PRIMORDIAL INFLATION WITH FLAT SUPERGRAVITY POTENTIALS

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ABSTRACT

A new class of Primordial Inflation Models (PRIMOs) is presented, based upon the possibility of getting naturally flat potentials in $N = 1$ supergravity. These models may be considered as the natural cosmological extension of some recently proposed no-scale particle physics models, where all mass scales are derived from the Planck mass (M_P). PRIMOs seem to satisfy all presently-known cosmological constraints and they are guaranteed (a) to be void of SUSY minima with negative cosmological constant, (b) of a stable (around the origin) potential form against finite temperature effects.

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The existence of a period of exponential expansion of the Universe, inflation, at the early stages of its development seems to be the only solution for long-standing cosmological mysteries related to the hot Big Bang model, and for problems raised by grand unified theories, supersymmetric or not. We refer to the flatness, homogeneity and isotropy problems, as well as the spectrum of energy density perturbations adequate for producing galaxies, and questions such as the possible large number of primordial monopoles and/or gravitinos, the existence of domain walls, etc.

The original version of the inflationary universe scenario presented by Guth¹⁾ [see also Ref. 2)] already contained the main desired qualitative features except for the necessary end of the inflationary period. Guth proposed that bubbles of a new phase in another metastable one were produced in a strongly first-order phase transition, in the early history of the universe. While the universe is trapped in the metastable phase it supercools, and the T^4 dependent radiation energy density, $\rho_{\text{rad}} = (\pi^2/30) g(T) T^4$ [$g(T)$ is the effective number of spin states of relativistic particles] becomes negligible with respect to the vacuum energy, V_0 . The expansion rate, H , the Hubble "constant" of the universe, becomes a constant given by

$$H \equiv \frac{\dot{R}}{R} = \left(\frac{8\pi}{3M_P^2} V_0 \right)^{1/2} \quad (1)$$

It yields an exponential expansion of the scale factor R of the Universe $R \sim e^{Ht}$. Here $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass. From now on, we work in units such that $M = M_P/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV $\equiv 1$, unless specified otherwise. In the new phase, $V_0 = 0$ and the standard cosmology is recovered inside the bubbles. The problem of the "graceful exit" arises because this phase transition is never completed. The nucleation and growth rate of the bubbles (in the standard Friedmann phase) is slow compared to the exponential expansion of the interspaces which remain in the metastable (de Sitter) phase. In the new inflationary scenario proposed by Linde³⁾ and also by Albrecht and Steinhardt⁴⁾, by considering the whole visible universe contained inside only one of the bubbles of the new phase, the necessity of completing the transition no longer exists. The original size of a bubble must be at most of the order of the horizon, the Hubble radius H^{-1} , at the moment at which the phase transition starts. The idea is, then, that a smooth region of this size (it could be picked up from a completely chaotic

environment) expands up to include all our observed Universe (whose radius is $\approx 10^{28}$ cm). For a characteristic value of $V_0 \approx (10^{15} \text{ GeV})^4$, the bubble must expand at least by a factor $\sim e^{65}$. That means that the transition to the new phase inside the bubble must not be instantaneous, but long enough to allow for that growth. In order to ensure the end of the inflation inside the bubble, the exponential expansion must, then, occur while the Universe "rolls down" slowly but surely towards the stable phase, once a certain kind of barrier has been overcome. Let us speak in terms of the order parameter of the transition, the vacuum expectation value of a certain Higgs field ϕ , which we call "inflaton". Due to the flatness of the inflaton potential near to the supercooled minimum, it must take many Hubble times (H^{-1}) for ϕ to increase significantly from its initial value. During this period, the energy density $\rho = V(\phi) + \frac{1}{2}(\partial\phi/\partial t)^2 + \rho_{\text{rad}}$ is dominated by the almost constant energy density of the inflaton potential, V_0 . The Universe approaches a de Sitter state, in which V_0 acts as the cosmological constant of an empty universe. During this period of inflation, R expands exponentially $R = e^{\int H dt} R_0$, and there is an effective temperature, called the Hawking temperature^{5),6)}, $T_H = H/2\pi$. As ϕ gets close to the stable minimum with $V = 0$ (we know that today the cosmological constant is $\lesssim 10^{-120} M_P^4$; that is, $V \lesssim 10^{-50} \text{ GeV}^4$), the potential must steepen in such a way that the rapid oscillations of ϕ lead to the conversion of the coherent inflaton energy into radiation. Through the coupling of ϕ with other fields, the Universe is "filled up" again. This period is called "reheating of the Universe".

This proposal is qualitatively correct, but all the specific models considered up to now failed quantitatively. In those based on $SU(5)$ ⁷⁾ Coleman-Weinberg⁸⁾ potentials, because the amount of inflation produced is very sensitive to the ϕ^4 coupling which, in turn, depends on the gauge coupling g , sufficient inflation occurs only when $g^2 \lesssim 10^{-2}$, instead of 0.3, otherwise quantum fluctuations produce too fast the ending of inflation⁹⁾ [see also Ref. 10)], the energy fluctuations are too large, $\delta\rho/\rho \sim 10$ [Ref. 11)] instead of 10^{-4} as is necessary to be consistent with the isotropy of the microwave background radiation ($\Delta T/T \sim 10^{-4}$) and, moreover, the transition of $SU(5)$ into $SU(3) \times SU(2) \times U(1)$ would not proceed directly, because other directions in the group space [e.g., $SU(4) \times U(1)$] are

favoured¹²⁾. These problems led several people^{10),13)-15)} to consider supersymmetric versions of the Coleman-Weinberg SU(5) potential. There, the ϕ^4 coupling can be made independent of the gauge coupling and, moreover, the potential is automatically very flat, therefore energy fluctuations of the correct order can be obtained¹⁴⁾. However, the potential is indeed too flat for enough reheating to be produced after the inflation. Besides, this approach does not solve the problem¹²⁾ of the wrong breaking patterns of SU(5). A way of avoiding the troubles caused by the identification of the inflaton with the Higgs field in the adjoint representation of SU(5) is to consider a primordial inflation^{13),14),16)-21)}; this means inflation independent of the GUT transition. The inflation must necessarily occur before the GUT transition for two important reasons: not only in order to preserve the baryon number asymmetry produced in the GUT transition, but also in order to erase any pre-existing random baryon asymmetry and anisotropies before the GUT transition. The inflaton must, then, be a singlet under the GUT group. In this case, the monopole problem is not necessarily solved by the inflation, but the number of monopoles can still be small enough if the GUT transition takes place at $T_c \leq 10^{10}$ GeV. When this delayed GUT transition happens, the horizons are already large enough, with respect to the size of the Universe, so that counting one monopole per horizon the number of monopoles per photon is $\eta_m/\eta_\gamma \sim (T_c/M_p)^3 \lesssim 10^{-27}$, in agreement with the minimal requirement that the mass density of monopoles cannot exceed that of baryons.

The idea of primordial inflation came about¹³⁾ because, in constructing supersymmetric inflationary scenarios, unnatural fine tunings of several parameters are necessary if the transition occurs at the GUT scale, while they are removed by moving the inflation closer to the Planck scale M_p [$\langle\phi\rangle \simeq M_p$, not the transition temperature]. Since the GUT phase transition scale cannot coincide with M_p , ϕ must be a GUT singlet. Near to the Planck scale, however, gravitational effects cannot be neglected. This observation immediately leads us to consider potentials dictated by $N = 1$ supergravity²²⁾ for producing primordial inflation¹⁶⁾⁻²¹⁾. There are many pitfalls in such kinds of approaches, and one has to be extremely careful in order to avoid them. We believe that in the present work we avoid all known problems, while all the desired features of inflation can be obtained. However, if minimal kinetic terms are taken for all the fields (Z , which

breaks supergravity, ϕ , the inflaton and y_i , the matter fields), inconsistencies are difficult to avoid. Either undesired supersymmetric minima may arise under debatable assumptions, as pointed out in Ref. 21), or, to avoid that, the reheating is lost, as in Ref. 21), because ϕ breaks supersymmetry in the global minimum and therefore its mass, m_ϕ , must be of the order of the gravitino mass, $m_{3/2}$: $m_\phi \sim m_{3/2} \sim 100$ GeV. The problem consists essentially of the presence of two very different scales, one for the inflation and one for the supersymmetry breaking.

Let us present carefully the standard approach and its potential problems in order to build up progressively the model which we believe to be correct. Take the three mentioned types of fields, Z , ϕ , y_i , (each one could stand for various fields) with minimal kinetic terms $G_{\alpha\beta} = \delta_{\alpha\beta}$. The $N = 1$ supergravity potential is²²⁾

$$V = e^G \left[\sum_{\alpha, \beta} G_{,\alpha} (G^{-1})^{\alpha}_{\beta} G_{,\beta} - 3 \right] + \frac{D^2}{2} \quad (2)$$

where α runs over all the fields ξ_α ($\xi_\alpha = Z, \phi, y_i$) and the derivatives have the usual meaning $G_{,\alpha} = \partial G / \partial \xi_\alpha$; $G_{,\alpha} = \partial G / \partial \xi_\alpha^*$. The last term is the "D-term", in which only gauge non-singlet matter fields appear²²⁾. The most general Kähler potential G is, then,

$$G = Z Z^* + y y^* + \varphi \varphi^* + \ln |f_T|^2 \quad (3)$$

The total superpotential f_T is an arbitrary analytic function of the fields Z , ϕ and y_i . One can think of two simple forms for f_T (and combinations of both): a separation in terms²³⁾, $f_T = f^{(1)}(Z) + f^{(2)}(\phi) + f^{(3)}(y)$, or in

factors²⁴⁾, $f_T = f^{(1)}(Z) \cdot f^{(2)}(\phi) \cdot f^{(3)}(y)$. Let us concentrate on the first form and define the scales m and μ , respectively associated to ϕ and to y, Z fields, as the magnitudes such that $|f^{(2)}(\phi)| \lesssim m^3$ and both $|f^{(1)}(Z)|$ and $|f^{(3)}(y)| \lesssim \mu^3$, for the relevant intervals of the fields ($|\phi| \lesssim M_P$, Z and y near to their minima) (remember that the dimension of the superpotentials is three). We know, because of our previous experience, that $m^6/M^2 \sim (10^{15} \text{ GeV})^4$ and $\mu^6/M^2 \sim (10^{10} \text{ GeV})^4$, i.e., $m \gg \mu$. The second scale corresponds to the desired splitting of superpartners of the normal matter. When supersymmetry is broken, the splittings are of the order of the gravitino mass $m_{3/2}$. Thus, in the true global minimum of the potential in which the Universe falls after the inflation, we want $m_{3/2} = \mu^3/M^2 \sim 10^2 \text{ GeV}$ (indeed, we should say we find $m_{3/2} \sim 10^2 \text{ GeV}$, since in our approach this scale is determined dynamically, as we will see later), but not $m_{3/2} \sim m^3/M^2 \sim 10^{12} \text{ GeV}$. However, the last value is needed at the false vacuum, i.e., during the inflation. We see that our problem consists of eliminating the large scale m at the global minimum, because if both m and μ are present, m will always dominate. When the separation in terms of f_T is used, our problem is solved if any dependence of the potential on m^3 is eliminated at the global minimum, that means $f^{(2)}(\phi) \approx 0$ and $f_{,\phi}^{(2)}(\phi) \approx 0$, that is, they are effectively zero with respect to m^3 , at that point. When the factorized form of f_T is used, the definition of the scales in this way is impossible. The two scales problem, in this case, is not as transparent as in the additive form, on which we will concentrate.

Because of the different scales, wherever $f(\phi) \neq 0$, then

$$f_T = f^{(1)}(Z) + f^{(2)}(\phi) + f^{(3)}(y) \simeq f^{(2)}(\phi) \quad (4)$$

and the potential

$$V = e^{|Z|^2 + |\phi|^2 + |y|^2} \left(|f_{,Z}^{(1)} + Z^* f_T|^2 + |f_{,\phi}^{(2)} + \phi^* f_T|^2 + |f_{,y}^{(3)} + y^* f_T|^2 - 3 |f_T|^2 \right) + \frac{D^2}{2} \quad (5)$$

reduces to

$$\tilde{V} = e^{|z|^2 + |y|^2 + |\varphi|^2} \left(|f_{,\varphi}^{(2)} + \varphi^* f_{(\varphi)}^{(2)}|^2 - 3 |f_{(\varphi)}^{(2)}|^2 \right) \quad (6)$$

$\tilde{V}$ is of order m^6 . Only at a global minimum with zero cosmological constant, $\tilde{V} = 0$, and with conserved supersymmetry, $f_{,\phi}^{(2)} + \phi^* f^{(2)} = 0$ (together they mean $f^{(2)} = 0$) the higher scale m disappears.

This approach has been used in Refs. 16) and 21), in which only $\tilde{V}(\phi, \phi^*)$ (taking the Z and y fields in their minima) has been studied. Primordial inflation occurs near to the Planck scale. Non-renormalizable terms cannot be strongly suppressed by the presence of the inverse powers of M_P necessary to reduce their dimensions to four. Since all terms possibly induced by radiative corrections could be present, $f^{(2)}$ can be chosen as a polynomial of any degree, or a series. However, the number of zeros of $(f_{,\phi}^{(2)} + \phi^* f^{(2)})$, that is, the number of supersymmetric minima, increases with the degree of $f^{(2)}$, and it has been argued²¹⁾ that, under certain assumptions, it may be indeed impossible to prevent the existence of at least one of them between the supercooled and the $V = 0$ minima. Since at the supersymmetric minima the potential is negative, $V < 0$, these minima are stable with respect to the $V = 0$ desired minimum [even if, strange as it sounds, the $V = 0$ is also stable against decay into the $V < 0$ ones²⁵⁾]. The Universe, then, inevitably ends up in one of the $V < 0$ minima. The solution proposed by Ovrut and Steinhardt consists of taking $f^{(2)} \neq 0$ and $f_{,\phi}^{(2)} + \phi^* f^{(2)} \neq 0$ at the $V = 0$ minimum. But in this case, as we have seen, the inflaton scale m^3 is not eliminated from the remaining potential, or, what amounts to the same thing, ϕ is now the field which breaks supersymmetry and therefore the masses of the degrees of freedom contained in ϕ , call it m_I (imaginary) and m_R (real) fulfil the sum rule $m_I^2 + m_R^2 = 4m_{3/2}^2$, which forces $m^3 \equiv \mu^3$. Thus at least there is not reheating, and probably no inflation either!

The first attempt to eliminate completely the danger of the negative cosmological constant supersymmetric minima which one could think of, is to try to use a positive definite potential for the inflaton, much in the same way as in global supersymmetries. This is possible by means of a recently proposed²⁶⁾ supergravity-breaking mechanism based on, for simplicity, one field Z (singlet of the GUT group) and whose Kähler potential, call it $G^{(1)}(Z, Z^*)$, is chosen to produce a flat vanishing potential,

$$e^{G^{(1)}} \left[\frac{G_{,Z}^{(1)} G_{,Z^*}^{(1)}}{G_{,ZZ^*}^{(1)}} - 3 \right] \equiv 0 \quad (7)$$

We see that $G^{(1)}$ must be of the form

$$G^{(1)} = -3 \ln [g(Z) + g^*(Z^*)] \quad (8)$$

where $g(Z)$ is any analytic function. All such theories defined by $g(Z)$ are in fact equivalent up to field redefinitions $Z \rightarrow g(Z)$, therefore one can choose, without loss of generality,

$$G^{(1)} = -3 \ln [Z + Z^*] \quad (9)$$

The Z field has a non-minimal kinetic term

$$G_{,ZZ^*}^{(1)} dZ dZ^* \quad (10)$$

corresponding to an Einstein-Kähler manifold with curvature given by

$$R_{,ZZ^*} = \frac{2}{3} G_{,ZZ^*} \quad (11)$$

It has been shown²⁶⁾ that supersymmetry is always broken ($G_{,Z} \neq 0$ and $e^{G(Z)} \neq 0$) in the whole positive kinetic energy domain defined by

$$\frac{1}{3} |G_{,Z}^{(1)}|^2 = G_{,ZZ^*}^{(1)} > 0 \quad (12)$$

The total Lagrangian, except for the gravitino mass term, is invariant under the isometric transformation on the Z -Einstein-Kähler manifold, namely²⁷⁾²⁸⁾

$$Z \rightarrow a^2 Z \quad (13a)$$

$$Z \rightarrow Z + ib \quad (13b)$$

$$Z \rightarrow \frac{\sin \theta Z + i \cos \theta}{i \cos \theta Z + \sin \theta} \quad (13c)$$

where a , b and θ are real constants.

These transformations form the three generators of the $SU(1,1)$ non-compact group. The first one, (13a), is the $U(1)$ dilatation transformation.

The simple choice of separate Kähler manifolds for the Z , ϕ and y fields gives rise to a semi-positive definite total potential. The further choice of minimal kinetic terms for the ϕ and y fields, i.e.,

$$G = G^{(1)}(Z, Z^*) + G^{(2)}(\phi, \phi^*) + G^{(3)}(y, y^*) \quad (14)$$

with $G_{\phi\phi^*} = 1$ and $G_{yy^*} = 1$, yields the potential

$$V = e^{G^{(1)} + G^{(2)} + G^{(3)}} \left(|G_{\phi}^{(2)}|^2 + |G_y^{(3)}|^2 \right) + \frac{D^2}{2} \quad (15)$$

where the gauge D-terms have the usual form, $D = g_a G_{yi} T_j^a y^{*j} = g_a y_i T_j^a y^{*j}$ in terms of the generators T^a of the gauge group, and g_a are gauge coupling constants.

As we will see, this potential cannot produce inflation. It has a very interesting property. The global minima have automatically zero cosmological constant (at the tree level). They are given by the conditions

$$G_{,\phi} = 0 \quad G_{,y_i} = 0 \quad (16)$$

At any of these minima, the gravitino mass, e^G , is undetermined. As is shown in Refs. 27) and 28), non-gravitational quantum corrections raise this degeneracy, and preferable values for the gravitino mass, as well as for the weak breaking scale are obtained dynamically,

$$m_{3/2} \sim M_W \sim M_P \exp\left(-\frac{\mathcal{O}(1)}{\alpha_2}\right) \sim 10^{-16} M \quad (17)$$

where α_2 is the weak coupling constant. Thus, the hierarchy of $m_{3/2} \approx M_W$ with respect to M_p is explained. These are nice features; however, to obtain an inflationary scenario, the existence of a local minimum with non-zero cosmological constant is necessary, except, maybe, in the inflaton directions. In this case, inflation may be produced just with a flat $T = 0$ inflaton potential if, due to finite temperature effects, the Universe is trapped for some time. We follow the first, more conventional approach. It is easy to see that the potential (15) never has such a minimum in the Z direction. Let us call V_0 the value of the potential at that minimum, $V_0 \neq 0$. The condition for having a minimum in the Z direction is

$$\frac{\partial V}{\partial Z} = G_{,Z}^{(1)} V_0 = 0 \quad (18)$$

For $V_0 \neq 0$ the condition is $G_{,Z}^{(1)} = 0$, which is in contradiction with the positive definiteness condition, Eq. (12), of the kinetic energy. [In the y_i directions, the conditions $G_{,y_i}^{(3)} = 0$, the same as in global minima, are harmless]. The only consistent solution is a minimum where $V_0 = 0$, i.e., with zero cosmological constant.

There is a possibility of bypassing this difficulty^(F1). The inflaton field ϕ must have a non-minimal kinetic term. We need to generalize the Z - ϕ Kähler manifold, but at the same time keeping the validity of the $SU(1,1)$ invariance (because it ensures an undetermined gravitino mass at low energies, at the tree level) and the positivity of the total potential.

A suitable solution [to Eq. (7)] is

$$G = -3 \left[\ln(Z + Z^* + h(\varphi, \varphi^*)) \right] + g(\varphi, \varphi^*, y, y^*) \quad (19)$$

where h and g are real functions.

The resulting potential is semipositive definite:

$$V = e^G \left[\frac{|g_{,\varphi}|^2}{(g_{,\varphi\varphi^*} + G_{,Z} h_{,\varphi\varphi^*})} + |g_{,y}|^2 \right] + \frac{D^2}{Z} \quad (20)$$

(remember that $D = g_a g_{yi} T^{aj} y^{*j} = g_a y_i T^{aj} y^{*j}$). The denominator of the first term in Eq. (20) defines the positive kinetic energy domain for the inflaton field. In fact, the product of the eigenvalues, λ 's, of the kinetic terms, is

$$\prod \lambda_z \lambda_\varphi \lambda_y = \text{Det } G_{,i}{}^j = G_{,zz^*} [g_{,\varphi\varphi^*} + G_{,z} h_{,\varphi\varphi^*}] G_{,yy^*} \quad (21)$$

The conditions for positive kinetic energy are

$$\lambda_z = G_{,zz^*} > 0 \quad (22a)$$

$$\lambda_\varphi = g_{,\varphi\varphi^*} + G_{,z} h_{,\varphi\varphi^*} > 0 \quad (22b)$$

$$\lambda_{y_i} = G_{,y_i y^{*j}} = \delta_i^j \quad (22c)$$

as is shown in Ref. 27).

The diagonalized kinetic terms for Z and ϕ are

$$\mathcal{L}_{kin} = \frac{1}{12} (\partial_\mu G^0)^2 + \frac{3}{4} e^{\frac{2}{3} G^0} I_\mu^z + (g_{,\varphi\varphi^*} + G_{,z} h_{,\varphi\varphi^*}) \partial_\mu \varphi \partial^\mu \varphi^*$$

$$\text{where } G^0 = -3 \ln (Z + Z^* + h(\varphi, \varphi^*)) \quad (23)$$

$$\text{and } I_\mu = i \left[\partial_\mu (Z - Z^*) + h_{,\varphi} \partial_\mu \varphi - h_{,\varphi^*} \partial_\mu \varphi^* \right]$$

Note that, due to the positivity of the potential, the global minima are given by the conditions

$$g_{,\varphi} = 0 \quad g_{,y_i} = 0 \quad (24)$$

The gravitino mass is again undetermined at the global minima at the tree level, and its value is fixed very close to the weak scale M_W when non-gravitational quantum correction are taken into account^{27),28)}. The main difference between the potentials (15) and (20) appears clearly when a local minimum with non-zero cosmological constant is considered. The existence of such a local minimum depends, for the potential (20), on the values of

$g_{\phi\phi^*}$ and $h_{\phi\phi^*}$ [while it is impossible for the previous potential of Eq. 15)]. The derivative of the potential (20) in the Z-directions is

$$V_{,Z^*} = \left[G_{,Z^*} - G_{,ZZ^*} \frac{h_{,\varphi\varphi^*}}{(g_{,\varphi\varphi^*} + G_{,Z} h_{,\varphi\varphi^*})} \right] V_0 \quad (25)$$

where $V_0 = e^G |g_{,\phi}|^2 / (g_{,\phi\phi^*} + G_{,Z} h_{,\phi\phi^*})$. Using the relations $G_{,ZZ^*} = G_{,Z} G_{,Z^*} / 3$ and $G_{,Z} = G_{,Z^*} = -3 / (Z + Z^* + h(\phi, \phi^*))$, we can write

$$V_{,Z} = G_{,Z} \frac{(g_{,\varphi\varphi^*} + \frac{2}{3} h_{,\varphi\varphi^*} G_{,Z})}{(g_{,\varphi\varphi^*} + G_{,Z} h_{,\varphi\varphi^*})} V_0 \quad (26)$$

The fact that $g_{,\phi\phi^*} + G_{,Z} h_{,\phi\phi^*} > 0$ and $G_{,Z} < 0$ [positivity of the kinetic energy, Eqs. (22)] implies that:

- I) the set of values $g_{,\phi\phi^*} < 0$ and $h_{,\phi\phi^*} \geq 0$ are excluded (they give negative kinetic energy).
- II) the values $g_{,\phi\phi^*} \geq 0$ and $h_{,\phi\phi^*} \leq 0$ (both equal signs excluded) yield to a monotonic potential with $V_{,Z} < 0$ always. Only global minima with $g_{,\phi} = 0$ and $g_{,\varphi} = 0$, zero cosmological constant and undetermined (at the tree level) gravitino mass can be present.
- III) when $g_{,\phi\phi^*} > 0$ and $h_{,\phi\phi^*} > 0$ the potential is monotonic, as in case II), the difference being that at the global minima the gravitino mass (even if undetermined) is bounded from below. In fact, because of condition (22b)

$$-G_{,Z} > \frac{g_{,\varphi\varphi^*}}{h_{,\varphi\varphi^*}} \quad (27)$$

or, written in terms of the gravitino mass $m_{3/2}$,

$$m_{3/2}^2 > \frac{1}{27} \left(\frac{g_{,\varphi\varphi^*}}{h_{,\varphi\varphi^*}} \right)^3 e^{g(\varphi, \varphi^*, y, y^*)} \quad (28)$$

IV) when $g_{,\phi\phi^*} < 0$ and $h_{,\phi\phi^*} < 0$, a local minimum with positive cosmological constant may exist. From Eq. (25), $V_{,Z} = 0$, there is a minimum for

$$-G_{,Z} = \frac{3}{2} \frac{g_{,\varphi\varphi^*}}{h_{,\varphi\varphi^*}} \quad (29)$$

with $g_{,\phi} \neq 0$ and $g_{,y} = 0$. Let us define the mass m_0 at that minimum as

$$m_0^2 = e^G = \left| \frac{1}{3} G_{,Z} \right|^3 e^g = \left(\frac{1}{2} \frac{g_{,\varphi\varphi^*}}{h_{,\varphi\varphi^*}} \right)^3 e^g \quad (30)$$

This is the gravitino mass at that point, $m_{3/2}^2 = m_0^2 (F^2)$. The value of the potential at that minimum is

$$V_0 = m_0^2 \frac{|g_{,\varphi}|^2}{\left(-\frac{1}{2} g_{,\varphi\varphi^*}\right)} \quad (31)$$

The four possibilities explained above are summarized in Fig. 1. Notice that the value of m_0 has no relation with the physical gravitino mass at present. We live in the global minimum at which the Universe arrives after the inflation, where the cosmological constant is zero at the classical level, and the gravitino mass is fixed, by radiative corrections, to be of order M_W (and, by the same procedure, a negative cosmological constant of order M_W^4 is settled). The mass e^G gets contributions from all the non-zero superpotentials, and that of the inflaton is dominant at the local minimum at which inflation occurs. Thus m_0 is related to the scale necessary for inflation (m_0 is of order 10^{13} GeV in the examples presented here). We

consider this point particularly interesting. This is an example of a situation in which the supersymmetry breaking parameter (or gravitino mass $m_{3/2}$) suffers a phase transition together with the inflaton field (F2).

The choice $g_{\phi\phi^*}|_{\phi=0} = -2$ yields a canonical kinetic term at $\phi = \phi^* = 0$ where the false vacuum will be (as we will see immediately). Around any other minimum, the correctly normalized classical inflaton field is $\sqrt{-\frac{1}{2}g_{\phi\phi^*}|_{\text{at the minimum}}} \times \phi$, which will always be of order one for us. The value of m_0 depends on a function of ϕ and ϕ^* which we take, for simplicity, to be a constant, a , of order one, $g_{\phi\phi^*}/h_{\phi\phi^*} = a$; thus

$$\left(G, z\right)_{\min} = -\frac{3}{2}a \quad \text{and} \quad m_0^2 = \left(\frac{a}{2}\right)^3 \left(e^g\right)_{\min} \quad (32)$$

We choose to include a $|\phi|^4$ term, thus g and h are

$$\begin{aligned} h &= -\frac{2}{a} \varphi \varphi^* + \frac{\beta}{2a} (\varphi \varphi^*)^2 \\ g &= -2 \varphi \varphi^* + \frac{\beta}{2} (\varphi \varphi^*)^2 + \ln |f_T|^2 + \gamma \gamma^* \end{aligned} \quad (33)$$

The $|\phi|^4$ term could be absent if we did not worry about the mean value of ϕ at high temperatures. This term creates a circle of singularities [see Eq. (31)]: $(-g_{\phi\phi^*}/2)^{-1} = (1 - \beta\phi\phi^*)^{-1}$ in the potential. Thus β fixes the radius of a disc $|\phi| < \beta^{-1/2}$ in which $g_{\phi\phi^*} < 0$, i.e., the kinetic term of ϕ is positive, and to which the inflaton is confined by an infinite cylindrical barrier. Then the false vacuum must be at the origin $\phi = \phi^* = 0$, because at very high temperatures T the average value of ϕ will be the centre of the disc.

Let us fix our attention on the total superpotential $f_T(\phi, y)$. We know, as we have already said, that the supersymmetry breaking induced by the inflaton has a scale $\sqrt{m_0 M}$, which we parametrize as $(m^3/M)^{1/2}$ (m^3 is such that the superpotential of ϕ is at most of order m^3). This scale is of the order $(m^3/M)^{1/2} \sim 10^{15}$ GeV. (It is eventually fixed by the requirements on the spectrum of density fluctuations.) It is very different from the breaking scale induced by the Z -field [or, equivalently, by the G^0 field defined in Eq. (23)] whose order is necessarily $(\mu^3/M)^{1/2} \approx \sqrt{M_W M} \approx 10^{10}$ GeV in order to

get breakings of order M_W at low energies. Then, we need two different terms in the superpotential

$$f_T = F^{(1)}(\varphi) + F^{(2)}(y) \quad (34)$$

Let us remember that the potential is [see Eqs. (20) and (31)]

$$\begin{aligned} V &= -e^{\frac{2}{3}G_{1z}} \left[\frac{|g_{1\varphi}|^2}{2(1-\beta\varphi\varphi^*)(-1-\frac{G_{1z}}{a})} + |g_{1y}|^2 \right] + \frac{D^2}{2} \\ &= -\left(\frac{G_{1z}}{3}\right)^3 e^{-2\varphi\varphi^* + \frac{\beta}{2}(\varphi\varphi^*)^2 + yy^*} \times \\ &\quad \times \left\{ \frac{|(F^{(1)}(\varphi) + F^{(2)}(y))(-2\varphi^* + \beta\varphi\varphi^{*2}) + F_{,\varphi}^{(1)}|^2}{2(-1-\frac{G_{1z}}{a})(1-\beta\varphi\varphi^*)} + \right. \\ &\quad \left. + |(F^{(1)}(\varphi) + F^{(2)}(y))y^* + F_{,y}^{(2)}|^2 \right\} + \frac{D^2}{2} \end{aligned} \quad (35)$$

It is very important to notice that here we consider for simplicity only the minimal case where the Kähler potential G does not contain interference terms between the Z and y fields. This situation corresponds to an effective low-energy theory where the supersymmetric breaking parameter A ²⁹⁾ takes the value $A = 3$, and in this case the Yukawa couplings are scaled by $m_{3/2}$. More general couplings, with $A \leq 3$, of the hidden sector and the low-energy matter sector can be obtained, and are indeed necessary to obtain gravitino mass-independent Yukawa couplings ²⁸⁾. For this purpose, one must assume that the low-energy matter fields transform under dilatation, and the requirement of a dilatation-invariant superpotential translates into trilinear terms in the superpotential like ²⁸⁾

$$q(\theta) e^{-G/2} y_i y_j y_k \quad (36)$$

Here we have introduced the scale function $q(\theta)$, depending only on $\theta = (1/2i) \ln(Z^*/Z)$ [G° is defined in Eq. (23)]. If $q(\theta)$ is a constant, the A parameter takes the value $A = 3/2$, but in general $3/2 \leq A \leq 3$ is permitted²⁸⁾. The superpotential $F^{(2)}(y)$ for non-gauge singlet fields must be at least bilinear in the fields, and thus the values $y_1 = 0$, $F_{,y}^{(2)} = 0$ correspond to a minimum, because $g_{,y} = 0$ (see that the contribution to V is just $|g_{,y}|^2$). Moreover, the minimum with $y = 0$ for all fields, including singlets which give mass to other fields, is always preferred at high T by temperature effects, because it contains the maximum number of light degrees of freedom. Let us remember that the T^4 contribution of the one-loop effective potential due to finite temperature corrections is negative, and proportional to the number of light degrees of freedom $n(T)$ at a given T ³⁰⁾:

$$\Delta V(T) \sim -n(T) T^4 \quad (37)$$

These considerations can always be applied for ranges of parameters and fields such that the potential is renormalizable, that is, terms of order higher than four in the fields can be neglected.

From (35), we see that the condition for a minimum in the y direction is always $g_{,y} = 0$ ($|g_{,y}|^2$, the contribution to V , is positive definite and this value also minimizes the D -term, $D \equiv g^a g_{,y} T^a y^* = 0$; g^a is a gauge coupling constant). Therefore, at high T (that is, up to the end of inflation), the minimum with all $y = 0$ and $F_{,y}^{(2)} = 0$ will be preferred. Before and during the inflation, then, we take y and G_Z in their minima and study the potential as a function of ϕ only. At the global, $V = 0$, minima, the situation is a bit more complicated. The quantity which must vanish is [see the first term in Eq. (35)]

$$(-2\varphi^* + \beta\varphi\varphi^{*2})(F_{(\varphi)}^{(1)} + F_{(y)}^{(2)}) + F_{,\varphi}^{(1)} = 0 \quad (38)$$

which means that ϕ does not break supersymmetry. We see, then, that there are two types of $V = 0$ minima:

- 1) $F_{\phi}^{(1)} \approx 0$ and $F^{(1)}(\phi) \approx 0$, i.e., they are zero or effectively zero at the scale m^3 . The scale m^3 disappears from the potential and all matter fields will remain light. There are two cases with these characteristics, which cannot be distinguished by studying only the inflaton potential. In one case, $F_{\phi}^{(1)} = 0$ and $F^{(1)}(\phi) + F^{(2)}(y) = 0$, thus the gravitino mass becomes exactly zero (since $e^g = 0$) at the global minimum. $m_{3/2}$ is not undetermined, but zero. Thus supersymmetry is not broken and a minimum of this kind does not correspond to our world. The other case occurs when both functions, $F_{\phi}^{(1)}$ and $F^{(1)}(\phi)$, are different from zero, but their scale is effectively zero with respect to m^3 . Then $e^g \neq 0$, which means that $m_{3/2}$ is undetermined, even if $V \equiv 0$. It is in this case that radiative corrections will create a minimum with $V < 0$ at a value $m_{3/2} \approx M_W$, fixing in this way the scale μ^3 (which up to this point has been undetermined) so that $F_{\phi}^{(1)} \approx \mu^3$ and $F^{(1)}(\phi) \approx \mu^3$.
- 2) $F_{\phi}^{(1)} \neq 0$ and $F^{(1)}(\phi) \neq 0$, i.e., they are both of order m^3 . Then, through the term of the potential

$$e^g |g, y|^2 \approx |(F_{\phi}^{(1)} + F^{(2)}(y)) y^* + F_{,y}^{(2)}|^2 \quad (39)$$

a mass is given to matter fields (this mass term is also present before, during all the period in which $F^{(1)}(\phi) \neq 0$):

$$|F(\phi)|^2 y y^* \lesssim \frac{m^6}{M^4} y y^* \quad (40)$$

That means that, at low temperatures, the scalar partners of the multiplets acquire masses of order $\lesssim m^3/M$, with a considerable reduction of the number of light degrees of freedom with respect to the type 1 minimum.

Therefore, among all the $V = 0$ minima, the type 1 will be preferred by finite T effects (unless there are barriers larger than T^4),

$$V_1(T) - V_2(T) \sim \left(\frac{7}{8} \Delta n_f + \Delta n_b\right) T^4 e^{-\left(\frac{\mu^3}{M}\right)^{1/2} \frac{1}{T}} \quad (41)$$

Δn_f and Δn_b are the difference in light fermionic and bosonic degrees of freedom between type 1 and type 2 minima. The heavy degrees of freedom are weighted by $e^{-(m^3/M)^{1/2}/T}$ and can be neglected for $T < (m^3/M)^{1/2} \sim 10^{15}$ GeV, while the temperature after the reheating is just $T_R \sim 10^{10} - 10^{11}$ GeV. If the barriers between type 1 and type 2 minima are larger than T_R^4 , different independent universes will develop in each of these minima. Our requirement for the inflation is, then, that the Universe in the type 1 minimum must be large enough to contain all our observable Universe. It is just enough that the barriers be smaller than V_0 to ensure that the coherent ϕ field oscillates many times throughout the entire global minimum before its energy is converted into radiation. The oscillations, of frequency $\omega^2 \sim \partial^2 V / \partial \phi^2 \sim m_\phi^2$, are damped due to particle creation in a time of order Γ^{-1} , where $\Gamma \sim m_\phi^3 / M^2$ as we will see later). We see that $\Gamma^{-1} \sim (M/m_\phi)^2 \omega^{-1}$; the field oscillates around 10^{10} times before decaying. Γ is larger in the type 1 minima because the number of open channels is larger than in those of type 2. Moreover, when the temperature of the radiation increases, the type 1 minima are preferred because of temperature effects.

We now have a complete view of our model for inflation. At very high T , the average value of ϕ will be $\phi = 0$, the centre of the allowed disc created by the β -term in g . We will impose on g the conditions necessary for inflation, and obtain in this way the development of g around $\phi = 0$ (it will be sufficient to take up to the third order in ϕ). Then, there will always be one or various global minima of type 1 where we want the inflation to end up. We can easily construct them by summing to g terms $\ln(\phi - \phi_i)^2$, where ϕ_i is the position of the minimum (we will take care not to destroy

the properties of g around $\phi = 0$). Of course, in this procedure, other minima of type 2 will be produced, but type 1 will be preferred at any $T > 0$ (and $T < \text{around } 10^{15} \text{ GeV}$, the mass of the heavy degrees of freedom, to be precise). At $T = 0$, both types of minima are degenerate with $V = 0$ (due to the semi-positive definiteness of the potential), and the type 1 minima are the ones which contain only light (at most of order $\sqrt{\mu^3/M}$) degrees of freedom. Thus, finite T -effects will lower type 1 minima with respect to those of type 2, and the Universe will necessarily end up in a type 1 minimum, unless the barrier is larger than T_R^4 . If that occurs, different Universes develop around each minimum, and ours is one around a type 1 minimum. We will only worry about producing, in a specific model, a path (that the Universe will follow from $\phi = 0$ to the final type 1 minimum) simple enough to be able to calculate all the parameters of the inflation. What follows is just an example of how the mechanism works.

Let us put into practice the previous programme. At $\phi = 0$, we ask for a non-zero cosmological constant $\Lambda \sim e^g |g, \phi|^2 \neq 0$, $V, \phi = 0$, and a positive curvature, large in any direction except one. For simplicity, we separate the scale m^3 from the superpotential $F^{(1)}(\phi) = m^3(F^{(1)}(\phi)/m^3)$ and write a term $(\ln m^6)$ in g . All the terms in the rest $(F^{(1)}(\phi)/m^3)$ do not include any large numbers. Then we ask for $e^g = m^6$ and $|g, \phi|^2 = \xi^2$ at the local minimum, where ξ is of order one [then, see Eq. (32), the gravitino mass is $m_0 = (a/2)^{3/2} m^3$]. A correct inflation can always be obtained if the potential is almost flat in one direction (we choose the real axis), with a small [with respect to the total potential $V(\phi=0) = V_0 \sim m^6$] curvature, $\sim \varepsilon m^6$, which provides the only scale around the false minimum, besides V_0 . The value $\varepsilon \neq 0$ creates a small maximum in the potential; therefore the barrier to be overcome depends directly on ε , and the "roll over" time scale (after the maximum but near to it) depends on ε , $\tau \sim 1/\varepsilon H$. A large curvature in the other direction ensures the initial path which will be followed. We obtain the development of g around $\phi = 0$, g_0 ,

$$\begin{aligned}
 g_0 = & -2\varphi\varphi^* + \frac{\beta}{2}(\varphi\varphi^*)^2 + \ln m^6 + \\
 & + \left\{ (-\xi\varphi + \frac{1}{2}(2-\xi^2)\varphi^2 - \frac{\xi^3}{3}\left(1 - \frac{\beta}{2\xi^2} + \frac{\varepsilon}{2}\right)\varphi^3 + \right. \\
 & \left. + h.c. \right\}
 \end{aligned} \tag{42}$$

(we have defined ξ as $-\xi \equiv g, \left. \phi \right|_{\phi=0}$). The masses around $\phi = 0$ are, for the real and imaginary components

$$\begin{aligned} m_R^2 &= m_0^2 \varepsilon \xi^4 \\ m_I^2 &= m_0^2 (16 + (-12 + 2\beta) \xi^2) - m_R^2 \end{aligned} \quad (43)$$

where we have used $m_0^2 = (a/2)^3 m^6$ as the gravitino mass.

Let us take two examples, the first with one minimum of type 1 at $\phi = \phi_0$, the second with two symmetric minima at $\phi = \pm\phi_0$:

$$I) \quad g = g_0 + \left\{ \ln(\varphi - \varphi_0)^2 - 2 \ln \varphi_0 + 2 \left(\frac{\varphi}{\varphi_0} \right) + \left(\frac{\varphi}{\varphi_0} \right)^2 + \frac{2}{3} \left(\frac{\varphi}{\varphi_0} \right)^3 + h.c. \right\} \quad (44a)$$

$$II) \quad g = g_0 + \left\{ \ln[(\varphi - \varphi_0)^2 (\varphi + \varphi_0)^2] - 4 \ln \varphi_0 + 2 \left(\frac{\varphi}{\varphi_0} \right)^2 + h.c. \right\} \quad (44b)$$

The only part which remains, by taking the development of g around $\phi = 0$ up to the third order, is g_0 given in Eq. (42). The conditions on the inflationary period depend only on g_0 while the reheating, directly related to the mass of the inflaton at ϕ_0 , depends on the behaviour of g around ϕ_0 . We will introduce a dependence, in principle unnecessary, of g_0 on ϕ_0 through the choice of β . We like to have β such that the disc of allowed values for ϕ is quite small, say $\beta = (2\phi^2)^{-1}$. We choose, for simplicity, $a/2 = 1$. The parameters ϕ_0 (position of the minimum on the real axis), m^6 (the scale of the potential is m^6/M^2) and ε (the parameter of the curvature at the origin which creates the false minimum and the barrier) are enough to fit the suitable values of the "roll over" time scale τ for the field ϕ after the maximum ($\tau H \sim \varepsilon^{-1}$), the probability of overcoming the barrier near to the origin (depends on ε and m), the density fluctuations $\delta\rho/\rho$ (depend on m and β , that is, ϕ_0), and the inflaton mass at the global minimum (depends on

m and ϕ_0). The values $m_R \approx 2 \times 10^{10}$ GeV [see Eq. (43)], $V_0 = (a/2)^3 m^6 \xi^2 = (2.3 \times 10^{15} \text{ GeV})^4$ and $\phi_0 = M_P = 1.2 \times 10^{19}$ GeV (that is, $\epsilon = 5 \times 10^{-4}$, $m^3 = 3 \times 10^{-6}$, $\phi_0 = 5$, $\xi = 0.32$), yield all the desired features of inflation. The roll-over time scale (the time necessary for a change e^1 in the scale factor of the Universe once the barrier has been overcome) is

$$\tau_H \approx \frac{\xi^2}{2\epsilon} \approx 10^2 > 65 \quad (45)$$

Following the proposal of Hawking and Moss³¹⁾, we estimate the probability of overcoming the barrier which confines ϕ near to the origin by evaluating the rate for a simultaneous jump of a region of space larger than the de Sitter horizon volume, from the local minimum V_0 to the local maximum $V_{\max}$ of the potential. The probability $P \sim e^{-B}$ must be small, therefore B must be large,

$$B \approx 24 \pi^2 \frac{(V_{\max} - V_0)}{V_0^2} \approx \frac{32}{9} \frac{\pi^2}{4 \beta^2} \frac{\epsilon^3 \xi^6}{m^6 \left(\frac{a}{2}\right)^3} \approx 3 \times 10^2 \quad (46)$$

This is maybe a too simplified way of evaluating the beginning of the "roll-over" period. However, we think that the only important parameter here is the curvature parameter ϵ . To get a large B , it is enough to have a large ϵ (which will also be the scale of the curvature after the maximum) and a small scale parameter m^6 , which is really fixed by the following two conditions (on $\delta\rho/\rho$ and m_ϕ). Quantitatively, only a very particular value of the energy density fluctuations $(\delta\rho/\rho)|_4 \sim 10^{-4}$ is suitable for the formation of galaxies³²⁾ and compatible with the isotropy of the background radiation. In our case (where the potential contains a cubic term)

$$\frac{\delta\rho}{\rho} = \sqrt{\frac{3}{2\pi^3}} \left(\frac{a}{2}\right)^{3/2} m^3 2\beta \left[\ln\left(\frac{m^3 \left(\frac{a}{2}\right)^{3/2} \xi}{\sqrt{3} k}\right) \right]^2 \quad (47)$$

for k^{-1} of the size of the galaxy, $k^{-1} = 10^{23} \text{ cm} = 5 \times 10^{36} \text{ GeV}^{-1}$:

$$\frac{\delta \mathcal{P}}{\mathcal{P}} \approx \frac{m^3}{\varphi_0^2} \quad 2.6 \times 10^3 \approx 3. \times 10^{-4} \quad (48)$$

The mass of the inflaton at the global minimum is fixed by phenomenological requirements¹⁷⁾ to be $m_\phi \approx 10^{13} \text{ GeV}$. In our case,

$$m_\phi^2 \approx 4 \frac{m^6}{\varphi_0^4} \left(\frac{a}{z}\right)^3 \frac{e^{\frac{\varphi_0^2}{4} + \dots}}{\left(-\frac{1}{2} g_{\varphi\varphi^*}\right)\big|_{\varphi=\varphi_0}} \approx 2. \times 10^{-11} \quad (49)$$

The dots "..." stand for $22/3 - 11 \times R/6 - R^2 - 2 \times R^3(1-\epsilon/2)/3 \approx -0.9$, where $R = \xi\phi_0 = 1.6$; that is, $m_\phi \approx 0.4 \times 10^{-5} M = 1 \times 10^{13} \text{ GeV}$. The factor $\left(-\frac{1}{2}(g_{\phi\phi^*})\big|_{\phi=\phi_0}\right)^{-1} = (1 - \beta\phi_0\phi_0)^{-1} = 2$ comes from the normalization of the kinetic term around the minimum. The potential obtained in this way is shown in Figs. 2 and 3. In this example, the barrier separating the type 1 and type 2 global minima is just $V_0/3$. In Eq. (47), the development around $\phi = 0$ of the potential just up to the third order and the approximation $\xi < 1$ have been taken. Smaller barriers could also be obtained. In order to arrive at barriers smaller than T_R^4 , the parameters are such that the fourth order should be taken into account.

Let us follow the history of the Universe once the inflaton arrives at the global minimum and loses its energy, decaying into all the particles to which it is coupled. During this period of reheating of the Universe, the inflaton is oscillating around the global minimum and, therefore, the scale m^3 is still present. The field is now $\tilde{\phi} = \phi - \phi_0$, the excitation around the minimum. From the piece of the potential

$$\left| \left(F^{(1)}(\varphi) + F^{(2)}(y) \right) (-z\varphi^* + \beta\varphi\varphi^{*2}) + F_{,\varphi}^{(1)} \right|^2 \quad (50)$$

and from the fact that

$$\left| F^{(1)}(\varphi) (-2\varphi^* + \beta \varphi \varphi^{*2}) + F_{,\varphi}^{(1)} \right|^2 \simeq m_\varphi^2 |\tilde{\varphi}|^2 \quad (51)$$

the term in (50) can be rewritten as

$$\left| m_\varphi \tilde{\varphi} + f(y) \right|^2 \quad (52)$$

where

$$f(y) = F^{(2)}(y) (-2\varphi_0 + \beta \varphi_0^3) \quad (53)$$

Therefore the most important couplings are of the type

$$m_\varphi \tilde{\varphi} f(y) \quad (54)$$

Let us write the superpotential for matter $f(y)$

$$f(y) = f_5 + f_B \quad (55)$$

where f_5 is responsible for the breaking of SU(5) and f_B for the generation of cosmological baryon asymmetry and the rest of the Yukawa couplings. f_5 could be, for example, a polynomial

$$f_5 = P(\Sigma) \quad (56)$$

where Σ is in the adjoint representation [24] of SU(5). The resulting potential would be

$$V_5 \simeq m_{3/2}^2 \sum_{\alpha=1}^{24} \left| \frac{\partial P}{\partial \Sigma^\alpha} + \sum_{\alpha}^+ P \right|^2 + \frac{1}{2} D^2 \quad (57)$$

thus $\langle \Sigma \rangle \approx 10^{+3} M$, and the mass of the GUT gauge bosons turns out to be $M_X \sim 10^{-3} M$. This value for $\langle \Sigma \rangle$ comes out naturally if one keeps the coefficients of $P(\Sigma)$ in the natural interval 1 to 10^{-1} . The value of M_X comes out due to the positive definiteness of the potential. This property implies that the potential may have many degenerate minima with $V = 0$. At very high temperature, the symmetric minimum at $\langle \Sigma \rangle = 0$ will be preferred, since it contains the larger number of light degrees of freedom. However, once $SU(5)$ enters into the confining phase³³⁾, the number of degrees of freedom in the symmetric phase decreases, and the nearest minimum, which is at $\langle \Sigma \rangle \approx 0(10^{-3} M) \approx 10^{15}$ GeV, becomes favoured. The scale $m_{3/2}^2$ in front of the potential yields a barrier, of order $m_{3/2}^2 M^2 \approx (10^{10} \text{ GeV})^4$, small with respect to the barriers which appear in non-supersymmetric GUTS. This is the origin of a low critical temperature T_c , $T_c \sim 10^{10}$ GeV, for the GUT spontaneous breaking.

In f_B , following Ref. 17), we will introduce a singlet field Y which acts as mediator between the inflaton and the fields H and $\bar{H}$, the $[5]$ and $[\bar{5}]$ Higgs multiplets. It is necessary to delay the production of H and $\bar{H}$ up to the moment when they are widely out of equilibrium, so that sufficiently large asymmetry $\eta_B/\eta_Y \sim 10^{-4} \Delta B$ (where ΔB is the net baryon number per decay) can be achieved. For that purpose, the coupling $YH\bar{H}$ must be of order $0(10^{-6})$. It could be generated by a term $\langle \Sigma \rangle \langle \Sigma \rangle / M^2$, for example. Moreover, the mechanism of Ref. 34) for splitting the doublet-triplet components of H by means of Higgs multiplets θ and $\bar{\theta}$ in the representations $[50]$ and $[\bar{50}]$ of $SU(5)$ is used. Then

$$\begin{aligned}
 f_B \approx & a_1 M \theta \bar{\theta} + \frac{a_2}{M} \bar{\theta} \Sigma^2 H + \frac{a_3}{M} \bar{H} \Sigma^2 \theta + \\
 & + a_4 \frac{\Sigma}{M} \frac{\Sigma}{M} Y \bar{H} H + a_5 \frac{\Sigma}{M} \Sigma Y^2 + \\
 & + a_6 Y^3 + \text{Yukawa couplings}
 \end{aligned} \tag{58}$$

The mass is $m_Y \sim a_5 \langle \Sigma \rangle / M \sim 10^{10}$ GeV, as is required. Actually, the terms in f_B should be multiplied by $e^{-G^0/2}$ [G^0 is defined in Eq. (23)], in order to obtain $m_{3/2}$ independent couplings. Notice that only f_B is rescaled by $e^{-G^0/2}$ but not f_5 [see Eq. (55)]. This is because the $m_{3/2}^2$ factor in V_5 produces naturally a delayed GUT phase transition at a low critical temperature $T_c \approx 10^{10}$ GeV. The coupling $e^{-G^0/2} f_B$ does not alter the inflation picture, and has been considered in detail in Ref. 28).

In order to make contact with the content of Ref. 28), let us write explicitly the form of the Kähler potential valid at low energies. As we said [see Eqs. (23), (33) and (34)],

$$\begin{aligned} G^0 + \ln |f_T|^2 + \dots &= \\ = G^0 + \ln \left| F^{(1)}(\varphi) + f_5(\Sigma) + e^{-G^0/2} f_B(y_i) \right|^2 + \dots \end{aligned} \quad (59)$$

Here we have separated Σ from the other matter fields y_i .

Let us denote by v^3 the value of $(F^{(1)}(\phi) + f_5(\Sigma))$ at the global minimum of the broken GUT phase. Finite temperature effects will choose the minimum where $v^3 \ll m^3$. Since we are interested in solutions with small v.e.v's for the matter fields y_i , we write the following expansion

$$\begin{aligned} G^0 + \ln |f_T|^2 + \dots &= \\ = G^0 + \ln |v^3|^2 + \left\{ \frac{F^{(1)}(\varphi) + f_5(\Sigma) - v^3}{v^3} + \left(v^3 e^{G^0/2} \right)^{-1} f_B(y_i) + h.c. \right\} \end{aligned} \quad (60)$$

We see that the gravitino mass is $m_{3/2} \approx e^{G/2} \approx v^3 e^{G^0/2}$. Radiative corrections fix the value of G^0 such that $m_{3/2} \approx M_W$ (28).

It is very important to realize that the introduction of the singlet Higgs field Y in f_B is only safe in the case which we are considering, that of a semi-positive definite potential. A v.e.v. $\langle Y \rangle \neq 0$ would yield a direct mass term for the field H of order $\langle Y \rangle \langle \Sigma \rangle \langle \Sigma \rangle / M^2$, which would completely destroy the splitting mechanism, leading to equal masses for doublets and triplets. On the other hand, it is not possible to forbid other feasible values $\langle Y \rangle \sim m_Y$; but in our case, these minima are definitely degenerate (both at $V = 0$) at $T = 0$. Thus finite temperature effects will always prefer the minimum with the maximum number of light degrees of freedom, and that happens when $\langle Y \rangle = 0$.

From Eqs. (54) and (58), we see that the dominant decay mode of $\tilde{\phi}$ is $\tilde{\phi} \rightarrow 3Y$, which occurs through the coupling $(m_\phi/M)\tilde{\phi}YYY$ (besides the modes coming from the Yukawa couplings). Thus the decay rate is given by

$$\Gamma_\phi \approx \frac{m_\phi^3}{M^2} \quad (61)$$

Given all the required elements, we can reformulate for our model the sequence of events which begin at the end of the inflation. During the inflation, the temperature is $T_H = H/2\pi \approx 10^{11}$ GeV, but at the completion of the roll-over, the vacuum energy density V_0 is converted into relativistic degrees of freedom. At this stage, the Universe is inflaton dominated. All the other particles have been diluted by inflation. It is worthwhile noticing that the scalar field partner of the Goldstino is the field G^0 of Eq. (23) [see Eq. (23)]. The potential [see Eq. (20)] in terms of G^0 is completely defined, and has a deep minimum. G^0 is always at the minimum of such a potential during all the inflation and still when ϕ is oscillating around its global minimum. The mass of G^0 around its minimum is of order $m_0 \sim 0(10^{13} - 10^{15} \text{ GeV})$. Only when ϕ is fixed at its minimum, ($\phi \equiv \phi_0$), does the potential of G become flat at the tree level and, because of non-gravitational radiative corrections, acquires a v.e.v. and a mass both of

order $M_W \sim 100$ GeV. It should be stressed here that we assume that the corrections to the vacuum energy due to the superheavy fields should not exceed $O(m_{3/2})$, as is obviously required for the success of the dynamical determination of $m_{3/2}$ and M_W . For any temperature $T < m_\phi$, we assume that G^0 is in its low energy minimum; thus, the problems which could be generated by G^0 are the same as those of the gravitino, since their interaction rates and masses are almost the same. The ϕ fields are non-relativistic for any $T \leq m_\phi = 10^{13}$ GeV, and decay massively at $T_{D\phi}$, when $\Gamma_\phi \sim m_\phi^3/M^2 \sim H \sim \sqrt{m_\phi T_{D\phi}^3}/M$. Thus, roughly speaking, $T_{D\phi} \sim 0.6 \times 10^8$ GeV. Notice that from now on, the history of the Universe only depends on the value of m_ϕ and the couplings of the ϕ field. After ϕ decay, the Universe will heat up marginally to a temperature $T_R \sim (m_\phi T_{D\phi}^3)^{1/4} \sim O(10^{10} - 10^{11})$ GeV, while the SU(5) breaking occurs at $T \sim O(10^9 - 10^{10})$ GeV. Mainly Y's have been produced, which decay when Γ_y [see Eq. (58)] $\sim H = \sqrt{m_y T_{Dy}^3}/M$, at $T_{Dy} \sim 10^7$ GeV, producing Higgs triplets H and $\tilde{H}$ of mass $O(10^{10})$ GeV. These H's decay (mainly into μK and/or νK), producing $\eta_B/\eta_Y \sim 10^{-4} \Delta B$ (ΔB is the net baryon number per decay). The gravitino, and the scalar partner of the Goldstino, whose masses are 100 GeV, could be a problem after their decay, but with $T_R \leq O(10^{13})$ GeV their relative abundance $Y_{3/2} = \eta_{3/2}/\eta_Y = \eta_G/\eta_Y$ is small enough. It is $Y_{3/2} \sim O(10^{-8})$ in our case.

In conclusion, we have presented what we believe to be the only consistent way of producing inflation with a potential derived from $N = 1$ supergravity and dynamical determination of the low energy scale. The scale of this potential is the gravitino mass, $V \sim m_{3/2}^2 M^2$. This scale is different in the local minimum before inflation, and in the global minimum after it. In the former minimum, $m_{3/2} \sim 10^{13}$ GeV is fixed by the tree level potential. In the last minimum, $m_{3/2}$ is undetermined at the classical level, but non-gravitational radiative corrections fix it, $m_{3/2} \sim M_W$. Therefore, $m_{3/2}$ suffers a phase transition associated with the inflaton. The potential is semi-positive definite, as in global supersymmetric theories, which leads to the fact that the global minimum into which the Universe goes at the end of inflation is chosen by T-effects.

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FIGURE CAPTIONS

Fig. 1 : Different properties of the potential in Eq. (20) according to the signs of the functions $g, \phi\phi^*$ and $h, \phi\phi^*$.

Fig. 2 : The potential of Eq. (34) with the function g of Eq. (44a) as shown in the regions $-0.8 < \text{Re}\phi < 1$ and $-0.25 < \text{Im}\phi < 0.25$. The position of the local minimum at $\phi = \phi^* = 0$, and of the type 1 local minimum are indicated by an arrow. We can see three other minima of type 2.

Fig. 3 : Detail of Fig. 2 around the local minimum at $\phi = \phi^* = 0$, in the regions $|\text{Re}\phi| < 5 \times 10^{-3}$ and $|\text{Im}\phi| < 1.5 \times 10^{-5}$. A cut has been done at $V-V_0 = 0.4 \times 10^{-9}V_0$ to see the minimum clearly.

FOOTNOTES

F1 : There is another possibility if one renounces determining the gravitino mass at the global minimum through radiative corrections. The addition of a real function $\Phi(Z, Z^*)$ inside the logarithm in the $G^{(1)}$ of Eq. (9), $G^{(1)} = -3\ln |Z + Z^* + \Phi(Z, Z^*)|$, yields a semipositive definite potential, if $\Phi_{ZZ^*} \geq 0$ [see Ref. 26)]. In this case, the inflation is possible even if the ϕ and y_i fields have minimal kinetic terms, for instance with a total Kähler potential $G = G^{(1)} + \phi\phi^* + y_i y_i^* + \ln |f_T(\phi, y_i)|^2$. The gravitino mass parameter suffers a transition from a large value during the inflation and reheating, to a value fixed at the tree level by the total potential at the global minimum.

F2 : To call $e^{G/2}$ the "gravitino mass" is an excess of language outside the global minimum with zero cosmological constant, $\Lambda = 0$. In fact, in a de Sitter space, as in the local minimum with $\Lambda > 0$, the gravitino mass parameter $m_{3/2} = e^{G/2}$ does not define the mass of the gravitino.

Fig. 1

Fig. 2

Fig. 3