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SPINORS : SPECIAL EMBEDDING OF COLOUR AND CHIRAL FERMIONS

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ABSTRACT

Special embedding of colour in an $SO(18)$ complex spinor is analyzed in an attempt to incorporate chiral fermion generations. We obtain five normal generations. In addition, there are $Q = 5/3, -4/3$ colour triplet quarks and a doubly charged lepton. We also have shiny quarks (colour sextets and octets). The unification scale is extremely low (3×10^6 GeV) and there exists a low mass magnetic monopole.

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One of the most difficult problems confronting particle physics theory today is understanding the chirality and the family structure of fermions in the low energy world. In gauge theories, the chirality is most conveniently expressed as complexity ¹⁾ of the fermion representation under the low energy gauge group $SU(3)_C \times SU(2) \times U(1)$. Not all complex representations of $SU(3)_C \times SU(2) \times U(1)$ become real under the exact gauge symmetry of the world, $SU(3)_C \times U(1)_{em}$. The reality under $SU(3)_C \times U(1)_{em}$ depends on the symmetry breaking pattern or the hypercharge assignment ²⁾. But spinor representations of orthogonal groups are always real under $SU(3)_C \times U(1)_{em}$ ²⁾.

Spinors of $SO(4n + 2)$ are the beloved fermion representations ³⁾⁻⁶⁾. They provide complexity under the gauge group $SO(4n + 2)$ and ensure the reality under $SU(3)_C \times U(1)_{em}$. Furthermore, these are the natural generalizations (for $n \geq 3$) of the minimal $SU(5)$ or $SO(10)$ spinor for a flavour unification. For normal quarks and leptons with the bare value of $\sin^2 \theta_W^0 = 3/8$, however, one would not get any light generations from $n \geq 3$ [i.e., from $SO(14)$, $SO(18)$, etc.].

Flavour unification with "one spinor" of $SO(4n + 2)$ needs unfamiliar quarks and leptons. One example is the existence of hypercoloured quarks and leptons ⁵⁾. The other example is through non-standard hypercharge assignment to preserve the chirality, and necessarily introduces unfamiliar quarks ($Q=5/3$, $-4/3$) and leptons ($Q=2$). The maximum number of generations obtained in this way is three lepton generations, two quark generations, and unfamiliar generations ⁴⁾. This is phenomenologically unsuccessful because the b quark is known to decay quite normally ^{*}.

This leads me to consider another possibility for embedding $SU(3)_C \times SU(2) \times U(1)$ in $SO(4n + 2)$. We pursue a "special embedding of colour" in $SO(4n + 2)$. With the regular embedding of colour, a fundamental representation of $SU(2n + 1)$, a subgroup of $SO(4n + 2)$, contains three slots for a fundamental representation of $SU(3)_C$. In this case, all quarks are normally charged, in the sense of $Q = \frac{2}{3} + \text{integer}$. With a special embedding of colour, a fundamental representation of $SU(2n + 1)$ contains more than three slots for the $SU(3)_C$ embedding. With the special embedding of colour, we would expect integer charge quarks for triality zero representations of $SU(3)_C$. In this case, the motivation for using spinors of $SO(4n + 2)$ to obtain the colour triplets and anti-triplets with fractionally charged quarks is weakened. The colour and charge of fermions are the consequence of spontaneous symmetry breaking.

^{*} For another attempt, see J.E. Kim (Ref. 7)

The simplest example of the special embedding of colour is identifying the $SU(6)$ subgroup of $SO(4n+2)$ as the colour gauge group. The $SU(6)$ is broken to $SO(6)$ by the vacuum expectation values of the second rank symmetric tensor $\phi_{AB} = 21$. The fundamental representation 6 of $SU(6)$ becomes the fundamental representation 6 of $SO(6)$. Since $SO(6)$ is isomorphic to $SU(4)$, we can treat the 6 of $SO(6)$ as the pseudo-real 6 of $SU(4)$. When $SU(4)$ breaks to $SU(3)_c \times U(1)_y$, the 6 splits into 3_y and 3_{-y}^* of $SU(3)_c$. Thus the fundamental representation of $SU(2n+1)$ contains 3 and 3^* of $SU(3)_c$.

The minimal $SO(4n+2)$ group to realize the above special embedding of colour is $SO(18)$ since we must pick up a large enough group to contain $SU(6) \times SU(2)$. Also, the $SO(18)$ is the maximal group if we impose the asymptotic freedom above the grand unification scale. Instead of discussing the $SO(18)$ spinor, we will discuss an $SU(9)$ gauge group with fermion representations

$$\psi_A + \psi^{AB} + \psi_{ABC} + \psi^{ABCD} = 9^* + 36 + 84^* + 126 \quad (1)$$

where the $SU(9)$ indices $A, B, \dots$, etc., are antisymmetrized, and an $SU(9)$ singlet is neglected.

With the special embedding of colour, the $SU(6)$ branching rule is as follows ⁸⁾;

$$\begin{aligned} SU(6) &\supset SU(4): \\ (10000) 6 &= (010) 6 \\ (01000) 15 &= (101) 15 \\ (00100) 20 &= (200) 10 + (002) 10^* \\ (10001) 35 &= (101) 15 + (020) 20' \end{aligned} \quad (2)$$

We also cite the well-known $SU(4)$ branching rule,

$$\begin{aligned} SU(4) &\supset SU(3)_c \times U(1) \\ (100) 4 &= (00) 1_{-1/2} + (10) 3_{+1/6} \\ (010) 6 &= (10) 3_{-1/3} + (01) 3_{+1/3}^* \\ (101) 15 &= (00) 1_0 + (10) 3_{+2/3} \\ &\quad + (01) 3_{-2/3}^* + (11) 8_0 \end{aligned} \quad (3)$$

In Eqs. (2) and (3), the Dynkin symbol is accompanied by the dimension of the representation. In Eq. (3), the U(1) hypercharges are denoted as subscripts.

Let us look at branching rules of SU(9) representations into SU(6) x SU(2) x U(1). SU(9) indices are A,B,C, etc., and the first six indices of them are assigned to SU(6) indices P,Q,R, etc.; the seventh index is a singlet with a U(1) hypercharge only, and the eighth and ninth indices account for the electroweak indices, i,j, etc.

$$SU(9) \supset SU(6) \times SU(2) \times U(1):$$

$$\psi_A = \psi_P + \psi_7 + \psi_i = \underline{(6^*, 1)_0} + (1, 1)_{+1} + (1, 2)_{-1/2} \quad (4)$$

$$\begin{aligned} \psi^{AB} &= \psi^{PQ} + \psi^{P7} + \psi^{Pi} + \psi^{7i} + \psi^{ij} \\ &= \underline{(15, 1)_0} + (6, 1)_{-1} + (6, 2)_{+1/2} + (1, 2)_{-1/2} + (1, 1)_{+1} \end{aligned} \quad (5)$$

$$\begin{aligned} \psi_{ABC} &= \psi_{PQR} + \psi_{PQ7} + \psi_{PQi} + \psi_{P7i} + \psi_{Pij} + \psi_{7ij} \\ &= \underline{(20, 1)_0} + (15^*, 1)_{+1} + (15^*, 2)_{-1/2} + (6^*, 2)_{+1/2} + (6^*, 1)_{-1} + \underline{(1, 1)_0} \end{aligned} \quad (6)$$

$$\begin{aligned} \psi^{ABCD} &= \psi_{PQ} + \psi^{PQR7} + \psi^{PQRi} + \psi^{PQ7i} + \psi^{PQij} + \psi^{P7ij} \\ &= \underline{(15^*, 1)_0} + (20, 1)_{-1} + (20, 2)_{+1/2} + (15, 2)_{-1/2} + (15, 1)_{+1} + \underline{(6, 1)_0} \end{aligned} \quad (7)$$

where the hypercharge is written as subscripts. The underlined representations form a real subset and hence are removed by the survival hypothesis. Even though we consider the branching rule to $SU(6) \times SU(3)$ for a spontaneous symmetry breaking reason, the reality of the subset is unchanged. Thus the light chiral fermions are reduced from 255 to 192.

Using the branching rules of (2) and (3), we can now show $SU(3)_C \times SU(2)_L \times U(1)$ representations of 255 fermions. The electroweak $U(1)$ is a direct sum of $U(1)$'s of Eq. (3) and Eqs. (4)-(7).

$$\psi_A = (3^*, 1, \frac{1}{3}) + (3, 1, -\frac{1}{3}) + (1, 1, 1) + \underline{(1, 2, -\frac{1}{2})} \quad (8)$$

$$\begin{aligned} \psi^{AB} = & (8, 1, 0) + (3, 1, \frac{2}{3}) + (3^*, 1, -\frac{2}{3}) \\ & + (1, 1, 0) + (3, 1, -\frac{4}{3}) + (3^*, 1, -\frac{2}{3}) \\ & + \underline{(3, 2, \frac{1}{6})} + \underline{(3^*, 2, \frac{5}{6})} + \underline{(1, 2, -\frac{1}{2})} \\ & + (1, 1, 1) \end{aligned} \quad (9)$$

$$\begin{aligned} \psi_{ABC} = & (6, 1, \frac{1}{3}) + (6^*, 1, -\frac{1}{3}) + (3, 1, -\frac{1}{3}) + (3^*, 1, \frac{1}{3}) \\ & + (1, 1, -1) + (1, 1, 1) + (8, 1, 1) + (3^*, 1, \frac{1}{3}) \\ & + (3, 1, \frac{5}{3}) + (1, 1, 1) + (8, 2, -\frac{1}{2}) + (3^*, 2, -\frac{7}{6}) \\ & + \underline{(3, 2, \frac{1}{6})} + \underline{(1, 2, -\frac{1}{2})} + \underline{(3^*, 2, \frac{5}{6})} + \underline{(3, 2, \frac{1}{6})} \\ & + (3^*, 1, -\frac{2}{3}) + (3, 1, -\frac{4}{3}) + (1, 1, 0) \end{aligned} \quad (10)$$

$$\begin{aligned}
 \psi^{ABCD} = & (8, 1, 0) + (3^*, 1, -\frac{2}{3}) + (3, 1, \frac{2}{3}) + (1, 1, 0) \\
 & + (6, 1, -\frac{2}{3}) + (6^*, 1, -\frac{4}{3}) + (3, 1, -\frac{4}{3}) + (3^*, 1, -\frac{2}{3}) \\
 & + (1, 1, -2) + (1, 1, 0) + (6, 2, \frac{5}{6}) + (6^*, 2, \frac{1}{6}) \\
 & + \underline{(3, 2, \frac{1}{6})} + \underline{(3^*, 2, \frac{5}{6})} + \underline{(1, 2, -\frac{1}{2})} + \underline{(1, 2, \frac{3}{2})} \quad (11) \\
 & + (8, 2, -\frac{1}{2}) + \underline{(3, 2, \frac{1}{6})} + (3^*, 2, -\frac{7}{6}) + \underline{(1, 2, -\frac{1}{2})} \\
 & + (8, 1, 1) + (3, 1, \frac{5}{3}) + (3^*, 1, \frac{1}{3}) + (1, 1, 1) \\
 & + (3, 1, -\frac{1}{3}) + (3^*, 1, \frac{1}{3})
 \end{aligned}$$

where the normal quark and lepton doublets are underlined. Note that we obtain five generations of quarks and leptons which cancel $SU(3)_C \times SU(2) \times U(1)$ anomalies among themselves.

In addition, there are exotic light fermions whose doublets are as follows:

$$\begin{aligned}
 & \begin{pmatrix} \frac{4}{3} \\ \frac{1}{3} \end{pmatrix}^6_{5/6}, \quad \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}^{6^*}_{1/6}, \quad 2 \times \begin{pmatrix} 0 \\ -1 \end{pmatrix}^8_{-1/2}, \\
 & 3 \times \begin{pmatrix} \frac{4}{3} \\ \frac{1}{3} \end{pmatrix}^{3^*}_{5/6}, \quad 2 \times \begin{pmatrix} -\frac{2}{3} \\ -\frac{5}{3} \end{pmatrix}^{3^*}_{-7/6}, \quad (12) \\
 & \begin{pmatrix} 2 \\ 1 \end{pmatrix}^1_{3/2}
 \end{aligned}$$

where the electromagnetic charges are shown inside brackets, and $SU(3)$ properties and hypercharges are represented as superscripts and subscripts respectively. All doublets are left-handed. As we have pointed out earlier, for a property of spinor representations, there must be singlets corresponding to Eq. (12) to make the fermions real under $SU(3)_C \times U(1)_{em}$. Also it is easy to see that the exotic fermions cancel $SU(3)_C \times SU(2) \times U(1)$ anomalies among themselves. Note that we have integer charge colour octet quarks in additions to the so-called⁹⁾ "shiny quarks", colour 6 and 6^* . This results from the special embedding of colour, even though we started from completely antisymmetric representations of Eq. (1).

The implications of colour 6 and colour 8 quarks have been previously discussed in the literature^{9),10)}. But our special embedding of $SU(3)_C$ in a spinor is the first example giving a reason why these exotic colours might appear in nature.

The dynamical role played by these exotic quarks has been nicely discussed by Maricano¹⁰⁾. Even though it is a conjecture*, it is worthwhile to repeat it here. Suppose that the chiral symmetry breaking for a representation R satisfies the relation

$$C_2(R) \alpha_s(\mu_R) = C_2(3) \alpha_s(\mu_3) \quad (13)$$

where $R = 3, 6, 8$, etc. Because $C_2(6)/C_2(3) = 5/2$ and $C_2(8)/C_2(3) = 9/4$, the chiral symmetry breaking for the colour sextets and octets occurs at much higher mass scale than that of colour triplets. For example¹⁰⁾, if we take $\alpha_s(\mu_3) = 0.2$, we obtain $\mu_6/\mu_3 = 839$ and $\mu_8/\mu_3 = 273$. For the colour sextets the breaking of the $SU(4) \times SU(4) \times U(1)$ global symmetry produce 15 Goldstone bosons. Also from chiral symmetry breaking of colour octets, we obtain 15 Goldstone bosons. Out of these 30 Goldstone bosons, three are absorbed to $W^\pm$ and Z^0 to render them massive. Because the representation (12) contains only doublets, the tree level relation $M_W = M_Z \cos \theta_W$ is preserved. The remaining 27 bosons are heavy pions.

Because the five light generations and those exotic quarks are embedded together in the spinor representation of $SO(18)$, the chiral symmetries of these light quarks are destroyed and we expect the light fermion masses of order¹²⁾ $(\mu_6^3 \text{ and/or } \mu_8^3)/M_G^2$. However, it seems difficult to obtain a realistic fermion mass matrix in this way only. We eventually need Higgs doublets for the fermion masses.

* However, there is a support from Monte Carlo lattice calculations. See Ref. 11).

For the estimate of the grand unification scale^{13),3)}, we need a knowledge of the symmetry breaking pattern. We take a simple route for illustration¹⁴⁾:

$$\begin{aligned}
 SU(9) &\longrightarrow SU(6) \times SU(3) \quad \text{by } \langle H^{789} \rangle = M_G \\
 &\longrightarrow SU(3)_c \times SU(2) \times U(1) \quad \text{by } \langle H^{\{AB\}} \rangle = M_1 \\
 &\quad \quad \quad = \langle H^{ABCD} \rangle \\
 &\longrightarrow SU(3)_c \times U(1)_{em} \quad \text{at } 80 \text{ GeV}
 \end{aligned} \tag{14}$$

In the calculation, we will neglect Higgs contributions, because it may be possible to break the second and third steps dynamically, and also because it is hard to include precisely the Higgs effects at present. We will also neglect the threshold effect. But we will take into account the fact that the $SU(3)_c$ coupling at M_1 is $1/\sqrt{2}$ times the $SU(6)$ coupling at M_1 . The reason is the following. At M_1 , the $SO(6)$ coupling descending from $SU(6)$ is describing the coupling constant of the vector (6) representation. The spinor representation of $SO(6)$ has the coupling constant $1/\sqrt{2}$ times that of the vector representation. The $SU(4)$ gauge coupling to the fundamental representation 4 of $SU(4)$ is the magnitude of the gluon coupling to the colour triplet quarks.

The one-loop renormalization group of $SU(3)_c$ equations gives the following relations:

$$\frac{1}{g_{6c}^2(M_1)} = \frac{1}{g^2(M_G)} + \frac{1}{8\pi^2} \left(-\frac{66}{3} + \frac{2}{3} \cdot 24 \right) \ln \frac{M_G}{M_1} \tag{15}$$

$$\frac{1}{g_{3w}^2(M_1)} = \frac{1}{g^2(M_G)} + \frac{1}{8\pi^2} \left(-\frac{33}{3} + \frac{2}{3} \cdot 32 \right) \ln \frac{M_G}{M_1} \tag{16}$$

with obvious notations. At M_1 , the hypercharges from $SU(6)$ and $SU(3)$ mix to form the electroweak hypercharge. Therefore, the electroweak hypercharge at M_1 is given by ¹⁵⁾

$$\frac{1}{g'^2(M_1)} = \frac{4/3}{g_{6c}^2(M_1)} + \frac{3}{g_{3W}^2(M_1)} = \frac{13/3}{g^2(M_G)} + \frac{1}{8\pi^2} \frac{215}{9} \ln \frac{M_G}{M_1} \quad (17)$$

At M_w , we obtain

$$\begin{aligned} \frac{1}{g_c^2(M_w)} &= \frac{1}{g^2(M_G)} + \frac{1}{8\pi^2} \left(-\frac{16}{3}\right) \ln \frac{M_G}{M_1} \\ &\quad + \frac{1}{8\pi^2} \left(-\frac{33}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot 48\right) \ln \frac{M_1}{M_w} \end{aligned} \quad (18)$$

$$\frac{1}{g_2^2(M_w)} = \frac{1}{g^2(M_G)} + \frac{1}{8\pi^2} \frac{31}{3} \ln \frac{M_G}{M_1} + \frac{1}{8\pi^2} \left(-\frac{22}{3} + \frac{2}{3} \cdot 32\right) \ln \frac{M_1}{M_w} \quad (19)$$

$$\frac{1}{g'^2(M_w)} = \frac{1}{g'^2(M_1)} + \frac{1}{8\pi^2} \left(\frac{2}{3} \text{Tr } Y^2\right) \ln \frac{M_1}{M_w} \quad (20)$$

With $\text{Tr } Y^2 = 128$. One solution to these equations is

$$M_G \cong 2.7 \times 10^6 \text{ GeV}, \quad M_1 \cong 730 \text{ GeV}$$

$$\sin^2 \theta_w(M_w) = 0.215, \quad \alpha_{em}^{-1}(M_w) = 128$$

$$\alpha_G \cong 0.11, \quad \alpha_6(M_1) \cong 0.81$$

$$\alpha_s(M_w) \cong 0.13$$

where we used $\alpha_s = \alpha_c/2$. Observe the strong coupling regime around M_1 where we naively used the one loop renormalization group equations.

Note that the bare value of $\sin^2\theta_W^0 = 1/(1 + C^2) = 3/16$ can be renormalized to 0.215. The colour gauge coupling constant is not asymptotically free between M_W and M_1 . It is asymptotically free above M_1 and also below M_W if exotic quarks are removed above M_W .

There are two critical consequences of the model. One is the low scale of the grand unification which is not attractive in view of the fast proton decay. We should forbid proton decay either by rotating it away (i.e., by the kinematical reasons) or by imposing a global symmetry. However, each of these alternatives is difficult to realize. Another problem is the low mass of the magnetic monopole predicted. The mass of the magnetic monopole is $\sim M_1/\alpha \sim 10^5$ GeV, which will be problematic in the standard cosmology.

In conclusions, we have pursued the special embedding of colour in the spinor representation of $SO(18)$ which is the only reasonable group for this study. We obtained five generations of quarks and leptons in addition to shiny quarks (colour 6, 6* and 8) and unfamiliar fermions, i.e., $Q = 5/3, -4/3$ quarks and a doubly-charged lepton. The most difficult problem unsolved at present is how to make sense out of fantastically low grand unification scales and low mass magnetic monopoles. Unless these problems are solved, it is likely that the attractive idea of flavour unification¹⁾ incorporating shiny quarks^{9),10)}, does not work in practice, or the spinor of $SO(4n + 2)$ is representing merely a classification scheme of these quarks and leptons and compositeness will be revealed even at shorter distances.

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