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KOGUT-SUSSKIND AND WILSON FERMIONS IN THE QUENCHED APPROXIMATION:

A MONTE CARLO SIMULATION

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A B S T R A C T

We present a comparison between Kogut-Susskind and Wilson formulations of lattice QCD fermions, by means of a MC simulation on a $10^3 \times 20$ lattice at $\beta = 6$ in the quenched approximation. The two methods reasonably agree in the evaluation of the scale of the lattice spacing in physical units, $a^{-1} \approx 2$ GeV, but disagree by about a factor two in the estimate of the quark mass values.

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1. - INTRODUCTION

Theoretical bases for pure lattice gauge theories¹⁾ are well settled: the main invariance of the theory (gauge invariance) is implemented on the lattice for all values of the coupling constant. When the theory is coupled to fermionic matter fields, on the contrary, the situation becomes very embarrassing: deciding a priori that one wants to preserve the locality of the theory (compulsory choice, if one is interested in performing numerical simulations), one is left with competing difficulties that cannot be simultaneously eliminated [Nielsen-Ninomya theorem²⁾].

The Wilson³⁾ and Kogut-Susskind⁴⁾ formulations of lattice QCD with fermions suffer respectively from the explicit breaking of chiral invariance and from the doubling phenomenon. These features are supposed to be harmless in the continuum limit of the theory; but, for a finite lattice spacing, they lead to different systematic errors: the purpose of this paper is to make a comparison between the two formulations in the quenched approximation⁵⁾.

We have studied the hadronic correlation functions in both cases on the same set of gauge field configurations. This is in order to minimize the effects of the statistical uncertainties on the comparison of the two formulations.

The first section contains the generalities on the gauge configurations used, the second concerns the results for the Kogut-Susskind approach and the third those for the Wilson one. The two sets of results are compared in the last section.

2. - GAUGE FIELD CONFIGURATIONS

The analysis of fermion masses has been made on 14 gauge field configurations generated at $\beta = 6$ with the standard Wilson action on a $10^3 \times 20$ lattice. We spent about a thousand sweeps for the thermalization and then we kept one configuration every hundred sweeps. We adopted the Metropolis updating procedure with ten hits per point.

Our average plaquette from the 1300 last sweeps is ^{*})

$$E = .40607 \quad (10) \quad (2.1)$$

(The error is computed with respect to 13 samples of 100 sweeps, and is likely to be somewhat underestimated.)

3. - RESULTS FOR KOGUT-SUSSKIND FERMIONS

The generalities on this formulation are well known^{4),7)-9)}. We just recall that two complex one-component variables, $\bar{\chi}(n)$ and $\chi(n)$ are defined on each lattice site n after the various spin components of the original Dirac spinor have been decoupled. As a result of this transformation, the gamma matrices reduce to a set of phases defined on the links.

The presence of an explicitly unbroken flavour non-singlet part of the continuous chiral symmetry is the most appealing feature of this formulation: a zero mass pion for a zero bare quark mass value is natural to the theory. The nice feature of an unbroken chiral symmetry has a counter part: "flavour" symmetry is broken on the lattice and is restored only in the continuum limit. A recent study⁸⁾ for the SU(2) gauge theory seems to indicate that the

^{*}) In previous simulations on a lattice of the same size⁶⁾, a value for the average plaquette higher than the one quoted here by roughly one per cent was obtained because of a computer bag in the updating procedure. This bag, however, does not affect the results published in Ref. 6) in any significant way, as can be seen by comparing them to the results for Wilson fermions in the present paper. We thank H. Lipps for his help in correcting the bag and G. Martinelli, G. Parisi and C. Rebbi for discussions on this matter.

restoration occurs at slightly higher values of β than those where the scaling behaviour for $\bar{\chi}\chi$ sets in.

We define the quark propagator as

$$G(0,n) \equiv \langle \chi(0) \bar{\chi}(n) \rangle \quad (3.1)$$

in terms of which one can construct the correlation functions of local operators carrying definite quantum numbers in the continuum limit. The correlation functions that we consider for the meson states are:

$$\begin{aligned} G_{\pi}(t) &= \sum_{\vec{x}} |G(0,\vec{x};t)|^2 \\ G_{\rho}(t) &= \sum_{\vec{x}} |G(0,\vec{x};t)|^2 [(-1)^{n_x} + (-1)^{n_y} + (-1)^{n_z}] \\ G_{\Lambda_1}(t) &= \sum_{\vec{x}} |G(0,\vec{x};t)|^2 [(-1)^{n_x+n_y} + (-1)^{n_x+n_z} + (-1)^{n_y+n_z}] \\ G_{\epsilon}(t) &= \sum_{\vec{x}} |G(0,\vec{x};t)|^2 [(-1)^{n_x+n_y+n_z}] \end{aligned} \quad (3.2)$$

whose asymptotic behaviour in t is expected to be:

$$\begin{aligned} G_{\pi}(t) &\sim A_{\pi} e^{-m_{\pi}t} \\ G_{\rho}(t) &\sim A_{\rho} e^{-m_{\rho}t} + (-1)^t A_{\rho} e^{-m_{\rho}t} \\ G_{\Lambda_1}(t) &\sim A_{\tilde{\rho}} e^{-m_{\tilde{\rho}}t} + (-1)^t A_{\Lambda_1} e^{-m_{\Lambda_1}t} \\ G_{\epsilon}(t) &\sim A_{\tilde{\pi}} e^{-m_{\tilde{\pi}}t} + (-1)^t A_{\epsilon} e^{-m_{\epsilon}t} \end{aligned} \quad (3.3)$$

On a periodic lattice (that is indeed our case), any exponential $\exp\{-mt\}$ is replaced by $[\exp\{-mt\} + \exp\{-m(N_{\tau}-t)\}]$, where N_{τ} is the lattice time length.

The identification of baryonic quantum numbers stays very unclear: we will discuss this point at the end of this section.

We computed the quark propagators for each of the 14 gauge configurations by means of a second-order over-relaxed iterative algorithm: a first-order type, like the Gauss-Seidel, would not converge for staggered fermions. The calculation is performed at three values of quark masses: $m_a = 0.3, 0.2, 0.1$. These are quite high values (as we shall see, the lowest measured pion mass is of the order of 1.5 GeV!). This choice^{*)} was partly due to the need for comparing our results with similar ones at the same value of β ⁷⁾, partly to avoid the increase of statistical errors that we would get by going to smaller masses and, last but not least, to economize on the computer time. We take a great advantage here from the property of chiral invariance of this formulation which provides an extra point for free: zero mass pion at zero quark mass. The convergence of the relaxation algorithm for the quark propagator has been checked through the validity of the sum rule

$$G(0,0) = m \sum_t G_\pi(0,t) \quad (3.4)$$

which has been imposed at the level of 10^{-6} (on the relative error).

We have first computed the four correlation functions for mesonic states defined in Eq. (3.1). The analysis of $G_\pi(t)$ is easy as the pion is very dominant, and no oscillating states are present in this channel. The "effective" pion mass

$$m_\pi(t) = - \ln \left[\frac{G_\pi(t)}{G_\pi(t-1)} \right] \quad (3.5)$$

converges rapidly as t increases. In the worst case, $m = 0.1$, the successive values of $m_\pi(t)$ are 1.12, 0.99, 0.95, 0.94, 0.81, 0.81, 0.79, 0.79 for t from 1 to 8. Values for larger t 's are slightly lower due to the periodicity of the lattice.

The analysis of $G_\rho(t)$, $G_A(t)$ and $G_\varepsilon(t)$ are more involved as two states contribute to any of these operators. We are thus forced to use a fitting procedure. We have minimized the χ^2

$$\chi^2 = \sum_t \left(\frac{G^{\text{TH}}(t) - G^{\text{EXP}}(t)}{\delta G^{\text{EXP}}(t)} \right)^2 \quad (3.6)$$

^{*)} See the careful discussion of Ref. 10).

where $G^{TH}(t)$ is the theoretical expression of Eq. (3.3) and $G^{EXP}(t) \pm \delta G^{EXP}(t)$ the measured values. t runs from some t_{min} to $20-t_{min}$. The convergence with t_{min} is very good. For example, the ρ mass estimated from $G_\rho(t)|_{m=0.1}$ takes the successive values 1.02, 0.98, 0.96, 0.95 and 0.95 for $t_{min} = 1, 2, 3, 4$ and 5.

The errors we quote for the hadronic masses are the values of δm needed to change the χ^2 of 4 (number of parameters). Our results for the various masses (computed at different quark masses) are reported in Table 1. A check of the significance of the result is provided by the estimate of the $\tilde{\pi}$ and $\tilde{\rho}$ masses [see Eqs. (3.3)]. An analogous check can be done in the Wilson formulation, by using two independent local operators in order to estimate the π and ρ masses. We will discuss this point in Section 4.

Note that the agreement between the values of the two " ρ "'s is very good, while it is a little worse for the two " π "'s. One of the causes could be that the particle oscillating over the $\tilde{\pi}_{\epsilon}$, a 0^{++} state, turns out to be strongly coupled (A_ϵ is one order of magnitude larger than A_π). Moreover, as $m_\rho \rightarrow 0$, $m_\pi \rightarrow m_\pi$; at $m_q = 0.1$, $m_\pi = m_\pi$ in our statistical precision. This effect can be understood from the Goldstone realization of the chiral symmetry: the pion is decoupling. Anyway, we note that if we impose $m_\pi = m_\pi$, and we fit together G_π and G_ϵ with five free parameters, the mass of the 0^{++} does not vary inside the estimated errors. It would be, on the contrary, highly biased if we were enforcing a spurious constraint. The pion mass obtained is equal to the value found previously.

Given the high values of quark masses used, the extrapolation to practically zero masses may introduce systematic errors. Indeed, the question of the dependence of the meson mass upon the quark mass can be settled only for smaller quark masses and for the pion from current algebra arguments. In our case, we adopt the dependence which is natural on the lattice:

$$\cosh(M_i a) = A^i(m_q a) + B^i \quad (3.7)$$

which reduces, for small quark masses, to the usual linear dependence upon quark masses for the square of the meson mass

$$(Mia)^2 \sim A^i \cdot (m_q a) + B^i \quad (3.8)$$

An important motivation for our choice is also that experimentally both $\cosh(m_\pi)$ and $\cosh(m_\rho)$ (and, with slightly worse accuracy, the cosh of the other mesonic masses) are exactly linear in m_q in our quark mass range, and within the limit of our statistical accuracy, while the square masses show a systematical deviation from linearity.

The results of such a fit for the parameters A^i and B^i are:

$$\begin{aligned} A^\pi &= 6.8 & B^\pi &\simeq -.02 \\ A^S &= 8.4 & B^S &= .13 \end{aligned} \quad (3.9)$$

The error on the ρ mass extrapolated at $m_q = 0$ is estimated to be of the order of 20%.

Note (see Table 1) that B^π is zero within the statistical fluctuations of $(m^\pi a)$: this is a good check of the fulfilment of the chiral invariance in the lattice K-S formulation. A linear fit for the ρ mass squared would definitely be much worse in the region of quark masses where we have made our calculations.

Concerning the other states in Eq. (3.3), we have a stable result only for the parameters of the 0^{++} state: again, the fit with the hyperbolic cosine seems to work rather well on our data points. We obtain

$$A^{0^{++}} = 10.9 \quad B^{0^{++}} = 0.05 \quad (3.10)$$

For the A^1 type resonance, we can only roughly estimate the ratio of its mass to the ρ mass at $ma = 0.3$ and $ma = 0.2$: the statistical noise is too large at $ma = 0.1$ to allow a stable determination. We get:

$$\frac{M^A}{M^S} \simeq 1.3$$

The values in lattice spacing can be translated into physical ones. If the pion and the ρ meson are used as input, we get

$$a_{\pi-\rho}^{-1} \sim 2.1 \text{ GeV}$$

$$m_{u,d} \cdot a = 0.0007 \quad (3.11)$$

Another possible approach is that of using the K and the K^* as input values. Of course, having obtained our result in a case of degenerate quark masses, a prediction for the K and K^* can only be obtained under the approximation of linear dependence. In this case, one can fit (in lattice spacing units)

$$M_K^2 = \frac{A^K}{2} (m_s + m_d) + B^K$$

$$M_{K^*}^2 = \frac{A^{K^*}}{2} (m_s + m_d) + B^{K^*}$$

where A_K , B_K , A_{K^*} , B_{K^*} are respectively equal to the coefficients A_π , B_π , A_ρ , B_ρ in Eq. (3.8). From this approach, we get:

$$a_{K-K^*}^{-1} = 1.95$$

$$(m_u + m_s) a = 0.02 \quad (3.12)$$

One can also consider states made of strange quarks only, provided that one extracts the value of the mass of the state π_s equivalent to the pion from standard SU(3) mass formulae: $m_{\pi_s} \cong 700 \text{ MeV}$. One gets:

$$a_{\phi-\pi_s}^{-1} = 1.85$$

$$m_s \cdot a = 0.02 \quad (3.13)$$

The values obtained for a^{-1} with the three procedures are in reasonable agreement with each other: however, as we will see, the values for the quark masses in lattice units are about half those coming from the Wilson analysis. This result may just indicate that the values of m_q , and consequently those

for the various slopes $A_\pi, \dots$, are heavily affected by the finite renormalization corrections which relate their values to those calculated in the continuum. That the correction is as big as a factor two is, however, surprising. The Susskind formalism being chiral invariant, we may estimate f_π . This is given by the equation:

$$m_\pi^2 f_\pi^2 = m_q \langle \bar{\psi} \psi \rangle \quad (3.14)$$

We need $\bar{\psi} \psi$ to extract f_π^2 : our values are:

$$\begin{aligned} m a = .3 & \quad \langle \bar{\psi} \psi \rangle = .453 \quad (8) \\ m a = .2 & \quad \langle \bar{\psi} \psi \rangle = .363 \quad (9) \\ m a = .1 & \quad \langle \bar{\psi} \psi \rangle = .228 \quad (7) \end{aligned} \quad (3.15)$$

They are obtained from the definition:

$$\begin{aligned} \langle \bar{\psi} \psi \rangle &= \langle \bar{\psi} \psi \rangle_{SU(3)}^{2 \text{ flavours}} = \frac{1}{2} \sum_{i=1}^3 q_{ii}(0,0) \\ &= \langle \bar{u} u + \bar{d} d \rangle \end{aligned} \quad (3.16)$$

where the factor $\frac{1}{2}$ is needed because we are treating an effective four-flavour theory through the fermion degeneracy (the doubling is $2^{d/2} = 4$ in $d = 4$).

The values in Eq. (3.15) seem to follow a quadratic dependence upon m_q ; by extrapolating with a quadratic fit to $m_q \sim 0$ we get:

$$f_\pi \approx .08 a^{-1}$$

which turns into $f_\pi \sim 160$ MeV by using the value for a^{-1} in Eqs. (3.11)-(3.13). This value is higher than the experimental value. The main responsibility for the disagreement may be attributed to the value of the pion slope A_π . The comparison of the value we find with the one at different values of β does not show the behaviour implied by the renormalization group for QCD. The analogous analysis⁸⁾ in the SU(2) case in the corresponding region of β ($2.1 \leq \beta \leq 2.3$)

shows that A_π measured in lattice units is actually independent from β and it remains roughly unchanged when adding to the action one adjoint part with a small negative β_{adjoint} ¹¹⁾.

Another element which may also help in understanding the large value of f_π^2 comes from the strong coupling expansion for f_π^2 : the leading terms give $f_\pi = 190$ MeV (with a^{-1} fixed by the ρ mass) and the first correction in β is positive: the turnover only occurs at a high β value. This pattern is opposite, for example, to the one found for $\bar{\psi}\psi$ and m_ρ ^{*}).

We conclude the section on Kogut-Susskind fermions by commenting on the analysis for the baryon states. The assignments of quantum numbers are not understood in general: we define the baryonic operators by analogy with the mesonic ones of Eq. (3.2), by substituting to $|G(0,x)|^2$

$$\epsilon^{abc} \epsilon^{a'b'c'} q_{aa'} q_{bb'} q_{cc'} \quad (3.17)$$

For each of the four operators in Eq. (3.2) we find that, for large t , a single state dominates: its mass values in lattice units are 2.40(5), 2.05(5), 1.50(5) at $m_q = 0.3, 0.2, 0.1$, respectively. This is, for us, the lightest state we can observe: the extrapolation of its mass to practically zero quark mass done by a simple linear fit would show that the points are not linear at all in our quark mass range: the extrapolated value would be ~ 2 GeV. If we stick to the same prejudices we had for the analysis of meson masses, the "natural" variable would then be the hyperbolic sine:

$$\sinh [M_B a] = A^B(m_q a) + B^B \quad (3.18)$$

The result of the extrapolation according to the above equation gives: $M_B \sim 1$ GeV (always assuming $a^{-1} = 2$ GeV), and a very good linearity.

The latter result is rather encouraging, even if largely dependent upon the extrapolation procedure used.

*) We thank A. Morel for pointing this fact out to us.

4. - WILSON FERMIONS

The analysis for Wilson fermions closely follows the one described in the previous section. We compute the quark propagator by means of a first-order iterative algorithm (Gauss-Seidel), and in terms of these propagators we form the correlation functions of local operators with definite quantum numbers.

$$\begin{aligned}
 O^{\pi_1} &\equiv \bar{\psi} \gamma_5 \psi & ; & & O^{\pi_2} &\equiv \bar{\psi} \gamma_0 \gamma_5 \psi \\
 O_i^{S_1} &\equiv \bar{\psi} \gamma_i \psi & ; & & O_i^{S_2} &\equiv \bar{\psi} \sigma_{0i} \psi \\
 O_i^{A_1} &\equiv \bar{\psi} \gamma_5 \gamma_i \psi & ; & & O^E &\equiv \bar{\psi} \psi \\
 O_i^B &\equiv \bar{\psi} \sigma_{\kappa\ell} \epsilon^{\kappa\ell i} \psi
 \end{aligned} \tag{4.1}$$

For t large enough, one should recover

$$C^S(t) \equiv \int d^3x \langle O^S(x) O^S(0) \rangle \xrightarrow[t \rightarrow \infty]{} D_s e^{-m_s t} \tag{4.2}$$

where m_s is the mass of the lowest-lying state carrying the quantum numbers of the operator O^S , and D_s is a time-independent constant.

For finite t , the correlation function (4.2) is a superposition of exponentials with different slopes. In the Wilson formulation, the value of the hopping parameter, $K = K_c$, at which there is a massless pseudoscalar state, must be found "experimentally": the chirality being explicitly broken, this does not happen when the bare mass is zero. The values of K at which we have calculated our quark propagators are $K = 0.150$ and 0.155 . We know from previous calculations^{7),6),13)} that they are close to K_c and that they correspond to physical quark masses in the range 100 to 400 MeV.

The quark values of masses explored are much smaller than for Kogut-Susskind fermions: this results in a smaller dependence upon the extrapolation procedure used, but also in larger instabilities as far as the statistical fluctuations are concerned.

As regards the systematic errors of these calculations, we have only been able to make tests for the behaviour of the correlation functions assumed in Eq. (4.2) and for the dependence of the value of the lowest-lying state upon the choice of the operator (in the case of π , ρ , proton and Δ states).

We studied the contamination from excited states by looking at the quantities

$$\tilde{m}^s(t) \equiv \ln \left[\frac{C^s(t)}{C^s(t+1)} \right],$$

which we show in Fig. 1 for the case of the pion and the ρ states, at $K = 0.150, 0.155$. To obtain the values shown in Fig. 1, we have corrected the deviation at large t from the simple exponential behaviour of $C^s(t)$ due to the lattice periodicity. The values reported in the Figure are obtained by averaging the results for $m(t)$, for $\tilde{t} = t$ and $\tilde{t} = T-t$: this compensates for the asymmetry for $t \gtrsim T/2$ of our meson correlation functions (they do not behave exactly as a superposition of hyperbolic cosines) produced by the limited size of our statistical sample of gauge configurations. For a sufficiently large set of configurations, the asymmetry has to vanish: for a finite set, one remains with a residual asymmetry. This fact is only marginally relevant when relatively heavy quark mass values are used, but it gets amplified when they become small. For example, if we take the pion correlation function for reference, the worse left-right asymmetry turns out to be of the order of one-two per cent for the Kogut-Susskind case, of about 10 per cent for Wilson fermions at $K = 0.150$ and it increases up to 50% at $K = 0.155^{*)}$. We have, of course, checked that this is not produced by a systematic bias of our programs: from one configuration to another the asymmetry changes sign but, for light quark masses, the fluctuations are so large that they sum up, with a restricted set of configurations, to a still sizeable value.

The above discussion is the basis of our interpretation of the results for the baryon states. Indeed, the behaviour of $m(t)$ does not change when different operators are used for the mesons (in particular pion and ρ): on the contrary, large differences are observed when the same analysis is done for the proton. In Fig. 2, we report the value of $m(t)$ for two different operators for the proton, at $K = 0.150$ and $K = 0.155$, with

*) This actually happens only for $t \sim 8-9$: the asymmetry is otherwise definitely smaller for $t < 8$.

$$O_1^\alpha = (\psi C \psi) \gamma_5 \psi^\alpha$$

where C is the charge-conjugation matrix, and where O_2^α is defined as the non-relativistic limit of the operator O_1 . One can see that, apart from the fact that the excited states contamination gets worse for smaller quark masses, the two operators at $K = 0.155$ lead to different results. In particular, they are compatible up to $t \sim 6$, but while $m_{0_2}(t)$ keeps decreasing, $m_{0_1}(t)$ drastically increases. We explain this behaviour as follows. For spin- $\frac{1}{2}$ "resonances", the periodicity of the lattice implies that if, say, the proton-like states are propagating in the forward t direction, in the backward one we have the propagation of the opposite intrinsic parity states. In other words, different physical states dominate the behaviour of the correlation function for $t \gg T/2$. Furthermore, their residues, i.e., the coefficients of the exponential such as in Eq. (4.2), have the opposite sign. Therefore, if the proton is propagating, say, from the right but, due to the final asymmetry of the set of configurations, the contribution of the opposite parity state coming from the left is enhanced, this will result in a precocious increase of the proton $m(t)$: due to the other state which is taking over it is clear that, for t large enough, such an increase appears anyway just before the value at which the correlation function changes sign and starts to be dominated by the opposite parity hadron propagation. The behaviour of the two different operators O_1 and O_2 seems to support our interpretation; indeed the operator which does not show the precocious increase of the proton mass is the "non-relativistic" one. This is naturally expected to have a smaller projection on the opposite parity hadrons whose wave functions, being p waves, vanish in the non-relativistic limit. The effects of these hadrons on the estimate of the proton mass are then less important for the operator O_2 and the corresponding proton "mass" $m_{0_2}(t)$ will keep decreasing up to larger t , where the influence of the excited states has dissolved. For the reason explained above, we based our best estimate of the proton mass on m_{0_2} .

In Table 2, we report a summary of the results for different meson and baryon masses at the two values of K we considered. The numbers are extracted from the asymptotic values (large t) of $m(t)$ for both mesons and baryons. An alternative derivation for the mesons leads to the numbers on the right which

are obtained by fitting to the whole correlation function with two hyperbolic cosines. The values quoted for the baryons are obtained from the correlations of the non-relativistic operator O_2 .

The values in the Table can be translated into physical units by using as input either the light meson masses (π and ρ) or the strange mesons (K and K^*), the latter under the hypothesis $m_s \gg m_u, m_d$. The results for the lattice spacing and the quark masses are :

$$\begin{aligned} a_{\pi-\rho}^{-1} &= 2.2 \text{ GeV} \\ m_{u,d} a &= 0.002 \end{aligned} \quad (4.3)$$

$$\begin{aligned} a_{K-K^*}^{-1} &= 2.2 \text{ GeV} \\ (m_s + m_u) a &= 0.05 \end{aligned} \quad (4.4)$$

where $m_q \equiv \frac{1}{2}(1/K - 1/K_c)$. The compatibility among the two sets of results shows that a good fit will be obtained for K and K^* if π and ρ are used as input and vice versa.

The above values have been obtained by using the parametrization:

$$\begin{aligned} M_M^2 &= A_M \cdot \frac{1}{K} + B_M \\ &= 2A_M \left(\frac{1}{2K} - \frac{1}{2K_c} \right) + \left(B_M + \frac{A_M}{K_c} \right) \\ &\equiv 2A_M \cdot (mq \cdot a) + \tilde{B}_M \end{aligned} \quad (4.5)$$

The values for the coefficients A and B in Eq. (4.5) are found to be

$$\begin{array}{llll}
 A_{\pi} = 1.23 & B_{\pi} = -7.82 & \Rightarrow & 2A_{\pi} = 2.45 \quad \tilde{B}_{\pi} = 0.0 \\
 A_{\delta} = 1.02 & B_{\delta} = -6.40 & & 2A_{\delta} = 2.05 \quad \tilde{B}_{\delta} = 0.12 \\
 A_{A_1} = 0.76 & B_{A_1} = -4.61 & & \\
 A_{0^{++}} = 1.10 & B_{0^{++}} = -7.30 & & \\
 A_B = .99 & B_B = -6.29 & & (4.6)
 \end{array}$$

and we estimate $K_c = 0.157$. With the above set of values, the corresponding masses in physical units for the A_1 , δ , B mesons are

$$\begin{array}{ll}
 m_{A_1} \simeq 1 \text{ GeV} \\
 m_{0^{++}} \simeq 0.6 \text{ GeV} \\
 m_B \simeq 0.5 \text{ GeV}
 \end{array} \quad (4.7)$$

These values are low with respect to the correct ones: an improvement is found, however, with respect to the results obtained on smaller lattices where they could not even be estimated.

When the translation in physical units is made for the baryons, we get: $m_p = 1 \text{ GeV}$, $m_{\Delta} = 1.15$. The Δ -p splitting event, if rather small, appears to be a systematic feature configuration by configuration.

It is rather hard to estimate the errors on our numbers; the limited statistics available are small for a fair error analysis. The statistical errors for mesons can be estimated of the order of 5 to 10 per cent. Some estimates can be made on the systematic errors by comparing the numbers in Table 1, in particular those for different operators carrying the same quantum numbers: the variation is certainly within the statistical accuracy of our results.

Statistical and systematic errors are even larger in the estimates of baryon masses: the difference between results obtained with different operators is the most clear example. We therefore consider the values extracted for the proton and the Δ as only indicative.

We believe that the results for mesons are to be considered reasonably stable and in particular our estimate of the lattice spacing units $a^{-1} \simeq 2.1 \text{ GeV}$.

5. - COMPARISON OF THE RESULTS

While the situation for the baryon spectrum is still uncertain because of both statistical and systematic uncertainties, both the Kogut-Susskind and the Wilson fermion formulations lead to reasonable estimates of the meson spectrum, once some input values have been given to determine the free parameters of the theory, Λ and the quark masses. The question of the agreement between the two approaches then turns into the question whether the free parameters have the same numerical values in the two cases.

Our analysis clearly indicates a convergence of the value of the lattice spacing in physical units: we recall the values:

$$a_{K-S}^{-1} \underset{\beta=6}{\sim} 1.9 \text{ GeV} \quad \quad \quad \bar{a}_W^{-1} \underset{\beta=6}{\sim} 2.2 \text{ GeV} \quad (5.1)$$

This has to be contrasted with the results found at lower values of β ($\beta = 5.7$)^{12),13)} which indicated a major discrepancy on this number between the two cases.

However, the scale of the quark masses seems to be widely different. We find, for example, for the strange quark:

$$m_s^{K-S} a \simeq 0.02 \quad \quad \quad m_s^W a \simeq 0.05 \quad (5.2)$$

Given the agreement found for a^{-1} , this discrepancy is directly implied by one of the coefficients like A_π in Eq. (3.8) and $(2.A_\pi)$ in Eq. (4.6). This suggests that a quantity free of the quark mass might be in better agreement. Such is the relation

$$M_V^2 = A_{V-PS} M_{PS}^2 + B_{V-PS} \quad (5.3)$$

which is valid if a linear relation of the meson mass squared upon the quark masses is assumed.

The values for A_{V-PS} and B_{V-PS} that we get in the two cases are

$$\begin{aligned} A_{V-PS}^{K-S} &= 1.25 & B_{V-PS}^{K-S} &= 0.13 \\ A_{V-PS}^W &= 0.85 & B_{V-PS}^W &= 0.12 \end{aligned} \quad (5.4)$$

The disagreement is reduced, indicating that at least a part of the discrepancy found between Wilson and Kogut-Susskind might be ascribed to the finite renormalization corrections affecting the two definitions of quark masses. It is clear that a good agreement on the value of the quark masses would be a strong indication on the irrelevance of the particular way the problem of the breaking of chiral invariance is solved in the two approaches. Conversely, a disagreement on those values might mean just the opposite. In spite of this, we consider that the results of Eqs. (5.1) and (5.4) are rather encouraging; in particular, those on the lattice spacing are in reasonable agreement with the recent string tension estimates and suggest that we are not that far from a "continuum" behaviour of lattice QCD. An important source of differences between the two approaches is the form of the non-scaling corrections dying when "a" goes to zero as a power of the lattice spacing. It would then certainly be instructive to repeat such a comparison with improved action à la Symanzik, which, improving also the approach to the chiral limit, might lead to a more precocious universal behaviour of the two fermion formulations.

This calculation has been performed on the CRAY1-S of CCVR in Paris. We give here the computer time we used: since the most limiting factor for such a computation is the I/O time (on a 1 M word machine) we will give the true CPU time and an effective time, obtained by rescaling the total cost of the run to the price of a run without I/O. We obtain, for the different parts of our simulation,

Monte Carlo:	5,5 CPU hours,	9 effective hours
Wilson propagators:	12,5 CPU hours,	52 effective hours
K-S propagators:	2,5 CPU hours,	20 effective hours.

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$m_q a$	$m_\pi a$	$m_{\pi'} a$	$m_\rho a$	$m_{\rho'} a$	$m_{0^{++}} a$	$m_N a$
0.3	1.32(1)	1.44(5)	1.50(1)	1.51(2)	1.64(2)	2.40(5)
0.2	1.10(1)	1.16(6)	1.27(2)	1.27(3)	1.38(3)	2.05(5)
0.1	0.79(1)	0.85(10)	0.96(3)	0.95(4)	1.03(4)	1.50(5)

Table 1 - Masses of mesonic states and of the lighter baryonic state versus the bare quark mass (Kogut-Susskind formulation). The results are averages of fits to our data for $t \geq 3$ and 4. The fit to $G_\pi(t)$ includes a π' (not oscillating). A one-particle fit would give the same result with a slightly larger estimated error. In all cases the χ^2 normalized to the number of degrees of freedom is of order 1.

K Particle	0.150		0.155	
$\pi(1)$	0.60	0.60	0.31	0.33
$\rho(1)$	0.65	0.65	0.44	0.45
$\pi(2)$	0.59	0.60	0.31	0.28
$\rho(2)$	0.64	0.65	0.44	0.46
B	0.56	0.61	0.36	0.35
δ	0.65	0.63	0.41	0.40
A_1	0.67	0.64	0.52	0.50
P	1.10		0.65	
Δ	1.15		0.70	

Table 2 - Masses of hadronic states (Wilson formulation). For the mesonic states the number on the left (right) at both K's comes from a one (two) hyperbolic cosines fit. The values quoted for baryons come from the non-relativistic operator (O_2 in the text).

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FIGURE CAPTIONS

Figure 1 π and ρ states (Wilson fermions): effective mass as a function of the time.

Figure 2 Proton time-dependent masses for the relativistic (dots) and the non-relativistic (triangles) operators, at $K = 0.150$ and 0.155 .

- Figure 1 -

- Figure 2 -