Ref.TH.3831-CERN

GAUGED GOLDSTONE BOSON EFFECTIVE ACTION
FROM DIRECT INTEGRATION OF BARDEEN ANOMALY

Namik K. Pak and Paolo Rossi

CERN -- Geneva

ABSTRACT

We derive the effective action which describes the anomalous Goldstone boson interactions in the presence of external flavour $SU(N)_L \times SU(N)_R$ gauge fields, by a direct integration of the Bardeen anomaly. Through comparison with other results, we are able to elucidate the relationship between anomalies based on different regularization schemes. We also give a compact representation of the anomalous part of the fermionic determinant allowing for a simpler rederivation of our results.

Ref.TH.3831-CERN
February 1984

1. - INTRODUCTION

The idea that the Skyrme model¹⁾ provides an effective description of the infra-red properties of Quantum Chromodynamics (QCD)²⁾ has recently been reviewed^{3),4)}, supported by the new information supplied by the large N analysis of QCD concerning the baryon masses⁵⁾. Clearly, with this qualification, the Skyrme model should be able to reproduce the low energy phenomenology explained by the (anomalous) Ward identities of the underlying fundamental theory, QCD.

In the ungauged (purely hadronic) case, the additional term that is related to the anomalies of QCD was correctly guessed by Witten to be the Wess-Zumino term⁶⁾ (rewritten in a topologically meaningful fashion) which vanishes in the two flavour case. Then, he proceeded to find the anomaly related additional terms in the gauged case starting with the Wess-Zumino-Witten term and using Noetherian methods. Apart from being ad hoc, this method is very misleading because all the arguments are based on a term which vanishes for the two flavour case. However, π^0 is known to decay into two photons independent of the number of flavours. Thus, the Skyrme model should have additional anomaly related terms even in the two flavour case when coupled to electromagnetism.

Later, another ad hoc method⁷⁾ which essentially consists in first gauging the Wess-Zumino action and then decomposing and dropping the terms which are gauge invariant on the five dimensional manifold, whose boundary is space-time, claimed disagreements with Witten's non-Abelian anomalous effective Lagrangian. Given this dispute on the non-Abelian result and conceptual confusion concerning the difference between the two and higher flavour cases, the need for a direct derivation of the anomalous part of the Skyrme model is clear.

The first step forward in the direction of computing the anomalous part of the Skyrme model starting from the Bardeen anomaly⁸⁾ was given in Ref. 9). They directly integrated the Wess-Zumino functional differential equations⁶⁾ in the purely strong (no flavour gauging) case and were able to derive the Wess-Zumino-Witten action.

In this work, we will use the same method to derive the anomalous part of the Skyrme model with an arbitrary subgroup of the $SU(N)_L \times SU(N)_R$ gauged.

One of the novel aspects of our work is that the effective action we obtain is the integral of the Bardeen anomaly, which is based on a regularization scheme which respects the vector symmetries, whereas the effective actions of Refs. 4) and 7) yield left-right anomalies under chiral variations.

This work is organized as follows: in Section 2, we review the direct global integration method presented in Ref. 9) and show how to extend it to integrate the (four-dimensional) Bardeen anomaly in the presence of external $SU(N)_L \times SU(N)_R$ flavour gauge fields. The result is compared to previously proposed effective Lagrangians and shown to have a simple relationship with that of Ref. 7). The result for the electromagnetic case is also explicitly depicted. In Section 3, we discuss the relationship between different anomalies, the left-right and the axial, in the context of the effective action formalism, and find a general constructive approach to this problem. In Section 4, we reinterpret and rederive our results in terms of a representation for the anomalous part of the fermionic determinant, whose finite variation is exactly our effective action when the variation parameters are identified with the Goldstone boson fields. An appendix is devoted to a comparative discussion of the two-dimensional case.

2. - GLOBAL INTEGRATION OF BARDEEN ANOMALY

The fundamental fields in the effective level are the pion fields, U , the Goldstone bosons of the underlying theory, and the external gauge fields coupled to the flavour indices (candidates for γ , $W^\pm$, Z , etc.).

To formulate the problem, we first introduce the anomalous one-particle irreducible vacuum functional $W(U,V,A)$. The Ward identities are compactly summarized by the invariance properties of W , which can be written as:

$$X W(U, V, A) \equiv \left(\partial_\mu \frac{\delta}{\delta V_\mu} + [V_\mu, \frac{\delta}{\delta V_\mu}] + [A_\mu, \frac{\delta}{\delta A_\mu}] + \dots \right) W(U, V, A) = 0 \quad (2.1)$$

$$\begin{aligned} Y W(U, V, A) &\equiv \left(\partial_\mu \frac{\delta}{\delta A_\mu} + [V_\mu, \frac{\delta}{\delta A_\mu}] + [A_\mu, \frac{\delta}{\delta V_\mu}] + \dots \right) W(U, V, A) = \\ &= -G(V, A) \end{aligned}$$

where the dots stand for the parts acting on the U field; $G(V, A)$ is the Bardeen anomaly for arbitrary flavour which in our anti-Hermitean matrix notation^{*)}, reads⁸⁾:

$$\begin{aligned} G = \frac{N_c}{3\pi^2} \varepsilon_{\mu\nu\rho\sigma} &\left[A_\mu A_\nu A_\rho A_\sigma + \frac{3}{32} (V_{\mu\nu} V_{\rho\sigma} + \frac{1}{3} A_{\mu\nu} A_{\rho\sigma}) \right. \\ &\left. - \frac{1}{4} (A_\mu A_\nu V_{\rho\sigma} + V_{\mu\nu} A_\rho A_\sigma + A_\mu V_{\nu\rho} A_\sigma) \right] \end{aligned} \quad (2.2)$$

Because in gauging the flavour indices the colour degrees of freedom play the role usually played by the flavour, we introduced the colour factor into the Bardeen's expression from the start. Here $V_{\mu\nu}$ and $A_{\mu\nu}$ are chiral covariant curls of the external gauge fields:

$$\begin{aligned} V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] + [A_\mu, A_\nu] \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [V_\mu, A_\nu] + [A_\mu, V_\nu] \end{aligned} \quad (2.3)$$

It is well known that⁶⁾ for G's satisfying a set of consistency conditions, Eq. (2.1) can be locally integrated. The global integration is our main concern because of the topological conservation laws. The local integrability guarantees global integrability only in the case when the space of maps for the fields in the theory is simply connected. The fundamental fields in our effective theory, U, are elements of the coset

^{*)}Our conventions are $V_\mu = -iT^a V_\mu^a$ $A_\mu = -iT^a A_\mu^a$
 $\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$

space $SU(N)_L \times SU(N)_R / SU(N)_V$. The crucial boundary condition for existence of kinks, which we identify with the baryons, is $U \xrightarrow{x \rightarrow \infty} I$; this compactifies the space-time into $S^1 \times S^3$ [in the static case^{1), 2)}], one is only interested in the compactification $R^3 \rightarrow S^3$ with a restricted boundary condition $U \xrightarrow{x \rightarrow \infty} I$. Then, clearly the space of maps $\{U: S^1 \times S^3 \rightarrow G \mid U \xrightarrow{x \rightarrow \infty} I\}$ is not simply connected, and the global integration of the WZ-equation requires special care. This method is described in Ref. 9). One introduces a sequence of chiral transformations $\Omega(x; \tau)$, elements of the coset space, labelled by a continuous parameter $\tau \in [0, 1]$ such that Ω interpolates between well-defined space-time fields

$$\Omega(x, \tau) = \begin{cases} I, & \tau = 0 \\ \Omega_\lambda(x), & \tau = 1 \end{cases} \quad (2.4)$$

The range of $\Omega(x, \tau)$ is now a five-dimensional manifold $Q = D \times S^3$ (D is the two-dimensional disk), whose boundary is the space-time.

The chiral transformations on the disk Q are:

$$\begin{aligned} U^\Omega &= \Omega^{-1} U \Omega^{-1} \\ (V+A)^\Omega &= \Omega (V+A) \Omega^{-1} + \Omega \partial \Omega^{-1} \\ (V-A)^\Omega &= \Omega^{-1} (V-A) \Omega + \Omega^{-1} \partial \Omega \end{aligned} \quad (2.5)$$

Following Ref. 9), we next consider the quantity which will be a solution of Eq. (2.1) on the disk

$$H(\tau) = W(U^\Omega, V^\Omega, A^\Omega) \quad (2.6)$$

The total derivative of H with respect to τ (group variation, keeping space-time fixed) is given by

$$\frac{dH(\tau)}{d\tau} = \int d^4x \text{Tr} \left\{ (\Omega^{-1} \partial_\tau \Omega - \partial_\tau \Omega \Omega^{-1}) (XW)^{-2} - (\Omega^{-1} \partial_\tau \Omega + \partial_\tau \Omega \Omega^{-1}) (YW)^{-2} \right\} \quad (2.7)$$

When the Wess-Zumino consistency conditions are satisfied, this equation can be integrated and the result will be independent of the τ -path.

Substituting Eq. (2.1) into Eq. (2.7), we obtain

$$W(U, V, A) = W(U = I, V^{\Omega_1}, A^{\Omega_1}) - \int_0^1 d\tau \int d^4x \text{Tr} \left\{ (\Omega^{-1} \partial_\tau \Omega - \partial_\tau \Omega \Omega^{-1}) G(V^{\Omega_1}, A^{\Omega_1}) \right\} \quad (2.8)$$

Let us first observe that the integrand in Eq. (2.8) is a function only of $\Omega^2(x, \tau)$. Thus, we introduce a new disk function $\hat{U}(x, \tau) = \Omega^2(x, \tau)$. This further allows us to make the identification $\hat{U}(x, \tau = 1) = U(x)$. The boundary conditions satisfied by $\hat{U}$ are

$$\hat{U}(x, \tau) = \begin{cases} I, & \tau = 0 \\ U(x), & \tau = 1 \\ I, & x_\mu \rightarrow \infty \end{cases} \quad (2.9)$$

Now, the integration constant $W(U = I, V^{\Omega_1}, A^{\Omega_1})$, with $\Omega_1 = U^{\frac{1}{2}}$, being invariant under local $SU(N) \times SU(N)$ transformations can be set equal to zero⁹⁾. Thus, the solution of the inhomogeneous equation¹⁾ is

$$W(U, V, A) = - \int_0^1 d\tau \int d^4x \text{Tr} \left\{ (\Omega^{-1} \partial_\tau \Omega - \partial_\tau \Omega \Omega^{-1}) G(V^{\Omega_1}, A^{\Omega_1}) \right\} \quad (2.10)$$

WZ

The formidable task in computing the right-hand side is the computation of the chirally rotated (on the disk Q) Bardeen anomaly. We find it, for the purpose of comparison with the other works, more convenient to express our result in the LR language. Then, the chiral transformation laws can be written as:

$$\begin{aligned}
 A_\mu^\Omega &= \frac{1}{2} \Omega (A_\mu^R - \hat{U}^{-1} A_\mu^L \hat{U} - \hat{R}_\mu) \Omega^{-1} \\
 V_{\mu\nu}^\Omega &= \frac{1}{2} \Omega (A_{\mu\nu}^R + \hat{U}^{-1} A_{\mu\nu}^L \hat{U}) \Omega^{-1} \\
 A_{\mu\nu}^\Omega &= \frac{1}{2} \Omega (A_{\mu\nu}^R - \hat{U}^{-1} A_{\mu\nu}^L \hat{U}) \Omega^{-1}
 \end{aligned} \tag{2.11}$$

which makes the dependence of the rotated anomaly on $\Omega^2(x, \tau) = U$ manifest. Here, we introduced the following group currents on the disk

$$\begin{aligned}
 \hat{R}_\mu &= \hat{U}^{-1} \partial_\mu \hat{U} \\
 \hat{L}_\mu &= \partial_\mu \hat{U} \hat{U}^{-1} = \hat{U} R_\mu \hat{U}^{-1} \\
 r_\mu &= \Omega^{-1} \partial_\mu \Omega \\
 l_\mu &= \partial_\mu \Omega \Omega^{-1} = \Omega r_\mu \Omega^{-1} \\
 v_\mu &= l_\mu + r_\mu
 \end{aligned} \tag{2.12}$$

There is a nice set of relationships between these two sets

$$\begin{aligned}
 \Omega^{-1} (r_\mu + l_\mu) \Omega &= \hat{R}_\mu \\
 \Omega (r_\mu + l_\mu) \Omega^{-1} &= \hat{L}_\mu
 \end{aligned} \tag{2.13}$$

Further, they satisfy the following integrability conditions

$$\begin{aligned}
 \partial_\mu \hat{L}_\nu - \partial_\nu \hat{L}_\mu + [\hat{L}_\mu, \hat{L}_\nu] &= 0 \\
 \partial_\mu \hat{R}_\nu - \partial_\nu \hat{R}_\mu - [\hat{R}_\mu, \hat{R}_\nu] &= 0
 \end{aligned} \tag{2.14}$$

and similar ones for ℓ and r . These and their generalizations involving a higher (even) number of group currents will be heavily used in converting the five-dimensional disk integrals into space-time integrals (except the Wess-Zumino-Witten term). For instance,

$$\begin{aligned} \varepsilon_{\mu\nu\rho\sigma} [\partial_\mu (\hat{L}_\nu \hat{L}_\rho \hat{L}_\sigma) + \hat{L}_\mu \hat{L}_\nu \hat{L}_\rho \hat{L}_\sigma] &= 0 \\ \varepsilon_{\mu\nu\rho\sigma} [\partial_\mu (\hat{R}_\nu \hat{R}_\rho \hat{R}_\sigma) - \hat{R}_\mu \hat{R}_\nu \hat{R}_\rho \hat{R}_\sigma] &= 0 \end{aligned} \quad (2.15)$$

First, we write the integral in (2.10) as a proper "area" integral over the five-dimensional disk, Q . Replacing the four-dimensional ε -tensor with the five-dimensional one requires special care for the terms involving the gauge fields. Because gauge fields do not depend on the τ -parameter, the correct combinatorial factor is determined by the number of group currents, v_μ , which appear in the decomposition of the integrand. The correct factors are $1/5$, $1/4$, $1/3$, $1/2$ and 1 for the 5, 4, 3, 2, 1, group currents respectively. We label the disk co-ordinates by $i, j, k, \dots$ ($i, j = 0, 1, 2, 3, 5$). We will use the same notation for the gauge fields on the disk, however, remembering that $A_i = (A_\mu, 0)$, and $\partial_i A_j = (\partial_\mu A_\nu, 0)$, $\partial_5 A_j = 0$ for all j . Next, we collect the terms in W containing the same number of external sources into groups labelling them by an index which counts the number of sources:

$$W(U, A^L, A^R) = -\frac{N_c}{48\pi^2} \int d^5x \, \epsilon_{ijklm} \, \text{Tr} (I_4 + I_3 + I_2 + I_1 + I_0)$$

$$I_4 = a_i a_j a_k a_e V_m + \frac{3}{2} b_{ij}^+ b_{ke}^+ V_m + \frac{1}{2} b_{ij}^- b_{ke}^- V_m \\ - a_i a_j b_{ke}^+ V_m - b_{ij}^+ a_k a_e V_m - a_i b_{jk}^+ a_e V_m$$

$$I_3 = -a_i a_j a_k V_e V_m - a_i a_j V_k a_e V_m + \frac{3}{2} d_{ij}^+ b_{ke}^+ V_m \\ + \frac{3}{2} b_{ij}^+ d_{ke}^+ V_m + \frac{1}{2} d_{ij}^- b_{ke}^- V_m + \frac{1}{2} b_{ij}^- d_{ke}^- V_m \\ - a_i a_j d_{ke}^+ V_m - d_{ij}^+ a_k a_e V_m - a_i d_{jk}^+ a_e V_m \\ + a_i V_j b_{ke}^+ V_m + b_{ij}^+ a_k V_e V_m + a_i b_{jk}^+ V_e V_m$$

$$I_2 = a_i a_j V_k V_e V_m + a_i V_j a_k V_e V_m + \frac{3}{2} d_{ij}^+ d_{ke}^+ V_m \quad (2.16) \\ + \frac{1}{2} d_{ij}^- d_{ke}^- V_m + a_i V_j d_{ke}^+ V_m + d_{ij}^+ a_k V_e V_m \\ + a_i d_{jk}^+ V_e V_m - b_{ij}^+ V_k V_e V_m$$

$$I_1 = -a_i V_j V_k V_e V_m - d_{ij}^+ V_k V_e V_m$$

$$I_0 = \frac{1}{5} V_i V_j V_k V_e V_m$$

where for notational compactness, we have defined

$$\begin{aligned}
 a_i &= \Omega A_i^R \Omega^{-1} - \Omega^{-1} A_i^L \Omega \\
 b_{ij}^{\pm} &= \Omega A_i^R A_j^R \Omega^{-1} \pm \Omega^{-1} A_i^L A_j^L \Omega \\
 d_{ij}^{\pm} &= \Omega \partial_i A_j^R \Omega^{-1} \pm \Omega^{-1} \partial_i A_j^L \Omega
 \end{aligned} \tag{2.17}$$

We immediately recognize the I_0 term as the Wess-Zumino-Witten action, using (2.13):

$$W_0 = -\frac{N_c}{240\pi^2} \int d^5x \epsilon_{ijklm} \text{Tr} (\hat{L}_i \hat{L}_j \hat{L}_k \hat{L}_l \hat{L}_m) \tag{2.18}$$

Again using (2.13)-(2.15), we reduce the other terms into the following form:

$$\begin{aligned}
 W_4 &= -\frac{N_c}{48\pi^2} \int_0^1 d\tau \int d^4x \epsilon_{\mu\nu\rho\sigma} \partial_\tau \text{Tr} \left[\frac{1}{2} A_\mu^R \hat{U}^{-1} A_\nu^L A_\rho^R \hat{U}^{-1} A_\sigma^L \hat{U} \right. \\
 &\quad \left. + A_\mu^R A_\nu^R A_\rho^R \hat{U}^{-1} A_\sigma^L \hat{U} + A_\mu^R \hat{U}^{-1} A_\nu^L A_\rho^L A_\sigma^L \hat{U} \right] \\
 W_3 &= -\frac{N_c}{48\pi^2} \int_0^1 d\tau \int d^4x \epsilon_{\mu\nu\rho\sigma} \partial_\tau \text{Tr} \left[A_\mu^R \hat{U}^{-1} A_\nu^L \partial_\rho A_\sigma^L \hat{U} \right. \\
 &\quad \left. + A_\mu^R \hat{U}^{-1} \partial_\nu A_\rho^L A_\sigma^L \hat{U} + \partial_\mu A_\nu^R A_\rho^R \hat{U}^{-1} A_\sigma^L \hat{U} + A_\mu^R \partial_\nu A_\rho^R \hat{U}^{-1} A_\sigma^L \hat{U} \right] \\
 &\quad - \frac{N_c}{48\pi^2} \int d^5x \epsilon_{ijklm} \partial_m \text{Tr} \left[A_i^R \hat{U}^{-1} A_j^L \hat{U} A_k^R R_\ell \right. \\
 &\quad \left. + A_i^L \hat{U} A_j^R \hat{U}^{-1} A_k^L \hat{L}_\ell + A_i^R A_j^R A_k^R \hat{R}_\ell + A_i^L A_j^L A_k^L \hat{L}_\ell \right]
 \end{aligned}$$

$$\begin{aligned}
 W_2 = & -\frac{N_c}{48\pi^2} \int d^3x \epsilon_{ijklm} \partial_m \text{Tr} [(A_i^R \partial_j A_k^R + \partial_i A_j^R A_k^R) \hat{R}_l \\
 & + (A_i^L \partial_j A_k^L + \partial_i A_j^L A_k^L) \hat{L}_l + \partial_i A_j^R \hat{U}^{-1} A_k^L \hat{L}_l \hat{U} \\
 & + \partial_i A_j^L \hat{U} A_k^R \hat{R}_l \hat{U}^{-1} + A_i^L \hat{U} A_j^R \hat{U}^{-1} \hat{L}_k \hat{L}_l \\
 & - A_i^R \hat{U}^{-1} A_j^L \hat{U} \hat{R}_k \hat{R}_l + \frac{1}{2} A_i^R \hat{R}_j A_k^R \hat{R}_l - \frac{1}{2} A_i^L \hat{L}_j A_k^L \hat{L}_l] \\
 W_1 = & \frac{N_c}{48\pi^2} \int d^5x \epsilon_{ijklm} \partial_m \text{Tr} [A_i^R \hat{R}_j \hat{R}_k \hat{R}_l + A_i^L \hat{L}_j \hat{L}_k \hat{L}_l] \quad (2.19)
 \end{aligned}$$

The τ -integrals are easily carried out by using the boundary conditions (2.9). The 5-integrals are carried out by using the five-dimensional Gauss theorem, and we finally obtain

$$W(U, A^L, A^R)_{WZ} = W_c - W_0 \quad (2.20)$$

where

$$\begin{aligned}
 W_c = & -\frac{N_c}{240\pi^2} \int_Q d^5x \epsilon_{ijk\ell m} \text{Tr} (\hat{L}_i \hat{L}_j \hat{L}_k \hat{L}_\ell \hat{L}_m) \\
 & -\frac{N_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} \text{Tr} \left\{ - (A_\mu^R R_\nu R_\rho R_\sigma + A_\mu^L L_\nu L_\rho L_\sigma) \right. \\
 & + \frac{1}{2} (A_\mu^R R_\nu A_\rho^R R_\sigma - A_\mu^L L_\nu A_\rho^L L_\sigma) - A_\mu^R U^\dagger A_\nu^L U R_\rho R_\sigma + A_\mu^L U A_\nu^R U^\dagger L_\rho L_\sigma \\
 & + [\partial_\mu A_\nu^R U^\dagger A_\rho^L U R_\sigma + \partial_\mu A_\nu^L U A_\rho^R U^\dagger L_\sigma + (A_\mu^R \partial_\nu A_\rho^R + \partial_\mu A_\nu^R A_\rho^R \\
 & + A_\mu^R A_\nu^R A_\rho^R + A_\mu^R U^\dagger A_\nu^L U A_\rho^R) R_\sigma + (A_\mu^L \partial_\nu A_\rho^L + \\
 & + \partial_\mu A_\nu^L A_\rho^L + A_\mu^L A_\nu^L A_\rho^L + A_\mu^L U A_\nu^R U^\dagger A_\rho^L) L_\sigma] \\
 & + [A_\mu^R U A_\nu^L \partial_\rho A_\sigma^L U + A_\mu^R U^\dagger \partial_\nu A_\rho^L A_\sigma^L U + \partial_\mu A_\nu^R A_\rho^R U^\dagger A_\sigma^L U \\
 & + A_\mu^R \partial_\nu A_\rho^R U^\dagger A_\sigma^L U + A_\mu^R A_\nu^R A_\rho^R U^\dagger A_\sigma^L U + A_\mu^R U^\dagger A_\nu^L A_\rho^L A_\sigma^L U \\
 & \left. + \frac{1}{2} A_\mu^R U^\dagger A_\nu^L U A_\rho^R U^\dagger A_\sigma^L U \right\} \quad (2.21)
 \end{aligned}$$

$$\begin{aligned}
 W_0 \equiv W_c (U=I, A_\mu) = \\
 \frac{N_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} \text{Tr} \left\{ (A_\mu^L A_\nu^R - A_\mu^R A_\nu^L) (\partial_\rho A_\sigma^L + \partial_\rho A_\sigma^R) \right. \\
 \left. + A_\mu^L A_\nu^L A_\rho^L A_\sigma^R - A_\mu^R A_\nu^R A_\rho^R A_\sigma^L - \frac{1}{2} A_\mu^R A_\nu^L A_\rho^R A_\sigma^L \right\} \quad (2.22)
 \end{aligned}$$

All the terms in W_c (except an important relative sign difference between the Wess-Zumino-Witten term, and the rest) agree with Ref. 7), but not with Ref. 4). It is worth emphasizing that although for the purpose of comparison with the existing results^{4),7)}, our result is expressed in terms of left-right source fields, it is the integral of the Bardeen anomaly where the vector current is non-anomalous, whereas the variations of the effective actions of Refs. 4) and 7) yield the left-right anomalies which are based on different regularization schemes. The relationship between different anomalies and corresponding effective actions will be studied in more detail in Section 3.

The structure of our effective action is also worth a comment of immediate physical interest. It can easily be seen to have the following symmetry property:

$$W(U^{-1}, A^R, A^L) = -W(U, A^L, A^R) \quad (2.23)$$

Recalling that the transformation $U \rightarrow U^{-1}$ corresponds to $\pi \rightarrow -\pi$ in terms of the physical pion fields, we immediately recognize that this (anti) symmetry is a reflection of the physical parity symmetry. The implementation of this symmetry selects an effective Lagrangian against others (like Witten's) that might differ for gauge invariant, but not parity invariant, additions. A physical consequence of (2.23) is the fact that when $A^R = A^L$, as in the pure vector flavour gauging case, only processes with an odd number of Goldstone bosons are involved^{*)}.

We next consider the Abelian limit of our effective action (2.20) which corresponds to the electromagnetism

$$A_\mu^L = A_\mu^R = -ia_\mu Q^{em} \quad (2.24)$$

For arbitrary flavour, we obtain

$$\begin{aligned} W(U, a^{em}) = & W_{WZW} + \frac{iN_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} a_\mu \\ & \text{Tr} [Q (R_\nu R_\rho R_\sigma + L_\nu L_\rho L_\sigma)] \\ & - \frac{N_c}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} f_{\mu\nu} a_\rho \text{Tr} [Q^2 (L_\sigma + R_\sigma)] \\ & + \frac{1}{2} (U Q U^\dagger Q L_\sigma + U^\dagger Q U Q R_\sigma) \end{aligned} \quad (2.25)$$

For the two-flavour case, then, the correct Skyrme Lagrangian coupled to the electromagnetism is

^{*)} Suggested by P. Di Vecchia.

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{SKYRME}}^{\text{GAUGED}} + \frac{i N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} a_\mu \text{Tr} [Q (R_\nu R_\rho R_\sigma + L_\nu L_\rho L_\sigma)] \\ - \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} f_{\mu\nu} a_\rho \text{Tr} [Q^2 (L_\sigma + R_\sigma) \\ + \frac{1}{2} (Q U^{-1} Q U R_\sigma + Q U Q U^{-1} L_\sigma)] \end{aligned} \quad (2.26)$$

3. - THE RELATIONSHIP BETWEEN DIFFERENT ANOMALIES

Our result in Eq. (2.20) was obtained by direct integration of the Bardeen (axial) anomaly. In order to compare with previous results which have appeared in the literature, we must elucidate the relationship between different anomalies.

Let us recall the expression for the left-right anomaly:

$$G_{R,L} = \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} [2 \partial_\mu A_\nu^{R,L} \partial_\rho A_\sigma^{R,L} + \partial_\mu (A_\nu^{R,L} A_\rho^{R,L} A_\sigma^{R,L})] \quad (3.1)$$

By rewriting the Bardeen anomaly, Eq. (2.2), in terms of the left and right fields, we get the following relationship:

$$G_B = G_L + G_R + \Delta G \quad (3.2)$$

where

$$\begin{aligned} \Delta G = \frac{N_c}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} [A_\mu^L A_\nu^R \partial_\rho A_\sigma^R + A_\mu^R A_\nu^L \partial_\rho A_\sigma^R + A_\mu^L \partial_\nu A_\rho^R A_\sigma^R \\ + A_\mu^R \partial_\nu A_\rho^L A_\sigma^L + \partial_\mu A_\nu^R A_\rho^L A_\sigma^L + \partial_\mu A_\nu^L A_\rho^R A_\sigma^R + A_\mu^L A_\nu^R \partial_\rho A_\sigma^L \\ + A_\mu^R A_\nu^L \partial_\rho A_\sigma^L + A_\mu^L \partial_\nu A_\rho^L A_\sigma^R + \partial_\mu A_\nu^L A_\rho^R A_\sigma^L \\ + \partial_\mu A_\nu^R \partial_\rho A_\sigma^L + A_\mu^R A_\nu^L A_\rho^R A_\sigma^L + A_\mu^L A_\nu^R A_\rho^L A_\sigma^R + A_\mu^R A_\nu^R A_\rho^R A_\sigma^L \\ + A_\mu^L A_\nu^L A_\rho^L A_\sigma^R + A_\mu^R A_\nu^L A_\rho^L A_\sigma^L] \end{aligned} \quad (3.3)$$

Now let us consider the variation of $W_{WZ}(U, A^L, A^R)$ under an arbitrary $SU(N)_L \times SU(N)_R$ transformation

$$\begin{aligned} U &\rightarrow a U b^{-1} \\ A_\mu^L &\rightarrow a (A_\mu^L + \partial_\mu) a^{-1} \\ A_\mu^R &\rightarrow b (A_\mu^R + \partial_\mu) b^{-1} \end{aligned} \quad \begin{aligned} a &= e^{i\theta_L} \\ b &= e^{i\theta_R} \end{aligned} \quad (3.4)$$

The variation of W_{WZ} reduces to

$$\delta_{L,R} W_{WZ} = \delta_{L,R} W_c(U, A_\mu^L, A_\mu^R) - \delta_{L,R} W_0(A_\mu^L, A_\mu^R) \quad (3.5)$$

and we already know from Ref. 7) that

$$\delta_{L,R} W_c = -i \int d^4x \operatorname{Tr} [\theta_L G_L - \theta_R G_R] \quad (3.6)$$

We can easily compute the variation of W_0 , finding

$$\begin{aligned} \delta_L W_0 &= i \int d^4x \operatorname{Tr} [\theta_L (G_R + \Delta G)] \\ \delta_R W_0 &= -i \int d^4x \operatorname{Tr} [\theta_R (G_L + \Delta G)] \end{aligned} \quad (3.7)$$

An immediate consequence of Eqs. (3.6) and (3.7) is the result

$$\delta_{L,R} W_{WZ} = -i \int d^4x (\theta_L - \theta_R) G_B \quad (3.8)$$

confirming the validity of our previous computation.

Actually, the relationship

$$W_{WZ} = W_c - W_0 \quad (3.9)$$

is not unexpected, and throws some light on the relationship between different anomalies. A basic property^{*)} of the differential operator X we introduced in Eq. (2.1) allows us to write

*) Suggested by G.C. Rossi.

$$\left. X W(U, A_\mu^L, A_\mu^R) \right|_{U=I} = X W(I, A_\mu^L, A_\mu^R) \quad (3.10)$$

and therefore the equation

$$X W_{WZ}(U, A_\mu^L, A_\mu^R) = 0 \quad (3.11)$$

expressing the vector invariance of our effective Lagrangian, follows immediately.

Equation (3.10) shows how different axial anomalies (corresponding to different regularization prescriptions) may be related to each other in the language of effective Lagrangians. Let us start from the solution of the left-right anomaly equations: $W_c(U, A_\mu^L, A_\mu^R)$. It is obvious that we are going to get a new solution of the same equations if we add to W_c a completely left-right invariant term $I(U, A_\mu^L, A_\mu^R)$ such that

$$\delta_{L,R} I = 0 \quad (3.12)$$

However, while

$$W_c(U, A_\mu^L, A_\mu^R) + I(U, A_\mu^L, A_\mu^R) \quad (3.13)$$

has the same L and R anomalies as W_c , when we construct the vector invariant combination according to Eq. (3.10)

$$\begin{aligned} & W_c(U, A_\mu^L, A_\mu^R) + I(U, A_\mu^L, A_\mu^R) - W_c(I, A_\mu^L, A_\mu^R) - I(I, A_\mu^L, A_\mu^R) \\ &= W_{WZ}(U, A_\mu^L, A_\mu^R) + I(U, A_\mu^L, A_\mu^R) - I(I, A_\mu^L, A_\mu^R) \end{aligned} \quad (3.14)$$

we find that obviously

$$-\delta_V I(I, A_\mu^L, A_\mu^R) = 0 \quad (3.15)$$

whereas

$$-\delta_A \mathcal{I}(\mathcal{I}, A_\mu^L, A_\mu^R) = -i \int d^4x (\theta_L - \theta_R) \delta G \neq 0 \quad (3.16)$$

and therefore the axial anomaly of the new action differs from the Bardeen anomaly, Eq. (3.8), by the quantity δG .

Therefore, in general, different solutions of the left-right anomaly equations (like Witten's solution) lead to different axial anomalies, and our construction is unique in generating exactly the Bardeen anomaly and the physical symmetries it incorporates.

In the next section, we shall relate the anomaly change, Eq. (3.16), to the change in the fermionic determinant due to changing the regularization scheme.

4. - A REPRESENTATION OF THE FERMIONIC DETERMINANT

The results presented in Section 2 allow us to give an alternative representation of the anomalous part of the fermionic determinant that makes more immediate contact with the language of forms.

We know from Ref. 10) that the left-right anomaly has a topological origin: it may be thought of as the variation of the five-dimensional functional

$$\Delta = N_c \int d^5x \epsilon_{ijk\ell m} (\pi_{ijk\ell m}^L - \pi_{ijk\ell m}^R) \quad (4.1)$$

where

$$\begin{aligned} \pi_{ijk\ell m} = -\frac{1}{240\pi^2} \text{Tr} [& A_i A_j A_k A_\ell A_m - 5(\partial_i A_j + A_i A_j) A_k A_\ell A_m \\ & + 10(\partial_i A_j + A_i A_j)(\partial_k A_\ell + A_k A_\ell) A_m] \end{aligned} \quad (4.2)$$

Under left-right infinitesimal transformations

$$\delta_{L,R} \Delta = -\delta_{L,R} W_c = i \int d^4x \text{Tr} [\theta_L G_L - \theta_R G_R] \quad (4.3)$$

Therefore, if we consider the combination

$$\Delta_{WZ} = \Delta + W_0 \quad (4.4)$$

the following property immediately follows:

$$\delta_{L,R} \Delta_{WZ} = -\delta W_{WZ} = i \int d^4x \text{Tr} (\theta_L - \theta_R) G_B \quad (4.5)$$

By construction, Δ_{WZ} is a function of the gauge fields alone, not involving the pion field U , at the expense of introducing a fifth gauge field component A_5 . Therefore, Δ_{WZ} is a formal representation of the anomalous part of the fermionic determinant

$$W(A_\mu^R, A_\mu^L) = N_c \text{Tr} \text{Ln} i \left(\not{\partial} + \cancel{A^R} \frac{1+\gamma_5}{2} + \cancel{A^L} \frac{1-\gamma_5}{2} \right) \quad (4.6)$$

for which Eq. (4.5) is known to hold, under the assumption that the fermionic theory is regularized in such a way as to preserve vector invariance.

Let us notice that the function $W_0 = \Delta_{WZ} - \Delta$ is exactly a fourth order polynomial in the gauge fields, as predicted by Gasser and Leutwyler in Ref. 11). The definition of Δ_{WZ} provides us with an alternative approach to the construction of the WZ Lagrangian. Let us consider the effect of a finite chiral transformation on Δ_{WZ} :

$$\Delta_{WZ} (a(A_\mu^L + \partial_\mu) a^{-1}, b(A_\mu^R + \partial_\mu) b^{-1}) = \Delta'_{WZ} (A_\mu^L, A_\mu^R, a^{-1}b) \quad (4.7)$$

Vector invariance ensures us that Δ'_{WZ} depends only on the combination $a^{-1}b$.

We can now identify $a^{-1}b$ with the pion field and define

$$\bar{W}_{WZ}(A_\mu^L, A_\mu^R, U) = \Delta'_{WZ}(A_\mu^L, A_\mu^R, U) - \Delta'_{WZ}(A_\mu^L, A_\mu^R, I) \quad (4.8)$$

By construction

$$\delta \bar{W}_{WZ} = \delta W_{WZ} \quad (4.9)$$

But we can also show that the more attractive relationship

$$\overline{W}_{WZ} = W_{WZ} \quad (4.10)$$

holds, as was known to happen in the two-dimensional case¹²⁾.

Actually, Eq. (4.10) is only a special case of the following general result holding for any given completely invariant functional $I(U, A_\mu^L, A_\mu^R)$ obeying Eq. (3.12). We consider the function of the gauge fields

$$I_0(A_\mu^L, A_\mu^R) = I(I, A_\mu^L, A_\mu^R) \quad (4.11)$$

and we perform a L-R rotation of the fields

$$I_0(a(A_\mu^L + \partial_\mu) a^{-1}, b(A_\mu^R + \partial_\mu) b^{-1}) = I'_0(A_\mu^L, A_\mu^R, a^{-1}b) \quad (4.12)$$

due to the already discussed vector invariance of I_0 . Now, it is trivial to show that

$$I'_0(A_\mu^L, A_\mu^R, a^{-1}b) \equiv I(a^{-1}b, A_\mu^L, A_\mu^R) \quad (4.13)$$

If we choose the left-right invariant combination

$$W_c(U, A_\mu^L, A_\mu^R) + \Delta(A_\mu^L, A_\mu^R) \quad (4.14)$$

we find that its value, when $U = 1$, is exactly

$$W_0 + \Delta = \Delta_{WZ} \quad (4.15)$$

and therefore

$$\begin{aligned} \Delta'_{WZ} - \Delta_{WZ} &= W_c(a^{-1}b, A_\mu^L, A_\mu^R) - W_c(I, A_\mu^L, A_\mu^R) = \\ &= W_{WZ}(a^{-1}b, A_\mu^L, A_\mu^R) \end{aligned} \quad (4.16)$$

More generally, referring to the observations at the end of the previous section, we notice that a change in the effective L-R action

$$W_c \rightarrow W_c + I \quad (4.17)$$

leads naturally, through Eqs. (4.7) and (4.8), to a change in the fermionic determinant

$$\Delta_{WZ} \rightarrow \Delta_{WZ} + I_0 \quad (4.18)$$

and using Eq. (3.16)

$$\delta_A (\Delta_{WZ} + I_0) = i \int d^4x \text{Tr} (\theta_L - \theta_R) (G_B + \delta G) \quad (4.19)$$

Now, the repetition of the argument given before leads naturally to the construction of the modified effective action

$$W_{WZ} + I - I_0$$

fully reproducing the results of Section 3.

A final physically relevant observation concerns the following property of the polynomial W_0 . When we choose to gauge the maximal anomaly-free subgroup $U(N)_V$, that is when we impose $A_\mu^R = A_\mu^L$,

$$W_0 (A_\mu^L = A_\mu^R) = 0$$

and since $\Delta(A_\mu^L = A_\mu^R)$ is trivially zero, we obtain

$$\Delta_{WZ} (A_\mu^L = A_\mu^R) = 0$$

consistent with the requirement of vanishing anomaly.

ACKNOWLEDGEMENTS

We are grateful to P. Di Vecchia and G.C. Rossi for extremely useful discussions. We also thank Y. Frishman and R. Stora for comments.

APPENDIX - THE TWO-DIMENSIONAL CASE

We think it is instructive to compare our results to the corresponding effects in the two-dimensional case. We shall recover the standard two-dimensional WZ Lagrangian within a formalism more general than the one commonly used (and typically tailored on some special properties of the two-dimensional gauge fields). Our starting point is the anomaly of the fermionic determinant

$$\text{Tr} \text{Ln} i (\not{\partial} + \not{V} + \not{A}) = \text{Tr} \text{Ln} i (\not{\partial} + \not{C}) \quad (\text{A.1})$$

where

$$C_\mu = V_\mu + \epsilon_{\mu\nu} A_\nu \quad (\text{A.2})$$

Notice the very important property that the two independent components of the gauge field C_μ are related to left and right components through

$$C_+ = A_+^L \quad C_- = A_-^R \quad (\text{A.3})$$

The standard axial anomaly is known to be

$$\begin{aligned} -\frac{1}{2\pi} \epsilon_{\mu\nu} [\partial_\mu C_\nu + C_\mu C_\nu] &= \frac{1}{4\pi} \epsilon_{\mu\nu} [\partial_\mu A_\nu^L + \partial_\mu A_\nu^R + A_\mu^R A_\nu^L + A_\mu^L A_\nu^R] \\ &+ \frac{1}{4\pi} [\partial_\mu A_\mu^R + A_\mu^L A_\mu^R - \partial_\mu A_\mu^L - A_\mu^R A_\mu^L] \end{aligned} \quad (\text{A.4})$$

For further reference, we also recall that the left and right anomalies respectively are

$$\frac{1}{4\pi} [\epsilon_{\mu\nu} + g_{\mu\nu}] \partial_\mu A_\nu^R \quad \frac{1}{4\pi} [\epsilon_{\mu\nu} - g_{\mu\nu}] \partial_\mu A_\nu^L \quad (\text{A.5})$$

Following the notation and techniques presented in the text, we obtain

$$W[V, A^L, A^R] = \frac{N_c}{4\pi} \int d^3x (I_2 + I_1 + I_0) \quad (\text{A.6})$$

$$\begin{aligned}
 I_2 &= \text{Tr} \left[-\epsilon_{\mu\nu\tau} \hat{U}^{-1} A_\mu^L \hat{U} A_\nu^R \hat{R}_\tau - \epsilon_{\mu\nu\tau} \hat{U} A_\mu^R \hat{U}^{-1} A_\nu^L \hat{L}_\tau \right. \\
 &\quad \left. - \hat{U}^{-1} A_\mu^L \hat{U} A_\mu^R \hat{R}_\tau + \hat{U} A_\mu^R \hat{U}^{-1} A_\mu^L \hat{L}_\tau \right] \\
 I_1 &= \text{Tr} \left[-\epsilon_{\mu\nu\tau} \partial_\mu A_\nu^R \hat{R}_\tau - \epsilon_{\mu\nu\tau} \partial_\mu A_\nu^L \hat{L}_\tau - \partial_\mu A_\mu^R \hat{R}_\tau + \partial_\mu A_\mu^L \hat{L}_\tau \right. \\
 &\quad \left. - \epsilon_{\mu\nu\tau} A_\mu^R (\hat{R}_\nu \hat{R}_\tau - \hat{R}_\tau \hat{R}_\nu) + \epsilon_{\mu\nu\tau} A_\mu^L (\hat{L}_\nu \hat{L}_\tau - \hat{L}_\tau \hat{L}_\nu) \right. \\
 &\quad \left. + A_\mu^R (\hat{R}_\mu \hat{R}_\tau - \hat{R}_\tau \hat{R}_\mu) + A_\mu^L (\hat{L}_\mu \hat{L}_\tau - \hat{L}_\tau \hat{L}_\mu) \right] \\
 I_0 &= \text{Tr} \left[\frac{1}{3} \epsilon_{\mu\nu\tau} \hat{R}_\mu \hat{R}_\nu \hat{R}_\tau - \partial_\mu \hat{R}_\mu \hat{R}_\tau \right]
 \end{aligned}$$

and explicitly performing the integration we have

$$\begin{aligned}
 W_{WZ}(U, A^L, A^R) &= \frac{1}{12\pi} \int d^3x \epsilon_{\mu\nu\tau} \hat{R}_\mu \hat{R}_\nu \hat{R}_\tau + \frac{1}{8\pi} \int d^3x \partial_\mu U^{-1} \partial_\mu U \\
 &\quad - \frac{1}{4\pi} \int d^3x \left[(\epsilon_{\mu\nu} - g_{\mu\nu}) A_\mu^R R_\nu - (\epsilon_{\mu\nu} + g_{\mu\nu}) A_\mu^L L_\nu \right. \\
 &\quad \left. - (\epsilon_{\mu\nu} - g_{\mu\nu}) A_\mu^R U^{-1} A_\nu^L U + (\epsilon_{\mu\nu} - g_{\mu\nu}) A_\mu^R A_\nu^L \right] \quad (A.7) \\
 &= W(U, A^L, A^R) - W(I, A^L, A^R)
 \end{aligned}$$

We next notice that

$$\delta_{L,R} W_{WZ} = \delta_{L,R} W - \delta_{L,R} W_0 \quad (A.8)$$

where now

$$W_0 = \frac{1}{4\pi} \int d^3x \text{Tr} A_-^R A_+^L = \frac{1}{4\pi} \int d^3x \text{Tr} C_- C_+ \quad (A.9)$$

We immediately notice that Eqs. (3.6) and (3.7) hold. In particular,

$$\delta_{L,R} W = -i \int d^3x \text{Tr} (\theta_L G_L - \theta_R G_R) \quad (A.10)$$

and Δ_G takes the value

$$(g_{\mu\nu} - \epsilon_{\mu\nu}) (A_\mu^R A_\nu^L + A_\mu^L A_\nu^R) = [A_+^L, A_-^R]$$

It is interesting to reformulate the problem in terms of the variation of three forms. A manipulation analogous to that presented in Ref. 7) allows us to identify

$$\Delta = -\frac{1}{8\pi} \int d^3x \text{Tr} (A_\mu^L A_\mu^L + A_\mu^R A_\mu^R) + \int d^3x \epsilon_{\mu\nu\rho} (\Pi_{\mu\nu\rho}^L - \Pi_{\mu\nu\rho}^R) \quad (\text{A.11})$$

$$\Pi_{\mu\nu\rho} = \frac{1}{12\pi} (A_\mu A_\nu A_\rho - 3(\partial_\mu A_\nu + A_\mu A_\nu) A_\rho) \quad (\text{A.12})$$

such that Eqs. (4.3)-(4.10) hold also in the two-dimensional case. The extremely attractive feature of the two-dimensional case is the fact, related to the existence of only two independent components Eq. (A.3), and two gauge freedoms, that it is possible without loss of information to choose the parametrization

$$A_\mu^R = B^{-1} \partial_\mu B \quad A_\mu^L = A^{-1} \partial_\mu A \quad (\text{A.13})$$

which enables us to re-express our results in the form

$$\begin{aligned} \Delta_{WZ} (A_\mu^R, A_\mu^L) &= \Delta_{WZ} (A, B) = \\ &= \frac{1}{8\pi} \int d^3x \text{Tr} \partial_\mu A \partial_\mu A^{-1} + \frac{1}{12\pi} \int d^3x \epsilon_{\mu\nu\rho} \text{Tr} A^{-1} \partial_\mu A A^{-1} \partial_\nu A A^{-1} \partial_\rho A \\ &+ \frac{1}{8\pi} \int d^3x \text{Tr} \partial_\mu B \partial_\mu B^{-1} - \frac{1}{12\pi} \int d^3x \epsilon_{\mu\nu\rho} \text{Tr} B^{-1} \partial_\mu B B^{-1} \partial_\nu B B^{-1} \partial_\rho B \\ &+ \frac{1}{4\pi} \int d^3x \text{Tr} (B^{-1} \partial_\mu B) (A^{-1} \partial_\mu A) = \\ &= \frac{1}{8\pi} \int d^3x \text{Tr} \partial_\mu (AB^{-1}) \partial_\mu (BA^{-1}) + \frac{1}{12\pi} \int d^3x \epsilon^{\mu\nu\rho} (BA^{-1}) \partial_\mu (AB^{-1}) (BA^{-1}) \partial_\nu (AB^{-1}) (BA^{-1}) \partial_\rho (AB^{-1}) \end{aligned} \quad (\text{A.14})$$

which is nothing but the Polyakov-Wiegmann and Alvarez¹³⁾ result for the two-dimensional fermionic determinant.

REFERENCES

- 1) T. Skyrme, Proc. Roy. Soc. London A260 (1961) 127; Nucl. Phys. B31 (1962) 556.
- 2) N.K. Pak and H.C. Tze, Ann. Phys. 117 (1979) 164.
- 3) A. Balachandran, V. Nair and J. Stern, Phys. Rev. D27 (1983) 1133.
- 4) E. Witten, Nucl. Phys. B223 (1983) 422, 433.
- 5) E. Witten, Nucl. Phys. B160 (1979) 57.
- 6) J. Wess and B. Zumino, Phys. Lett. 37B (1970) 13.
- 7) K.C. Chou, H.Y. Guo, K. Wu and X.C. Song, Beijing preprint AS-ITP-83-025 (1983).
- 8) W. Bardeen, Phys. Rev. 184 (1969) 1848.
- 9) G.C. Rossi, M. Testa and K. Yoshida, Phys. Lett. 134B (1983) 78.
- 10) B. Zumino, Y.S. Wu and A. Zee, Seattle preprint 40048-18P3 (1983).
B. Zumino, Les Houches Lectures Notes, Berkeley preprint LBL-16747 (1983)
R. Stora, Cargèse Lectures, LAPP preprint LAPP-TH-94 (1983)
- 11) J. Gasser and H. Leutwyler, CERN preprint TH.3689 (1983);
see also P. Di Vecchia and P. Rossi, CERN preprint TH.3808 (1984).
- 12) A. D'Adda, A. Davis and P. Di Vecchia, Phys. Lett. 121B (1982) 335.
- 13) A. Polyakov and A.S. Wiegmann, Phys. Lett. 131B (1983) 121;
O. Alvarez, Berkeley preprint (1983).