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NO-SCALE SUPERSYMMETRIC GUTs

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A B S T R A C T

We construct locally supersymmetric GUTs in which radiative corrections determine all the mass scales which are hierarchically smaller than the Planck mass: $m_{3/2} = O(m_W) = \exp(-O(1)/\alpha_t)m_P$, etc. Such no-scale GUTs are based on a hidden sector with a flat potential guaranteed by $SU(1,1)$ conformal invariance. This is extended to include observable chiral fields in an $SU(n,1)/SU(n) \times U(1)$ structure reminiscent of $N \geq 5$ extended supergravity theories. Tree-level supersymmetry breaking is present only for the gravitino, and for the light gaugino masses through non-minimal kinetic terms reminiscent of $N \geq 4$ extended supergravity theories. Radiative corrections generate squark and slepton masses which are phenomenologically acceptable, and the right value of m_W is obtained if $m_t \approx 50$ GeV in the simplest such model.

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1. INTRODUCTION

Most supergravity GUTs contain interaction scales (m_W , $m_{3/2}$; m_X ?) which are fixed by hand¹⁾ to be hierarchically smaller than m_P , although scenarios have been proposed²⁻⁵⁾ in which m_W and/or m_X may be determined dynamically. A complete "solution" to the hierarchy problem in which $m_{3/2}$ is determined dynamically³⁾ as well as m_W requires⁴⁾ the existence of a hidden sector with a flat potential guaranteed by approximate $SU(1,1)$ invariance⁶⁾. Observable low-mass fields have to be combined with the hidden sector in a non-trivial way⁴⁾ if there is to be a non-trivial limit as $m_{3/2}/m_P \rightarrow 0$. The underlying $SU(1,1)$ symmetry is broken by the gravitino mass which is not invariant under dilatations and conformal transformations. Light fields should have non-zero conformal weights if their superpotential terms are not to vanish when $m_{3/2}/m_P \rightarrow 0$ ⁴⁾. Thus far we have not considered the high-mass fields which would appear in a no-scale SUSY GUT, associated, for example, with the first scale of GUT gauge symmetry breaking. It is difficult to see how the first stage of GUT symmetry breaking, and hence m_X , could be determined dynamically by radiative corrections in a manner directly analogous to the previous determination of m_W . Previously it was found^{3,4)} that dynamics forced $m_{3/2} = O(m_W)$: it is difficult to see how one can avoid finding $m_{3/2} = O(m_X)$ if previous models are extended to include heavy fields.

Our previous models^{3,4)} used a non-compact global $SU(1,1)$ symmetry⁶⁾ of the type found in $N = 4$ extended supergravity⁷⁾ to guarantee the flatness of the potential for different values of the gravitino mass. This flatness enabled radiative corrections of $O(m_{3/2}^2 m_W^2)$ to fix $m_{3/2} = O(m_W)$, with m_W determined by dimensional transmutation: $m_W = m_P \exp[-O(1)/\alpha_t]$. All the particles in our previous models had masses of order m_W , but particles with much heavier masses $O(m_X)$ or $O(m_P)$ exist in GUTs. In the presence of SUSY breaking, radiative corrections involving these heavy particles would be expected to change the vacuum energy by $O(m_{3/2}^2 m_X^2)$ or $O(m_{3/2}^2 m_P^2)$, thus overwhelming the $O(m_{3/2}^2 m_W^2)$ contribution of the light fields and preventing them from determining $m_{3/2}$ to be $O(m_W) \ll m_X$ or m_P . Instead, one would expect the heavy dynamics to fix $m_{3/2} = O(m_X \text{ or } m_P)$. To avert this disaster, one must arrange

that the heavy fields' contribution to the vacuum energy is also at most $O(m_{3/2}^2, m_W^2)$. Since the only known symmetry which can cancel contributions to the vacuum energy is SUSY⁸⁾, it seems that we must banish SUSY breaking from the mass spectrum of the heavy sector of the GUT.

Remarkably, there is a geometrically elegant class of models in which this occurs naturally. They have the structure of non-linear chiral models for the heavy sector, based on non-compact manifolds $SU(n,1)/SU(n) \times U(1)$ where n is the number of heavy chiral fields, and contain the previously advertized $SU(1,1)/U(1)$ as a submanifold. Analogous structures appear in $N \geq 5$ extended supergravity theories⁹⁾, for example $n = 5$ if $N = 5$. Since at least some of the light Higgs fields are members of split GUT multiplets which also contain heavy fields, it is necessary to include these in the non-linear chiral model structure and it is natural to include all other light chiral fields. In this case, there are *no* SUSY breaking masses for chiral superfields, so what is the origin of the SUSY breaking required phenomenologically at low energies? Here again, extended supergravity theories may provide the answer, since all the $N \geq 4$ theories contain non-minimal kinetic terms for the gauge fields^{7,9)}, including non-trivial chiral functions which may generate tree-level gaugino masses of $O(m_{3/2})$. We assume these are present for the light gauginos, but absent for the heavy gauginos in order to avoid the $O(m_{3/2}^2, m_X^2)$ or $m_{3/2}^2, m_P^2$ contributions to the vacuum energy mentioned previously. This asymmetry is easy to arrange when the GUT symmetry is spontaneously broken, though we are not yet able to provide a natural theoretical justification for this phenomenological ansatz. We presume that this ansatz will be justified by a final theory of everything (TOE) which may be based on extended supergravity.

Once SUSY breaking is introduced in this way into the observable fields through light gauginos, radiative corrections will feed it to all observable fields. This is desirable for Higgses, squarks and sleptons, but is potentially calamitous if the contagion spreads to the heavy fields. Our previous assumption of zero SUSY breaking for the heavy sector is violated by radiative corrections and hence technically unnatural as far as the renormalizable low energy ($\mu \lesssim m_P$) theory is

concerned. Our response is to postulate it as a boundary condition when the theory is renormalized at a renormalization scale $\mu = 0(m_X)$, and to appeal to the TOE to justify this assumption when all the supersymmetric heavy degrees of freedom are integrated over. It could be a technically natural boundary condition as far as the TOE is concerned, if corrections to the vacuum energy are finite as may be the case for extended supergravities. Postulating the absence of SUSY breaking for heavy fields appears to be a consistent assumption only if there is but one heavy scale, and accordingly we identify $m_X = 0(m_P)$ in this paper. The hierarchically lighter mass scales $m_W, m_{3/2}$ are however, generated by radiative corrections in a manner similar to that described previously^{3,4)}, but with the novel initial renormalization conditions

$$m_{\tilde{q}} = m_{\tilde{l}} = m_H \Big|_{\mu=0(m_P)} = 0 \quad ; \quad m_{\tilde{\nu}} \Big|_{\mu=0(m_P)} = 0(m_{3/2}) \quad (1.1)$$

The tree-level $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gaugino masses $m_{\tilde{V}}$ may be quite unequal when the GUT symmetry is broken at $m_X \approx m_P$. They drive the scalar (mass)² positive at $\mu < m_P$, ensuring the stability of the GUT spontaneous breaking scale $m_X = 0(m_P)$. At a sufficiently low scale $\mu_0 \ll m_P$ the t quark Yukawa coupling asserts itself as usual^{10,2-4)} and drives m_H^2 negative so that spontaneous breaking of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ occurs, $m_W \neq 0$ and $m_{3/2} = 0(m_W)$. The necessary t quark mass is free of the previous uncertainties associated with the choice of $m_{\tilde{q}}/m_{\tilde{W}}$ and of the SUSY breaking A parameter¹¹⁾. There are new uncertainties in the ratios of gaugino masses, but the $SU(3)_C$ and $SU(2)_L$ gaugino contributions usually dominate, so that these uncertainties are relatively minor. There is a residual uncertainty due to Higgs mixing. If we assume this is small, we find in the simplest model that

$$m_t \approx 50 \text{ GeV} \quad (1.2)$$

In this scenario, the lightest spin zero particles are predicted to be the right-handed sleptons.

This paper is organized as follows. In Section 2 we demonstrate that models whose Kähler potential has the form of a non-linear chiral model based on a non-compact $SU(n,1)/SU(n) \times U(1)$ manifold yield a globally supersymmetric effective field theory even if $m_{3/2} \neq 0$. We also show that non-minimal kinetic terms of the type found in $N \geq 4$ extended supergravity theories^{7,9)} can yield tree-level masses $m_{\tilde{V}} = O(m_{3/2})$ for the light gauginos with masses $\ll m_P$, without generating tree-level masses for the gaugino partners of the heavy gauge bosons with masses $= O(m_P)$. Our analysis is illustrated by an $SU(5)$ GUT embodying the missing partner mechanism¹²⁾. In Section 3, we analyse the renormalization of parameters in the effective theory of particles with low masses $\ll m_P$. The tree-level results of Section 2 are interpreted as renormalization conditions at $\mu = O(m_P)$ which provide boundary conditions for the integration of the renormalization group equations for parameters of the low-energy effective theory. We then integrate these renormalization group equations and demonstrate that a t quark with mass about 50 GeV can trigger $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ gauge symmetry breaking with the correct W boson mass. We also give representative examples of sparticle spectra for this model. Problems and prospects for future study are discussed in Section 4.

2. NON-COMPACT MANIFOLDS AND EFFECTIVELY SUPERSYMMETRIC THEORIES

2.1 Globally supersymmetric supergravity models

A dynamical mechanism for determining the gravitino mass $m_{3/2}$ requires a flat potential in the direction responsible for the super Higgs effect. It has been shown^{3,4)} that the radiative corrections of the low-energy effective theory destroy the flatness of the scalar potential by driving $\det m_{Higgs}^2 < 0$ at some scale $\mu_0 = m_P \exp[-O(1)/\alpha_t]$. This triggers $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ breaking: $m_W = O(\mu_0)$, and the dynamically preferred value of $m_{3/2}$ is also $O(m_W) = O(\mu_0)$ if the super Higgs potential was flat at the tree level. The resulting vacuum energy is negative and $O(m_{3/2}^4) = O(m_{3/2}^2 m_W^2) = O(m_W^4)$. Such a dynamical generation of $m_{3/2}$ can only work if the superheavy particles of the theory do not generate a vacuum energy of $O(m_{3/2}^2 m_{X,P}^2)$. However, if the heavy supermultiplets have mass splittings of order $m_{3/2}$, then as mentioned in Section 1 they naturally produce a vacuum energy of this order:

$$E_V \approx (m_{X,P}^2 + m_{3/2}^2)^2 - m_{X,P}^4 = O(m_{3/2}^2 m_{X,P}^2) \quad (2.1)$$

when either gravitational loops¹³⁾ or the one-loop effective potential for the renormalizable low-energy effective theory are computed. This makes the dynamical generation of $m_{3/2}$ problematic in GUTs with a very large scale m_X or m_P . The only possible solution is to construct a GUT in which heavy supermultiplets do not feel the SUSY breaking. We will now show that this is possible when a particularly symmetric Kähler potential is chosen. The resulting effective theory has a supersymmetric structure with possible soft SUSY breaking terms originating solely via the light SU(3), SU(2) and U(1) gaugino mass parameters.

Let us first assume the following Kähler potential G of n complex scalar fields z and ϕ^i : $i = 1, 2, \dots, n-1$:

$$G = -3 \log (f(z) + f^*(z^*) + g(\phi_i^+, \phi^i)) \quad (2.2)$$

where $f(z)$ is an analytic function of the field z which is an SU(5) gauge singlet, and $g(\phi_i^+, \phi^i)$ is a real function of the non-singlet matter fields ϕ_i^+ and ϕ^i . Included among the ϕ^i are the squarks, sleptons, $\underline{5}$ and $\bar{\underline{5}}$ of Higgses H and $\bar{H}$, $\underline{24}$ of Higgses Σ responsible for $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ breaking, and possibly other complex scalar fields. We will see shortly that the structure (2.2) is necessary, because of Eq. (2.1), for all the heavy fields including at least parts of the $\underline{5}$, $\bar{\underline{5}}$ and $\underline{24}$ Higgs multiplets. The SU(5) GUT invariance therefore requires all of these multiplets to appear in the form (2.2). While not actually necessary, it is natural to treat squarks and sleptons in the same way as the Higgses. To characterize the observable field couplings we must also specify an analytic chiral function $f_{ab}(z, \phi^i)$ multiplying the gauge kinetic term $W^a W^b$. Initially, we take $f_{ab} \sim \delta_{ab}$, i.e. field-independent, which ensures that no tree-level gaugino masses are generated after SUSY breaking. We will modify this assumption in Section 2.2.

The general form¹⁴⁾ of the scalar potential is then given by the Kähler potential G and its derivatives^{*)}:

$$G^i \equiv \frac{\partial G}{\partial y_i^+}, \quad G_i \equiv \frac{\partial G}{\partial y^i}, \quad G_i^j \equiv \frac{\partial G}{\partial y^i} \frac{\partial G}{\partial y_j^+} : \quad y^i \equiv \{z, \phi^i\} \quad (2.3)$$

in terms of which

$$V = e^G \left[G_i (G^{-1})^i_j G^j - 3 \right] + \frac{1}{2} D_a D_a \quad (2.4)$$

where

$$D^a = g \left(G_i (T^a)^i_j \phi^j \right) \quad (2.5)$$

with g the SU(5) gauge coupling and $(T^a)^i_j$ the representation matrix for the generator a . In Eq. (2.4) $(G^{-1})^i_j$ is the inverse of the matrix G_i^j of Eq. (2.3). The Kähler potential (2.2) gives zero when substituted into the first term of the potential in Eq. (2.4), as we will now show. It is convenient to define

$$G_f \equiv \frac{\partial G}{\partial f} = \frac{\partial G}{\partial f^+}, \quad G_{ff} \equiv \frac{\partial^2 G}{\partial f^2} \quad (2.6)$$

which obey

$$\frac{G_f^2}{G_{ff}} = 3 \quad (2.7)$$

*) We use similar notations for other derivatives of G .

In terms of G_f and G_{ff} , we have

$$G_i = G_f (f_i + g_i), \quad G^i = G_f (f^{i+} + g^i) \quad (2.8)$$

$$G_i^j = G_{ff} (f_i + g_i)(f^{j+} + g^j) + G_f g_i^j = \frac{1}{3} G_i G^j + G_f g_i^j$$

where we have used (2.7) to rewrite the last term of Eq. (2.8). It is easy to check using (2.7) that

$$G^i (G^{-1})_i^j = \frac{G_f}{G_{ff}} \frac{f^{j+}}{|f|^2} \quad (2.9)$$

is a projector onto the z direction, so that

$$G^i (G^{-1})_i^j G_j = \frac{G_f G_f}{G_{ff}} = 3 \quad (2.10)$$

which ensures the vanishing of the square bracket [...] in Eq. (2.4). Thus, the full scalar potential is just given by the D term

$$V = \frac{1}{2} D_a D_a \quad (2.11)$$

where

$$D_a = g G_f g_i (T^a)_j^i \phi^j \quad (2.12)$$

Let us also calculate the Yukawa couplings of the theory which are also defined¹⁴⁾ by G [Eq. (2.2)]:

$$\mathcal{L}_Y \equiv - \bar{\chi}_i \tau^{ij} \psi_j e^{G/2} \quad (2.13)$$

where $T^{ij} = G^{ij} + G^i G^j - G^k (G^{-1})_k^l G_l^{ij}$. Using Eqs. (2.8) and (2.9) and the identities:

$$\left. \begin{aligned} G^{ij} &= \frac{1}{3} G^i G^j + G_f g^{ij} \\ G_{ik}^{ij} &= \frac{2}{9} G_{ik} G^i G^j + \frac{G_f}{3} [g^{ij} G_{ik} + g_{ik}^i G^j + g_{ik}^j G^i] + G_f g_{ik}^{ij} \end{aligned} \right\} \quad (2.14)$$

one can show after some algebra that

$$\gamma^{ij} = \frac{2}{3} G^i G^j \quad (2.15)$$

Using the fact that the Goldstino field¹⁴⁾

$$\eta = G^i \psi_i \quad (2.16)$$

[recall that $D^a = 0$ at the minimum of the potential (2.11)], we see that all Yukawa interactions (2.13) and (2.15) involve the Goldstino η which is eaten by the gravitino after local SUSY breaking

$$- \bar{\psi}_i \gamma^{ij} \psi_j e^{G/2} = - \frac{2}{3} \bar{\eta} \eta e^{G/2} \quad (2.17)$$

Thus, no mass splitting between the bosonic and fermionic degrees of freedom is generated when SUSY is broken.

To understand better this result, we calculate the curvature tensor R_i^j of the manifold G (2.2):

$$R_i^j = \frac{\partial}{\partial y^i} \frac{\partial}{\partial y^j} \log \det G_k^l \quad (2.18)$$

which appears in the supertrace formula¹⁵⁾

$$\text{Str } \mathcal{M}^2 = \sum_j (-1)^{2j} (2j+1) \mathcal{M}_j^2 = 2m_{3/2}^2 (n-1 - G_i (G^{-1})^i_j R_k^j (G^{-1})^k_l G^l) \quad (2.19)$$

Using Eq. (2.7) and the expression (2.8) for G_i^j , one can show that

$$\det G_i^j = G_{ff} (-G_f)^{n-1} \det(-g_k^l) f_z f_z^* \quad (2.20)$$

It follows that

$$R_i^j = \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \phi^j} \log \det(-g_k^l) + \left(\frac{n+1}{3}\right) G_i^j \quad (2.21)$$

which we can now use to evaluate the supertrace formula (2.19). The projection property (2.9) of $G_i(G^{-1})_j^i$ means that there is no contribution from the first $\det(-g_k^l)$ term in Eq. (2.21). We therefore find that

$$\text{Str } \mathcal{M}^2 = 2m_{3/2}^2 \left(n-1 - \frac{n+1}{3} \frac{G_{ff} G_{ff}}{G_{ff}} \right) = -4m_{3/2}^2 \quad (2.22)$$

which shows that the only net contribution to the supertrace formula comes from the gravitino mass. This confirms our previous observation that there is no mass splitting between the bosonic (B) and fermionic (F) components of the chiral superfields:

$$\sum_B m_B^2 - \sum_F m_F^2 = 0 \quad (2.23)$$

This result is a natural generalization of the flatness⁶⁾ of the SU(1,1) model potential, for which the boson masses vanish.

The structure (2.21) of R_i^j is suggestive. In the case where $(-g_i^j)$ is field-independent: $-g_i^j \propto \delta_i^j$, the Kähler manifold is a maximally symmetric space defining an Einstein space with curvature $R = (n+1)/3$:

$$R_i^j = \frac{n+1}{3} G_i^j \quad (2.24)$$

In the case $g(\phi^i, \phi_j^+) \propto -\phi_i^+ \phi^i$, the kinetic terms of the scalar fields are invariant under the isometric transformations of the space, which form a non-compact SU(n,1) group^{*}). To see this, we make the following analytic redefinition of the field variables $(z, \phi^i) \rightarrow y^i$ ($i = 0, \dots, n-1$):

^{*}) We discard the irrelevant U(1) group of over-all phase transformations $\phi^i \rightarrow e^{i\theta} \phi^i$.

$$\left. \begin{aligned} f(z) &= \frac{1}{2} \frac{1 - y_0/\sqrt{3}}{1 + y_0/\sqrt{3}} \\ \phi^i &= \frac{y^i/\sqrt{3}}{1 + y_0/\sqrt{3}}, \quad i = 1, \dots, n-1 \end{aligned} \right\} \quad (2.25)$$

In terms of the redefined field variables, the Kähler potential G takes the symmetric form

$$G = -3 \log \left(1 - \frac{y^i y_i^+}{z} \right) + 3 \log \left| \left(1 + \frac{y_0}{\sqrt{3}} \right) \right|^2 \quad (2.26)$$

The metric G_i^j is defined by the first term in (2.26), from which it follows that the scalar fields parametrize the coset space $SU(n,1)/SU(n) \times U(1)$. As discussed in Ref. 4, the non-compact nature of the symmetry implies that the potential is not only constant but actually identical to zero. Conventional GUT gauge interactions and superpotential terms break the full non-compact symmetry of (2.26) down to a non-compact $SU(1,1)$ symmetry acting on the gauge singlet field (z or y_0), keeping its potential flat. The remaining spin-zero fields ($y_i: i = 1, \dots, n-1$) are pseudo-Goldstones which acquire masses from radiative corrections as we will discuss shortly. The Kähler metric term in (2.26) is not⁴⁾ invariant under the dilatations subgroup of $SU(1,1): G \rightarrow G + \text{constant}$, which means⁴⁾ that the gravitino mass $e^{G/2}$ breaks $SU(1,1) \rightarrow U(1)$. It is this breaking of $SU(1,1)$ which enables the real component of the complex field z to acquire a mass through the radiative corrections which determine dynamically m_W and $m_{3/2}$, as discussed in Refs. 2 and 3 and in Section 3. This mechanism presents cosmological problems¹⁶⁾ which we do not tackle in this paper. The as yet unbroken $U(1)$ is⁴⁾ the group of imaginary translations $f(z) \rightarrow f(z) + ib$. If it were not broken⁴⁾ by other interactions (e.g. in the superpotential, or in non-minimal chiral functions $f_{ab}(y^i)$ which could give gaugino masses), there would be a massless spin-zero Goldstone field. This would, however, be sufficiently weakly coupled to conventional matter to be phenomenologically acceptable.

The most convenient choice of scalar field representation is dictated by the scalar kinetic terms of the theory:

$$\mathcal{L}_{\text{K.T.}} = -G_i^j \partial_\mu y^i \partial^\mu y_j^+ \quad (2.27)$$

which can be rewritten using (2.8) as

$$\mathcal{L}_{\text{K.T.}} = -\frac{1}{3} G_i \partial_\mu y^i G^j \partial^\mu y_j^+ - G_5 g_i^j \partial_\mu \phi^i \partial^\mu \phi_j^+ \quad (2.28)$$

If we make the minimal choice $g(\phi^i, \phi_i^+) = -\phi^i \phi_i^+ / 3$ and make the analytic field redefinition $f(z) \rightarrow z$, we find

$$G = -3 \log(z + z^+ - \frac{1}{3} \phi^i \phi_i^+) \quad (2.29a)$$

and

$$G_5 = G_2 = -3e^{G/3} \quad (2.29b)$$

This then yields for the kinetic terms (2.28):

$$\begin{aligned} \mathcal{L}_{\text{K.T.}} = & -\frac{1}{12} (\partial_\mu G)^2 - \frac{3}{4} e^{\frac{2}{3}G} \left| \partial_\mu (z - z^+) + \frac{1}{3} (\phi_i^+ \partial_\mu \phi^i - \phi^i \partial_\mu \phi_i^+) \right|^2 \\ & - e^{\frac{1}{3}G} \left| \partial_\mu \phi^i \right|^2 \end{aligned} \quad (2.30)$$

We see from (2.30) that the normalization of the fields depends on the value of G which specifies the gravitino mass: $m_{3/2} = e^{G/2}$. This is a consequence of the Einstein structure of the manifold which necessitated a field-dependent metric G_i^j . Therefore, it is impossible to make an *analytic* field redefinition which sets the coefficient of $\partial^\mu \phi^i \partial_\mu \phi_i^+$ to unity. It is clear that the product of these coefficients must be of order $e^{(n+1)G/3}$ in order to generate the proportionality (2.24). This is indeed the case for the coefficients exhibited in Eq. (2.30):

$$\det G_i^j \sim e^{\frac{2}{3}G} \times (e^{\frac{1}{3}G})^{n-1} = e^{\frac{n+1}{3}G} \quad (2.31)$$

which leads directly to Eq. (2.24). Given a (dynamically determined) value $\langle G \rangle$ of G , one can then obtain normalized fields Y_i : by making the rescalings

$$\left. \begin{aligned} Y_0^R &= \frac{1}{\sqrt{6}} G, & Y_0^I &= \lambda^2 \sqrt{\frac{3}{2}} (z - z^*) \\ Y^i &= \lambda y^i, & i &= 1, 2, \dots, n-1 \end{aligned} \right\} \quad (2.32)$$

where the dilatation parameter

$$\lambda = e^{-\langle G \rangle / 6} \quad (2.33)$$

Note that the gravitino mass $m_{3/2}$ is expressible in terms of the exponential of Y_0^R only:

$$m_{3/2} = e^{G/2} = e^{\sqrt{\frac{3}{2}} Y_0^R} \quad (2.34)$$

and does not depend on the other fields. The total scalar potential is given by Eq. (2.11). The D-term takes its canonical form when expressed in terms of the normalized fields (2.32), since the dependence (2.12) on $G_F = -3e^{G/3}$ is absorbed by the field redefinitions (2.32):

$$D_a = g Y_i^\dagger (T^a)^i_j Y^j \quad (2.35)$$

Thus, the "hidden" z field indeed disappears from the tree-level potential (2.4). The normalization of the fermionic fields is also dictated by their kinetic terms

$$\mathcal{L}_{F.K.} = G_i{}^j \bar{\psi}_j \not{\partial} \psi^i \quad (2.36)$$

which become¹⁴⁾

$$\mathcal{L}_{F.K.} = \underbrace{e^{\frac{1}{3}G} \bar{\psi}_i \not{\partial} \psi^i}_{i=1,2,\dots,n-1} + \underbrace{\frac{1}{3} (G_i \bar{\psi} \not{\partial} \psi^i + \text{h.c.})}_{i=0,1,2,\dots,n-1} \quad (2.37)$$

after using Eqs. (2.8) and (2.16). The goldstino term is eaten by the gravitino term and the remaining kinetic terms for the (n-1) physical "observable" fields χ^i ($i = 1, 2, \dots, n-1$)

$$\mathcal{L}_{F.K.} = e^{\frac{1}{3}G} \bar{\psi}_i \not{\partial} \psi^i \quad (2.38)$$

are normalized correctly if we make the same dynamically determined rescaling by $\lambda = \langle e^{-G/6} \rangle$ (2.32) which worked for the scalar fields. After this, the gaugino-chiral fermion-scalar couplings take their canonical form, as do any interactions derived from superpotential terms involving the observable fields. The structure of the potential and of the gaugino-chiral fermion-scalar interactions shows clearly that if the 24 field Σ takes a vacuum expectation value breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$, it gives supersymmetric masses corresponding to the broken directions via the derivatives of the D-terms as in a globally supersymmetric theory. No SUSY breaking mass splitting between the gauge bosons, the scalar bosons of the 24 and the Dirac gaugino/chiral fermion mixing masses is generated, because $D^a = 0$ at the minimum of the potential (2.11).

However, the 24 Higgs potential is completely flat in the $SU(3) \times SU(2) \times U(1)$, $SU(4) \times U(1)$ and other directions, and no energetically favourable value exists for the 24 field. Also, the renormalizable dimension ≤ 4 terms in the theory, being supersymmetric, do not change the vacuum structure of the theory in any order of perturbation theory. Obviously we cannot say anything conclusive about the gravitational and other non-renormalizable interactions. However, we remind the reader that our stated¹⁷⁾ philosophy is to regard $N = 1$ supergravity models as *phenomenological* low-energy effective theories, analogous to the strong interaction phenomenological chiral Lagrangians of yore. As such, they are supposed already to include all effects of gravitational (c.f. strong interaction) radiative corrections. It would be double-counting to evaluate these and add them to the Ansatz (2.26). It is, of course, possible that such gravitational radiative corrections might be calculable, and if they suggest that the (2.26) is "naturally" stable, we would not look the gift horse in the mouth. We are free to hypothesize

that the tree-level structure is stable, perhaps because of the highly symmetric structure of the Kähler manifold. If this is not by itself sufficient, one could look to the underlying TOE which may well be an extended supergravity theory, whose structure⁹⁾ is already approached by our Ansatz (2.26) for the Kähler manifold.

2.2 Gaugino masses and gauge couplings

Clearly the Ansatz (2.26) is not completely realistic, since it does not impart any SUSY breaking to the low-energy spectrum. Also, we have yet to introduce some superpotential couplings for the light observable fields which would yield the correct fermion mass spectrum. Let us first discuss a possible way to communicate SUSY breaking to the light sector of the theory, and to fix the vacuum expectation value of the $\underline{24}$ field at a scale close to the unique mass scale $M \equiv m_p/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV associated with gravitational interactions. We have seen that the absence of SUSY breaking in the chiral superfield sector is a natural consequence of adopting an $SU(n,1)/SU(n) \times U(1)$ structure reminiscent of $N \geq 5$ extended supergravity theories. This analogy does not necessarily mean that SUSY breaking is absent from all the observable sector, since such theories also suggest^{7,9)} non-minimal kinetic terms¹⁴⁾ for the gauge supermultiplets. These may be characterized by an analytic function f_{ab} of the chiral superfields ϕ^i :

$$f_{ab}(\phi^i) V_a V_b \quad (2.39)$$

Non-trivial functions f_{ab} can generate non-zero gaugino masses which are of order the gravitino mass and proportional in our case to $f_{ab,z} \equiv \partial/\partial z f_{ab}$. We recall that for us $G^1(G^{-1})_1^k$ is a projector onto the z sector (2.9), so that the gaugino mass terms take the form

$$\begin{aligned} & \frac{1}{4} e^{G/2} G^l (G^{-1})_l^k f_{ab,k} \bar{\lambda}^a \lambda^b + (h.c.) \\ &= \frac{1}{4} e^{G/2} \frac{G_{ff}}{G_{ff}} f_{ab,z} \bar{\lambda}^a \lambda^b + (h.c.) \end{aligned} \quad (2.40)$$

If the original GUT group is broken down to a product of gauge subgroups by $\langle 0|Y_i|0\rangle \neq 0$, the non-trivial kinetic terms induced by (2.39) lead in general to *different* gauge couplings¹⁸⁾ for the different subgroups if $\langle 0|f_{ab}|0\rangle \neq \delta_{ab}$, even before renormalization effects are included. Thus, one could have, for example, $g_3(m_X) \neq g_2(m_X)$, and the difference could be *large* if $m_X = O(m_P)$ as in our scenario.

For the reason (2.1) already discussed, which is developed further in Section 3, we wish to ensure vanishing SUSY breaking masses for the gaugino spartners of massive gauge bosons with masses $O(m_X)$. Thus, in the case of $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ we must require

$$\langle 0|f_{ab,z}|0\rangle = 0 \quad \text{for } a,b = 9,10,\dots,20 \quad (2.41)$$

Since a naïve renormalization group extrapolation of the low energy $SU(3)$ and $SU(2)$ couplings gives $g_3(M) \approx g_2(M) \stackrel{?}{\approx} g_1(M)$, one may also wish to impose

$$\langle 0|f_{ab}|0\rangle \propto \delta_{ab} \quad \text{for } a,b = \begin{cases} 1,2,\dots,8 \\ 21,22,23,24 \end{cases} \quad (2.42)$$

It is trivial to find simple examples of models which obey both the conditions (2.41) and (2.42), if so desired. In the $SU(5)$ SUSY GUT developed previously in this section, the only chiral superfield which can make non-trivial contributions to $\langle 0|f_{ab}|0\rangle$ or $\langle 0|f_{ab,z}|0\rangle$ is the 24 Σ which is to have a non-zero vacuum expectation value of order M . We, therefore, study the analytic function $f_{ab}(\sigma,z)$ where σ is related to the normalized field Σ by

$$\Sigma = \lambda\sigma = \langle e^{-\eta/6} \rangle \sigma \quad (2.43)$$

as in Eq. (2.32). If we choose, for example, f_{ab} to be a function of $\hat{\Sigma} \equiv (\sigma/\sqrt{2z})$ alone:

$$f_{ab}(\sigma,z) = f_{ab}(\hat{\Sigma}) = f_{ab}\left(\frac{\sigma}{\sqrt{2z}}\right) \quad (2.44)$$

then

$$f_{ab,z} = - f'_{ab}(\hat{\Sigma}) \times \frac{\sigma}{(2z)^{3/2}} \quad (2.45)$$

The particular choice (2.44) has the property that the field $\hat{\Sigma}$ is almost normalized correctly:

$$\hat{\Sigma} = \frac{\sigma}{\sqrt{2z}} = C^{\frac{1}{2}} \Sigma : C = \left\langle \frac{z+z^* - \phi_i^+ \phi_i}{2z} \right\rangle \quad (2.46)$$

The factor C (2.46) depends on the imaginary part of z and takes the maximal value

$$|C| = \left| \left\langle \frac{z+z^*}{2z} \right\rangle \right| \left| \left\langle \left(1 - \frac{1}{3} \gamma_i^+ \gamma_i\right) \right\rangle \right| \leq \left| \left\langle 1 - \frac{1}{3} \gamma_i^+ \gamma_i \right\rangle \right| \approx \frac{2}{3} \quad (2.47)$$

as in our previous paper⁴⁾. Substituting Eq. (2.45) into the expression (2.40) for the gaugino masses we get

$$m_{ab} = - e^{g/2} \frac{g_f}{g_{ff}} \frac{1}{(2z)} \hat{\Sigma} f'_{ab}(\hat{\Sigma}) \approx e^{g/2} C^2 \hat{\Sigma} f'_{ab}(\hat{\Sigma}) \quad (2.48)$$

As an example, let us make the simplest possible Ansatz:

$$f_{ab}(\hat{\Sigma}) = \delta_{ab} (1 + \alpha \text{Tr} \hat{\Sigma}^2) + d_{abc} \hat{\Sigma}_c (\beta + \gamma \text{Tr} \hat{\Sigma}^2) \quad (2.49)$$

from which we deduce (assuming $\langle \hat{\Sigma} \rangle = M$)

$$\langle 0 | \frac{\partial f_{ab}}{\partial \hat{\Sigma}_c} | 0 \rangle = 2\alpha M \delta_{ab} + d_{ab24} (\beta + 3\gamma M^2) \quad (2.50)$$

To satisfy the gaugino mass cancellation condition (2.41) we must impose:

$$2\alpha M - \frac{1}{2\sqrt{5}} (\beta + 3\gamma M^2) = 0 \quad (2.51)$$

If we also choose to require equality of the low-energy gauge couplings (2.42), we must also impose

$$\beta + \gamma M^2 = 0 \quad (2.52)$$

It is, of course, possible to combine the two conditions (2.51) and (2.52):

$$\beta = -2\sqrt{5}\alpha M, \quad \gamma = 2\sqrt{5}\alpha/M \quad (2.53)$$

In this case, the absolute values of the light gaugino masses are not fixed, since they depend on the magnitude of α , etc., but their ratios $\xi_i \equiv M_i/m_{3/2}$ ($i = 3, 2, 1$) are specified:

$$\xi_3 : \xi_2 : \xi_1 = 5 : -5 : -1 \quad (2.54)$$

in this simplest possible model. There is no reason to expect these particular ratios to emerge from any more sophisticated or complete model, however. At the moment, we do not have any deep understanding how the TOE ensures that the necessary (2.41) and possibly desirable condition (2.42) are satisfied. All we know is that the condition (2.41) is a necessary condition for our philosophy of dynamical determination of $m_{3/2}$ to be valid. We defer its deeper understanding to future efforts¹⁹⁾. Note that in the presence of the SU(3), SU(2) and U(1) gauginos, the tree level supertrace formula (2.22) becomes

$$\text{Str } M^2 = (-4 - 16\xi_3^2 - 6\xi_2^2 - 2\xi_1^2) m_{3/2}^2 \quad (2.22')$$

which is subject to radiative corrections.

The presence of $SU(3) \times SU(2) \times U(1)$ gaugino masses proportional to the vacuum expectation value of $\hat{\Sigma}$ introduces radiative quantum corrections to the tree-level potential (2.11), which take the general form

$$\Delta V^R = \sum_J \frac{1}{64\pi^2} (-1)^{2J} (2J+1) \mathcal{M}_J^4 \log \frac{\mathcal{M}_J^2}{\mu^2} \quad (2.55)$$

in leading order. The non-zero gaugino masses make a contribution

$$\Delta V^R(\hat{\Sigma}) \simeq -\frac{1}{64\pi^2} \zeta m_{3/2}^2 \frac{|\hat{\Sigma}|^4}{M^2} \log \frac{|\hat{\Sigma}|^2}{M^2} \quad (2.56)$$

where the coefficient ζ depends on the ratios ξ_i (2.54), and we have chosen a scale $\mu = M$ to specify our renormalization conditions. (See Section 3 for more discussion of this point.) We see from Eq. (2.56) that $\Delta V^R > 0$ for any value of $|\hat{\Sigma}| < M$, whereas ΔV^R is unbounded below for $|\hat{\Sigma}| > M$. We do not believe that the radiative quantum corrections for $|\hat{\Sigma}| > M$ can be meaningfully approximated by the expression (2.55): physics beyond M is in the hands of the TOE. We are content that (2.55) disfavors minima with $|\hat{\Sigma}| < M$, and leave the rest to the TOE. We assume that the minimum at $|\hat{\Sigma}| = M$ has cosmological constant $\ll m_{3/2}^2 M^2$, as may be natural from the point of view of the TOE -- see Section 3. The SUSY breaking light gaugino masses feed masses of order $m_{3/2}$ to the light spin-zero components of the 24 multiplet and to other scalar fields by radiative corrections.

2.3 Superpotential couplings

To obtain a realistic theory we need Yukawa couplings for the light degrees of freedom, and we must split the $\underline{5} + \bar{\underline{5}}$ of Higgses into light doublet and heavy triplet components. These can easily be done by adding to the function G of Eq. (2.26) an analytic superpotential function, so that the full Kähler potential is

$$\mathcal{G} = G + F(\phi^i) + F^\dagger(\phi_i^\dagger) \quad (2.57)$$

Such an addition does not modify the geometric properties of the manifold as expressed by the metric $\mathcal{G}_i^j = G_i^j$ and the corresponding curvature $\mathcal{R}_i^j = R_i^j$. In parti-

cular, the supertrace formula (2.22) or (2.22') is unchanged and the low-energy spectrum remains supersymmetric. After some algebra, one can show that

$$G_i (G^\dagger)^i_j G^j = \frac{G_f G_{\bar{f}}}{G_{ff}} + e^{-\frac{1}{3}G} F_i^\dagger F_i \quad (2.58)$$

where $F_i = \partial F / \partial \phi^i$, etc., so that the total scalar potential (2.11) becomes

$$V_T = e^G e^{-\frac{1}{3}G} F_i F_i^\dagger + \frac{1}{2} D^2 \quad (2.59)$$

The potential (2.59) is positive semi-definite and its minimum is given by $F^i = D^a = 0$ and has zero cosmological constant at the tree level. The modification of the Yukawa terms due to F can easily be deduced from Eq. (2.13):

$$\gamma^{ij} = \frac{2}{3} (G^i + F^i)(G^j + F^j) + F^{ij} + \frac{1}{3} F^i F^j \quad (2.60)$$

Subtracting the goldstino field $\eta = (G^i + F^i)\psi_i$ we obtain the residual couplings

$$- \bar{\chi}_i (F^{ij} + \frac{1}{3} F^i F^j) e^{G/2} \chi_j \quad (2.61)$$

Taking into account the correct normalization [(2.38) *et seq.*] of the fermion kinetic terms we arrive at the fermion mass matrix

$$\mathcal{M}_F^{ij} = e^{-G/3} F^{ij} e^{G/2} \quad (2.61a)$$

and the bosonic (mass)² matrix

$$(\mathcal{M}_B^2)^i_j = e^{-G/3} F^{ik} F_{kj}^\dagger e^{-G/3} e^G \quad (2.61b)$$

which manifests the equality

$$(\mathcal{M}_B^2)^i_j = \mathcal{M}_F^{ik} \mathcal{M}_{kj}^\dagger \quad (2.62)$$

as in the case of global supersymmetry. If F is only trilinear in the fields ϕ^i :

$$F = c_{ijk} \phi^i \phi^j \phi^k \quad (i=1,2,\dots,n-1) \quad (2.63)$$

and we rescale them as in Eq. (2.32) then the full potential V_T of Eq. (2.59) takes the form:

$$V_T = \sum_{i=1}^{n-1} |c_{ijk} \gamma^j \gamma^k|^2 e^{F+F^\dagger} + \frac{1}{2} g^2 (\gamma_i^\dagger (T^a)_j^i \gamma^j)^2 \quad (2.64)$$

Therefore, the effective quartic couplings of the theory are independent of the value of $\mathcal{G}$ and hence do not⁴⁾ scale with the gravitino mass $e^{g/2}$: neither do the rescaled Yukawa couplings (2.61). There is no SUSY breaking for the chiral fields at the tree-level, and in particular no SUSY breaking parameters A or B ¹¹⁾. These are, of course, generated by radiative corrections as discussed in Section 3.

Before passing to this subject, we make a final comment concerning the doublet-triplet Higgs mass splitting. This can easily be arranged^{12,20)} by assuming the existence of $\underline{50}+\overline{\underline{50}}$ representations θ and $\bar{\theta}$ which couple to the $\underline{5}+\overline{\underline{5}}$ (H and $\bar{H}$) and the $\underline{24}$ fields $\hat{\Sigma}$, in such a way as to leave the doublet components of H and $\bar{H}$ massless, while their triplet components are eaten by the triplet components of θ and $\bar{\theta}$, acquiring very large masses. For consistency of the assumption that there are no SUSY breaking mass splittings for any heavy fields, despite the effects of radiative corrections, it appears necessary that the massive triplets have masses $O(\langle \hat{\Sigma} \rangle) = O(M)$. This can be arranged by choosing superpotential terms

$$F = \sqrt{\text{Tr} \sigma^2} \bar{\theta} \theta + \frac{1}{\sqrt{\text{Tr} \sigma^2}} (\bar{H} \sigma^2 \theta + \bar{\theta} \sigma^2 H) \quad (2.65)$$

Although this non-holomorphic dependence on σ is unfamiliar, it is not original²¹⁾, does not appear to pose any fundamental theoretical problems, and in fact appears in extended supergravity theories.

3. RENORMALIZATION, SUSY AND GAUGE SYMMETRY BREAKING

We have seen in the previous section how an $SU(n,1)/SU(n) \times U(1)$ structure inspired by extended supergravity theories can provide us with spontaneous $SU(5)$ GUT gauge symmetry breaking at a scale of $O(m_p)$ and a low-energy sector which is completely supersymmetric apart from SUSY breaking masses for the light gauginos and for the gravitino. The $SU(n,1)/SU(n) \times U(1)$ flavour symmetry which guaranteed the absence of SUSY breaking masses for squarks, sleptons and Higgses is violated by the Yukawa couplings of the theory and by the gauging of an $SU(5)$ GUT subgroup of the flavour symmetry. Accordingly, we would expect radiative corrections to generate a more general form of SUSY breaking, including squark, slepton and Higgs masses. Before computing these, we first recall our philosophical basis for using the $SU(n,1)/SU(n) \times U(1)$ structure as a starting point.

As was mentioned in the Introduction, we wish to avoid contributions to the vacuum energy which are $O(m_{3/2}^2 m_X^2 \text{ or } m_{3/2}^2 m_P^2)$. We would expect these to be generated by gravitational and/or non-gravitational radiative corrections if any of the heavy particles with masses $O(m_X \text{ or } m_P)$ acquire SUSY breaking masses of $O(m_{3/2})$. The analysis of the previous section guarantees that these are absent at the tree level, but the remarks of the previous paragraph imply that SUSY breaking returns to haunt us at the loop level. If there is just one large ($\gg m_W$) mass scale M , then it is feasible to cancel out the $O(m_{3/2}^2 M^2)$ contributions by specifying that the renormalized SUSY breaking mass parameters of the heavy sector vanish when the theory is renormalized at some scale $\mu = O(M)$. This is, however, not feasible if there are two or more heavy scales, for example, if $m_W \ll m_X \ll m_P$, since radiative corrections would generate SUSY breaking at a renormalization scale $\mu = O(m_X)$ even if it were absent at $\mu = O(m_P)$. Therefore, we have focused our attention on theories with a single large mass-scale, and identified $m_X = O(m_P)$. Of course, specifying the absence of SUSY breaking in the heavy sector when it is renormalized at $\mu = O(m_P)$ is not natural from the point of view of the renormalizable low-energy theory. However, if the ultimate theory (TOE) operating at scales $\mu \gtrsim O(m_P)$ is actually

finite^{*}), as may well be the case for an ($N = 8$?) extended supergravity theory, then setting SUSY breaking mass parameters and the vacuum energy of the heavy sector to a small value is natural for renormalization scales $\mu \gtrsim 0(m_P)$. We have no idea how our effective $N = 1$ SUSY theory may be extracted^{22,23}) from an underlying extended supergravity theory or other TOE, but assume as a working hypothesis that this is possible. Thus, we interpret Section 2 as providing an effective low-energy theory where:

$$\left. \begin{aligned} m_{\tilde{q}}^2(\mu) &= m_{\tilde{l}}^2(\mu) = m_H^2(\mu) = 0 \\ M_3(\mu) &\equiv m_g(\mu), M_2(\mu) \equiv m_{\tilde{w}}(\mu), M_1(\mu) \equiv m_{\tilde{B}}(\mu) \\ &= O(m_{3/2}) \end{aligned} \right\} \quad (3.1)$$

for some $\mu = (M)$, and we use (3.1) as boundary conditions for integrating the renormalization group equations of the low-energy effective theory.

In the particular $SU(5)$ model of Section 2, there are also components of the 24 of Higgs fields Σ which also have zero mass at the tree level:

$$m_{\Sigma_{1,2,\dots,8}}^2 = m_{\Sigma_{21,22,23,24}}^2 = 0 \quad (3.2)$$

These reflect the fact that the assumed form and scale

$$\langle 0 | \Sigma | 0 \rangle = M \lambda_{24}, \quad \langle 0 | \text{other spin-zero} | 0 \rangle = 0 \quad (3.3)$$

of the $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ breaking vacuum expectation value is undetermined at the tree level. It is an important check on the self-consistency of our philosophy that the effective potential is positive for $\langle 0 | \Sigma | 0 \rangle$ somewhat below m_P as we saw in Section 2.2. As mentioned there, we do not care about checking stability for $\langle 0 | \Sigma | 0 \rangle > M$, since that is a domain where the non-renormalizability of

^{*}) We note parenthetically that there is no prospect of "solving" the hierarchy problem by using radiative corrections to generate dynamically the weak interaction scale in any theory which yields a finite effective theory at scales $\ll m_P$.

the theory precludes any meaningful calculation, and we assume that the ultimate TOE prevents $\langle 0|\Sigma|0\rangle$ from drifting into values $> M$. It is easy to check using the renormalization group equations that since the only SUSY breaking parameters present at $\mu = O(M)$ are the light gaugino masses, all the initially vanishing spin-zero masses (3.1) and (3.2) immediately become positive at $\mu < M$. This ensures that our assumed vacuum (3.3) is indeed stable with respect to small decreases in $\langle 0|\Sigma|0\rangle$, as was already argued in Section 2.2. This does not prevent the light doublet Higgses from acquiring hierarchically smaller vacuum expectation values $\langle 0|H, \bar{H}|0\rangle = O(m_W)$ as we shall now see.

The one-loop renormalization group equations for the SU(3), SU(2) and U(1) gauge couplings are²⁴⁾

$$\mu \frac{d\alpha_i}{d\mu} = - \frac{\alpha_i^2}{2\pi} [b_i] + \dots \quad (3.4a)$$

where

$$\left. \begin{aligned} b_1 &= -2N_G - \frac{3}{10} N_D - \frac{3}{5} N_S \\ b_2 &= 6 - 2N_G - \frac{1}{2} N_D - 2N_3 \\ b_3 &= 9 - 2N_G - 3N_8 \end{aligned} \right\} \quad (3.4b)$$

where $N_G = 3$ is the number of quark and lepton generations, $N_D = 2$ is the number of light Higgs doublets, N_S is the number of light singlets from split $\underline{10} + \overline{\underline{10}}$ representations (see later), which we take as $N_S = 0$ for the moment, and $N_3, N_8 = 1$ are the numbers of light Higgs triplets $\Sigma_{21,22,23}$ and octets $\Sigma_{1,\dots,8}$ respectively. With these values

$$b_1 = -6.6, \quad b_2 = -3, \quad b_3 = 0 \quad (3.4c)$$

and if we take the representative initial values

$$\alpha_3^{-1}(m_W) = 10, \quad \alpha_2^{-1}(m_W) = 27, \quad \alpha_1^{-1}(m_W) = 60, \quad \alpha_{em}^{-1}(m_W) = 127 \quad (3.5)$$

we find that

$$\alpha_3^{-1}(M) = 10, \alpha_2^{-1}(M) = 9, \alpha_1^{-1}(M) = 20 \quad (3.6a)$$

These couplings are unequal at $\mu = 0(m_p)$, which we can interpret in either of two ways. We could add in extra low-energy particles which bring $\alpha_1(M)$ into line with the others, for example, two light $SU(3) \times SU(2)$ singlets from a $\underline{10} + \overline{\underline{10}}$ of $SU(5)$ yield $N_S = 2$ and hence:

$$b_1 \rightarrow -7.8 \Rightarrow \alpha_1^{-1}(M) = 13 \quad (3.6b)$$

while leaving b_3 and b_2 and hence $\alpha_3^{-1}(M)$ and $\alpha_2^{-1}(M)$ unchanged. Alternatively, we can exploit the observation made at the end of Section 2 that the $SU(3)$, $SU(2)$ and $U(1)$ gauge couplings may take significantly different values at m_X , if $m_X = 0(M)$ and we have non-minimal kinetic terms for the gauge fields. We will take (3.4, 3.5, 3.6a and 3.6b) as representative models for the gauge couplings at scales $\mu \lesssim 0(M)$.

Introducing the notations

$$\langle \alpha_i^n \rangle_\mu \equiv \frac{1}{4\pi} \int_{\mu^2}^{M^2} d \log \mu'^2 \frac{\alpha_i^n(\mu')}{\alpha_i^{n-1}(M)} \quad (3.7a)$$

and

$$\tilde{z}_i \equiv \frac{M_i(M)}{m_{3/2}} \quad (3.7b)$$

the renormalization group equations for the low energy SUSY breaking masses can be integrated using the initial conditions (3.1) to yield (where not specified, we use the notation of Ref. 25):

$$m_{E_i}^2(\mu) = m_{3/2}^2 \times \frac{12}{5} \langle \alpha_i^3 \rangle_\mu \tilde{z}_i^2 \quad (3.8a)$$

$$m_{L_i}^2(\mu) = m_{3/2}^2 \times \left[3 \langle \alpha_i^3 \rangle_\mu \tilde{z}_i^2 + \frac{3}{5} \langle \alpha_1^3 \rangle_\mu \tilde{z}_1^2 \right] \quad (3.8b)$$

$i=1,2,3$

$$m_{Q_i}^2(\mu) = m_{3/2}^2 \times \left[\frac{16}{3} \langle \alpha_3^3 \rangle_\mu \bar{z}_3^2 + 3 \langle \alpha_2^3 \rangle_\mu \bar{z}_2^2 + \frac{1}{15} \langle \alpha_1^3 \rangle_\mu \bar{z}_1^2 \right] \quad (3.8c)$$

$$m_{U_i}^2(\mu) = m_{3/2}^2 \times \left[\frac{16}{3} \langle \alpha_3^3 \rangle_\mu \bar{z}_3^2 + \frac{16}{15} \langle \alpha_1^3 \rangle_\mu \bar{z}_1^2 \right] \quad (3.8d)$$

$$m_{D_i}^2(\mu) = m_{3/2}^2 \times \left[\frac{16}{3} \langle \alpha_3^3 \rangle_\mu \bar{z}_3^2 + \frac{4}{15} \langle \alpha_1^3 \rangle_\mu \bar{z}_1^2 \right] \quad (3.8e)$$

$$m_{Q_3}^2(\mu) = m_{Q_i}^2(\mu) - r^2 (m_{Q_3}^2(\mu) + m_{U_3}^2(\mu) + m_2^2(\mu) - m_4^2(\mu) + m_{3/2}^2 A^2(\mu)) \quad (3.8f)$$

$$m_{U_3}^2(\mu) = m_{U_i}^2(\mu) - 2r^2 (m_{Q_3}^2(\mu) + m_{U_3}^2(\mu) + m_2^2(\mu) - m_4^2(\mu) + m_{3/2}^2 A^2(\mu)) \quad (3.8g)$$

$$m_{D_3}^2(\mu) = m_{D_i}^2(\mu) \quad i=1,2 \quad (3.8h)$$

$$m_1^2(\mu) = m_{L_i}^2(\mu) + m_4^2(\mu) \quad (3.8i)$$

$$m_2^2(\mu) = m_1^2(\mu) - 3r^2 \left[m_{Q_3}^2(\mu) + m_{U_3}^2(\mu) + m_2^2(\mu) - m_4^2(\mu) + m_{3/2}^2 A^2(\mu) \right] \quad (3.8j)$$

$$m_{\Sigma_8}^2(\mu) = m_{3/2}^2 \times 12 \langle \alpha_3^3 \rangle_\mu \bar{z}_3^2 \quad (3.8k)$$

$$m_{\Sigma_3}^2(\mu) = m_{3/2}^2 \times 8 \langle \alpha_2^3 \rangle_\mu \bar{z}_2^2 \quad (3.8l)$$

$$m_{\Sigma_1}^2(\mu) = O(m_{3/2}^2) \quad (3.8m)$$

$$M_j(\mu) = \frac{\alpha_j(\mu)}{\alpha_j(M)} \bar{z}_j m_{3/2} \quad : j=1,2,3 \quad (3.8n)$$

$$A(\mu) = \frac{1}{1+6r^2} \left(\frac{16}{3} \langle \alpha_3^2 \rangle_\mu \bar{z}_3 + 3 \langle \alpha_2^2 \rangle_\mu \bar{z}_2 + \frac{13}{19} \langle \alpha_1^2 \rangle_\mu \bar{z}_1 \right) \quad (3.8o)$$

where

$$r^2 = \frac{\int_{\mu^2}^M d \log \mu'^2 \left(\frac{h_t}{4\pi} \right)^2 (m_{Q_3}^2 + m_{U_3}^2 + m_2^2 - m_4^2 + m_{3/2}^2 A^2)}{(m_{Q_3}^2 + m_{U_3}^2 + m_2^2 - m_4^2 + m_{3/2}^2 A^2)_\mu} \approx \int_{\mu^2}^M d \log \mu'^2 \left(\frac{h_t}{4\pi} \right)^2, \quad (3.9)$$

It is possible to relate r^2 to the t quark mass:

$$\left(\frac{m_t}{m_W}\right)^2 = N \frac{r^2}{1+6r^2} 2\sin^2\theta : \tan\theta = \frac{v_2}{v_1} \quad (3.10)$$

where

$$N(\mu) = \frac{\frac{16}{3} \alpha_3(\mu) + 3\alpha_2(\mu) + \frac{13}{15} \alpha_1(\mu)}{(1 - e^{-c_H}) \alpha_2(\mu)} \quad (3.11)$$

with

$$C_H = \frac{16}{3} \langle \alpha_3 \rangle_\mu + 2 \langle \alpha_2 \rangle_\mu + \frac{13}{15} \langle \alpha_1 \rangle_\mu \quad (3.12)$$

For orientation, with the inputs (3.5) and (3.6a), we find

$$N(m_W) = 18.15 \quad (3.13)$$

and hence

$$r^2 = \frac{1}{2} \left(\frac{m_t}{m_W}\right)^2 \frac{1}{18.15 \sin^2\theta - 3 \left(\frac{m_t}{m_W}\right)^2} \quad (3.14)$$

The above equations differ from those of Ref. 25 because of the different initial conditions (3.1). It is, however, easy to see from the Eqs. (3.8) how the basic mechanism of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ gauge symmetry breaking can still work as usual. Equations (3.8a-e), (3.8h,i) and (3.8k-m) tell us that most spin-zero particles are guaranteed to have positive mass squared and hence cannot develop vacuum expectation values. Comparing Eqs. (3.8f,g and j), we see that the t quark Yukawa coupling r^2 (3.9) makes a negative contribution to m_2^2 (and to a lesser extent $m_{Q_3}^2$, $m_{U_3}^2$). This may^{10,2-4)} be large enough to make m_2^2 vanish at some scale μ_0 : $m_2^2(\mu_0) = 0$. Thereafter, $m_2^2(\mu)$ is negative at renormalization scales $\mu \leq \mu_0$ and $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ breaking can occur at a renormalization scale $\mu \leq \mu_0 \ll m_P$.

To study this possibility numerically, we have taken the idealized limit $m_u \rightarrow 0$ corresponding to the absence of $H-\bar{H}$ mixing. It is necessary²⁾ to have a non-zero value of m_u in order to avoid an unacceptable light axion, but $m_u \ll m_{3/2}$ is nevertheless possible. In this case, $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ symmetry breaking and the dynamical determination of the gravitino mass occur just as in Ref. 3. We have the full potential

$$V = V_0(\mu) + \delta V(\mu) + \dots \quad (3.15)$$

where the tree-level potential

$$V_0(\mu) = \frac{g_2^2(\mu) + g_1^2(\mu)}{8} (|H|^2 - |\bar{H}|^2)^2 + m_1^2(\mu) |H|^2 + m_2^2(\mu) |\bar{H}|^2 \quad (3.16)$$

and the one-loop radiative corrections^{*)}

$$\delta V(\mu) = \frac{1}{64\pi^2} \sum_J (-1)^{2J} (2J+1) m_J^4(\mu) \left[\ln \frac{m_J^2(\mu)}{\mu^2} + c \right] \quad (3.17)$$

where c is a renormalization prescription dependent quantity. As we see later, physical quantities are independent of c . The minimum of $V_0(\mu)$ is when

$$\langle 0 | \bar{H} | 0 \rangle^2 = - \frac{4m_2^2(\mu)}{g_2^2(\mu) + g_1^2(\mu)} \equiv \frac{v^2}{2} \quad (3.18a)$$

$$V_0^{\min}(\mu) = \frac{-2m_2^4(\mu)}{g_2^2(\mu) + g_1^2(\mu)} \quad (3.18b)$$

In the neighbourhood of μ_0 we have

$$m_2^2(\mu) = C_0 \log \mu^2 / \mu_0^2 \quad (3.19a)$$

where

$$C_0 = 3 \frac{\alpha_t}{4\pi} (m_{Q_3}^2(\mu_0) + m_{U_3}^2(\mu_0) + A^2(\mu_0) m_{3/2}^2) - \frac{3\alpha_2}{4\pi} M_2^2(\mu_0) - \frac{3\alpha_1}{20\pi} M_1^2(\mu_0) \quad (3.19b)$$

*) Here we assume that terms like $\text{Str} \mathcal{M}^2 \times \Lambda_{\text{cutoff}}^2$ are not generated in the effective theory, or are at least independent of the $\bar{Y}_0^R$ field.

Using Eqs. (3.8), it is easy to see that $\delta V(\mu)$ (3.17) vanishes for some renormalization scale $\mu = O(m_{3/2})$. In the toy model of section 2.2, where $\xi_3^2 = \xi_2^2 = 25\xi_1^2 \equiv \xi^2$, for instance, we have

$$\delta V(\mu) = 8 \cdot 2 \cdot 3^4 m_{3/2}^4 \left(\log \frac{\xi^2 m_{3/2}^2}{\mu^2} + 1.6 + c - \frac{0.002}{3^4} \log \frac{m_{3/2}^2}{\mu^2} \right) \quad (3.20a)$$

We choose μ such that $\delta V(\mu) = 0$, or

$$\log \frac{\xi^2 m_{3/2}^2}{\mu^2} = \frac{(-1.6 - c - \frac{0.002}{3^4} \log \xi^2)}{(1 - \frac{0.002}{3^4})} \equiv L \quad (3.20b)$$

Renormalizing the theory at this scale $\mu \propto m_{3/2}$ (Y_0^R), we see from (3.18) and (3.19a) that

$$V_0^{\min}(t) = -\frac{2}{g_2^2 + g_3^2} \left(\frac{C_0}{\xi^2} \right)^2 e^{2t} (t - L)^2 \quad (3.21)$$

where $t \equiv \ln m_{3/2}^2 \xi^2 / \mu_0^2 = \sqrt{6} Y_0^R + \ln M^2 \xi^2 / \mu_0^2$ in the notation (2.32). The minimum value of $V_{\min}(t)$ (3.21) is obtained when

$$t = L - 1 \quad \text{ie. } \frac{\xi^2 m_{3/2}^2}{\mu_0^2} = \mu_0^2 e^{L-1} \quad (3.22)$$

and hence [using $m_W^2 = g_2^2 v^2 / 4$ and Eq. (3.18a)]:

$$m_W^2 = 2 \cos^2 \Theta_W \left[\frac{3\alpha_t}{4\pi} (m_{\nu_3}^2 + m_{Q_3}^2 + A^2 m_{3/2}^2) - \frac{3\alpha_2}{4\pi} M_2^2 - \frac{3\alpha_1}{20\pi} M_1^2 \right]_{\mu_0} \quad (3.23)$$

Note that no dependence on the constant c appears in the relation (3.23) between physical quantities. Using the $SU(2) \times U(1)$ breaking condition $m_2^2(\mu_0) = 0$, the relation (3.23) becomes [using eq. (3.10)]:

$$m_W^2 = 2 \cos^2 \Theta_W \frac{\alpha_2}{4\pi} \left[\frac{N m_1^2}{1 + 6r^2} - 3M_2^2 - \frac{3}{5} \frac{\alpha_1}{\alpha_2} M_1^2 \right] \quad (3.23a)$$

which is approximately

$$m_W^2 \approx \frac{m_{3/2}^2}{30} \left[\xi_2^2 + 0.1 \xi_1^2 \right] \quad (3.23b)$$

where we have used the fact that the numerical value of $N/(1+6r^2) \approx 16.88$. Note that the minimization with respect to Y_0^R or t fixes only the product $m_{3/2}^2 (\xi_2^2 + 0.1 \xi_1^2)$ and not $m_{3/2}^2$ alone. This enables us also to construct models²⁶⁾ in which the gravitino mass is fixed to be $O(m_P)$, while the gaugino masses are fixed dynamically.

It is clear that the value of the t quark mass which gives the correct value (3.23) of m_W is almost independent of the magnitudes of ξ_3 , ξ_2 , and ξ_1 , but does depend on their ratios. From Eq. (3.8j), $m_2^2(\mu_0) = 0$ when

$$r^2 = \frac{0.44\bar{\xi}_2^2 + 0.04\bar{\xi}_1^2}{3(15.4\bar{\xi}_3^2 + 0.4\bar{\xi}_2^2 + 0.04\bar{\xi}_1^2 + 3.8\bar{\xi}_3\bar{\xi}_2 + 0.9\bar{\xi}_3\bar{\xi}_1 + 0.1\bar{\xi}_2\bar{\xi}_1)} \quad (3.24)$$

The coefficients $\langle \alpha_i^n \rangle_{\mu=O(m_W)}$ are almost the same in the minimal model (3.4c, 3.5 and 3.6a) and in the modified model (3.4c, 3.5 and 3.6b). Taking as an illustration the values of the gaugino mass ratios found in the simplest model discussed in Section 2:

$$\bar{\xi}_3 : \bar{\xi}_2 : \bar{\xi}_1 = 5 : -5 : -1 \quad (3.25)$$

we find from (3.24) and (3.14) that

$$m_t \approx 53 \text{ GeV} \quad (3.26)$$

Even within the stated assumptions, this estimate must be allowed a 20% error because of higher order uncertainties. Varying the stated assumptions, the value of m_t is clearly sensitive to the model-dependent values of the ξ_i (3.25) inserted into Eq. (3.24). The effects of varying the low-energy spectrum so as to change $\alpha_i^{-1}(m_P)$ (3.6a,b) are probably less severe. In general, we would expect the value of m_t (3.26) to be decreased if $H-\bar{H}$ mixing is included [$m_4 \neq 0$ in Eqs. (3.8)]. Notice, however, that some of the ambiguities present in previous estimates of the t quark mass have now disappeared. We do not have any ambiguity in the initial value of A ¹¹⁾, since the theory is supersymmetric when renormalized at the Planck

scale, and the renormalized value of A (3.80) is determined by the ξ_i through r^2 (3.9). Also, we do not have any ambiguity in the squark or slepton to gaugino mass ratios: they are zero at the Planck scale. The same calculation that led to the t quark mass estimate (3.26) also predicts the entire low-energy sparticle spectrum, with the results given in the Table. We see that the $SU(2)$ -singlet charged sleptons E_i are expected to be the lightest spin-zero particles. This is because all SUSY breaking masses for the first two generations come from their gauge interactions, and the $SU(2)$ -singlet charged sleptons E_i only experience the relatively weak $U(1)$ gauge interactions. The spectrum of spin- $\frac{1}{2}$ sparticles (gaugino/Higgsino mixtures) is more difficult to predict since it depends on the magnitudes of the ξ_i as well as on their ratios, and also varies with the magnitude of $H-\bar{H}$ mixing. In the Table we give their values for $\xi_3 : \xi_2 : \xi_1 = 5 : -5 : -1$, and in the limit of small $H-\bar{H}$ mixing ϵ .

4. PROBLEMS AND PROSPECTS

In this paper, we have posed the problem how to determine the gravitino mass dynamically, and thus "solve" the hierarchy problem^{3,4)}, in the presence of heavy fields. We have proposed a class of solutions which exploit non-compact Kähler manifolds of the form $SU(n,1)/SU(n) \times U(1)$, reminiscent of those found⁹⁾ in $N \geq 5$ extended supergravity theories. The resulting low-energy theory is globally supersymmetric, apart from SUSY breaking gaugino masses which can be induced if the gauge superfields have non-trivial kinetic terms as in $N \geq 4$ extended supergravity theories^{7,9)}. The global SUSY of the low-energy chiral fields is not maintained by radiative corrections, and we impose it as an initial renormalization condition on the spin-zero masses when the low-energy theory is renormalized at a scale $\mu = O(m_P)$. We have in the backs of our minds the existence of a Theory of Everything (TOE), probably based on extended supergravity, which may provide these initial conditions in a natural way because the TOE is finite.

We find it suggestive that our efforts to "solve" the hierarchy problem repeatedly lead us back to features found in extended supergravity theories: non-compact global symmetries and coset spaces, non-minimal gauge kinetic terms, etc. At the moment, our use of these features in phenomenological $N = 1$ supergravity theories is not justified by any derivation from extended supergravities of an effective theory incarnating these desirable properties and avoiding undesirable ones. For example, what to make of the phenomenological tragedy that all extended supersymmetric theories contain real, not chiral, fermion spectra? This is an ever more pressing problem which does not seem to be nearing solution. Radical ideas have been proposed, which include the suggestion²²⁾ that all our observable fields are composites made out of preons which are the fields appearing in the original formulation of $N = 8$ extended supergravity. It has even been suggested²³⁾ that the physical spectrum might contain infinite-dimensional representations of a non-compact global symmetry. These ideas have never been supported by dynamical calculations, and there are suggestions²⁷⁾ that global non-compact symmetries such as those found in extended supergravities may actually inhibit the formation of bound states. An entirely different rôle^{3,4)} for non-compact global symmetry has been developed in this paper, and its compatibility with previously proposed ideas is dubious. It is by no means clear how our non-compact coset manifold could emerge in a composite scenario, even if the preons had an analogous structure. In addition to these conundra, we have had to appeal to a TOE based on extended supergravity theory as a *deus ex machina* which obligingly guarantees the correct initial renormalization conditions at $\mu = 0(m_P)$.

Further rummaging through the attic of extended supergravity may bring us more serendipitous discoveries. However, there is an urgent need to bring it down to ground with more meaningful dynamical calculations.

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Table: Mass spectrum in toy model

Particle	Mass in GeV
t	53
$\tilde{e}, \tilde{\mu}, \tilde{\tau}_L$	300
$\tilde{\nu}_{e,\mu,\tau}$	290
$\tilde{e}, \tilde{\mu}, \tilde{\tau}_R$	57
$\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}_L$	840
$\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}_R$	790
$\tilde{b}_L$	820
$\tilde{b}_R$	790
$\tilde{t}_1$	740
$\tilde{t}_2$	830
Σ_3	1200
Σ_2	480
$H^\pm$	300
H^0	290, 290, 92
$\tilde{g}$	440
$\tilde{W}^\pm/\tilde{H}^\pm$	190, $\epsilon \geq 20$
$\tilde{\gamma} + \dots$	50
$\tilde{Z} + \dots$	120
$\tilde{H}_{1,2}$	20, $\epsilon \geq 20$