begin{pmatrix} -8\rho_1^{5/2} \rho_2^{5/2} I_\mu \sigma^a \Sigma_\mu \sigma_R^a & 4\sqrt{2} \rho_1^{5/2} \rho_2^{3/2} J_{\mu\nu} \sigma^a \Sigma_\mu \sigma_R^a \bar{\Sigma}_\nu \\ -4\sqrt{2} \rho_1^{3/2} \rho_2^{5/2} I_{\mu\nu} \bar{\Sigma}_\mu \sigma^a \Sigma_\nu \sigma_R^a & -4\rho_1^{3/2} \rho_2^{3/2} I_{\mu\nu\alpha} \bar{\Sigma}_\mu \sigma^a \Sigma_\nu \sigma_R^a \bar{\Sigma}_\alpha \end{pmatrix}, \quad (10)$$

where

$$I_\mu; I_{\mu\nu}; I_{\mu\nu\alpha} = \frac{1}{\pi^2} \int d^4x \frac{(x-x_1)_\mu; (x-x_1)_\mu (x-x_2)_\nu; (x-x_1)_\mu (x-x_2)_\nu (x-x_2)_\alpha}{[(x-x_1)^2 + \rho_1^2]^2 [(x-x_2)^2 + \rho_2^2]}$$

and

$$J_{\mu\nu} = \frac{1}{\pi^2} \int d^4x \frac{(x-x_1)_\mu (x-x_2)_\nu}{[(x-x_1)^2 + \rho_1^2]^3 [(x-x_2)^2 + \rho_2^2]^2} \quad (11)$$

All these integrals are only a function of $\Delta^2 \equiv (x_1 - x_2)_\mu (x_1 - x_2)_\mu$.

Equation (10) can be recast as

$$K_0 = \det \begin{pmatrix} 8\rho_1^{5/2} \rho_2^{5/2} I_1 \sigma^a \not{A} \sigma_R^a & 4\sqrt{2} \rho_1^{5/2} \rho_2^{3/2} J' \sigma^a \not{A} \sigma_R^a \bar{A} \\ -4\sqrt{2} \rho_1^{3/2} \rho_2^{5/2} I' \bar{A} \sigma^a \not{A} \sigma_R^a & -4\rho_1^{3/2} \rho_2^{3/2} (I_2 \bar{A} \sigma^a \not{A} \sigma_R^a \bar{A} + I_3 \bar{\Sigma}_\mu \sigma^a \not{A} \sigma_R^a \bar{\Sigma}_\mu) \end{pmatrix}, \quad (12)$$

where $\not{A} = \Sigma_\mu \Delta_\mu$, $\bar{A} = \bar{\Sigma}_\mu \Delta_\mu$ and

$$I_1; I_2; I'; J' = \int_0^1 dz \frac{z^2(1-z)^2; z^3(1-z)^3; z^2(1-z)^3; z^3(1-z)^2}{[z^2\Delta^2 - z(\Delta^2 + \rho_1^2 - \rho_2^2) - \rho_2^2]^3} \quad (13)$$

and

$$I_3 = \frac{1}{4} \int_0^1 dz \frac{z^2(1-z)^2}{[z^2\Delta^2 - z(\Delta^2 + \rho_1^2 - \rho_2^2) - \rho_2^2]^2}$$

Calling $\sigma^a \otimes \sigma^a_R = K (= \Sigma_\mu L_\mu)$, and $I_1 I_2 = I' J' = A$, $I_1 I_3 = B$,

and squaring up the determinant, we have

$$K_0^2 = (32 g_1^4 g_2^4)^4 \det [K (A \bar{K} K \bar{K} + B \bar{\Sigma}_\mu K \bar{\Sigma}_\mu)] \\ \cdot \det [\bar{K} (A \bar{K} \bar{K} K + B \Sigma_\nu \bar{K} \Sigma_\nu)]$$

It is useful to note that $\bar{\Sigma}_\mu K \bar{\Sigma}_\mu = -2\bar{K}$, $\Sigma_\mu \bar{K} \Sigma_\mu = -2K$
and $(\bar{K} K)^2 + (\bar{K} \bar{K})^2 = 4(\Delta \cdot L)^2 - 2\Delta^2 L^2$ so that

$$K_0 = (32 g_1^4 g_2^4)^2 L^2 [(A \Delta^2 + 2B)^2 - 8AB(\Delta \cdot L)^2] \quad (14)$$

In order to perform the integration over the relative orientation of the anti-instanton in Eq. (8), we parametrize the orientation matrix R_{ab} in terms of a unit vector $u_4^2 + \vec{u}^2 = 1$ in the group space

$$R_{ab} = \delta_{ab} + 2 [\epsilon_{abc} u_c u_4 + (u_a u_b - \delta_{ab} \vec{u}^2)]$$

Then $L^2 = \Delta^2 + 8(u \cdot \Delta)^2$ and $\Delta \cdot L = -\Delta^2 + 4(u \cdot \Delta)^2$.

Using

$$\int d\Omega_R u_\mu u_\nu u_\alpha u_\beta = \frac{\pi^2}{12} [\delta_{\mu\nu} \delta_{\alpha\beta} + \delta_{\mu\alpha} \delta_{\nu\beta} + \delta_{\mu\beta} \delta_{\nu\alpha}]$$

we have,

$$\int d\Omega_R K_0 = (32 g_1^4 g_2^4)^2 \pi^2 \Delta^4 [26(A \Delta^2 + 2B)^2 - 80\Delta^2 AB] \quad (15)$$

Since we are working in the approximation of sufficiently separated instanton and anti-instanton, we need to keep only the leading terms in Δ^2 in A and B. These can be seen to be as

follows after evaluating the various integrals in Eq. (13) :

$$A \approx - \frac{1}{4(\Delta^2 + \rho_1^2 + \rho_2^2)^6} \left[21 - 10 \ln \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1 \rho_2} \right)^2 + \ln \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1^2} \right)^2 \ln \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_2^2} \right)^2 \right]$$

$$B = - \frac{1}{4(\Delta^2 + \rho_1^2 + \rho_2^2)^5} \left[\ln \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1 \rho_2} \right)^2 - 3 \right] , \quad (16)$$

so that equation (15) reduces to

$$\int d\Omega_R K_0 = \frac{1}{8(\Delta^2 + \rho_1^2 + \rho_2^2)^8} \left[45 \times 121 - 510 \ln^2 \frac{\rho_2^2}{\rho_1^2} + 13 \ln^4 \frac{\rho_2^2}{\rho_1^2} \right. \\ \left. + \left(248 \ln^2 \frac{\rho_2^2}{\rho_1^2} - 40 \times 129 \right) \ln \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1^2 \rho_2^2} \right)^2 \right. \\ \left. + 26 \left(67 - \ln^2 \frac{\rho_2^2}{\rho_1^2} \right) \ln^2 \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1^2 \rho_2^2} \right)^2 \right. \\ \left. - 248 \ln^3 \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1^2 \rho_2^2} \right)^2 + 13 \ln^4 \left(\frac{\Delta^2 + \rho_1^2 + \rho_2^2}{\rho_1^2 \rho_2^2} \right)^2 \right] . \quad (17)$$

This has to be substituted into Eq. (8). Next we perform the integration over the instanton and anti-instanton locations x_1, x_2 .

Calling the nearest distance of approach between instanton and anti-instanton Δ_0 , we obtain

$$\langle \theta | H | \theta \rangle_{\text{I}\bar{\text{I}}} = - R^3 N \int \frac{d\rho_1}{\rho_1^5} \frac{d\rho_2}{\rho_2^5} \frac{(\rho_1 \mu)^6 (\rho_2 \mu)^6}{(\rho_1^2 + \rho_2^2 + \Delta_0^2)^6} \rho_1^8 \rho_2^8 f(\rho_1^2, \rho_2^2) , \quad (18)$$

where

$$N = \frac{2}{24\pi^2} \left(\frac{8\pi^2}{g^2(\mu)} \right)^8 e^{-16\pi^2/g^2(\mu)}$$

Here we have replaced g^2 by $g^2(\mu)$ in the forefactor N and used the renormalization group equation

$$\frac{g^2}{g^2(\mu)} = \frac{g^2}{g^2(\mu)} - 6 \ln(g\mu) \quad (19)$$

in the exponent. The function $f(\rho_1^2, \rho_2^2)$ is given by

$$f(\rho_1^2, \rho_2^2) = \sum_{n=0}^4 A_n \ln^n \left(\frac{\rho_1^2 + \rho_2^2 + \Delta_0^2}{\rho_1 \rho_2} \right) + \sum_{k=1}^2 \left[B_k \ln^{2k} \frac{\rho_2^2}{\rho_1^2} + C_k \ln^2 \frac{\rho_2^2}{\rho_1^2} \ln^k \left(\frac{\rho_1^2 + \rho_2^2 + \Delta_0^2}{\rho_1 \rho_2} \right) \right], \quad (20)$$

and the numerical constants are, $A_0 \approx 3091.0$, $A_1 \approx -6789.0$, $A_2 \approx 5305.4$, $A_3 \approx -1726.5$, $A_4 = 208$, $B_1 \approx -371.5$, $B_2 = 13$, $C_1 \approx 431.6$ and $C_2 = -104$.

Finally, before doing integration over the scale size in Eq. (18), we have to make a choice for the nearest distance of approach between the instanton and anti-instanton, Δ_0 , as a function of their scales. As has been argued recently by Dynakov and Petrov¹¹, instanton-anti-instanton interactions introduce a hard core repulsive potential already at a separation of about two or three times the instanton size. In particular, they suggest that it happens at a distance $\kappa(\rho_1^2 + \rho_2^2)$, with $\kappa = 3.37$. We take this to be our Δ_0^2 .

The ρ_1, ρ_2 integration in Eq. (18) is ultraviolet finite, but diverges for large instanton and anti-instanton sizes. However, we integrate over ρ_1, ρ_2 up to some average instanton and anti-instanton scale, ρ_c . To do this we write

$\rho_1 \mu = v \cos \alpha$, $\rho_2 \mu = v \sin \alpha$ and integrate over v to obtain :

$$\langle \theta | H | \theta \rangle_{\text{II}} = -\frac{1}{8} R^3 V \frac{\mu^4 (\mu \rho_c)^4}{(1+\kappa)^6} \left[\sum_{n=0}^4 a_n A_n + \sum_{\ell=1,2} (b_\ell B_\ell + c_\ell C_\ell) \right], \quad (21)$$

where

$$\begin{aligned} a_n &= 2 (-1)^n \int_0^{\pi/4} d\alpha \cos \alpha \sin^9 \alpha \ln^n \left(\frac{\sin \alpha \cos \alpha}{1+\kappa} \right), \quad n=0, 1, 2, 3, 4 \\ b_n &= 2 \int_0^{\pi/4} d\alpha \cos \alpha \sin^9 \alpha \ln^{2\ell} (\tan^2 \alpha), \quad \ell=1, 2 \\ c_\ell &= 2 (-1)^\ell \int_0^{\pi/4} d\alpha \cos \alpha \sin^9 \alpha \ln^2 (\tan^2 \alpha) \ln^\ell \left(\frac{\sin \alpha \cos \alpha}{1+\kappa} \right), \quad \ell=1, 2. \end{aligned} \quad (22)$$

Calculating the object in the square brackets in (21) numerically, we obtain the vacuum energy from sufficiently separated instanton-anti-instanton pairs as :

$$\langle \theta | H | \theta \rangle = -C R^3 \left(\frac{8\pi^2}{g^2} \right)^8 \rho_c^{-4} (\mu \rho_c)^{12} \exp \left[-\frac{16\pi^2}{g^2(\mu)} \right], \quad (23)$$

where $C = 3 \times 10^{-4} \left(\frac{2}{21\pi^2} \right)$ for $\kappa = 3.37$. Using renormalization group equation (19), we finally obtain the energy density of the vacuum as

$$E_\theta / R^3 = -C \left(\frac{8\pi^2}{g^2(\rho_c)} \right)^8 \rho_c^{-4} \exp \left[-\frac{16\pi^2}{g^2(\rho_c)} \right]. \quad (24)$$

We have thus shown that instanton-anti-instanton fluctuations do contribute to the vacuum energy in supersymmetric QCD. The vacuum energy is proportional to the fourth power of the inverse of the

scale cut-off ρ_c , as expected. We may take $\rho_c \sim \Lambda_{\text{QCD}}^{-1}$, the strong interaction scale of the $SU(2)$ gauge theory.

This vacuum energy, no doubt, is small. The calculation presented here, indeed, can be improved upon by taking into account the instanton-anti-instanton interaction. Whether that would change our result or if other non-perturbative contributions wipe out the effect of instanton-anti-instanton contributions to the vacuum energy is not clear. If this does not happen the vacuum energy (24) would suggest an explicit breakdown of supersymmetry. Further investigations in this direction have to be made.

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Figure caption

Separated instanton-anti-instanton configuration. Regions I and II contain instanton and anti-instanton respectively, Region III is essentially pure vacuum.