Ref.TH.3835-CERN

1/N² EXPANSION OF MEAN FIELD FOR LATTICE CHIRAL MODELS

Y. Brihaye

CERN - Geneva

A B S T R A C T

For lattice chiral models in $U(N)$ and $SU(N)$, we study in detail the $1/N^2$ and $1/N^4$ corrections to the large- N mean field prediction. We use a systematic expansion of the single link group integrals; special attention is paid to the free energy and the two-point correlation functions of the models and the main differences between $U(N)$ and $SU(N)$ are pointed out.

Ref.TH.3835-CERN

February 1984

Among the techniques developed for the study of lattice models, mean-field¹⁾ (m.f.) is a very attractive one because it is non-perturbative in the coupling constant and it furnishes good information on the phase diagram of spin and gauge models.

The implications of m.f. for lattice chiral models were first studied by Kogut et al.²⁾; more recently improved m.f. was obtained by several groups^{3),4)}. Most of the information is available for a few finite values of N ($N=2,3,4,5$) or in the large N limit. Improved m.f. for finite N requires a good knowledge of the so-called "single link group integral" which has been studied by several authors⁵⁾. Here we will use the single link integral under the form of an $1/N^2$ expansion. This new form, recently obtained⁶⁾ for $U(N)$ and $SU(N)$, makes the computations of the $1/N^2$ (and even higher order) corrections straightforward in improved m.f.. It simplifies a technique developed earlier by Müller and Rühl⁷⁾ and used more recently in Ref. 4).

In the first part of the paper, we focus on the free energy of the lattice chiral model, exhibiting the large N , $1/N^2$ and $1/N^4$ corrections for the $U(N)$ and $SU(N)$ symmetry groups. In this respect, our results are complementary to those of Refs. 3) and 4). We work with an arbitrary number of spacetime dimensions, all quantities appearing in terms of the variable βd . We check, up to our best knowledge, the agreement of our formula with the standard $1/\beta$ expansion and point out some restrictions in the domain of validity of the corrections.

In the second part, we show how m.f. can provide an expression for the two-point correlation functions. Afterwards, some conclusions and hopes are drawn. We summarize in an Appendix the one-link group-integral formula of interest for the paper.

In order to apply the mean-field technique to the chiral model on the lattice, one changes constrained variables into unconstrained ones by use of a random field transform:

$$Z_N = \int dU_m \exp N\beta \sum_{\mu, m} \text{tr} (U_m U_{m+\mu}^+ + U_{m+\mu} U_m^+) \quad (1)$$

$$= \int dV_m dA_m \exp \sum_m \left\{ \beta N \sum_{\mu} \text{tr} (V_m V_{m+\mu}^+ + V_{m+\mu} V_m^+) \right. \\ \left. - N \text{tr} (A_m V_m^+ + V_m A_m^+) + N \mathcal{W}(A_m, A_m^+) \right\} \quad (2)$$

$$W(A, A^\dagger) = \frac{1}{N} \ln \int dU \exp N \operatorname{tr} (AU^\dagger + UA^\dagger) \quad (3)$$

Here, U_n denotes matrices in $U(N)$ or $SU(N)$, dU_n is the Haar measure on this group and A_n, V_n are arbitrary complex $N \times N$ matrices. The integration over U_n in Eq. (3) is the single link integral studied by many authors⁵⁾.

Following the standard m.f. approach, one searches for a configuration of A_n, V_n which minimizes the integrand in Eq. (2) by doing the ansatz that the saddle point is translational invariant and proportional to the identity matrix. So the problem consists in maximizing the following effective action:

$$-\frac{S_{\text{eff}}}{N^2 V} = 2\beta d v^2 - 2a v + \frac{w(a, a^*)}{N}$$

$$A_m = a \mathbb{1}, \quad V_m = v \mathbb{1}. \quad (4)$$

for which the saddle point equations are

$$a = 2\beta d v \quad (5a)$$

$$v = \frac{1}{2N} \frac{\partial w}{\partial a} \quad (5b)$$

We will first study the $U(N)$ case; when A is proportional to the identity, the form of w of interest [see Eq. (A.2)] reduces to the expansion below [first discovered in Ref. 8)].

$$\frac{w}{N} \Big|_{\lambda_i = a^2} = 2a - \frac{1}{2} \ln 2a - \frac{3}{4} - \frac{1}{8N^2} \ln \left(1 - \frac{1}{2a}\right) \quad (6)$$

$$+ \frac{3}{2^7 N^4} \cdot \frac{1}{(2a-1)^3} + \dots, \quad \text{if } a > \frac{1}{2}$$

for $a > \frac{1}{2}$. Then, the saddle point equations (5) can be arranged in the following form:

$$a = 2\beta d v \quad (7a)$$

$$\frac{a^2}{2\beta d} - a + \frac{1}{4} = \frac{-1}{N^2 \cdot 16(2a-1)} + \frac{-9a}{N^4 2^7(2a-1)^4} + \dots \quad (7b)$$

assuming that the function a admits an $1/N^2$ expansion, i.e.,

$$a = a_0 + \frac{1}{N^2} a_1 + \frac{1}{N^4} a_2 + \dots \quad (8)$$

we can recursively determine the quantities a_i

$$a_0 = \beta d \left(1 \pm \sqrt{1 - \frac{1}{2\beta d}} \right) \quad (9a)$$

$$a_1 = \frac{-a_0}{8(2a_0-1)^2} \quad (9b)$$

$$a_2 = \frac{-a_0(26a_0-1)}{2^7(2a_0-1)^5} \quad (9c)$$

Obviously, solutions (9) are acceptable only in the region $\beta d \geq \frac{1}{2}$ (weak coupling); moreover, the solution corresponding to the -1 sign in Eq. (9a) has to be rejected because it does not obey the condition $a > \frac{1}{2}$ in Eq. (6). This solution can be considered as an analytic continuation of the $+1$ solution. After substitution of solution (9) in the effective action (4) we have obtained

$$\frac{F_{\text{eff}}}{N^2 V} = \left(a_0 - \frac{1}{2} - \frac{1}{2} \ln 2a_0 \right) - \frac{1}{8N^2} \ln \left(1 - \frac{1}{2a_0} \right) + \frac{1}{2^5 N^4 (2a_0-1)^3} + \dots \quad (10)$$

The agreement of Eq. (10) with the standard weak coupling expansion can be checked by expanding a_0 in powers of $1/\beta d$. Neglecting an irrelevant additive constant, we found

$$\frac{F_{eff}}{VN^2} = 2\beta d - \frac{1}{2} \ln 2\beta d + \frac{N+1}{2^5 N^2} \frac{1}{\beta d} + \frac{N^2+2}{2^8 N^2} \left(\frac{1}{\beta d}\right)^2 + \frac{10N^4+25N^2+6}{3 \cdot 2^{12} N^4} \left(\frac{1}{\beta d}\right)^3 \quad (11)$$

to be compared with the one-loop calculation presented in Ref. 3).

While the large N limits of a (i.e., a_0) and F are defined on the interval $\beta d \geq \frac{1}{2}$, the $1/N^2$ corrections (a_1, a_2) become infinite when βd approaches the critical point ($\beta d = \frac{1}{2}$). The singularities are stronger while increasing the order in $1/N^2$. Therefore, the corrections have to be considered seriously in a smaller interval $\frac{1}{2} + \varepsilon \leq \beta d \leq \infty$, where ε cannot be determined as far as one considers a finite order expansion. It is obvious that if we could treat $w(a)$ exactly, no such singularity problem would occur. This problem was already raised [see Refs. 7) and 8)]; it is related to the fact that the expansion (A.2) is unable to reproduce the function w around the phase transition point arising in the large N limit.

The one-loop correction to the m.f. approximation (10) can be computed by expanding A_n and V_n in (2) around the saddle point (9); it is convenient to split the fluctuations into Hermitian and anti-Hermitian parts:

$$A_m = a \mathbb{1} + B_m + i C_m, \quad B_m = B_m^+, \quad C_m = C_m^+ \quad (12a)$$

$$V_m = v \mathbb{1} + D_m + i E_m, \quad D_m = D_m^+, \quad E_m = E_m^+ \quad (12b)$$

Then, substituting (12) into the effective action (4), we have to identify the quadratic form in the fluctuation. The derivation is the same as in the large N limit case presented in Ref. 3); therefore, we give here only the final result in a form suitable for the inclusion of $1/N^2$ corrections (or more).

$$\begin{aligned} \mathcal{M} = \sum_m \left\{ 2\beta \sum_{\mu} t E_m E_{m+\mu} - 2t E_m C_m + f(a) t C_m C_m \right. \\ \left. + 2\beta \sum_{\mu} t D_m D_{m+\mu} - 2t B_m D_m + f(a) t B_m B_m \right. \\ \left. + 2 \sum_{i,j} B_m^{ii} B_m^{jj} (g(a) \delta_{ij} + \frac{h(a)}{N}) + 2 k(a) \sum_{i \neq j} B_m^{ij} B_m^{ji} \right\} \quad (13) \end{aligned}$$

where f, g, h, k are defined by

$$f(a) = \left. \frac{\partial}{\partial \lambda_k} w \right|_{\lambda_k = a^2} \quad (14)$$

$$h(a) = \left. \frac{1}{\lambda_k - \lambda_l} \left(\frac{\partial}{\partial \lambda_k} - \frac{\partial}{\partial \lambda_l} \right) w \right|_{\lambda_k = \lambda_l = a^2} \quad (15)$$

$$g(a) \delta_{kl} + \frac{1}{N} h(a) = \left. \frac{\partial^2}{\partial \lambda_k \partial \lambda_l} w \right|_{\lambda_k = \lambda_l = a^2} \quad (16)$$

Using the formula (A.2) one can compute the above quantities f, g, k, h .

$$f(a) = \frac{1}{a} \left(1 - \frac{1}{4a} \right) - \frac{1}{16N^2} \cdot \frac{1}{a^2(2a-1)} - \frac{9}{2^{10}N^4} \cdot \frac{1}{a^2(2a-1)^4} + \dots \quad (17)$$

$$g(a) = h(a) = \frac{1}{2a} \left(\frac{3}{2a} - 1 \right) + \frac{3}{32N^2} \cdot \frac{1}{a^2(2a-1)} + \frac{9}{2^{10}N^4} \cdot \frac{(5a-1)}{a^2(2a-1)^4} + \dots \quad (18)$$

$$h(a) = \frac{1}{16a^2} + \frac{1}{32N^2} \cdot \frac{1}{a^2(2a-1)^2} + \frac{9}{2^{10}N^4} \cdot \frac{6a-1}{a^2(2a-1)^5} + \dots \quad (19)$$

The equations of motion (7) imply $f(a) = v/a = 1/2\beta d$; then, after use of the standard technique, the effect of the determinant on the effective action can be put in the form

$$\frac{F_{eff}}{VN^2} \text{ (one loop)} = \frac{1}{2} \left\{ F_d(1) + \frac{N^2-1}{N^2} F_d(2\beta d(f+k)) + \frac{1}{N^2} F_d(2\beta d(f+k+h)) \right\} \quad (20)$$

with the notation of Ref. 4)

$$F_d(x) = - \int_{-\pi}^{\pi} \left(\frac{dp}{2\pi} \right)^d \ln \left(1 - \frac{x}{d} \sum_{\mu=1}^d \cos p_{\mu} \right) \quad (21)$$

The large N limit of (20) is in agreement with the result of Refs. 3) and 4). The special combinations of f , k , h appearing in Eq. (21) are the λ_i functions obtained in Ref. 7), here derived in a simpler way. They can be expressed in terms of a_0 in the following manner

$$f+k = \frac{1}{8a_0^2} \left[1 + \frac{1}{N^2} \cdot \frac{8a_0-3}{4(2a_0-1)^2} + o\left(\frac{1}{N^4}\right) \right] \quad (22a)$$

$$f+k+h = \frac{1}{4a_0^2} \left[1 + \frac{1}{N^2} \cdot \frac{a_0}{(2a_0-1)^2} + o\left(\frac{1}{N^4}\right) \right] \quad (22b)$$

The first non-trivial term in the coupling constant obtained from (20) is of order β^{-2} and takes the form

$$\frac{F_{eff}}{VN^2} \text{ (one loop)} = \left(1 + \frac{3}{N^2}\right) \left(\frac{1}{2^{11} \beta^2 d^3} + o(\beta^{-3} d^{-4}) \right) + \frac{1}{N^4} \left(\frac{1}{2^{12} \beta^3 d^4} + o(\beta^{-4} d^{-5}) \right) \quad (23)$$

The SU(N) case

The single link group integral for SU(N) is given in Eq. (A.3). To do naïve m.f., we need only the case where $\lambda_i = a^2$. Furthermore, the saddle point equation with respect to θ is simply solved by choosing a to be real ($\theta = 0$), so the formula of interest takes the form

$$\left. \frac{\tilde{w}}{N} \right|_{\substack{\lambda_i = a^2 \\ \theta = 0}} = w_\infty + \frac{1}{N^2} w_1 + \frac{1}{N^4} w_2 + \dots \quad (24)$$

where

$$w_1(a) = \ln \left(\sum_{\ell=-\infty}^{+\infty} \left(1 - \frac{1}{2a}\right)^{\frac{7}{2}(\ell^2 - \frac{1}{4})} \right) \equiv -\frac{1}{8} \ln \left(1 - \frac{1}{2a}\right) + s(a) \quad (25)$$

$$w_2(a) = \frac{1}{24(2a-1)^3} \cdot \frac{\sum_{\ell} (\ell^2 - \frac{1}{4})(\ell^2 - \frac{9}{4}) \left(1 - \frac{1}{2a}\right)^{\frac{\ell^2}{2}}}{\sum_{\ell} \left(1 - \frac{1}{2a}\right)^{\frac{\ell^2}{2}}} \quad (26)$$

Then the solution of the saddle point equation becomes rapidly tedious but a formal solution can be given: obviously, a_0 is as in (9a), and

$$a_1 = \frac{a_0^2}{2a_0 - 1} \dot{w}_2(a_0) \quad (27)$$

$$a_2 = \frac{a_0^2}{2a_0 - 1} \left\{ \dot{w}_2(a_0) + a_1 \ddot{w}_2(a_0) - \frac{a_1^2}{2a_0^3} \right\} \quad (28)$$

Further, the effective energy can be shown to be

$$\begin{aligned} \frac{F_{eff}}{VN^2} = & a_0 - \frac{1}{2} - \frac{1}{2} \ln 2a_0 + \frac{1}{N^2} w_1(a_0) \\ & + \frac{1}{N^4} \left(\frac{a_0^2}{2(2a_0-1)} (\dot{w}_1(a_0))^2 + w_2(a_0) \right) + o\left(\frac{1}{N^6}\right) \end{aligned} \quad (29)$$

For technical reasons, we will only discuss the $1/N^2$ correction. At this level, the only difference between $U(N)$ and $SU(N)$ is in the appearance of the function $s(a)$ defined in Eq. (25). This function behaves straightforwardly around $a = \frac{1}{2}$ and its asymptotic behaviour ($a \rightarrow \infty$) can be identified after one recognizes that⁹⁾:

$$\sum_{n=-\infty}^{+\infty} e^{-n^2 \pi Z} = \sqrt{\frac{\pi}{Z}} + O(|Z|)^{\gamma}; (Z \rightarrow 0) \quad (30)$$

where γ is arbitrarily large. This approximation was also used numerically by the authors of Ref. 4). Then, putting

$$Z = -\frac{1}{2\pi} \ln\left(1 - \frac{1}{2a}\right) \quad (31)$$

in Eq. (30) leads to the following approximation of $s(a)$:

$$s(a) \Big|_{a \text{ large}} = -\frac{1}{2} \ln \left[-\frac{1}{2\pi^2} \ln\left(1 - \frac{1}{2a}\right) \right] + \ln(1 + O(|Z|^{\gamma})). \quad (32)$$

If we expand the first term of Eq. (32) in powers of $1/\beta d$, neglecting the second term which is a non-perturbative one, and substitute into the effective action (29), the result is

$$\frac{F_{\text{eff}}}{VN^2} = 2\beta d - \frac{N^2-1}{2N^2} \ln 2\beta d + \frac{N^2-3}{2^5 N^2} \frac{1}{\beta d} + \frac{N^2-14}{2^8 N^2} \left(\frac{1}{\beta d}\right)^2 + \dots \quad (33)$$

This formula coincides (up to $1/N^4$ terms) with the one-loop calculation of Ref. 3).

So far, the one-loop correction to m.f. approximation (33) can be obtained using the same method as for $U(N)$ but taking into account the θ variable. Since this result was obtained in Ref. 4) (using a different technique) we do not present it again here. Instead, we just write explicitly the first non-trivial order appearing in the effective action, to be compared with its analogue in $U(N)$ [Eq. (23)].

$$\frac{F_{\text{eff}}}{VN^2} (\text{one loop}) = \frac{1}{2^4 \beta^2 d^3} \left(1 + \frac{1}{N^2} \left(7 + \frac{1}{144} \right) \right) + O(\beta^{-3} d^{-4}) \quad (34)$$

One can also use the m.f. technique to compute an approximation to the correlation function. In the spirit of m.f. the integrals over random fields are dominated by the saddle point configuration. In particular, for the two-point correlation function, we have

$$\begin{aligned}
 \langle \frac{\hbar}{N} (U_m U_m^\dagger + U_m, U_m^\dagger) \rangle &= \frac{\int dV_p dA_p \frac{\hbar}{N} (V_m V_m^\dagger + V_m, V_m^\dagger) \exp S_{eff}}{\int dV_p dA_p \exp S_{eff}} \\
 &= 2 v^2 + \left(\frac{1}{d} - \text{corrections} \right) \quad (35)
 \end{aligned}$$

So the naïve m.f. approximation is provided by v^2 and does not depend on the separation. Using our knowledge of v [see Eqs. (7a), (9), (27) and (28)], one can stress the difference between $U(N)$ and $SU(N)$. In the weak coupling region, we have obtained

$$\langle \frac{\hbar}{N} (U_m U_m^\dagger + U_m, U_m^\dagger) \rangle \Big|_{m.f.} = \frac{1}{2} \left(1 + \sqrt{1 - \frac{1}{2\beta d}} \right)^2 \left[1 + \frac{1}{N^2} \left(-\frac{7}{2^7 \beta^2 d^2} + \dots \right) \right]^2 \quad (36a)$$

in $U(N)$

$$= \frac{1}{2} \left(1 + \sqrt{1 - \frac{1}{2\beta d}} \right)^2 \left[1 + \frac{1}{N^2} \left(\frac{1}{8\beta d} + \frac{7}{2^7 \beta^2 d^2} + \dots \right) \right]^2 \quad (36b)$$

in $SU(N)$

Furthermore, the correction in (35) can be computed in m.f. perturbation. Using the mass matrix (13), the one-loop corrections take the form

$$\langle \frac{\hbar}{N} (U_m U_m^\dagger + U_m, U_m^\dagger) \rangle \Big|_{one\ loop} = \hat{F}_d(\tilde{f}) + \frac{N-1}{N^2} \hat{F}_d(\tilde{f} + 2\tilde{k}) + \frac{1}{N^2} \hat{F}_d(\tilde{f} + 2\tilde{k} + 2\tilde{h}) \quad (37)$$

in $U(N)$

$$\begin{aligned}
 &= \frac{N-1}{N^2} \left[\hat{F}_d(\tilde{f}) + \hat{F}_d(\tilde{f} + 2\tilde{k}) \right] + \frac{1}{N^2} \left[\hat{F}_d(\tilde{f} - \tilde{\ell}) \right. \\
 &\quad \left. + \hat{F}_d(\tilde{f} + 2\tilde{k} + 2\tilde{\ell}) \right] \quad (37)
 \end{aligned}$$

in $SU(N)$

where $\hat{F}_d(x)$ is defined by

$$\hat{F}_d(x) \equiv \hat{F}_d(x, \vec{n}, \vec{n}') = \int_{-\pi}^{\pi} \left(\frac{dP}{2\pi} \right)^d \frac{x \exp i \vec{p}(\vec{n} - \vec{n}') + \beta \frac{\partial}{\partial \beta} \left(\frac{x}{d} \right) \sum_{\mu=1}^d \cos p_{\mu}}{1 - (2\beta d) \frac{x}{d} \sum_{\mu=1}^d \cos p_{\mu}} \quad (38)$$

f, k, h are defined in Eqs. (14)-(16) respectively, and $\tilde{f}, \tilde{k}, \tilde{h}$ obey the same definition with w replaced by $\tilde{w}$ [see Eq. (A.3)], and finally

$$\tilde{\ell}(a) = \frac{2a-1}{a} \frac{\partial \beta}{\partial a} \quad (39)$$

The first term in Eq. (38) comes from the expectation values of the form $\langle D_n D_{n'} \rangle$ appearing in (35) while the second comes from $\langle v D_n \rangle$. The latter does not vanish due to the presence of higher orders in the development of w ; it can be estimated by using the classical equations (5) together with the Schwinger-Dyson equation of the field theory governing the fluctuations in (12)¹⁰.

Putting $(n-n')_{\mu} = (\eta-1)\delta_{1\mu}$ the non-trivial terms in η take the form:

$$\begin{aligned} & \langle \frac{1}{N} (U_n U_{n'}^{\dagger} + U_{n'} U_n^{\dagger}) \rangle = \langle \frac{1}{N} (U_n U_{n'}^{\dagger} + U_{n'} U_n^{\dagger}) \rangle_{A_5} = \\ & \frac{1}{\beta (2^5 \beta d^2)^{\eta}} \left[1 + o(\beta^{-1}) + \frac{1}{N^2} \{ 2^{\eta} - 1 + o(\beta^{-1}) \} + \frac{1}{N^4} \left\{ \frac{\eta (2^{\eta-1} - 1)}{4 \beta d} + \dots \right\} + \dots \right]_{U(N)} \quad (40a) \end{aligned}$$

$$\frac{1}{\beta (2^5 \beta d^2)^{\eta}} \left[1 + o(\beta^{-1}) + \frac{1}{N^2} \left\{ 2^{\eta} + 2^{\eta-1} - 1 + \frac{1}{12^{\eta}} + o(\beta^{-1}) \right\} + \dots \right]_{SU(N)} \quad (40b)$$

In Eq. (40a), we have explicitly written the $1/N^4$ term in order to exhibit that this term is depreciated by a factor $1/\beta d$ with respect to the $1/N^2$ term, like in (23).

Concluding remarks

We have obtained the $1/N^2$ correction to m.f. for lattice chiral models. It is pleasant to see that our method is able to reproduce the one-loop result obtained from standard perturbation technique with the right $1/N^2$ coefficient. For reasons explained in the text, the method does not allow to say something on the intermediate coupling (critical point) region.

The second saddle point solution [corresponding to -1 in (9a); let us call it a^-] even if not obeying the condition in Eq. (6) is supported by the mass matrix (20) in a very limited interval in the large N limit:

$$\begin{aligned} \mathcal{M} \Big|_{\substack{a^- \\ N \rightarrow \infty}} \text{ is positive definite} & \quad \text{iff} \quad 1 - 2\beta d [f + 2h](a^-) \geq 0 \\ & \quad \text{iff} \quad \frac{1}{2} \leq \beta d \leq \frac{9}{16} \end{aligned} \quad (41)$$

Originally, we thought that this second phase could be the origin of the strange behaviour of the twisted chiral model discovered in Ref. 11). Indeed, the authors have found numerically two different values for the free energy in the region $0.45 \leq \beta \leq 0.5$ (for $N = 24$, $d = 2$), i.e., a region which agrees qualitatively with the interval (41). However, quite recently, an explanation of Kogut's result has been proposed¹²⁾ as an instanton effect.

Acknowledgements

I am grateful to P. Rossi for many stimulating discussions and to W. Grimus for a helpful conversation.

APPENDIX

Here, we present the functions $w(A, A^+)$ [see definition in (3)] in the form of an $1/N^2$ expansion. The large N limit⁵⁾ of w (let us call it w_∞) is the same for $U(N)$ and $SU(N)$ and depends only on the eigenvalues λ_i of AA^+ :

$$Nw_\infty(\lambda_i) \equiv 2 \sum_{i=1}^N \lambda_i^{1/2} - \frac{1}{2N} \sum_{i,j=1}^N \ln(\lambda_i^{1/2} + \lambda_j^{1/2}) - \frac{3}{4} N \quad (A.1)$$

For $U(N)$, the corrections are, to the best of our knowledge,

$$\begin{aligned} w(A, A^+) = Nw_\infty(\lambda_i) - \frac{1}{8N} \ln(1-t) + \frac{3}{2^7 N} \frac{t^{(3)}}{(1-t)^3} \\ + \frac{3^2 \cdot 5}{2^{10} N} \frac{t^{(5)}}{(1-t)^5} + \frac{3^2 \cdot 7}{2^{10} N} \frac{(t^{(3)})^2}{(1-t)^6} + O\left(\frac{1}{N^7}\right), \\ t < 1 \end{aligned} \quad (A.2)$$

with
$$t^{(i)} \equiv \sum_{j=1}^N \left(\frac{1}{2N \lambda_j^{1/2}} \right)^i, \quad t \equiv t^{(1)}.$$

To obtain more accuracy is just a question of work by using the method presented in Ref. 6).

For $SU(N)$, the corrections depend on the determinant of A as well as on the eigenvalues of AA^+ ; the constraint $\det U = 1$ makes the function w more complicated. Actually, it takes the form

$$\begin{aligned}\tilde{w}(A, A^+) &= \tilde{w}(\lambda_i, \theta) \\ &= N \cdot w_\infty(\lambda_i) + \frac{1}{N} \ln \left[\sum_{\ell=-\infty}^{\infty} e^{iN\ell\theta} (1-t)^{\frac{1}{2}(\ell^2 - \frac{1}{4})} \right] \\ &\quad + \frac{t^{(3)}}{24N} \frac{\sum_{\ell} e^{iN\ell\theta} (\ell^2 - \frac{1}{4})(\ell^2 - \frac{9}{4})(1-t)^{\frac{1}{2}(\ell^2 - 6)}}{\sum_{\ell} e^{iN\ell\theta} (1-t)^{\frac{1}{2}\ell^2}} + O\left(\frac{1}{N^2}\right)\end{aligned}\tag{A.3}$$

where

$$e^{2iN\theta} = \frac{\det A}{\det A^+}$$

REFERENCES

- 1) E. Brezin and J.-M. Drouffe - Nucl.Phys. B200 [FS4] (1982) 93.
- 2) J.B. Kogut, M. Snow and M. Stone - Nucl.Phys. B200 [FS4] (1982) 211.
- 3) Y. Brihaye and P. Rossi - "The Weak Coupling Phase of Lattice Spin and Gauge System", CERN Preprint TH. 3590 (1983), to appear in Nucl.Phys.
- 4) A. Guha and S.-C. Lee - "Improved Mean Field Studies of SU(N) Chiral Models and Comparison with Numerical Simulation", ITP-SB-83-34 (1983).
- 5) R. Brower and M. Nauenberg - Nucl.Phys. B180 [FS2] (1980) 221;
R. Brower, P. Rossi and C.-I. Tan - Phys.Rev. D23 (1981) 942;
For a complete reference list, see Ref. 6).
- 6) Y. Brihaye and P. Rossi - "On the One-Link Integral Over Compact Groups", CERN Preprint TH. 3762 (1983), to appear in Lett.Math.Phys.
- 7) V.F. Müller and W. Rühl - Nucl.Phys. B210 [FS6] (1982) 289.
- 8) Y.Y. Goldschmidt - J.Math.Phys. 21 (1980) 1842.
- 9) G. Doetsch - Handbuch der Laplace Transformationen (Birkhauser Verlg., Basle 1955), Vol. II, p. 122.
- 10) P. Rossi - Private communication.
- 11) S. Das and J. Kogut - "Twisted Reduced Chiral Model at Finite N", Fermilab-Pub-83/97-THY (1983).
- 12) F.R. Klinkhamer - "Saddle Points in the Twisted Reduced Chiral Model", Santa Barbara Preprint, NSF-ITP 84-01 (1984).