MONTE CARLO RENORMALIZATION GROUP STUDY
OF ϕ^4 FIELD THEORY

David J. E. Callaway

and

Roberto Petronzio

CERN -- Geneva

A B S T R A C T

A previously proposed general method for evaluating block renormalized coupling constants within the framework of the Monte Carlo renormalization group (MCRG) is applied to ϕ^4 field theory. The flow diagrams, fixed points, and critical exponents are determined in two, three, and four dimensions. Results in four dimensions are consistent with the idea that ϕ^4 field theory is trivial (non-interacting) in the continuum limit. The possibility of using MCRG techniques to ascertain whether a general non-asymptotically free theory is trivial or not is also discussed.

1 - PROLEGOMENA

The concept of particle mass generation by spontaneous symmetry breaking [1] has become one of the most useful ideas in field theory. Typically this symmetry breaking is produced by the introduction of a scalar field. The new field supposedly interacts with itself in such a fashion (via a ϕ^4 coupling) as to generate a ground state which lacks a symmetry of the Lagrangian. At the classical level, gauge particles propagating in the asymmetric vacuum may then appear to be massive, while the underlying theory remains renormalizable. It is generally assumed that no significant changes occur in this picture due to quantum effects such as renormalization.

Such an assumption may not be correct, however. A large body of analytical [2] and numerical [3] evidence has been presented which indicates that pure ϕ^4 field theory is "trivial", i.e., non-interacting in the infinite cut-off limit in four spacetime dimensions. This triviality seems to persist for all values of the bare coupling constant, and presumably therefore precludes the existence of spontaneous symmetry breaking in pure ϕ^4 field theory.

Clearly it is necessary to understand the phenomenon of triviality in detail. In particular, a non-trivial theory is a necessary precursor for spontaneous symmetry breaking by elementary scalar particles. Additionally, if pure ϕ^4 field theory is trivial but becomes non-trivial by the addition of gauge fields [as with, e.g., the $O(N)$ gauge model in the large N limit], phenomenological constraints on the Higgs mass may result [4,5].

At the present time such a demonstration of non-triviality can be performed only in perturbation theory, which typically limits the analysis to asymptotically free theories. Since realistic theories are not necessarily asymptotically free, a more general method for determining whether or not a theory is trivial must be found.

The Monte Carlo renormalization group (or MCRG) is a natural candidate for such an analysis. By dealing directly with the scaling properties of the various fields and operators under consideration, it allows an economical (albeit numerical) formulation of the problem. Additionally, by restricting the analysis to a few carefully chosen operators, the MCRG admits of an accurate and systematically improvable approximation method. As is shown below, the combination of numerical analysis with judicious approximation can lead to techniques of some utility in the non-perturbative analysis of quantum field theory.

Here the MCRG is applied to pure ϕ^4 field theory on a hypercubical lattice in four (Euclidean) dimensions. No additional non-Gaussian fixed points of the recursion relation are found, consistent with the concept that the theory is non-interacting or "trivial". For completeness, ϕ^4 field theory is also studied in two and three dimensions. In all cases the flow diagram, fixed points, and critical exponents are determined.

2 - DESCRIPTION OF THE PROCEDURE

The Monte Carlo renormalization group [6] is a combination of the principles of Monte Carlo simulation [7] with those of the real-space renormalization group [8]. The system under consideration is divided into "blocks" and a set of block variables is calculated by averaging in some fashion over the site variables. This new system is governed by a block renormalized action in the same way that the site system is governed by a site action. By studying the way in which the site action "flows" into the block renormalized action the critical properties of the system can be determined. Our general procedure for such a study has been presented previously [9]; this method is reviewed and applied to ϕ^4 theory below.

Consider a system of site variables $\{\phi\}$ governed by a site action $A\{\phi\}$. A block renormalized action $A'\{\phi'\}$ is determined from the site action by the use of an appropriate projection operator $P[\{\phi'\},\{\phi\}]$:

$$\exp[-A'\{\phi'\}] = \text{Tr}_{\{\phi\}} P[\{\phi'\}, \{\phi\}] \cdot \exp[-A\{\phi\}] \quad (1)$$

where the trace (or functional integral) is only over the site variables $\{\phi\}$. The requirement that the renormalization group transformation preserve the partition function of the system imposes the constraint

$$\text{Tr}_{\{\phi'\}} P[\{\phi'\}, \{\phi\}] = 1 \quad (2)$$

The projection operator is otherwise arbitrary.

The site action $A\{\phi\}$ for ϕ^4 theory is parameterized in the form

$$A\{\phi\} = T\{\phi\} + \sum_i V(\phi_i) \quad (3a)$$

$$T\{\phi\} = -K \sum_{\langle ij \rangle} \phi_i \phi_j \quad (3b)$$

$$V(\phi_i) = \lambda (\phi_i^2 - f)^2 \quad (3c)$$

where $\langle ij \rangle$ denotes a sum over all nearest neighbor pairs in the lattice. Each of the (real) $\{\phi\}$ range from negative to positive infinity, and in the limit of large and positive λ the theory approaches the Ising model. [Note that if each of the $\{\phi\}$ is rescaled by a constant factor, the Jacobian matrix in the path integral adds only a constant term to the action. Thus there is a one-parameter scale invariance to the theory].

For the sake of simplicity the projection operator is usually taken to be a product of delta functions,

$$P[\{\phi'\}, \{\phi\}] = \prod_i \delta[\phi'_i - g_i\{\phi\}] \quad (4)$$

where the $g_i\{\phi\}$ are some appropriately chosen functions of the $\{\phi\}$. However we find it very advantageous to use instead a "smeared" projection operator,

$$P[\{\phi'\}, \{\phi\}] = N(\beta) \cdot \prod_i \exp[-\beta (\phi'_i - g_i\{\phi\})^2] \quad (5)$$

where $N(\beta)$ is a normalization factor which depends only on β . With the use of the projection operator of Eq. (5) it is easy to do a simulation while holding some of the $\{\phi'\}$ fixed. It is more difficult to fix some of the $\{\phi'\}$ when the projection operator of Eq. (4) is used instead because of the problems associated with inverting a delta function. For sufficiently large β , the two projection operators are of course equivalent.

It is worth emphasizing the utility of the choice Eq. (5). The advantage of this "smeared" projection operator is that it allows each of the variables $\{\phi'\}$ separately either to be generated [by first producing a configuration $\{\phi\}$ and then generating one or more of the $\{\phi'\}$ using Eq. (5)] or to be fixed beforehand [in which case the appropriate term(s) in the exponential Eq. (5) are treated as part of the site action $A\{\phi\}$ when the $\{\phi\}$ are generated]. Some of the block variables can be fixed in this fashion while others are allowed to fluctuate.

As regards the choice of $g_i\{\phi\}$, good results can be obtained with the choice,

$$g_i\{\varphi\} = \text{sign}(\sum \varphi_i) [b^{-d} \sum_i \varphi_i^2]^{1/2} \quad (6a)$$

which reduces to the well-known (and highly successful) "majority rule" [8] projection operator in the Ising limit. Here,

$$\text{Sign}(x) = 1 \quad (x \geq 0) \quad (6b)$$

$$\text{Sign}(x) = -1 \quad (x < 0)$$

while the sum in Eq. (6a) is over all b^d site fields in a given hypercubical block of side length b in d dimensions.

With the above description in mind, the prescription for calculating the block renormalized couplings is straightforward. First all of the block variables $\{\phi'\}$ but one (denoted by ϕ_0') are fixed in a fashion prescribed below. The remaining variable is allowed to fluctuate with a distribution given by Eq. (5).

Both the site and block actions can be expanded in terms of appropriate basis functions $\{S\}$ and coupling constants $\{K\}$ and $\{K'\}$ respectively as follows:

$$A'\{\varphi'\} = \sum_{\alpha} k'_{\alpha} S_{\alpha}\{\varphi'\} \quad (7a)$$

$$A\{\varphi\} = \sum_{\alpha} k_{\alpha} S_{\alpha}\{\varphi\} \quad (7b)$$

The $\{\phi'_i, i \neq 0\}$ are fixed in such a fashion that all but a small number of the $\{S\}$ (e.g. one, denoted by S_0) which involve ϕ_0' are zero in the block system. Then the probability that the lone fluctuating variable assumes the value ϕ_0' is proportional to

$$\mathcal{P}(\phi_0') \propto \exp[k'_0 S_0\{\varphi'\}] \quad (8)$$

which only depends upon the chosen coupling K_0' and the known function $S_0\{\phi'\}$. It is thus a trivial matter to determine K_0' by measuring the function $\rho(\phi_0')$. In this fashion all of the couplings of the theory can be evaluated systematically.

It is of course not always possible to choose a single block configuration in which all of the $\{S\}$ but the chosen one are zero. However, by suitable combinations of the results from several block configurations it is possible to separate the couplings. Additionally, in practice only a small number of couplings are large, and it is often possible to separate these with a single block configuration.

The block renormalized action $A'\{\phi'\}$ is in general an arbitrarily complicated function of the $\{\phi'\}$, while clearly in practice it is possible to determine only a limited number of couplings in the theory. Thus a systematic approximation scheme is required in which the remaining undetermined couplings are parameterized in an effective fashion. The approximation scheme used here is essentially a generalization of the one used in Ref. 9. The kinetic $T'\{\phi'\}$ and potential $V'\{\phi'\}$ terms in the block renormalized action are parameterized in terms of three effective couplings, the object being to express $A'\{\phi'\}$ in the form,

$$A'\{\phi'\} = \sum_{\langle ij \rangle} T'(\phi_i', \phi_j') + \sum_i V'(\phi_i') \quad (9a)$$

with

$$T'(\phi_i', \phi_j') = -K' \phi_i' \phi_j' \quad (9b)$$

$$V'(\phi_i') = \lambda' (\phi_i'^2 - f')^2 \quad (9c)$$

The parametrization eq.(9a) in terms of only the three constants appearing in eqs. (9b) and (9c) is an approximation. However, with the projection operator Eq.(6), the probability distribution for the renormalized system is sharply peaked around two maxima. The effective couplings K', λ', f' are determined by requiring a good description of the distribution around these maxima (i.e. for the most likely configurations) and are therefore expected to provide reliable information about the structure of the block renormalized action.

First, the block renormalized potential $V'(\phi'_0)$ is extracted. All of the $\{\phi'\}$ (excepting ϕ'_0) are set to a background pattern designed to cancel most of the interactions of ϕ'_0 with the other $\{\phi'\}$. The magnitude of each of the $\{\phi'\}$ is fixed to its most probable value $\sqrt{f'}$ (since f' is always positive whenever the projection operator Eqs.(6) is used). This fact is no surprise, for if a ϕ' is zero, all the $\{\phi\}$ in its block also must equal zero. Such a configuration is very unlikely to occur, and certainly cannot be the most probable configuration as is needed if f' were to be less than or equal to zero. In fact, since the block renormalized distribution is sharply peaked about its maximum, f' (calculated by means described below) equals f to high accuracy. Because of the scale invariance mentioned above, no information is therefore lost by setting f (and thus f') equal to unity. If f' had not so fortunately been equal to f , it could of course have been determined recursively. That is, fix a background configuration, determine f' , use the new f' to produce an improved background configuration, and so on.

Thus the magnitude of the background fields is fixed, and it remains to choose their sign. Several choices of background pattern are clearly possible. The one used here is a simple generalization of the pattern used in Ref. 9. The signs of the $\{\phi'\}$ (except as usual ϕ'_0) are chosen for the two-dimensional case as in the pattern Fig. 1. In each higher dimension every field in the pattern changes sign as the origin is crossed in the new direction. The structure of the two-dimensional pattern thus endures, although the overall sign changes.

In this fashion a large fraction of the multiple-field interactions involving ϕ'_0 are cancelled. Most of the remaining interactions [such as $\phi'_0{}^2\phi'_i{}^2$] are even in ϕ'_0 and are thus included in an effective fashion. The sole fluctuating variable ϕ'_0 is measured, and the block renormalized potential $V'(\phi'_0)$ is determined from the binned distribution $\rho(\phi'_0)$ by use of the relation,

$$V'(\phi'_0) = -\ln [\rho(\phi'_0)] + \text{constant} \quad (10)$$

In Eq. (10), the constant is determined by requiring $V'(\phi'_0)$ to equal zero at its minimum.

The reader may wonder why the potential is not extracted by setting all of the $\{\phi'\}$ (except of course ϕ'_0) to zero. In that case, all

of the interactions of ϕ'_0 with other fields vanish, and only $V'(\phi'_0)$ remains. There are two problems with this approach. First, with the form of $g_i\{\phi\}$ given by Eqs. (7), setting a ϕ' to zero also typically forces all the $\{\phi\}$ in its block to zero, which magnifies problems of finite size and convergence. Secondly, as stated above, setting all the other $\{\phi'\}$ to zero implies that information about terms like $\phi'^2_0\phi'^2_i$ is lost.

If the potential $V'(\phi'_0)$ were exactly given by

$$V'(\phi'_0) = \lambda' (\phi'^2_0 - f')^2 \quad (11)$$

then the coupling constants λ' and f' would be determined by the moment equations,

$$f' = \frac{\langle \phi'^6_0 \rangle - 3 \langle \phi'^4_0 \rangle \langle \phi'^2_0 \rangle}{\langle \phi'^4_0 \rangle - 3 \langle \phi'^2_0 \rangle^2} \quad (12a)$$

$$\lambda' = \frac{1}{4} \frac{1}{\langle \phi'^4_0 \rangle - f' \langle \phi'^2_0 \rangle} \quad (12b)$$

More generally, effective parameters f' and λ' can be defined via Eqs. (12) and the resulting "phenomenological" potential Eq. (11) can be compared with the "exact" potential given by Eq. (10). As can be seen in Fig. 2, the agreement is excellent for small λ and, as expected, improves as λ increases. Therefore, rather than the more difficult chore of studying the block renormalization of a function $V(\phi)$ [as was done in a different approximation scheme in Ref. 10] it is possible to study just the renormalization group trajectories of λ and f . It should be pointed out that in the new formalism it is also possible to study the renormalization of $V(\phi)$ directly, simply by using the V' generated by Eq. (11) to generate a distribution in ϕ by histogram. We in fact tried this, using a V to generate a V' which was then used as input to the next round.

It remains to calculate the effective kinetic term T' in the block renormalized action. The $\{\phi'\}$ surrounding ϕ'_0 are set to a background pattern, formed by taking the same background pattern as in the calculation of V' except for the 2^d nearest neighbors, which are all set

to $\sqrt{f'}$. The number of times p_+ [p_-] that ϕ'_0 is positive [negative] is measured, and an effective two-field coupling K' is determined by,

$$K' = 2^{-(d+1)} \ln \left[\frac{p_+}{p_-} \right] \quad (13)$$

(note that $f=f'=1$).

Numerically such a definition is equivalent to the result obtained by taking p_+ and p_- to be the higher and lower of the two relative minima of the potential when the 2^d nearest neighbors are arranged in the prescribed fashion. This latter definition is in turn equivalent to taking the leading term in the expansion of the kinetic term about the minima of V' . Both definitions approach the two-spin nearest neighbor coupling in the Ising limit.

Of course, the precise determination of the nearest neighbour coupling would require a careful disentangling of other couplings, e.g., the four field couplings. In this respect, the nearest neighbour coupling defined here is an effective one; this fact does not disturb the comparison of ϕ^q with the Ising model, provided the same background pattern is arranged for both cases.

Derivatives of the various block renormalized couplings are calculated directly by taking correlations of various block renormalized operators with site operators. The method allows each derivative to be evaluated separately, and is a generalization of the approach given in [9]. The derivatives of f' and λ' with respect to f, λ , and K are extracted by measuring the correlations arising from analytical differentiation of Eqs. (12). These derivatives are obtained by using the first background pattern. The derivatives of K' are found using the second pattern. The expectation value of the derivatives of $A\{\phi\}$ with respect to the chosen coupling is measured when ϕ'_0 is positive, and the expectation value when ϕ'_0 is negative is subtracted. This difference gives the derivative of K' with respect to the chosen coupling. The latter procedure was also applied in [9].

Given the renormalized couplings and their derivatives at some point, it is a simple matter to locate nearby fixed points $\{K^*\}$ in a recursive fashion by use of the first order Taylor series expansion,

$$K'_\alpha - K^*_\alpha = \frac{\partial K'_\alpha}{\partial K_\beta} (K_\beta - K^*_\beta) \quad (14)$$

which is valid near a fixed point. Therefore by using both the flows and derivatives it is possible in principle to locate rapidly all fixed points of a theory without sampling every point. It is also remarkable how rapidly the procedure Eq. (14) converges. The known values of $\{K\}$, $\{K'\}$, and their derivatives are inserted, and a new trial value of $\{K^*\}$ is found. Typically in one or two iterations the procedure is stable to within statistical accuracy.

At the fixed point $K'=K=K^*$ the critical exponents are calculated by diagonalizing the matrix of partial derivatives of the $\{K'\}$ with respect to the $\{K\}$. In the present case the (3x3) derivative matrix is essentially block diagonal; thus only the derivative of K' with respect to K is required to determine the thermal exponent y_T . In general, if λ_e is the largest eigenvalue of this matrix for all even-parity interactions, the thermal exponent is given by,

$$y_T = \ln \lambda_e / \ln b \quad (15)$$

where b is the linear scale factor of the transformation. For the results given here, b equals either two or three.

Lastly, the method used to update the lattice is presented. Periodic boundary conditions are used throughout. For values of λ less than or equal to unity, the standard Metropolis et. al. [11] method is used. When λ is larger than one, a heat bath routine is devised as follows. Note that the distribution

$$\exp [-\lambda (\phi^2 - f)^2] \quad (16)$$

is Gaussian in ϕ^2 . Gaussian random numbers can be generated by a closed form expression. This serendipity suggests the following procedure:

- Generate two uniformly distributed random numbers $0 < r_1 < 1$, $0 < r_2 < 1$. Use these to generate ϕ^2 with a Gaussian distribution using the standard formula:

$$\phi^2 = f + \cos(2\pi r_1) \sqrt{-(\ln r_2)/\lambda} \quad (17)$$

- Reject ϕ^2 if it is less than zero,
- Calculate ϕ by taking the square root of ϕ^2 and assigning the sign randomly,
- Apply the Metropolis et. al. method to the remaining parts of the action:

$$A_{\text{heat}} = T\{\phi\} + \frac{1}{2} \ln \phi^2 \quad (18)$$

using the ϕ generated in the above fashion for the tentative update. The extra logarithm in Eq. (18) comes from the Jacobian factor arising through the change of variables implicit in the first step above. The procedure above is equivalent to the standard method, but is far more efficient in the limit of large λ .

3 - RESULTS

The flow diagrams for ϕ^4 field theory in two, three, and four dimensions are structurally very similar. As discussed at length in section 2, f' is always positive and essentially equals f when both are larger than zero. When f is less than or equal to zero, the renormalization group transformation defined here behaves like a "prefacing transformation", that is, it maps the system into a new one with f positive. Because no fixed point with f negative or zero can therefore exist, and because of the scale invariance of the theory, no generality is lost by considering systems with f equal to one, as is done here. Thus only the flows in the (K, λ) plane are shown.

As can be seen in Fig.3, the structure in these three cases consists of an Ising ferromagnetic fixed point at $(K, \lambda) = (K^*, \infty)$ and a decoupling fixed point at $(K, \lambda) = (0, \infty)$. [In principle an antiferromagnetic fixed point (of no field-theoretic significance) at $(K, \lambda) = (-K^*, \infty)$ should also exist. This occurs because the Ising limit is symmetric under the simultaneous change of the sign of fields on one of the two nearest neighbour sublattices, and the change of the sign of K . However, the

projection operator eq.(6) is not symmetric under this interchange and so the fixed point symmetry is lost]. The ferromagnetic fixed point is repulsive and the $K=0$ fixed point attractive as expected. No additional fixed points were found in the region explored ($K < 0.2$ for $d=4$). Critical exponents for the ferromagnetic (Ising) fixed point are shown in the Table. The computer program was checked by comparison with results obtained by a direct simulation of the Ising model. Results are obtained over l iterations following an equilibration period of $l/10$ iterations (the value of l for each case is listed in the Table). By suitable combinations of different background patterns, the fixed point can be located in the multiparameter space of coupling constants and thus the critical exponents may be systematically improved.

4 - CONCLUSIONS

A general method proposed previously [9] for studying real space renormalization group flows was successfully applied to ϕ^4 field theory in four, three and two dimensions. The fixed point structure and flow diagrams were determined and the critical exponents calculated. The critical temperature and exponents agreed well with known results for the Ising model. In a wide region of coupling constant values no evidence for an additional non-Gaussian fixed point in four dimensions was seen, consistent [10] with the notion that the theory is "trivial" or non-interacting in the continuum limit.

It was also seen that reasonable results could be obtained with a rather small lattice and relatively few iterations. Accordingly, we are optimistic about our method and are presently extending the above analysis to gauge theories coupled to scalar fields (such as the Abelian Higgs model [12]). Such an analysis may be of some phenomenological importance, as discussed in the first section and in Refs. 4 and 5.

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TABLE

Ising limit critical exponents ($\lambda = 10^9$, $f = 1$)

Lattice length L	Scale factor b	Number of iterations I	K^*	y_T
<u>d = 2</u> $y_T = 1$ (exact), $K^* = 0.44$				
45	3	10 000	0.47	1.0
15	3	20 000	0.48	0.79
10	2	50 000	0.71	2.0
<u>d = 3</u> $y_T \cong 1.59$, $K^* = 0.22$				
15	3	500	0.25	1.8
10	2	4000	0.27	1.6
9	3	15 000	0.24	1.3
6	2	15 000	0.29	1.4
<u>d = 4</u> $y_T = 2$ (mean field), $K^* = 0.15$				
9	3	15 000	0.162	1.75
6	2	15 000	0.184	1.6

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FIGURE CAPTIONS

Fig. 1: Two-dimensional cross section of the first "background pattern". Each $+$ $[-]$ represents a ϕ' field set to $+\sqrt{f'}$ $[-\sqrt{f'}]$. The unconstrained field ϕ_0 is denoted by 0.

Fig. 2: Plot of the block renormalized potential $V'(\phi')$ calculated using Eq. (10) [circles] and the function $V'_{\text{mom}} = \lambda'(\phi'^2 - f')^2$. For V'_{mom} , λ' and f' are determined with the use of the moment equations, Eqs. (12). Error bars are purely statistical, and only two are shown for the purpose of illustration.

Fig. 3: Plot of the flow diagram for the four-, three- and two-dimensional Ising model in the (K, λ) plane.

Fig. 1

Fig. 2

Fig. 3