QCD SUM RULES AND HYBRID MESONS

J. Govaerts

Department of Physics
University of Texas at Austin

F. de Viron, D. Gusbin

Institut de Physique Théorique
Université Catholique de Louvain
Louvain-La-Neuve

and

J. Weyers^{*)}

CERN - Geneva

A B S T R A C T

A general background field method is presented to compute operator product expansions in the framework of QCD sum rules. Using it for operators coupling to hybrid mesons, we predict the masses and couplings of quark-antiquark-gluon states with $J^{PC} = 1^{-+}, 1^{+-}, 0^{++}, 0^{--}$ and $I = 0, 1$.

^{*)} On leave of absence from the University of Louvain.

1. - INTRODUCTION

The QCD sum rules invented several years ago by Shifman, Vainshtein and Zakharov¹⁾ (SVZ) have led to an impressive number of very satisfactory results on the hadronic spectrum and its properties.

Using the fundamental parameters of the QCD Lagrangian (i.e., the quark masses and the strong coupling constant) and taking non-perturbative effects into account via various vacuum matrix elements, mesonic and baryonic masses and couplings^{1),2)}, electromagnetic form factors³⁾, magnetic moments⁴⁾, partial hadronic widths⁵⁾, have been computed for many hadronic states, and the results are quite remarkable⁶⁾.

More precisely, a basic assumption in the derivation¹⁾ of the QCD sum rules is that the operator product expansion⁷⁾ (OPE) holds in the non-trivial physical vacuum for T-ordered products of operators with definite quantum numbers. Vacuum matrix elements of these operators are then used as a parametrization of the long range non-perturbative effects, which eventually turn out to induce the hadronic structure, while the corresponding Wilson coefficients are calculated in perturbation theory, i.e., in the asymptotically free regime of the theory at short distances.

Although the method is very appealing physically, since it corresponds to a simple picture¹⁾ where quarks and gluons propagate through the physical vacuum and interact with the large scale field fluctuations, it is not clear that the prescription is indeed correct.

For example, it is well known¹⁾ that the OPE breaks down for operators with high enough dimension because of instanton effects in the vacuum. However, as long as the contributions of these higher dimensional operators are kept small enough, it may be reasonable to consider the first few terms in the OPE as a reliable approximation to the exact n-point functions.

The next step is to write down dispersion relations for the invariant functions appearing in the decomposition of the n-point functions and to saturate them with resonances and continuum states. In this way, one obtains sum rules which relate hadronic parameters to fundamental QCD parameters and vacuum matrix elements. By using the moment method¹⁾ or by making a Borel transformation¹⁾ of these sum rules, in order to enhance the contribution of the lowest-lying resonance as well as to decrease the effect of the higher

dimensional operators in the OPE, one finally ends up with the so-called "QCD sum rules" which provide information on the properties of hadronic states coupling to the corresponding operators.

In other words, the QCD sum rules are a "semi-phenomenological" method to predict hadronic properties in the framework of QCD. It is worth emphasizing that except for the lattice approximation which is plagued with problems⁸⁾ of its own, the SVZ approach is the only one which allows to relate the hadron spectrum to the fundamental Lagrangian of QCD.

In view of the successes of this approach in the calculation of properties of hadronic states also described by the quark model, it becomes interesting to apply the same techniques to predict properties of "unusual" hadrons such as glueballs or hybrid states (bound states of quarks and gluons)⁹⁾. The experimental discovery of such states is of the utmost importance in establishing QCD as the theory of strong interactions and obviously some knowledge of where and how to look for these states should be a valuable help in trying to find them.

It is the purpose of this paper to study the masses and couplings of hybrid quark-antiquark-gluon ($q\bar{q}g$) states with $J^{PC} = 1^{-+}, 0^{++}, 1^{+-}, 0^{--}$; $I = 0, 1$; and containing u and d quarks only.

The 1^{-+} and 0^{--} states are particularly interesting since no ordinary meson with these exotic quantum numbers can be formed from a valence quark-antiquark pair. Therefore, there are no mixing effects of these states with ordinary mesons, as there will be for the 1^{+-} and 0^{++} $q\bar{q}g$ states.

On the other hand, there are no 1^{-+} or 0^{--} glueball states formed from two gluons only (in the 1^{-+} sector, this is due to the Landau-Yang selection rule) and one expects therefore small mixing effects between these glueball and $q\bar{q}g$ hybrid states.

All this makes the study of these exotic hybrid states quite exciting since one expects a clear theoretical situation in that sector of the QCD spectrum, as opposed to the rather confused situation in the glueball sector [see the recent attempts at identifying the $\psi(1440)$ and $\theta(1700)$ as glueballs¹⁰⁾].

On the other hand, the 1^{+-} and 0^{++} hybrids will mix with ordinary $q\bar{q}$ mesons. We will not compute these effects here but nevertheless present the results for these states since they are obtained as "by-products" of our calculations and give anyway an idea of what the hybrid spectrum in that sector looks like.

Some of the results presented here have already been published¹¹⁾. Unfortunately, some mistakes were made in Ref. 11) which do affect the predictions for hybrid state masses. In this paper we present a corrected and extended report of our calculations. Let us point out that we still disagree with a previous¹²⁾ estimate of the mass of the $1^{+-} q\bar{q}g$ state from QCD sum rules.

The paper is organized as follows. In Section 2, we present a general method¹¹⁾ to compute operator product expansions useful for QCD sum rules, which simplifies the calculations, especially in the case of two-point functions where one can work in configuration space. This is done by expanding the quark and gluon fields around classical background fields which satisfy the equations of motion of QCD and account for the vacuum matrix elements of scalar and gauge invariant quantum operators. Although this idea is already used in a more or less systematic way in the literature^{13),14)}, as we realized only after completion of our first paper, we think that our technique provides a complete and consistent framework for the use of that idea.

By using that method in the case of two-point functions of operators coupling to the hybrid states considered here, the corresponding OPE's are computed and their expressions are given in Section 3.

These results are then applied to physical states, for which the masses and couplings are predicted. This is done in Section 4.

2. - THE OPE FOR QCD SUM RULES: A BACKGROUND FIELD METHOD

From the Lagrangian of QCD

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{i}{2} [\bar{\psi} \not{D} \psi - (\not{D} \bar{\psi}) \psi] - m \bar{\psi} \psi \quad (2.1)$$

one derives the equations of motion

$$i \gamma^\mu \mathcal{D}_\mu \psi = m \psi \quad (2.2a)$$

$$i(\overline{\mathcal{D}_\mu \psi}) \gamma^\mu = -m \bar{\psi} \quad (2.2b)$$

$$\mathcal{D}_{ab}^\nu F_{\mu\nu}^b = g \bar{\psi} \gamma_\mu T_a \psi \quad (2.2c)$$

In these expressions, A_μ^a and ψ are the gluon and quark fields [a sum over flavours is understood in Eq. (2.1)] transforming as the adjoint and fundamental representations of SU(3), $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$ is the gluon field strength, and g, m are the strong coupling constant and the quark mass.

The generators T^a in the fundamental representation as well as the structure constants f^{abc} are normalized by:

$$[T^a, T^b] = if^{abc} T^c \quad (2.3a)$$

$$\text{Tr } T^a T^b = \frac{1}{2} \delta^{ab} \quad (2.3b)$$

and the gauge covariant derivatives in the fundamental and adjoint representations are:

$$D_\mu = \partial_\mu - i g A_\mu^a T^a \quad (2.4a)$$

$$D_\mu^{ab} = \partial_\mu \delta^{ab} - g f^{abc} A_\mu^c \quad (2.4b)$$

In principle, the Lagrangian (2.1) contains the information to describe all the properties of the hadronic world. For example, the existence of a scalar state of mass m_s coupling to an operator $O(x)$ [such as $\bar{\psi}(x)\psi(x)$] manifests itself as a pole at $q^2 = m_s^2$ in the two-point function:

$$i \int d^4x e^{iq \cdot x} \langle 0 | T O(x) O(0) | 0 \rangle \quad (2.5)$$

However, it is not possible in QCD to compute that function exactly, because of non-perturbative phenomena responsible for confinement and chiral symmetry breaking and leading to the existence of hadrons, and one has to introduce an approximation.

The one used in the QCD sum rule approach is to assume that the OPE is still valid in the physical vacuum:

$$i \int d^4x e^{iq \cdot x} \langle 0 | T O(x) O(0) | 0 \rangle = \sum_{k \rightarrow -\infty} C_k(q^2) \langle 0 | O_k | 0 \rangle \quad (2.6)$$

where the local (Lorentz) scalar and gauge invariant operators O_k are ordered by dimension and the Wilson coefficients $C_k(q^2)$ fall off by the corresponding powers of q^2 .

The matrix elements $\langle 0 | O_k | 0 \rangle$ (for O_k different from the identity operator) are used as a parametrization of the long-range non-perturbative effects responsible for the hadronic spectrum, while the Wilson coefficients, computed in perturbation theory as series in $\alpha_s = g^2/4\pi$, correspond to the QCD dynamics at short distances.

These coefficients may be computed by considering the T-product $i \int d^4x e^{iqx} T(O(x)O(o))$ in different quark and gluon states, so as to select the corresponding operator O_k in the OPE. We propose here another way for doing that, which finds its motivation in the physical picture at the basis of the whole QCD sum rule approach.

The operators $O(o)$ and $O(x)$, acting on the vacuum, create and annihilate quark and gluon fields which, while propagating through the vacuum, may interact with long-range non-perturbative fluctuations. These effects give the non-vanishing matrix elements $\langle 0 | O_k | 0 \rangle$ in the expansion (2.6).

This simple picture suggests to represent these vacuum fluctuations by some classical background fields, satisfying the equations of motion, around which all the quark and gluon fields are expanded as quantum fluctuations over which the functional integral will be done (in other words, we simply shift the fields by the amount corresponding to the background fields).

Then, by computing the left-hand side of Eq. (2.6) as a short-distance expansion in the background fields, we recover the OPE (2.6) in the following way: each term in that expansion containing a scalar and gauge invariant functional O_k of the external fields corresponds to the term $C_k(q^2) \langle 0 | O_k | 0 \rangle$ in the OPE, while any other term has to be discarded since either it will not be a scalar or gauge invariant functional of the external fields (for example A_μ^a) or it will also be a functional of the quantum fluctuations, and these have vanishing vacuum matrix elements.

In this way, each term in the short-distance expansion in Eq. (2.6) is easily computed, and, as it turns out, this method is very useful in the case of light quarks because it is then possible to work in configuration space where calculations are much simpler than in momentum space.

However, we want to insist on the fact that there is no physical significance to these background fields; they are merely a technical device used to simplify the calculation of the OPE needed in the QCD sum rule approach.

We make the following substitution in the Lagrangian (2.1) and in all Green's functions:

$$A_\mu^a(x) \longrightarrow A_\mu^a(x) + \phi_\mu^a(x) \quad (2.7a)$$

$$\psi(x) \longrightarrow \psi(x) + \eta(x) \quad (2.7b)$$

where $A_\mu^a(x)$ and $\psi(x)$ are the classical gluon and quark background fields, satisfying Eqs. (2.2), and $\phi_\mu^a(x)$, $\eta(x)$ are their quantum fluctuations over which the functional integral will be performed.

The Lagrangian (2.1) is then transformed into:

$$\begin{aligned} \mathcal{L}_{eff} = & \mathcal{L}(A, \psi, \bar{\psi}) + \mathcal{L}(ghost) + \bar{\eta} [i \not{D}_\mu - m] \eta \\ & + \frac{1}{2} \phi_\mu^a [g^{\mu\nu} D_{ac}^2 - (1 - \frac{1}{2})(D^\mu D^\nu)_{ac} + 2g f_{abc} G_b^{\mu\nu}] \phi_\nu^c \\ & + g (\bar{\psi} \not{D}_\mu \phi_\mu^a T^a \eta + \bar{\eta} \not{D}_\mu \phi_\mu^a T^a \psi) - g^2 f^{abc} f_{adef} A_\mu^b \phi_\nu^c \phi_d^e \phi_f^a \\ & - g f_{abc} (\partial^\mu \phi_a^\nu) \phi_\mu^b \phi_\nu^c - \frac{1}{4} g^2 f^{abc} f_{adef} \phi_\mu^b \phi_\nu^c \phi_d^e \phi_f^a \\ & + g \bar{\eta} \not{D}_\mu \phi_\mu^a T^a \eta \end{aligned}$$

(2.8)

where $\mathcal{L}(A, \psi, \bar{\psi})$ is the QCD Lagrangian evaluated for the background fields, $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$ is the background gluon field strength, and D_μ^{ab} are the background gauge covariant derivatives (functions of A_μ^a but not ϕ_μ^a).

We used in Eq. (2.8) the background gauge for the gauge fluctuations $\phi_\mu^a(x)$, i.e., $D_\mu^{ab}\phi_b^\mu = 0$, by adding to the Lagrangian the gauge fixing term $(-1/2\alpha)(D_\mu^{ab}\phi_b^\mu)^2$. $\mathcal{L}(\text{ghost})$ is the ghost contribution corresponding to that choice of gauge.

Let us note that several a priori possible terms, such as $g\bar{\psi}\gamma^\mu\phi_\mu^a T^a\psi$, do not appear in the effective Lagrangian (2.8). This follows from the use of the equations of motion for the background fields, which leads to several cancellations for these terms and leaves us with the usual gauge and quark couplings and couplings of the quantum fluctuations to one background field only.

By integrating over the quantum fluctuations, the quadratic terms in η and ϕ_μ^a give the propagators $S_F(x, y)$ and $S_{\mu\nu}^{ab}(x, y)$ of the quark and gluon fluctuations in the background fields, defined through the equations:

$$[i \not{D} - m] S_F(x, y) = i \delta^{(4)}(x - y) \quad (2.9a)$$

$$[g^{\mu\nu} D_{ac}^2 + 2g f_{abc} G^{\mu\nu}_b] S_{\nu\rho}^{cd}(x, y) = i \delta_\rho^\mu \delta_a^d \delta^{(4)}(x - y) \quad (2.9b)$$

(we take the gauge parameter $\alpha = 1$).

In order to do short-distance expansions in n-point functions, it is very convenient to fix the gauge freedom of the background field $A_\mu^a(x)$ by use of the Schwinger or fixed-point gauge¹⁵⁾:

$$x^\mu A_\mu^a(x) = 0 \quad (2.10)$$

since we then obtain¹⁶⁾:

$$\begin{aligned} A_\mu^a(x) &= \int_0^1 d\alpha \, \alpha x^\nu G_{\nu\mu}^a(\alpha x) \\ &= \sum_{n=0}^{\infty} \frac{1}{(n+2)n!} x^{\alpha_1} \dots x^{\alpha_n} x^\nu (D_{\alpha_1} \dots D_{\alpha_n} G_{\nu\mu})^a(0) \end{aligned} \quad (2.11a)$$

which leads at once to gauge covariant results in the OPE.

Equations (2.9) can now be solved as a perturbation series in the background fields $G_{\mu\nu}^a(0)$ and their covariant derivatives $(D \dots DG)(0)$, as well in configuration space as in momentum space.

We find^{*)} ($z = x-y$ in the expressions, and an ultra-violet cut-off is understood in the $\ln(-z^2)$ factors):

$$\begin{aligned} S_{\mu\nu}^{ab}(x,y) &= \frac{1}{4\pi^2} \frac{g_{\mu\nu} \delta^{ab}}{z^2 - i\varepsilon} + \frac{1}{8\pi^2} g f^{abc} G_{\mu\nu}^c(0) \ln(-z^2) \\ &\quad - \frac{1}{8\pi^2} g f^{abc} g_{\mu\nu} G_{\lambda\sigma}^c(0) \frac{x^\lambda y^\sigma}{z^2 - i\varepsilon} + \dots \end{aligned} \quad (2.11b)$$

^{*)} These results do of course agree with some expressions already given in the literature^{13),14)}.

$$S_{\mu\nu}^{ab}(x, y) = \int \frac{d^4 p}{(2\pi)^4} e^{-i p \cdot (x-y)} \left[-\frac{i g_{\mu\nu} \delta^{ab}}{p^2 + i\varepsilon} + \right. \\ \left. + 2i g f^{abc} G_{\mu\nu}^c(0) \frac{1}{(p^2 + i\varepsilon)^2} + g f^{abc} g_{\mu\nu} G_{\lambda\sigma}^c(0) \frac{x^\lambda p^\sigma}{(p^2 + i\varepsilon)^2} + \dots \right] \quad (2.11c)$$

$$S_F(x, y) = \frac{i}{2\pi^2} \frac{z_\mu j^\mu}{[z^2 - i\varepsilon]^2} - \frac{m}{4\pi^2} \frac{1}{z^2 - i\varepsilon} + \frac{i m^2}{8\pi^2} \frac{z_\mu j^\mu}{z^2 - i\varepsilon} \\ - \frac{1}{16\pi^2} g G_{\mu\nu}^a(0) \frac{T^a j^\mu z_\alpha j^\alpha j^\nu}{z^2 - i\varepsilon} + \frac{1}{4\pi^2} g G_{\mu\nu}^a(0) \frac{T^a z^\mu y^\nu z_\alpha j^\alpha}{[z^2 - i\varepsilon]^2} \\ + \frac{i}{8\pi^2} m g G_{\mu\nu}^a(0) \frac{T^a z^\mu y^\nu}{z^2 - i\varepsilon} - \frac{1}{32\pi^2} m g G_{\mu\nu}^a(0) T^a \sigma^{\mu\nu} \ln(-z^2) \\ + \frac{1}{48\pi^2} g (D^\nu G_{\nu\mu})^a(0) T^a j^\mu \ln(-z^2) \\ + \frac{1}{12\pi^2} g (D_\alpha G_{\nu\mu})^a(0) T^a \frac{1}{[z^2 - i\varepsilon]^2} z^\nu y^\mu (z^\alpha + y^\alpha) z_\rho j^\rho \\ - \frac{1}{144\pi^2} g (D_\alpha G_{\nu\mu})^a(0) T^a \frac{1}{z^2 - i\varepsilon} \{ (2z^\nu + y^\nu) j^\alpha z_\rho j^\rho j^\mu - \\ - (z^\nu + 2y^\nu) j^\mu z_\rho j^\rho j^\alpha - 2z^\alpha z^\mu j^\nu + 3(z^\alpha + y^\alpha) j^\nu z_\rho j^\rho j^\mu + \\ + g^{\alpha\nu} z_\rho j^\rho j^\mu z_\sigma j^\sigma \}$$

$$\begin{aligned}
 & + \frac{i}{32\pi^2} g^2 G_{\nu\mu}^a(0) T^a G_{\alpha\beta}^b(0) T^b \left\{ -\frac{1}{12} \frac{(2x^\nu + y^\nu)}{z^2 - i\varepsilon} z_\sigma j^\sigma j^\mu j^\alpha z_\sigma j^\sigma j^\beta + \right. \\
 & \quad + \frac{1}{24} z_\sigma j^\sigma j^\mu j^\alpha j^\nu j^\beta \ln(-z^2) - \frac{1}{6} g^{\mu\alpha} j^\beta (2x^\nu + y^\nu) \ln(-z^2) - \\
 & \quad \left. - \frac{1}{12} j^\nu j^\mu j^\alpha z_\sigma j^\sigma j^\beta \ln(-z^2) \right\} \\
 & - \frac{i}{16\pi^2} g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b \frac{1}{[z^2 - i\varepsilon]^2} x^\mu y^\nu x^\alpha y^\beta z_\sigma j^\sigma \\
 & - \frac{i}{192\pi^2} g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b \frac{y^\beta}{z^2 - i\varepsilon} \left\{ (2x^\nu + y^\nu) j^\alpha z_\sigma j^\sigma j^\mu - (x^\nu + 2y^\nu) j^\mu z_\sigma j^\sigma j^\alpha \right. \\
 & \quad \left. - 2x^\alpha z^\mu j^\nu + 2x^\alpha j^\nu z_\sigma j^\sigma j^\mu - 2j^\mu z_\sigma j^\sigma j^\nu + g^{\nu\alpha} z_\sigma j^\sigma j^\mu z_\sigma j^\sigma \right\} \\
 & + \frac{i}{64\pi^2} g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b y^\beta j^\mu \ln(-z^2) \\
 & + \dots
 \end{aligned}$$

(2.12a)

$$S_F(x, y) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \{ * \}$$

$$\{ * \} = \frac{i}{\not{p} - m} - \frac{1}{2} g G_{\mu\nu}^a T^a \frac{1}{\not{p} - m} j^\mu \frac{1}{\not{p} - m} j^\nu \frac{1}{\not{p} - m}$$

$$+ \frac{1}{2} i g G_{\mu\nu}^a(0) T^a x^\nu \frac{1}{\not{p} - m} j^\mu \frac{1}{\not{p} - m}$$

$$+ \frac{1}{3} i g (D_\alpha G_{\nu\mu})^a(0) T^a \left\{ \frac{1}{\not{p} - m} j^\alpha \frac{1}{\not{p} - m} j^\nu \frac{1}{\not{p} - m} j^\mu \frac{1}{\not{p} - m} + \frac{1}{\not{p} - m} j^\nu \frac{1}{\not{p} - m} j^\alpha \frac{1}{\not{p} - m} j^\mu \frac{1}{\not{p} - m} \right\}$$

$$- \frac{1}{3} g (D_\alpha G_{\nu\mu})^a(0) T^a \left\{ 2x^\alpha \frac{1}{\not{p} - m} j^\nu \frac{1}{\not{p} - m} j^\mu \frac{1}{\not{p} - m} + x^\nu \frac{1}{\not{p} - m} j^\alpha \frac{1}{\not{p} - m} j^\mu \frac{1}{\not{p} - m} \right\}$$

$$\begin{aligned}
& -\frac{1}{3} i g^2 (D_\alpha G_{\nu\mu})^a(0) T^a x^\alpha x^\nu \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \\
& -\frac{1}{4} i g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b \left\{ \frac{1}{\not{p}-m} \not{x}^\nu \frac{1}{\not{p}-m} \not{x}^\beta \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} + \right. \\
& \quad \left. + \frac{1}{\not{p}-m} \not{x}^\beta \frac{1}{\not{p}-m} \not{x}^\nu \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} + \frac{1}{\not{p}-m} \not{x}^\nu \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\beta \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} \right\} \\
& + \frac{1}{4} g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b x^\nu \left\{ \frac{1}{\not{p}-m} \not{x}^\beta \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} \right. \\
& \quad \left. + \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\beta \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} \right\} \\
& + \frac{1}{4} g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b x^\beta \frac{1}{\not{p}-m} \not{x}^\nu \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} \\
& + \frac{1}{4} i g^2 G_{\mu\nu}^a(0) T^a G_{\alpha\beta}^b(0) T^b x^\nu x^\beta \frac{1}{\not{p}-m} \not{x}^\mu \frac{1}{\not{p}-m} \not{x}^\alpha \frac{1}{\not{p}-m} \\
& + \dots
\end{aligned}$$

(2.12b)

(the $+i\epsilon$ prescription is understood in (2.12b)).

Note that the quark propagator in configuration space is given as an expansion in the quark mass (we only computed the m , m^2 and $mG(0)$ terms). This means that Eq. (2.12a) can only be used for light quarks. For heavy quarks, one has to go to momentum space and use Eq. (2.12b).

Although only the expressions in configuration space are used in this paper, we give Eqs. (2.11b) and (2.12b) since they have to be known for the OPE of three-point functions.

Let us also remark that the choice of the Schwinger gauge for $A_\mu^a(x)$ breaks translation invariance; this is why terms in x^μ or y^μ appear in the propagators. Translation invariance is restored at the end of the calculations since the OPE for gauge invariant operators gives a gauge invariant result.

This method for short-distance expansions in the framework of QCD sum rules should by now be clear. We used it to obtain the results of the next section. In Appendix A, we show how it works in the simpler case of the vector current $\bar{\psi}\gamma_\mu\psi$.

3. - THE SHORT-DISTANCE EXPANSIONS

To compute sum rules for the hybrid mesons that we consider in this paper, we have to use operators with the same quantum numbers as these resonances. The simplest Hermitian gauge invariant operators coupling to $q\bar{q}g$ states with $I = 0, 1$ and $J^{PC} = 1^{-+}, 0^{++}$ and to $q\bar{q}g$ states with $I = 0, 1$ and $J^{PC} = 0^{--}, 1^{+-}$ are respectively:

$$J_{(+)\mu}^n(x) = g \bar{\psi}(x) \Lambda^n \gamma^\alpha G_{\alpha\mu}^a(x) T^a \psi(x) \quad (0 \leq n \leq 3) \quad (3.1a)$$

$$J_{(-)\mu}^n(x) = g \bar{\psi}(x) \Lambda^n \gamma^\alpha \gamma_5 G_{\alpha\mu}^a(x) T^a \psi(x) \quad (0 \leq n \leq 3) \quad (3.1b)$$

Λ^n are the $U(2)$ flavour generators in the fundamental representation, normalized by $\text{Tr} \Lambda^n \Lambda^m = \frac{1}{2} \delta^{nm}$, and $\phi(x)$ is the quark isospin doublet $\begin{pmatrix} u \\ d \end{pmatrix}$ (substitution (2.7) has of course not yet been made).

Since we choose here to work in the limit of exact isospin symmetry ($m_u = m_d = m$), there will be no mixings between states of definite isospin.

Hence it suffices to consider only flavour diagonal two-point functions, which we take to be those with $n = 0$ and $n = 3$.

For non-conserved vector currents, the two-point function has the structure:

$$i \int d^4x e^{iqx} \langle 0 | T J_\mu^+(x) J_\nu(0) | 0 \rangle = \left(q_\mu q_\nu - g_{\mu\nu} \right) \Pi_V(q^2) + \frac{q_\mu q_\nu}{q^2} \Pi_S(q^2) \quad (3.2)$$

where the imaginary parts of Π_V and Π_S receive contributions from the corresponding vector and scalar intermediate states.

In the following, we will denote these spectral functions by Π_V^+ , Π_V^- , Π_S^+ and Π_S^- for the 1^- , 1^+ , 0^{++} and 0^{--} states respectively, and by π in general.

We have computed the OPE for these Green's functions with operators of dimension 0 to 8. Each of their coefficients is computed at lowest order in α_s in perturbation theory and we are only interested in their leading behaviour as $(q^2 \rightarrow -\infty)$. This leaves us with the diagrams shown in Fig. 1. These calculations were done using the background field method, and our results are the following.

$$\begin{aligned} \Pi_V^+(q^2) = & \frac{-d_S}{480\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[\frac{-d_S}{18\pi} m \bar{\psi}\psi + \frac{-1}{72} \frac{\alpha_S}{\pi} G_{\mu\nu}^a G_{\mu\nu}^{a'} \right] q^2 \ln(-q^2/\mu^2) \\ & + \frac{\pi d_S}{2} \bar{\psi} \Lambda^\alpha j^\alpha T^a \psi \bar{\psi} \Lambda^\beta j^\beta T^a \psi + \left[\frac{-1}{128\pi^2} g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c + \right. \\ & + \frac{5d_S}{192\pi} m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \left. \right] \ln(-q^2/\mu^2) + \frac{1}{864\pi^2} \left[-15 O_1 + 7 O_2 + 3 O_3 - O_4 \right] \frac{1}{q^2} \ln(-q^2/\mu^2) \\ & - \frac{1}{12} g^3 \left\{ 3 f^{abc} \bar{\psi} \Lambda^\alpha j^\alpha T^c \psi G_{\alpha\beta}^a + 2 G_{\alpha\mu}^a (\bar{\psi} \Lambda^\alpha T^a T^b \sigma^{\alpha\mu} j^\beta \psi + \right. \\ & \left. + \bar{\psi} \Lambda^\alpha T^b T^a j^\beta \sigma^{\alpha\mu} \psi) \right\} (\bar{\psi} \Lambda^\alpha j^\alpha T^b \psi) \frac{1}{q^2} \quad (3.3) \\ & + \left\{ \frac{i}{72} \left[4 \bar{\psi} j^\alpha j^\beta j^\gamma G_{\alpha\mu} D_\rho (G_{\rho\mu} \psi) + \bar{\psi} j^\alpha j^\mu j^\beta G_{\alpha\nu} D_\nu (G_{\beta\mu} \psi) + \bar{\psi} j^\alpha j^\mu j^\beta G_{\alpha\mu} D_\nu (G_{\beta\nu} \psi) \right] \right. \\ & \left. + \frac{\pi^2}{4} m \bar{\psi} [j^\alpha j^\beta T^a T^b + j^\beta j^\alpha T^b T^a] \psi \frac{\alpha_S}{\pi} G_{\alpha\mu}^a G_{\beta\mu}^b \right\} \frac{1}{q^2} \end{aligned}$$

$$\begin{aligned}
 \Pi_S^+(q^2) = & \frac{-\alpha_s}{960\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[\frac{\alpha_s}{128\pi} m\bar{\psi}\psi + \frac{1}{48} \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a \right] q^2 \ln(-q^2/\mu^2) \\
 & + \frac{3\pi\alpha_s}{2} \bar{\psi} \Lambda^\alpha j^\alpha T^a \psi \bar{\psi} \Lambda^\alpha j^\alpha T^a \psi + \left[\frac{1}{128\pi^2} g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \right. \\
 & \left. - \frac{5\alpha_s}{192\pi} m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \right] \ln(-q^2/\mu^2) + \frac{1}{864\pi^2} [18O_1 + 30O_2 - 9O_3 - 3O_4] \frac{1}{q^2} \ln(-q^2/\mu^2) \\
 & + \frac{1}{4} g^3 \left[5f^{abc} \bar{\psi} \Lambda^\alpha j^\alpha T^c \psi G_{\alpha\beta}^a + 2G_{\mu}^a (\bar{\psi} \Lambda^\alpha T^a T^b \sigma^{\alpha\mu} j^\beta \psi + \bar{\psi} \Lambda^\alpha T^b T^a j^\beta \sigma^{\alpha\mu} \psi) \right] (\bar{\psi} \Lambda^\alpha j^\alpha T^b \psi) \frac{1}{q^2} \\
 & + \left\{ \frac{-i}{24} \left[2\bar{\psi} j^\alpha j^\beta j^\beta G_{\alpha\mu} D_\beta (G_{\beta\mu} \psi) - \bar{\psi} j^\alpha j^\mu j^\beta G_{\alpha\mu} D_\nu (G_{\beta\mu} \psi) - \bar{\psi} j^\alpha j^\mu j^\beta G_{\alpha\mu} D_\nu (G_{\beta\nu} \psi) \right] \right. \\
 & \left. - \frac{\pi^2}{4} m \bar{\psi} [j^\alpha j^\beta T^a T^b + j^\beta j^\alpha T^b T^a] \psi \frac{\alpha_s}{\pi} G_{\alpha\mu}^a G_{\beta\mu}^b \right\} \frac{1}{q^2} \quad (3.4)
 \end{aligned}$$

$$\begin{aligned}
 \Pi_V^-(q^2) = & \frac{-\alpha_s}{480\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left\{ \frac{-\alpha_s}{9\pi} m\bar{\psi}\psi - \frac{1}{72} \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a \right\} q^2 \ln(-q^2/\mu^2) \\
 & + \frac{\pi\alpha_s}{2} \bar{\psi} \Lambda^\alpha j^\alpha T^a \psi \bar{\psi} \Lambda^\alpha j^\alpha T^a \psi + \left\{ -\frac{1}{128\pi^2} g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \right. \\
 & \left. - \frac{13\alpha_s}{192\pi} m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \right\} \ln(-q^2/\mu^2) + \frac{1}{864\pi^2} [-15O_1 + 7O_2 + 3O_3 - O_4] \frac{1}{q^2} \ln(-q^2/\mu^2) \\
 & - \frac{1}{12} g^3 \left\{ 3f^{abc} \bar{\psi} \Lambda^\alpha j^\alpha T^c \psi G_{\alpha\beta}^a \right. \\
 & \left. + 2G_{\mu}^a [\bar{\psi} \Lambda^\alpha T^a T^b \sigma^{\alpha\mu} j^\beta \psi + \bar{\psi} \Lambda^\alpha T^b T^a j^\beta \sigma^{\alpha\mu} \psi] \right\} (\bar{\psi} \Lambda^\alpha j^\alpha T^b \psi) \frac{1}{q^2} \\
 & + \left\{ \frac{i}{72} [4\bar{\psi} j^\alpha j^\beta j^\beta G_{\alpha\mu} D_\beta (G_{\beta\mu} \psi) + \bar{\psi} j^\alpha j^\mu j^\beta G_{\alpha\mu} D_\nu (G_{\beta\mu} \psi) + \bar{\psi} j^\alpha j^\mu j^\beta G_{\alpha\mu} D_\nu (G_{\beta\nu} \psi)] \right. \\
 & \left. - \frac{\pi^2}{4} m \bar{\psi} [j^\alpha j^\beta T^a T^b + j^\beta j^\alpha T^b T^a] \psi \frac{\alpha_s}{\pi} G_{\alpha\mu}^a G_{\beta\mu}^b \right\} \frac{1}{q^2} \quad (3.5)
 \end{aligned}$$

$$\begin{aligned}
 \Pi_S^-(q^2) = & \frac{-\alpha_s}{960\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[\frac{-\alpha_s}{12\pi} m \bar{\psi} \psi + \frac{1}{48\pi} \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a \right] q^2 \ln(-q^2/\mu^2) \\
 & + \frac{3\pi\alpha_s}{2} \bar{\psi} \Lambda^a j_{\mu}^a T^a \psi \bar{\psi} \Lambda^a j_{\mu}^a T^a \psi + \left[\frac{1}{128\pi^2} g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \right. \\
 & + \frac{13\alpha_s}{192\pi} m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \left. \right] \ln(-q^2/\mu^2) + \frac{1}{864\pi^2} [18O_1 + 30O_2 - 9O_3 - 3O_4] \frac{1}{q^2} \ln(-q^2/\mu^2) \\
 & + \frac{1}{4} g^3 [5f^{abc} \bar{\psi} \Lambda^a j_{\mu}^a T^c \psi G_{\alpha\beta}^a + 2G_{\alpha\mu}^a (\bar{\psi} \Lambda^a T^a T^b \sigma^{\alpha\mu} j_{\beta}^b \psi + \bar{\psi} \Lambda^a T^b T^a j_{\beta}^b \sigma^{\alpha\mu} \psi)] (\bar{\psi} \Lambda^a j_{\mu}^a T^b \psi) \frac{1}{q^2} \\
 & + \left\{ \frac{-i}{24} \left\{ 2\bar{\psi} j^{\alpha} j^{\beta} j^{\gamma} G_{\alpha\mu} D_{\rho} (G_{\rho\mu} \psi) - \bar{\psi} j^{\alpha} j^{\mu} j^{\beta} G_{\alpha\nu} D_{\nu} (G_{\rho\mu} \psi) - \bar{\psi} j^{\alpha} j^{\mu} j^{\beta} G_{\alpha\mu} D_{\nu} (G_{\rho\nu} \psi) \right\} \right. \\
 & \left. + \frac{g^2}{4} m \bar{\psi} [j^{\alpha} j^{\beta} T^a T^b + j^{\beta} j^{\alpha} T^b T^a] \psi \frac{\alpha_s}{\pi} G_{\alpha\mu}^a G_{\beta\mu}^b \right\} \frac{1}{q^2}
 \end{aligned}
 \tag{3.6}$$

In these expressions, μ is some typical hadronic mass and we defined:

$$G_{\mu\nu} = g G_{\mu\nu}^a T_a \tag{3.7a}$$

$$O_1 = \text{Tr}(G_{\mu\nu} G_{\nu\rho} G_{\rho\sigma} G_{\sigma\mu}) \quad O_2 = \text{Tr}(G_{\mu\nu} G_{\rho\sigma} G_{\nu\rho} G_{\sigma\mu})$$

$$O_3 = \text{Tr}(G_{\mu\nu} G_{\mu\nu} G_{\rho\sigma} G_{\rho\sigma}) \quad O_4 = \text{Tr}(G_{\mu\nu} G_{\rho\sigma} G_{\mu\nu} G_{\rho\sigma}) \tag{3.7b}$$

It is understood that these amplitudes are renormalized by a subtraction at μ^2 which, to be completely consistent, should take into account the anomalous dimensions of the operators and the currents $J_{(\pm)\mu}^n(x)$. Indeed, only vacuum matrix elements of renormalization group invariant operators have a physical meaning.

However, since the anomalous dimensions of the currents $J_{(\pm)\mu}$ and of most of the operators in Eqs. (3.3) to (3.6) are not known, we will assume that they introduce only small corrections to our results and we will neglect them in the following. This means that α_s has to be evaluated at $\alpha_s(\mu)$ via the renormalization group, i.e., $\alpha_s(\mu) = 4\pi/9 \ln(\mu^2/\Lambda^2)$, whenever it is not absorbed in the vacuum matrix elements (as it is for example in $\langle \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} \rangle$).

Only a few vacuum matrix elements of the operators appearing in the OPE's are known, and one has to introduce some approximation to evaluate the others. We used here the vacuum dominance assumption which leads then to the following results.

$$\begin{aligned}
 \Pi_V^+(q^2) = & \frac{-d_s}{480\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[\frac{-d_s}{18\pi} \langle m \bar{\psi} \psi \rangle - \frac{1}{72} \langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \rangle \right] q^2 \ln(-q^2/\mu^2) \\
 & - \frac{\pi d_s}{36} \langle \bar{\psi} \psi \rangle^2 + \left[-\frac{1}{128\pi^2} \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle + \right. \\
 & \left. + \frac{5d_s}{192\pi} \langle m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \right] \ln(-q^2/\mu^2) \\
 & - \frac{\pi^2}{324} \langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \rangle^2 \frac{1}{q^2} \ln(-q^2/\mu^2) \\
 & + \left[\frac{\pi^2}{18} \langle m \bar{\psi} \psi \rangle \langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \rangle + \frac{19\pi d_s}{2592} \langle \bar{\psi} \psi \rangle \langle g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \right] \frac{1}{q^2}
 \end{aligned}
 \tag{3.8}$$

$$\begin{aligned}
 \Pi_S^+(q^2) = & \frac{-d_s}{960\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[\frac{d_s}{12\pi} \langle m \bar{\psi} \psi \rangle + \frac{1}{48} \langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \rangle \right] q^2 \ln(-q^2/\mu^2) \\
 & - \frac{\pi d_s}{18} \langle \bar{\psi} \psi \rangle^2 + \left[\frac{1}{128\pi^2} \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle + \right.
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{5d_s}{192\pi} \langle m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \Big] \ln(-q^2/\mu^2) \\
 & + \frac{37\pi^2}{3456} \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle^2 \frac{1}{q^2} \ln(-q^2/\mu^2) \\
 & + \left[-\frac{\pi^2}{12} \langle m \bar{\psi} \psi \rangle \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle - \frac{107\pi d_s}{864} \langle \bar{\psi} \psi \rangle \langle g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \right] \frac{1}{q^2}
 \end{aligned} \tag{3.9}$$

$$\begin{aligned}
 \Pi_V^-(q^2) = & -\frac{d_s}{480\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[-\frac{d_s}{9\pi} \langle m \bar{\psi} \psi \rangle - \frac{1}{72} \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle \right] q^2 \ln(-q^2/\mu^2) \\
 & + \frac{\pi d_s}{36} \langle \bar{\psi} \psi \rangle^2 + \left[-\frac{1}{128\pi^2} \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle + \right. \\
 & - \frac{13d_s}{192\pi} \langle m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \Big] \ln(-q^2/\mu^2) \\
 & - \frac{\pi^2}{324} \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle^2 \frac{1}{q^2} \ln(-q^2/\mu^2) \\
 & + \left[-\frac{\pi^2}{9} \langle m \bar{\psi} \psi \rangle \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle - \frac{203\pi d_s}{2592} \langle \bar{\psi} \psi \rangle \langle g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \right] \frac{1}{q^2} \tag{3.10}
 \end{aligned}$$

$$\Pi_S^-(q^2) = -\frac{d_s}{960\pi^3} (q^2)^3 \ln(-q^2/\mu^2) + \left[-\frac{d_s}{12\pi} \langle m \bar{\psi} \psi \rangle + \frac{1}{48} \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle \right] q^2 \ln(-q^2/\mu^2)$$

$$\begin{aligned}
& + \frac{\pi d_s}{12} \langle \bar{\psi} \psi \rangle^2 + \left[\frac{1}{128 \pi^2} \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle + \right. \\
& \quad \left. + \frac{13 d_s}{192 \pi} \langle m g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \right] \ln(-q^2/\mu^2) \\
& + \frac{37 \pi^2}{3456} \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle^2 \frac{1}{q^2} \ln(-q^2/\mu^2) \\
& + \left[\frac{\pi^2}{12} \langle m \bar{\psi} \psi \rangle \left\langle \frac{d_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} \right\rangle + \frac{7 \pi d_s}{864} \langle \bar{\psi} \psi \rangle \langle g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle \right] \frac{1}{q^2}
\end{aligned}$$

(3.11)

Let us remark that the vacuum dominance approximation implies flavour-independent results (the two-point functions are the same for $n = 0$ and $n = 3$) which means that the $I = 0$ and $I = 1$ states are degenerate.

For convenience, we will write in the following Eqs. (3.8) to (3.11) under the general form:

$$\begin{aligned}
\Pi(q^2) = & -A (q^2)^3 \ln(-q^2/\mu^2) - B q^2 \ln(-q^2/\mu^2) + \tilde{B} \\
& - C \ln(-q^2/\mu^2) - D \frac{1}{q^2} \ln(-q^2/\mu^2) - E \frac{1}{q^2}
\end{aligned}$$

(3.12)

This defines the coefficients A through E in each case.

4. - THE QCD SUM RULES AND THEIR PREDICTIONS

For the spectral functions $\Pi(q^2)$, dispersion relations can be written down, with perhaps a certain number of unknown subtraction terms:

$$\text{Re } \Pi(q^2) = \frac{1}{\pi} \int_0^{\infty} ds \left(\frac{q^2}{s} \right)^n \frac{\text{Im } \Pi(s)}{s - q^2} + \sum_{k=0}^{n-1} a_k (q^2)^k \quad (4.1)$$

These subtractions can be removed by taking the appropriate number of derivatives of $\Pi(q^2)$. The higher the derivative the bigger the relative contribution of the lowest-lying resonance to the dispersion integral and the further one moves away from the asymptotically free region, which can be compensated by choosing a larger $(-q^2)$. For light quark systems, one usually considers the limiting procedure for that method, or the Borel transformation L_M of Eq. (4.1) defined as¹⁾:

$$L_M = \lim_{\substack{-q^2, n \rightarrow \infty \\ -q^2/n = M^2}} \frac{1}{(n-1)!} (q^2)^n \left(-\frac{d}{dq^2} \right)^n \quad (4.2)$$

Applying L_M to Eq. (4.1) we get:

$$L_M \text{Re } \Pi(q^2) = \frac{1}{\pi M^2} \int_0^{\infty} ds e^{-s/M^2} \text{Im } \Pi(s) \quad (4.3)$$

By using in the left-hand side of that relation our results of the last section, and in the right-hand side some simple model to saturate $\text{Im}\Pi(s)$ by one narrow resonance and a continuum with threshold s_0 in form of a θ -function, we obtain QCD sum rules from which predictions for masses and couplings can be made. More precisely, we take for $\text{Im}\Pi(s)$:

$$\text{Im } \Pi(s) = \pi g_R^2(m_R^2)^4 \delta(s - m_R^2) + \pi(A s^3 + B s) \theta(s - s_0) \quad (4.4)$$

m_R is the mass of the resonance, g_R its coupling to the vacuum (defined by the matrix element $\langle 0 | J_\mu(0) | q\bar{q}g \rangle$), and A, B are given by Eq. (3.12).

We then obtain the following QCD sum rules for the $q\bar{q}g$ hybrid states considered in this paper^{*}:

$$g_R^2(m_R^2)^4 e^{-m_R^2/M^2} = (M^2)^4 \left\{ 6A \left[1 - e^{-r} \left(1 + r + \frac{1}{2}r^2 + \frac{1}{6}r^3 \right) \right] + B \frac{1}{(M^2)^2} \left[1 - e^{-r} (1 + r) \right] + C \frac{1}{(M^2)^3} + \left[D \left(\ln \frac{M^2}{\mu^2} - \gamma \right) + E \right] \frac{1}{(M^2)^4} \right\} \quad (4.5)$$

with $r = s_0/M^2$.

In order to extract the physical information contained in (4.5), we made a least-square best fit of the right-hand side by the function $g_R^2(m_R^2)^4 e^{-m_R^2/M^2}$ of M^2 with g_R^2 , m_R^2 as free parameters. This is done for different values of s_0 on a reasonable interval of values of M^2 , so that neither the contributions of the higher power corrections nor those of the continuum are too big.

The value of s_0 for which the mean relative deviation from the equality (4.5) is the smallest is considered as the best value for s_0 , and g_R^2 , m_R^2 are then known from the best fit.

^{*}) One has: $L_M(q^2)^n \ln(-\alpha^2 q^2) = -n! (M^2)^n$ $L_M\left(\frac{1}{q^2}\right)^n = \frac{(-1)^n}{(n-1)!} \left(\frac{1}{M^2}\right)^n$
 $L_M\left(\frac{1}{q^2}\right)^n \ln -\alpha^2 q^2 = \frac{(-1)^n}{(n-1)!} \left(\frac{1}{M^2}\right)^n \left[\ln(\alpha^2 M^2) + \left(1 + \frac{1}{2} + \dots + \frac{1}{n-1} - \gamma\right) \right]$

where γ is Euler's constant ($\gamma = 0.5772157$).

then known from the best fit.

Our results are given in Table 1. We used:

$$\begin{aligned}
 \Lambda &= 100 \text{ Mev} & m_u &= m_d = m = 5.5 \text{ Mev} \\
 \langle \bar{\psi} \psi \rangle &= \langle \bar{u} u + \bar{d} d \rangle = 2 (-.25 \text{ Gev})^3 \\
 \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\alpha}^{\mu\nu} \rangle &= (0.33 \text{ Gev})^4 \\
 \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle &= (0.60 \text{ Gev})^6 \\
 \langle g G_{\mu\nu}^a \bar{\psi} \sigma^{\mu\nu} T^a \psi \rangle &= m_0^2 \langle \bar{\psi} \psi \rangle \text{ with } m_0^2 = 0.5 \text{ to } 1.0 \text{ Gev}^2 \text{ (17)} \\
 \mu &= 700 \text{ Mev}
 \end{aligned}
 \tag{4.6}$$

Taking $m = 0$, varying m_0^2 from 0.5 to 1.0 GeV^2 or changing $\langle G^3 \rangle$ by 20% leaves these results essentially unchanged. Their dependence on μ and Λ is weak, especially for the 1^{++} state, as can be seen from Table 2.

Taking the ratio of the derivative of (4.5) with respect to $(-1/M^2)$, and (4.5), we obtain:

$$m_R^2 = M^2 \frac{\mathcal{N}}{\mathcal{D}}$$

(4.7a)

where

$$\begin{aligned} \mathcal{N} = & 24 A \left[1 - e^{-t} \left(1 + t + \frac{1}{2} t^2 + \frac{1}{6} t^3 + \frac{1}{24} t^4 \right) \right] \\ & + 2 B \frac{1}{(M^2)^2} \left[1 - e^{-t} \left(1 + t + \frac{1}{2} t^2 \right) \right] \\ & + \frac{C}{(M^2)^3} + \frac{D}{(M^2)^4} \end{aligned} \quad (4.7b)$$

and $\mathcal{D}$ being the term in brackets which is multiplied by $(M^2)^4$ in Eq. (4.5).

This defines another way to predict masses (and couplings) provided the sum rule (4.5) and its derivative are reliable in a wide enough interval of M^2 values.

In the case of the scalar states, it is found that the ratio (4.7a) as a function of M^2 stabilizes for some definite value of s_0 and the predictions obtained from it are identical to those given in Table 1. For example, in Fig. 2, we show the situation for the 0^{--} state.

On the other hand, for the vector states, the ratio (4.7a) does not stabilize for any value of s_0 , as is shown in Fig. 3 for the 1^{+-} state.

This means that the sum rule obtained as the derivative of (4.5) is not reliable for these states. Several explanations can be offered for this fact: the most likely one is that higher dimensional operators are needed for Eq. (4.7) to stabilize. We believe, however, that within the framework of the QCD sum rules, the best fit values to Eq. (4.5) do provide a reliable approximation to the physical situation. We have checked that this is indeed the case in various channels such as 1^{--} states (the ρ meson) or 1^{+-} states (the B meson) where a best fit to the analogue of Eq. (4.5) does lead to a correct mass prediction without using the analogue of Eq. (4.7).⁽⁶⁾

The results given in Table 1 are expected to hold with a precision of the order of 10% for the masses and 20% for the couplings since this is typically the case for QCD sum rule predictions in the non-exotic light quark hadronic spectrum.

As mentioned in the Introduction, our prediction for the mass of the 1^{+-} state disagrees with the result of Balitsky et al.¹²⁾: the discrepancy can be

traced back to the different Wilson coefficients that we obtain for the operators of dimension 6 and 8 in the OPE.

Predictions for the 1^{-+} hybrid states have also been made using the bag model^{18),19)}: they range from 1.4 GeV to 2.2 GeV. We believe of course that the QCD sum rule approach is much more reliable than the bag model. Nevertheless, it is rather remarkable that both approaches predict fairly light vector hybrid states. Let us also remark that our result completely disagrees with a recent calculation of Cornwall and Tuan²⁰⁾.

As far as the 1^{+-} hybrid states are concerned, the bag model predicts them around 2 GeV¹⁹⁾. A definite prediction based on the QCD sum rules should await a detailed study of mixing effects of these states with "normal" $q\bar{q}$ mesons with the same quantum numbers such as the B(1235).

A rather unexpected conclusion of our best fit to the QCD sum rules for hybrid states is thus that the scalar states are predicted to lie extremely high, making them essentially unobservable, while the vector states turn out to be surprisingly light and hence should be accessible experimentally.

5. - CONCLUSION

In this paper, we have used the QCD sum rule technique to make predictions for the masses and couplings of hybrid $q\bar{q}g$ mesons with $J^{PC} = 1^{-+}$, 1^{+-} , 0^{++} and 0^{--} containing u and d quarks.

We first presented a general background field method useful in the calculation of operator product expansions for QCD sum rules.

Using that method, we computed the OPE for two-point functions of operators coupling to the hybrid states considered here. From these results, we made predictions for the masses and couplings of these states; they are given in Table 1.

The masses of the scalar states are heavy, much heavier than those of the vector states which seem to be surprisingly light when compared to other calculations.

Our most interesting result is perhaps the prediction of the 1^{-+} state at 1.3 GeV, since this should be experimentally accessible. As pointed out in Refs. 12) and 19), these states could be produced in the decay of heavy quarkonia $1^{--} \rightarrow \gamma + \text{anything}$ and in $p\bar{p}$ collisions, and, because of their low mass, the dominant two-body decay modes of the $I = 1$ states are $\pi\eta$, $\pi\eta'$ and $\pi\rho$, while the $I = 0$ state is kinematically stable for two-body decays into ordinary mesons. We intend to come back to that important question in the near future.

ACKNOWLEDGEMENTS

The authors would like to thank Professor J. Pestieau, Dr. L.J. Reinders and Professor H. Rubinstein for useful discussions. The work of J.G. was supported in part by the Robert A. Welch Foundation and a NSF grant PHY83-04629.

	$m_R^2(\text{GeV})^2$	$m_R(\text{GeV})$	g_R^2	g_R	$s_o(\text{GeV})^2$	$\sqrt{s_o}(\text{GeV})$
1^{-+}	1.7	1.3	1.2×10^{-4}	1.1×10^{-2}	2.5	1.6
1^{+-}	1.0	1.0	5.6×10^{-4}	2.4×10^{-2}	2.0	1.4
0^{++}	11.4	3.4	5.3×10^{-6}	2.3×10^{-3}	14	3.7
0^{--}	11.5	3.4	5.1×10^{-6}	2.3×10^{-3}	14	3.7

Table 1 - QCD sum rule predictions for $q\bar{q}g$ hybrid mesons.

	$\mu(\text{MeV})$	$\Lambda(\text{MeV})$	$m_R(\text{GeV})$	g_R	$s_o(\text{GeV}^2)$
1^{-+}	300	100	1.29	1.19×10^{-2}	2.5
	500	100	1.28	1.15×10^{-2}	2.5
	700	100	1.28	1.13×10^{-2}	2.5
	1000	100	1.28	1.11×10^{-2}	2.5
	700	200	1.29	1.14×10^{-2}	2.5
1^{+-}	300	100	0.94	3.5×10^{-2}	2.0
	500	100	0.99	2.7×10^{-2}	2.0
	700	100	1.02	2.4×10^{-2}	2.0
	1000	100	1.04	2.1×10^{-2}	2.0
	700	200	0.97	3.1×10^{-2}	2.0

Table 2 - Dependence of our predictions on μ and Λ .

APPENDIX A

Here, we illustrate the use of the background field method in the simple case of the vector operator $J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ (for one quark flavour). The two-point function is:

$$i \int d^4x e^{iqx} \langle 0 | T J_\mu(x) J_\nu(0) | 0 \rangle = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi(q^2) \quad (\text{A.1})$$

so that

$$\Pi(q^2) = -\frac{1}{3q^2} i \int d^4x e^{iqx} \langle 0 | T J_\mu(x) J^\mu(0) | 0 \rangle \quad (\text{A.2})$$

By doing the substitution (2.7) in $J_\mu(x)$, it is clear that the Wilson coefficient of the operator $\mathbb{1}$ is given by:

$$\begin{aligned} i \int d^4x e^{iqx} \overline{\eta(x)} \not{x}_\mu \overline{\eta(x)} \not{x}_\nu \eta(0) \not{x}^\nu \eta(0) &= \\ &= -i \int d^4x e^{-iqx} \text{Tr} [S_F(x,0) \not{x}^\mu S_F(0,x) \not{x}_\mu] \end{aligned} \quad (\text{A.3})$$

where the quark propagator is taken at zeroth order in the background fields, and the trace is understood to be over Dirac and colour indices.

By Eq. (2.12a), this reduces easily to

$$\frac{6i}{\pi^4} \int d^4x e^{iqx} \frac{x^2}{[x^2 - i\varepsilon]^4} \quad (\text{A.4})$$

and through Eqs. (B.2) given in Appendix B, this is simply:

$$\frac{3}{4\pi^2} q^2 \ln(-a^2 q^2) \quad (\text{A.5})$$

so that the contribution of the operator $\mathbb{1}$ to the OPE of (A.1) is

$$\overline{\Pi}(q^2)_{\mathbb{1}} \underset{q^2 \rightarrow -\infty}{=} -\frac{1}{4\pi^2} \ln(-a^2 q^2) \quad (\text{A.6})$$

Let us now consider the operator $\bar{\psi}\psi$; the corresponding Wilson coefficient is proportional to m (by dimensional considerations) and will be obtained from the following terms:

$$\begin{aligned} & i \int d^4x e^{iqx} \bar{\psi}(x) \gamma_\mu \overline{\eta(x)\eta(0)} \gamma^\mu \psi(0) \\ & + i \int d^4x e^{iqx} \overline{\eta(x) \gamma_\mu \psi(x) \bar{\psi}(0) \gamma^\mu \eta(0)} \end{aligned} \quad (\text{A.7})$$

where the background fields $\psi(x)$ have to be expanded at $x = 0$. Therefore, the terms in $m\bar{\psi}(0)\psi(0)$ are obtained from the term in m in $S_F(x,y)$ and the term in $\partial_\alpha \psi(0)$ (together with the equations of motion) in that expansion.

The first contribution is:

$$\begin{aligned} & i \int d^4x e^{iqx} \bar{\psi}(0) \gamma_\mu \left(\frac{-m}{4\pi^2} \right) \frac{1}{x^2 - i\varepsilon} \gamma^\mu \psi(0) + i \int d^4x e^{iqx} \bar{\psi}(0) \gamma_\mu \left(\frac{-m}{4\pi^2} \right) \frac{1}{x^2 - i\varepsilon} \psi(0) = \\ & = -\frac{8m}{q^2} \bar{\psi}(0) \psi(0) \end{aligned} \quad (\text{A.8})$$

and the second:

$$\begin{aligned}
 & i \int d^4x e^{iqx} \partial_\rho \bar{\psi}(0) x^\rho \gamma_\mu \left(\frac{i}{2\pi^2} \right) \frac{x^\mu}{[x^2 - i\varepsilon]^2} \gamma^\mu \psi(0) + i \int d^4x e^{iqx} \bar{\psi}(0) \gamma_\mu \left(\frac{i}{2\pi^2} \right) \frac{-x^\mu}{[x^2 - i\varepsilon]^2} \gamma^\mu x^\rho \partial_\rho \psi(0) \\
 &= \frac{1}{\pi^2} \left[\partial_\rho \bar{\psi}(0) \gamma_\rho \psi(0) - \bar{\psi}(0) \gamma_\rho \partial_\rho \psi(0) \right] \int d^4x e^{iqx} \frac{x^\rho x^\rho}{[x^2 - i\varepsilon]^2} \\
 &= \frac{1}{\pi^2} \left[\partial_\rho \bar{\psi}(0) \gamma_\rho \psi(0) - \bar{\psi}(0) \gamma_\rho \partial_\rho \psi(0) \right] (-i)^2 \frac{\partial}{\partial q_\rho} \frac{\partial}{\partial q_\rho} \int d^4x e^{iqx} \frac{1}{[x^2 - i\varepsilon]^2} \\
 &= \frac{-2i}{(q^2)^2} \left[\partial_\rho \bar{\psi}(0) \gamma_\rho \psi(0) - \bar{\psi}(0) \gamma_\rho \partial_\rho \psi(0) \right] (q^\rho q^\rho - 2 q^\rho q^\rho)
 \end{aligned}$$

(A.9)

which, by symmetrizing over q (since we consider only the limit $q^2 \rightarrow -\infty$) and using Eqs. (2.2), reduces to

$$\frac{2m}{q^2} \bar{\psi}(0) \psi(0)$$

(A.10)

The total contribution of $\bar{\psi}\psi$ to $\Pi(q^2)$ is therefore:

$$\Pi(q^2) / \bar{\psi}\psi \stackrel{q^2 \rightarrow -\infty}{=} \frac{2m}{(q^2)^2} \bar{\psi}(0) \psi(0)$$

(A.11)

The Wilson coefficients for all the other operators can be obtained following the same procedure (for the coefficient of $G_{\mu\nu}^a G_a^{\mu\nu}$, see for example Ref. 13)), but terms in the background fields in $S_F(x,y)$ will now contribute (for example, using the equations of motion, the terms in (DG) give the coefficient of $(\bar{\psi} \gamma_\mu T_a \psi)(\bar{\Sigma} \bar{q} \gamma^\mu T^a q)$).

The results do of course agree with those of SVZ¹⁾.

APPENDIX B

From the propagator of a scalar particle, we have the relations²¹⁾:

$$\begin{aligned} \int \frac{d^4 x}{(2\pi)^4} \frac{e^{iqx}}{x^2 - a^2 \pm i\varepsilon} &= -\frac{1}{4\pi^2} \delta(q^2) \\ &+ \frac{a^2}{8\pi} \theta(q^2) \left[\frac{J_1(a\sqrt{q^2})}{a\sqrt{q^2}} \mp i \frac{N_1(a\sqrt{q^2})}{a\sqrt{q^2}} \right] \\ &\mp i \frac{a^2}{4\pi^2} \theta(-q^2) \frac{K_1(a\sqrt{-q^2})}{a\sqrt{-q^2}} \end{aligned} \quad (\text{B.1})$$

Taking derivatives of these identities with respect to a^2 , and expanding the Bessel functions at $(a^2 = 0)$ with $q^2 \neq 0$, it is possible to show that the leading short-distance singularities are given by:

$$\int d^4 x e^{iqx} \frac{1}{[x^2 - a^2 \pm i\varepsilon]} \underset{\substack{q^2 < 0 \\ a^2 \rightarrow 0}}{=} (\mp i)(2\pi)^2 \frac{1}{-q^2 \pm i\varepsilon} \quad (\text{B.2a})$$

$$\int d^4 x e^{iqx} \frac{1}{[x^2 - a^2 \pm i\varepsilon]^{n+1}} \underset{\substack{q^2 < 0 \\ a^2 \rightarrow 0 \\ n \geq 1}}{=} (\mp i)(2\pi)^2 \frac{(-q^2)^{n-1}}{2^{2n} n! (n-1)!} \ln(-a^2 q^2) \quad (\text{B.2b})$$

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FIGURE CAPTIONS

- Fig. 1 Diagrams contributing at lower order in α_s , in the limit ($q^2 \rightarrow -\infty$), to the OPE with operators of dimension 0 to 8.
- Fig. 2 a) The ratio (4.7a) for the 0^{--} state. The number next to each curve is the corresponding value of s_0 . All numbers are in GeV or GeV².
b) The ratio (4.7a) for the 0^{++} state.
- Fig. 3 The ratio (4.7a) for the 1^{--} state. The number next to each curve is the corresponding value of s_0 . All numbers are in GeV or GeV².

Fig.1

- fig. 2a. -

- fig. 2b. -

Fig.3