THE NON-LINEAR SUPERSYMMETRIC σ MODEL ON $E_6/SO(10) \times U(1)$

Y. Achiman^{*)}

CERN - Geneva

and

S. Aoyama⁺⁾ and J.W. van Holten

Physics Department, University of Wuppertal

A B S T R A C T

We give the explicit $SO(10) \times U(1)$ decomposition of the E_6 algebra. On this basis we find a non-linear realization of the $E_6/SO(10) \times U(1)$ coset space. We obtain the Kähler potential of this manifold in closed form and use it to construct the corresponding supersymmetric σ model. It involves a 16 family of quasi-Goldstone fermions whose four-fermion interactions are effectively those of an $SO(10)$ GUT. Possible applications are briefly discussed.

^{*)} Permanent address: Physics Department, University of Wuppertal, Postfach 100 127, D-5600 Wuppertal 1, Germany.

⁺⁾ Research Fellow of the Alexander von Humboldt Foundation.

One of the important problems that arise in the construction of preon models and models of strong interactions ¹⁾ is to determine the low-energy properties of the theory; that is, to find the spectrum of light particles and their interactions at energies that are small compared to the scale of binding of the interactions considered. A useful method in this respect is the construction of effective lagrangians.²⁾ Although in principle an effective action can be constructed ab initio, from the dynamics of the underlying model, this is often too difficult in practice. However, once the correct low-energy degrees of freedom are known, the form of the effective action is usually severely restricted by the symmetries of the theory, and in some cases even determined uniquely. For this reason the study of non-linear realizations of symmetry groups has become of considerable importance in elementary particle physics. It goes without saying, that the most difficult problem is to find out what the low-energy degrees of freedom are. No general method exists for finding them, but there are several useful results, such as 't Hooft's anomaly-matching conditions ¹⁾ and Goldstone's theorem that give constraints on the spectrum of light particles. Massless (composite) Goldstone bosons (GB's) arise whenever a global symmetry G is broken down to a subgroup H . It was first suggested by Buchmüller, Peccei and Yanagida³⁾ that light fermions can appear as supersymmetric partners of such GB's, the so-called quasi Goldstone fermions (QGF's). This idea has been applied in many recent models.⁴⁾

In $N=1$ supersymmetry, non-linear lagrangians for scalars and spinors always describe fields taking their values in a Kähler manifold.⁵⁾ Thus if a theory exhibits a break-down $G \rightarrow H$, either G/H is a Kähler manifold itself (in which case there are no other light scalars except the Goldstone particles); or it can be embedded in some Kähler manifold, in which case there are extra light scalars

required by supersymmetry, but not by Goldstone's theorem. In this case the number of light particles is not fixed uniquely and their interactions involve arbitrary functions.

From the point of view of constructing preon models, as well as that of grand unification, a group of considerable interest is E_6 .⁶⁾ In this paper we study its non-linear realization on the coset space $E_6/SO(10) \times U(1)$,^{3), 7)} which is of interest for several reasons. First of all this is a Kähler manifold and therefore admits a unique minimal supersymmetric realization. Secondly, the chiral Goldstone superfields involved in the breaking $E_6 \rightarrow SO(10) \times U(1)$ transform according to a representation 16 of $SO(10)$ that can describe exactly a family of quarks and leptons, and is frequently used in grand unification. Moreover, in E_6 preon models three families of fermions arise naturally.⁸⁾

Finally we note that $E_7/E_6 \times U(1)$ is a Kähler manifold as well, with the QGF's transforming as a 27 family of E_6 . Hence E_6 can arise from a broken supersymmetric E_7 invariance at a higher energy scale. This is of considerable interest for attempts at superunification using $N=8$ supergravity.⁹⁾

In constructing the $E_6/SO(10) \times U(1)$ coset space it is important to have a manifestly $SO(10) \times U(1)$ -covariant decomposition of the E_6 Lie-algebra at our disposal. Such a decomposition has the following form. We split the generators of E_6 into 45 generators $H_{mn} = -H_{mn}^+$ of $SO(10)$, as $SO(10)$ -scalar generator $Y = Y^+$ for $U(1)$, and 16 complex generators X_α (as well as their conjugates $\bar{X}^\alpha = X_\alpha^+$) that transform as chiral spinors under $SO(10)$. This determines most of the commutation relations:

$$\begin{aligned}
 [H_{mn}, H_{kl}] &= \delta_{nk} H_{ml} + \delta_{ml} H_{nk} - \delta_{nl} H_{mk} - \delta_{mk} H_{nl}, \\
 [H_{mn}, Y] &= 0, \\
 [Y, X_\alpha] &= X_\alpha, \quad [Y, \bar{X}^\alpha] = -\bar{X}^\alpha, \\
 [H_{mn}, X_\alpha] &= -\frac{1}{2} (\Gamma_{mn}^+)_{\alpha}{}^{\beta} X_{\beta}, \quad [H_{mn}, \bar{X}^\alpha] = \frac{1}{2} \bar{X}^{\beta} (\Gamma_{mn}^+)_{\beta}{}^{\alpha},
 \end{aligned} \tag{1}$$

Here $\frac{1}{2} \Gamma_{mn}^{\pm}$ are the generators of the chiral spinor representation of $SO(10)$:

$$\Gamma_{mn}^{\pm} = \frac{1}{4} (1 \pm \Gamma_{11}) [\Gamma_m, \Gamma_n], \tag{2}$$

and the Γ_m are the ten-dimensional Dirac-matrices. We use the chiral representation of these matrices, in which Γ_{11} is diagonal:

$$\Gamma_{11} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Then the chiral $\Gamma_m^{\pm}$ and their products can be written as (16x16)-dimensional matrices, corresponding to the unit- and Dirac-matrices in $d=9$.

In order to complete the algebra (1) to E_6 , we must determine the commutators of X and $\bar{X}$ in such a way, that the algebra closes and the Jacobi identities are satisfied. This leads to the result:

$$\begin{aligned}
 [X_\alpha, \bar{X}^\beta] &= (\Gamma_{mn}^+)_{\alpha}{}^{\beta} H_{mn} + 3 \delta_{\alpha}^{\beta} Y, \quad ^+) \\
 [X_\alpha, X_\beta] &= 0, \quad [\bar{X}^\alpha, \bar{X}^\beta] = 0.
 \end{aligned} \tag{3}$$

^{+) One may also take the negative of the r.h.s.:}

$$[X_\alpha, \bar{X}^\beta] = -[(\Gamma_{mn}^+)_{\alpha}{}^{\beta} H_{mn} + 3 \delta_{\alpha}^{\beta} Y]$$

This corresponds to the non-compact form of E_6 .

The Jacobi identities are satisfied because of the following remarkable identity:

$$\frac{1}{2} (\Gamma_{mn}^+)_{\alpha} \gamma (\Gamma_{mn}^+)_{\beta}^{\delta} - 3 \delta_{\alpha}^{\gamma} \delta_{\beta}^{\delta} = \frac{1}{2} (\Gamma_{mn}^+)_{\beta} \gamma (\Gamma_{mn}^+)_{\alpha}^{\delta} - 3 \delta_{\beta}^{\gamma} \delta_{\alpha}^{\delta}, \quad (4)$$

which is only true in $d=10$. This identity can be proved by the methods described in ref. (10), appendix. Thus the quantity

$$M_{\alpha\beta}^{\gamma\delta} = -\frac{1}{2} (\Gamma_{mn}^+)_{\alpha} \gamma (\Gamma_{mn}^+)_{\beta}^{\delta} + 3 \delta_{\alpha}^{\gamma} \delta_{\beta}^{\delta} \quad (5)$$

has the symmetry properties

$$M_{\alpha\beta}^{\gamma\delta} = M_{\beta\alpha}^{\gamma\delta} = M_{\beta\alpha}^{\delta\gamma} = M_{\alpha\beta}^{\delta\gamma}. \quad (6)$$

Another important property of the quantities M concerns the symmetry of their product. Namely, if $(\alpha\beta\gamma)$ denotes complete symmetrization over the indices enclosed, then

$$M_{\rho\sigma}^{\lambda(\alpha} M_{\tau\lambda}^{\beta\gamma)} = M_{\sigma\tau}^{\lambda(\alpha} M_{\rho\lambda}^{\beta\gamma)} = M_{\tau\rho}^{\lambda(\alpha} M_{\sigma\lambda}^{\beta\gamma)}. \quad (7)$$

Unfortunately, the direct proof of this property is too long to present here.

Using the results (1)-(7) it is now possible to find a non-linear realization of E_6 . It involves 16 complex fields ϕ_{α} and their conjugates $\bar{\phi}^{\alpha}$, transforming as chiral spinors under $SO(10) \times U(1)$, which span the coset space $E_6/SO(10) \times U(1)$. Their complete E_6 - transformation properties read

$$\begin{aligned} [\gamma, \phi_{\alpha}] &= \phi_{\alpha}, & [\gamma, \bar{\phi}^{\alpha}] &= -\bar{\phi}^{\alpha}, \\ [H_{mn}, \phi_{\alpha}] &= -\frac{1}{2} (\Gamma_{mn}^+)_{\alpha}^{\beta} \phi_{\beta}, & [H_{mn}, \bar{\phi}^{\alpha}] &= \frac{1}{2} \bar{\phi}^{\beta} (\Gamma_{mn}^+)_{\beta}^{\alpha}, \\ [\bar{X}^{\beta}, \phi_{\alpha}] &= \delta_{\alpha}^{\beta}, & [\bar{X}^{\beta}, \bar{\phi}^{\alpha}] &= \frac{1}{2} M_{\gamma\delta}^{\alpha\beta} \bar{\phi}^{\gamma} \bar{\phi}^{\delta}, \\ [X_{\beta}, \phi_{\alpha}] &= -\frac{1}{2} M_{\alpha\beta}^{\gamma\delta} \phi_{\gamma} \phi_{\delta}, & [X_{\beta}, \bar{\phi}^{\alpha}] &= -\delta_{\beta}^{\alpha}. \end{aligned} \quad (8)$$

The closure of the E_6 algebra on ϕ_α may be proved by checking the Jacobi identities involving two generators of E_6 and one ϕ -field. The only non-trivial one is

$$[X_\alpha, [X_\beta, \phi_\gamma]] + [X_\beta, [\phi_\gamma, X_\alpha]] + [\phi_\gamma, [X_\alpha, X_\beta]] = 0, \quad (9)$$

for which one needs the property (7).

The fields ϕ_α can be interpreted as the Goldstone fields corresponding to the spontaneously broken symmetries generated by $X, \bar{X}$. To construct a non-linear lagrangian for these Goldstone bosons, containing no more than two derivatives, is straightforward in principle. Let g_α^β denote the metric tensor of the manifold $E_6/SO(10) \times U(1)$; then the lagrangian

$$\mathcal{L} = -g_\alpha^\beta \partial_\mu \bar{\phi}^\alpha \partial_\mu \phi_\beta \quad (10)$$

is invariant under general co-ordinate transformations on the manifold. Now the transformations (8) represent Killing vectors of the manifold and therefore leave g_α^β form-invariant. Hence (10) is form-invariant as well.

Because $E_6/SO(10) \times U(1)$ is known to be a Kähler manifold, there must exist a real Kähler potential $K[\phi, \bar{\phi}]$, such that

$$g_\alpha^\beta = \frac{\partial^2 K}{\partial \bar{\phi}^\alpha \partial \phi_\beta} \quad (11)$$

This Kähler potential can be used at the same time as a superfield lagrangian for the supersymmetric extension of (10).⁵⁾ It must have the property, that under the transformations (8) it changes only by the real part of an analytic function

$$\delta K[\phi, \bar{\phi}] = f[\phi] + \bar{f}[\bar{\phi}]. \quad (12)$$

This guarantees the invariance of g_α^β , as follows from (11). The solution for the $E_6/SO(10) \times U(1)$ coset space is given by

$$K[\phi, \bar{\phi}] = \bar{\phi} [Q^{-1} \ln(1+Q)] \phi \quad (13)$$

where $Q_\alpha^\beta = \frac{1}{2} M_{\alpha\gamma}^{\beta\delta} \bar{\phi}^\gamma \phi^\delta$.

It is easy to show that for $\phi \neq 0$, Q is positive definite; namely, for arbitrary ψ_α :

$$\bar{\psi}^\alpha Q_\alpha^\beta \psi_\beta = -\frac{1}{4} (\bar{\psi} \Gamma_{mn}^+ \phi)^2 + \frac{3}{2} (\bar{\psi} \phi)^2 > 0,$$

because Γ_{mn}^+ is anti-hermitean. Hence Q^{-1} exists. Even in the limit $\phi \rightarrow 0$ is $K[\phi, \bar{\phi}]$ itself well-defined, as is clear from its series expansion:

$$\begin{aligned} K[\phi, \bar{\phi}] &= \bar{\phi}^\alpha \left(\delta_\alpha^\beta - \frac{1}{2} Q_\alpha^\beta + \frac{1}{3} Q_\alpha^\gamma Q_\gamma^\beta + \dots \right) \phi_\beta \\ &= \bar{\phi}^\alpha \phi_\alpha - \frac{1}{2!} [X_\alpha, \phi_\beta] \bar{\phi}^\alpha \bar{\phi}^\beta + \frac{1}{3!} [X_\alpha, [X_\beta, \phi_\gamma]] \bar{\phi}^\alpha \bar{\phi}^\beta \bar{\phi}^\gamma + \dots \end{aligned} \quad (14)$$

The last expression is very useful in determining the transformation properties of $K[\phi, \bar{\phi}]$. First we note that multiple Jacobi identities of ϕ with X guarantee the complete symmetry of

$$[X_{\alpha_1}, [X_{\alpha_2}, \dots [X_{\alpha_n}, \phi_{\alpha_n}] \dots]]$$

in the indices $(\alpha_1, \dots, \alpha_n)$. This is a direct generalization of the result (7). If we now perform an E_6 -transformation on $K[\phi, \bar{\phi}]$ with

$$\Lambda = \omega^{mn} H_{mn} + i\lambda Y - \bar{\epsilon}^\alpha X_\alpha + \epsilon_\alpha \bar{X}^\alpha,$$

one finds

$$[\Lambda, K] = \bar{\epsilon}^\alpha \phi_\alpha + \epsilon_\alpha \bar{\phi}^\alpha \quad (15)$$

This has indeed the form (12).

As we have mentioned already, K can be used as a superfield lagrangian for the supersymmetric extension of (10), if we interpret $\phi, \bar{\phi}$ as chiral superfields. Its component form can be expressed entirely in terms of geometric quantities: the metric $g_{\alpha\beta}$, the connection $\Gamma^{\alpha\beta}_{\gamma}$ and the curvature tensor $K_{\alpha\beta\gamma}^{\delta}$, as was shown by Zumino.⁵⁾ The component lagrangian is the $(\bar{\theta}\theta)^2$ -term of the real superfield $K[\phi, \bar{\phi}]$ and has the form

$$\mathcal{L} = -g_{\alpha\beta}(\partial_{\mu}\bar{\phi}^{\alpha}\partial_{\mu}\phi^{\beta} + \bar{\psi}_{\beta}^{+}\not{\partial}\psi^{-\alpha}) + K_{\alpha\beta\gamma}^{\delta}\bar{\psi}_{\beta}^{+}\psi_{\delta}^{+}\bar{\psi}^{-\gamma}\psi^{-\alpha}, \quad (16)$$

where the $\psi^{\pm}$ are chiral spinors: $\gamma_5\psi^{\pm} = \pm\psi^{\pm}$, $\bar{\psi}^{\pm} = \bar{\psi}^{\mp}$, and the covariant derivatives, connection and curvature are defined by

$$\begin{aligned} \mathcal{D}_{\mu}\psi_{\alpha}^{+} &= \partial_{\mu}\psi_{\alpha}^{+} + \Gamma^{\beta\gamma}_{\alpha}(\partial_{\mu}\phi_{\beta})\psi_{\gamma}^{+}, \\ \Gamma^{\beta\gamma}_{\alpha} &= g^{-1}_{\delta\beta}g_{\alpha}^{\delta,\gamma}; \quad K_{\alpha\beta\gamma}^{\delta} = g_{\kappa}^{\delta}\Gamma^{\kappa\beta}_{\alpha,\gamma}. \end{aligned} \quad (17)$$

From eqs. (11), (14) one finds that

$$\begin{aligned} \frac{\delta K}{\delta\phi_{\alpha}} &= [\bar{\phi}(1+Q)^{-1}]^{\alpha}, \\ g_{\alpha\beta} &= \delta_{\alpha\beta} - M_{\alpha\gamma}^{\beta\delta}\bar{\phi}^{\gamma}\phi_{\delta} + \frac{1}{4}M_{\lambda\beta}^{(\alpha e}M_{\gamma\delta}^{\sigma)}\bar{\phi}^{\gamma}\bar{\phi}^{\delta}\phi_e\phi_{\sigma} + \dots \end{aligned} \quad (18)$$

Now one can show that

$$\Gamma^{\beta\gamma}_{\alpha} = -M_{\alpha\lambda}^{\beta\gamma}\frac{\delta K}{\delta\phi_{\lambda}}, \quad (19)$$

and as a result, using (17):

$$K_{\alpha\beta\gamma}^{\delta} = -M_{\kappa\lambda}^{\beta\delta}g_{\alpha}^{\kappa}g_{\gamma}^{\lambda}. \quad (20)$$

To prove (19), note the identities

$$\frac{\delta^2 K}{\delta \phi_\alpha \delta \phi_\gamma} = -\frac{1}{2} M_{\kappa\lambda}^{\alpha\gamma} \frac{\delta K}{\delta \phi_\kappa} \frac{\delta K}{\delta \phi_\lambda},$$

and

$$g_{\beta}^{\alpha,\gamma} = \frac{\delta^3 K}{\delta \bar{\phi}^\beta \delta \phi_\alpha \delta \phi_\gamma}.$$

(21)

Eq.(20) is of considerable interest, because together with (5) it shows, that the four-fermion interaction in (16) can, after a Fierz rearrangement, be written as a current x current interaction of $SO(10) \times U(1)$ with the current in the adjoint representation $\underline{45} + \underline{1}$. Note that in principle $\underline{16} \times \underline{16} = \underline{1} + \underline{45} + \underline{210}$, but the $\underline{210} \times \underline{210}$ term is absent here, even though it is allowed by $SO(10)$ invariance.

$$\begin{aligned} M_{\alpha\gamma}^{\beta\delta} \bar{\psi}_\beta^+ \psi_\delta^+ \bar{\psi}^{-\alpha} \psi^{-\gamma} &= \frac{1}{2} M_{\alpha\gamma}^{\beta\delta} \bar{\psi}_\beta^+ \gamma_\mu \psi^{-\alpha} \bar{\psi}_\delta^+ \gamma_\mu \psi^{-\gamma} \\ &= -\frac{1}{4} (\bar{\psi}^+ \Gamma_{mn}^+ \gamma_\mu \psi)^2 + \frac{3}{2} (\bar{\psi}^+ \gamma_\mu \psi)^2. \end{aligned}$$

(22)

Thus the currents carry the quantum numbers of $SO(10) \times U(1)$ gauge fields. This makes it natural to promote the global symmetries to local ones, as for example in $SO(10)$ grand unification.¹¹⁾ The chiral fermions then carry exactly the right quantum numbers for describing a family of quarks and leptons.

In view of the possible phenomenological applications, it is of interest to discuss briefly the decomposition of E_6 under the maximal subgroup $SU(3) \times SU(3) \times SU(3)$. We distinguish the mutually commuting generators T_I of these $SU(3)$ groups by subscripts $I=L, R$ and C . The remaining 54 generators are denoted Q , $\bar{Q}$ and transform under the $SU(3)$ subgroups according to the representation $(3, 3, 3)$ and its conjugate.⁶⁾ In this decomposition the non-trivial part of the E_6 algebra consists of the commutators of Q and $\bar{Q}$. They read:

$$\begin{aligned} [Q^{ai\alpha}, Q^{bj\beta}] &= -\epsilon^{abc} \epsilon^{ijk} \epsilon^{\alpha\beta\gamma} Q_{ck\gamma}, \\ [Q^{ai\alpha}, Q_{bj\beta}] &= \delta_b^a \delta_j^i T_{Q\beta}^\alpha + \delta_b^a \delta_\beta^\alpha T_{Rj}^i + \delta_\beta^\alpha \delta_j^i T_{Lb}^a. \end{aligned} \quad (23)$$

If one compares with the $SO(10) \times U(1)$ decomposition, the generators X , $\bar{X}$ turn out to be linear combinations of the following T_I 's and Q 's:

$$\begin{aligned} X &= \{ Q^{i3c}, Q_{3ic}, T_{L3}^i, T_{Ri}^3 \}, \\ \bar{X} &= \{ Q^{3ic}, Q_{i3c}, T_{Li}^3, T_{R3}^i \}, \quad i=1,2; c=1,2,3. \end{aligned} \quad (24)$$

If the corresponding symmetries are broken, leaving only $SO(10) \times U(1)$ invariant, the $SU_C(3) \times SU_L(3) \times SU_R(3)$ subgroup is broken down to $SU_C(3) \times SU_L(2) \times SU_R(2) \times U(1) \times U(1)$. In this basis we can denote the Goldstone fields by $\Phi_\alpha = \{ q^{i3c}, q_{3ic}, t_{L3}^i, t_{Ri}^3 \}$. The non-linear part of their E_6 transformations are given by

$$\begin{aligned} [Q^{i3c}, q_{3jb}] &= -q^{i3c} q_{3jb} + \delta_b^c (q^{i3a} q_{3ja} + t_{L3}^i t_{Rj}^3), \\ [Q^{i3c}, q_{j3b}] &= -q^{i3b} q_{j3c} + \epsilon^{3ij} \epsilon^{k3l} \epsilon^{acb} q_{3ka} t_{Rl}^3, \\ [Q^{i3c}, t_{L3}^k] &= -q^{k3c} t_{L3}^i - \frac{1}{2} \epsilon^{3ik} \epsilon^{l3j} \epsilon^{acb} q_{3jb} q_{3la}, \\ [Q^{i3c}, t_{Rk}^3] &= -q^{i3c} t_{Rk}^3, \\ [Q_{3ic}, q^{jab}] &= -q_{3ic} q^{jab} + \delta_c^b (q_{3ia} q^{ja3} + t_{L3}^j t_{Ri}^3), \\ [Q_{3ic}, t_{Rk}^3] &= -q_{3kc} t_{Ri}^3 - \frac{1}{2} \epsilon_{j3l} \epsilon_{3ik} \epsilon_{acb} q^{l3b} q^{ja3}, \\ [Q_{3ic}, q_{3jb}] &= -q_{3jc} q_{3ib} + \epsilon_{k3l} \epsilon_{3ij} \epsilon_{acb} t_{L3}^l q^{k3a}, \\ [Q_{3ic}, t_{L3}^k] &= -q_{3ic} t_{L3}^k, \end{aligned} \quad (25)$$

$$[T_L^i{}_3, q^{j3c}] = -t_L^i{}_3 q^{j3c} + \frac{1}{2} \epsilon^{3ij} \epsilon^{k3l} \epsilon^{acb} q_{3ka} q_{3lb} ,$$

$$[T_L^i{}_3, q_{3jc}] = -t_L^i{}_3 q_{3jc} ,$$

$$[T_L^i{}_3, t_L^j{}_3] = -t_L^i{}_3 t_L^j{}_3 ,$$

$$[T_L^i{}_3, t_R^3{}_j] = q^{i3b} q_{3jb} ,$$

$$[T_R^3{}_i, q^{k3c}] = -t_R^3{}_i q^{k3c} ,$$

$$[T_R^3{}_i, q_{3kc}] = -t_R^3{}_k q_{3ic} + \frac{1}{2} \epsilon_{3ik} \epsilon_{j3l} \epsilon_{acb} q^{j3a} q^{l3b} ,$$

$$[T_R^3{}_i, t_L^j{}_3] = q^{j3c} q_{3ic} ,$$

$$[T_R^3{}_i, t_R^3{}_j] = -t_R^3{}_i t_R^3{}_j ,$$

$$a, b, c = (1, 2, 3);$$

$$i, j, \dots = (1, 2) .$$

Also we have, as before:

$$[\bar{X}^\alpha, \phi_\beta] = \delta^\alpha_\beta . \quad (26)$$

A long but straightforward calculation then shows that the algebra (25), (26) closes indeed. Eqs. (25) define the components of the quantities M , eq. (5), in the transformed basis and the properties (6) can be verified. We also checked eq. (22) in this representation and found the current x current interactions to be exactly those of the $SO(10)$ GUT.¹¹⁾

Summarizing, we constructed explicitly $SO(10) \times U(1)$ covariant decomposition of the E_6 algebra and the corresponding non-linear representation of the coset $E_6/SO(10) \times U(1)$. We found the Kähler potential of this manifold in closed form and used it to calculate the effective supersymmetric interactions of the chiral Goldstone fields. The fact that $E_6/SO(10) \times U(1)$ is a Kähler manifold induced a minimal set of quasi-Goldstone fermions in a 16 family of $SO(10)$,

with effective four-fermion interactions which are those of the $SO(10)$ GUT. In view of possible phenomenological applications, we presented also a non-linear realization of the manifold in a different representation based on the $[SU(3)]^3$ maximal subgroup of E_6 .

These results allow for interesting phenomenological applications as the coset fits naturally into E_6 as well as $SO(10)$ grand unification schemes. Apart from the applications mentioned in the introduction, we can think of the following possibilities: One can gauge the $SO(10)$ subalgebra or alternatively hope that composite gauge bosons will be dynamically generated.^{9),12)} In this case an $SO(10)$ GUT will be generated. Another application will be to fit the coset $E_6/SO(10) \times U(1)$ into an E_6 preon model.⁸⁾

Details of our calculations and a discussion of the applications will be presented elsewhere.

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