SPECTRAL FUNCTION SUM RULES FOR GLUONIC CURRENTS

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A B S T R A C T

We determine the masses and the decay amplitudes of the 2^{++} , 0^{-+} and 0^{++} gluonia in an $SU(3)_c$ Yang-Mills theory. We estimate the so-called topological susceptibility of the $U(1)$ sector. We discuss the $SU(3)_F$ breaking effects to the $\eta'(896)$ mass-relation from current algebra and predict the η' parameter values.

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1. - INTRODUCTION

There is nowadays increasing evidence that quantum chromodynamics (QCD) is a good candidate theory of the strong interactions dynamics. As the eight gluons which mediate such interactions can interact between themselves, it appears natural to expect gluonium states built from colour singlet local gluon operators¹⁾, as is the case for hadrons built with local quark-colour singlet operators. At the present time, gluonium (or glueball) candidates have been reported²⁾ from π^-p or pp scattering experiments, as well as from the analysis of the ϕ radiative decays. The experimental analysis is based on the observation of copious Zweig forbidden reactions, such as $\pi^-p \rightarrow \phi\phi(\eta\eta)+n$ or $\phi \rightarrow \gamma\delta\pi$. These data stimulate a much more careful analysis of the theoretical predictions of the gluonium parameters.

An amount of work from the lattice Monte Carlo calculations has been done in order to determine the gluonium masses³⁾ for an $SU(3)_c$ gauge group from a 8^4 size lattice. Also new results are available from the MIT bag model⁴⁾. In our opinion, previous analysis from QCD sum rules⁵⁾, based on the two-point function of the gluonic currents^{*)}:

$$J_{\mu\nu} = : -\vec{F}_{\mu\alpha} \vec{F}_{\nu\alpha} + \frac{1}{4} g_{\mu\nu} \vec{F}_{\alpha\beta} \vec{F}_{\alpha\beta} : \quad , \quad (1.1)$$

$$J_- = : \left(\frac{3\alpha_s}{4\pi} \right) \vec{F}_{\mu\nu} \vec{\tilde{F}}_{\mu\nu} : \quad (1.2)$$

$$J_+ = : \alpha_s \vec{F}_{\mu\nu} \vec{F}_{\mu\nu} : \quad (1.3)$$

(where the sum over colour is understood), having respectively the quantum numbers $J^{PC}=2^{++}$, 0^{-+} and 0^{++} gluonia ($F_{\mu\nu}$ is the gluon field strength), still

*) The currents J_- and J_+ enter respectively into the divergence of the $U(1)_A$ current and the trace of the energy momentum tensor θ_μ^μ . They have been shown^{6a,b)} to be ultra-violet finite in the world without quarks to two-loops in the $\overline{MS}$ scheme^{6c)}. Then, they become convenient quantities to use in the sum rule analyses. The finiteness of $J_{\mu\nu}$ in Eq. (1.1) has not yet been studied. However, we shall see later on that our analysis will be limited to the leading term in the expansion without radiative corrections. In this way, there will be no reliance on the renormalization group invariance of the currents in Eqs. (1.1)-(1.3).

need to be reconsidered, as no firm predictions of the gluonium parameters have been obtained in the available literature. Actually,

- the numerical analysis done in the confrontation of the theoretical (QCD) and experimental (resonance) parts of the sum rules has not yet been performed very carefully; so, it becomes difficult to compare the available QCD sum rule results⁵⁾ to those from other theoretical approaches^{3),4)};
- the sum rules used previously stress the unreliable value of the real part of the gluonic two-point function at zero momentum transfer:

$$\psi(0) = i \int d^4x \langle 0 | T J_G(x) (J_G(0))^{\dagger} | 0 \rangle, \quad (1.4)$$

where $J_G(x)$ is the gluonic current defined in Eqs. (1.1)-(1.3). Actually, the dependence on the sum rule on the value of $\psi(0)$ appears when one works for example with various derivatives of $(\psi(q^2) - \psi(0))/q^2$. We shall see later on in Section 6 how we shall determine such a subtraction constant following the methods in Ref. 7).

In the first parts of this paper, our aim is to extract the gluonium parameters using an improved numerical analysis and using sum rules which do not depend on the subtraction constant $\psi(0)$. In so doing, we shall use the following ingredients.

A) - We shall work with the moments^{8),9a)}:

$$R(\tau \equiv \frac{1}{M^2}) = -\frac{d}{d\tau} \log \int_{t_0}^{\infty} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \psi(t), \quad (1.5a)$$

which have been discussed in the context of non-relativistic potential models in Ref. 8) and which have been used (in a slightly variant form) for example in Refs. 9a) and 10) for the analysis of the meson spectrum. The main feature of R is that in the limit $\tau \rightarrow \infty$, R approaches the ground state mass squared and so it becomes an interesting battle horse for hunting the hadron masses.

The τ variable can be introduced by applying to the subtracted (three times as we shall see later)^{*)} dispersion relation:

^{*)}The QCD evaluation of $\psi(q^2)$ in the case of currents quoted in Eqs. (1.1) and (1.3) leads to the asymptotic form: $\psi_{\infty}(q^2) \sim (q^2)^2 \log -q^2/v^2$, with v , a subtraction scale which characterizes the renormalization scheme used.

$$\begin{aligned} \psi(q^2 \equiv -Q^2) = & -(Q^2)^3 \int_{t_0}^{\infty} \frac{dt}{t^3} \frac{1}{t+Q^2} \frac{1}{\pi} \text{Im} \psi(t) + \\ & \psi(0) + q^2 \psi'(0) + \frac{1}{2} (q^2)^2 \psi''(0), \end{aligned} \quad (1.5b)$$

(with $-q^2 \equiv Q^2 \geq 0$ and t_0 is the particle threshold production), obeyed by the two-point function $\psi(q^2)$ of the T-product of two gluonic currents, the Laplace operator⁹⁾:

$$\mathbb{L} \equiv \lim_{\substack{N \rightarrow \infty \\ Q^2 \rightarrow \infty}} \frac{(-1)^N (Q^2)^N}{(N-1)! (\partial Q^2)^N} \frac{\partial^N}{\partial Q^2} \quad (1.5c)$$

Then, the resulting sum rule will be less dependent on $\psi(0)$ as the dependence on $\psi(0)$ could be due only to its imaginary part^{*}). In the non-relativistic case, the τ variable plays the role of an imaginary time variable, while the minimum of R represents the optimal upper bound on the square of the ground state mass. We conjecture that such a result holds as well for the relativistic case where we expect that the approximate form of the sum rule R given by QCD is not too far from its exact form in the region where we work. So, we shall consider the minimum of R to be also an upper bound to the lowest ground state mass squared.

^{*}) Actually, we know from perturbative result that the subtraction constants $\psi(0)$, $\psi'(0)$ do not have an imaginary part. We also know for $SU(n)_F$ that $\psi(0)$ of the pion is proportional to $m\langle\bar{\psi}\psi\rangle$ and has no imaginary part. We assume that $\text{Im} \psi(0)$ is zero for the gluonic current and so $\psi(0) \approx \text{Re} \psi(0)$. The vanishing of $\text{Im} \psi(0)$ could be interpreted as due to the absence of massless particle state which should not exist due to the confinement of gluon and quarks.

B) - Our parametrization of the spectral function $\text{Im } \psi(t)$ will be organized as follows: we introduce one resonance contribution via its coupling f_G to the gluonic current:

$$\langle 0 | J_G | G \rangle = \sqrt{2} M_G^2 f_G. \quad (1.7)$$

This resonance can be an effective resonance which can be the barycentre of many quasi-degenerate resonances. We shall use a narrow width approximation, as in any case the correction due to a finite width Γ_G to the sum rule is about Γ_G/M_G (as one can easily inspect in writing a Breit-Wigner form of the gluonium propagator) and becomes negligible. The continuum contribution to the spectral function is parametrized by the leading QCD contribution to the spectral function. It is a Θ function starting from a threshold $\sqrt{t_c}$. The value of $\sqrt{t_c}$ should not be directly interpreted as the threshold of multiparticle states, as our model of the continuum is simply an average of the possible contributions of higher resonance masses to the spectral integral.

In practice, M_G^2 , t_c and f_G will be the three free parameters of the analysis, but we shall see later on that one can relate f_G to the former two by requiring the coincidence of the theoretical and experimental sides of R in the small τ region.

C) - We shall perform our fitting procedure by confronting the theoretical and experimental sides of R in the range of τ values belonging to the interval $[\tau_{\min}, \tau_{\max}]$, where $\tau_{\min}$ is around zero and $\tau_{\max}$ is the maximal value of τ where the approximate expression of R given by the few terms of the operator product expansion of the two-point function is expected to still be a good approximation of the exact form of R . In practice, we shall limit ourselves to the value of $\tau_{\max}$ less than τ_0 at which the approximate form of R presents a minimum, as beyond such values of $\tau_{\max}$, there is no reason to believe the validity of the approximation used in the QCD evaluation of R . The fit will be done using the FUMILI program of the CERN library¹¹⁾ which minimizes after each iteration:

$$\chi^2 = \sum_{j=1}^N \left| \frac{R_{th}^j - R_{exp}^j}{\Delta R_{th}^j} \right|^2, \quad (1.8a)$$

where R_{exp}^j will be characterized by the set of free parameters (M_G^2, t_c) . R_{th}^j is the input QCD expression of the moment at the point τ_i . Actually, we give as input to the program, N points τ_j with $\tau_j - \tau_{j+1} = 0.01 \text{ GeV}^{-2}$ and N discrete values of R_{th}^j . To each value of R_{th}^j , we associate a vertical error bar ΔR_{th}^j which we assume to contain the unknown terms of the QCD series expansion of R_{th}^j . In fact, it is natural to expect that for higher values of τ_j , the approximation used for R_{th}^j tends to be bad, as the contributions of the unknown non-perturbative terms to R_{th}^j become more and more important. In the following analysis, we shall use the parametrization:

$$\Delta R_{\text{th}}^j \simeq \Delta a (\tau_j)^n (-\log \tau_j \Lambda^2), \quad (1.8b)$$

where the τ -dependence is the typical non-perturbative leading contributions to the sum rules as we shall see later, Δa is obtained from an error calculation induced by the uncertainties in the determination of the non-perturbative parameters $(\alpha_s F^2, g f_{\text{abc}} F_{\text{abc}}^3, \dots)$. The error bars in Eq. (1.8b) introduced in this way may be interpreted as correlated errors and the χ^2 in Eq. (1.8a) could not be the true χ^2 (in the sense of statistics).

In our analysis, we shall only be very qualitative as the behaviour of χ^2 per points, plotted in various figures (see, e.g., Fig. 2), serves only as a guide for fixing the values of τ_{min} and τ_{max} which we obtain when the χ^2 per points is stable or presents a minimum. In this way, we shall obtain a range of values of M_G^2 and t_c , and we shall not attach any importance to the values of the error bar given by the program to each value of M_G^2 and t_c as, in any case, the program will not provide the full error.

In the second step of the analysis, we use the above values of τ_{min} and τ_{max} and we do a separate fit of the two extremal values of R_{th}^j (see, e.g., Fig. 1) by introducing a smaller value Δa in order to get the strongest constraint in the fitting procedure. Then, we get two values of the set (M_G^2, t_c) from the analysis of the lower and upper curves. Our final values of (M_G^2, t_c) will be in the largest range of values corresponding to the minimum or to the stability of χ^2/NDF , obtained from the first and second steps of analysis.

D) - The input R_{th} is obtained via the evaluation of the QCD perturbative and non-perturbative diagrams, shown in Table 1, which contribute to the gluonic two-point function $\phi(-q^2 \gg \Lambda^2)$. Then, R_{th} will be parametrized by:

- the QCD scale $\Lambda_{\overline{MS}}$ of the $\overline{MS}$ renormalization scheme^{6c)} which is known to be¹²⁾:

$$\Lambda \simeq (100 - 200) \text{ MeV} . \quad (1.9a)$$

In our analysis, which concerns the world without quarks, we shall use, however, this phenomenological value of Λ and consistently the value of the coefficient of the β function for $SU(3)_C \times SU(3)_F$, i.e.^{6c)}:

$$\beta \equiv \left(\frac{\alpha_s}{\pi} \right) \left(\beta_1 = -\frac{9}{2} \right) + \left(\frac{\alpha_s}{\pi} \right)^2 \left(\beta_2 = -8 \right) + \dots \quad (1.9b)$$

We shall see later on that this choice of Λ and β will not affect our numerical analysis as the sum rules depend very weakly on the value of Λ and β because the radiative corrections to R_{th} in the unit operator are less important in the ratios of the Laplace sum rule (1.5a) than in the Laplace sum rule itself. Actually, the corrections introduced in the uses of $\Lambda_{\overline{MS}}$ rather than Λ_{NQ} in the world with no quarks are typically of the order of $(\log \Lambda_{NQ}/\Lambda_{\overline{MS}})/\log Q/\Lambda_{\overline{MS}}$, as are other radiative corrections;

- the value of the gluon condensate which we decide to fix as:

$$\alpha_s \langle F^2 \rangle = (0.066 \pm 0.030) \text{ GeV}^4, \quad (1.10a)$$

where the central value comes from a recent $SU(3)_C$ gauge 8^4 lattice Monte Carlo calculation³⁾, as we shall be concerned here with a calculation in the hypothetical world without quarks. The error bar is a generous error in order to accommodate the phenomenological estimate of $\alpha_s \langle F^2 \rangle$ ¹³⁾ and the more speculative estimate of $\alpha_s \langle F^2 \rangle$ in the absence of quarks^{5a)}. It will also be used as an estimate of Δa entering in ΔR_{th}^j in Eq. (1.8b);

- the value of $g^3 \langle F^3 \rangle$ will be taken to be:

$$\begin{aligned} g^3 \frac{1}{4abc} \langle F_a^{\mu\nu} F_b^{\nu\rho} F_c^{\rho\mu} \rangle &\equiv g^3 \langle F^3 \rangle \\ &= (1.1 \pm 0.2) \text{ GeV}^2 \alpha_s \langle F^2 \rangle, \end{aligned} \quad (1.10b)$$

by assuming that the ratio of $(g^3 \langle F^3 \rangle)^{1/6}$ and $(\alpha_s \langle F^2 \rangle)^{1/4}$ obtained for an $SU(2)_c$ gauge group from a 4^4 lattice¹⁴⁾ is maintained for an $SU(3)_c$ and a larger lattice.

2. - THE 2^{++} GLUONIUM

We shall be concerned with the two-point function $\phi(q^2)$ defined as:

$$\begin{aligned} T_{\mu\nu,\rho\sigma} &= i \int d^4x e^{iqx} \langle 0 | T J_{\mu\nu}(x) (J_{\rho\sigma}(0))^{\dagger} | 0 \rangle \\ &\equiv \left(\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho} - \frac{2}{3} \eta_{\mu\nu} \eta_{\rho\sigma} \right) \psi(q^2), \end{aligned} \quad (2.1a)$$

with

$$\eta_{\mu\nu} \equiv \left(g_{\mu\nu} - q_{\mu} q_{\nu} / q^2 \right). \quad (2.1b)$$

The operator production expansion (OPE) of $\phi(q^2)$ has been studied in Ref. 5a). It has been shown that the leading non-perturbative contribution comes from dimension eight operators. The moment which is independent of the value of the two-point function at zero momentum reads:

$$\begin{aligned} R_{2+} &\equiv - \frac{d}{d\tau} \log \int_{t_0}^{\infty} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \psi(t) \\ &\simeq \frac{3}{\tau} \left\{ 1 + \frac{200}{9} \pi^3 \alpha_s \langle 0 | \left(f^{abc} F_{\mu\nu}^b F_{\alpha\rho}^c \right)^2 - \right. \\ &\quad \left. - 2 \left(f^{abc} F_{\mu\alpha}^b F_{\nu\alpha}^c \right)^2 | 0 \rangle \tau^4 \right\} \end{aligned} \quad (2.2a)$$

Actually, there is not yet an available accurate estimate of the matrix elements in Eq. (2.2a). Then, we use as in Ref. 5a) the factorization hypothesis which is expected to be only good in the large colour number N_c value limit:

$$\langle 0 | \left(\frac{1}{\sqrt{2}} f^{abc} F_{\mu\nu}^b F_{\alpha\beta}^c \right)^2 - 2 \left(\frac{1}{\sqrt{2}} f^{abc} F_{\mu\alpha}^b F_{\nu\alpha}^c \right)^2 | 0 \rangle \simeq \frac{3}{16} \langle F^2 \rangle^2 . \quad (2.2b)$$

Using the central value of $\alpha_s \langle F^2 \rangle$ given in Eq. (1.10a), and the value of the coefficient of the β function given in Eq. (1.9b), we get:

$$R_{2+} \simeq \frac{3}{\tau} \left\{ 1 - (0.4 \pm 0.2) \text{ GeV}^8 (\log \tau \Lambda^2) \tau^4 \right\}, \quad (2.2c)$$

where we have assumed a 50% uncertainty in the use of Eq. (2.2b). Actually, such an error might be an underestimate of its actual value because of the uncertainty of the validity of Eq. (2.2b), but we shall see later on that a change of the strength of the dimension eight operator contribution by a factor 2 will not affect our numerical conclusion. We give the τ behaviour of Eq. (2.2c) in Fig. 1. The presence of the minimum signals to us the maximum value of τ up to which one should trust the approximation used in Eq. (2.2). We parametrize the phenomenological side of the sum rule using the discussion in Section 1 B). Then, we get:

$$R_{2+} \simeq \frac{2 M_G^6 \frac{1}{6} e^{-M_G^2 \tau} + \left(\frac{3}{10\pi^2} \right) e^{-p_c} \tau^{-4} \left(1 + p_c + \frac{p_c^2}{2!} + \frac{p_c^3}{3!} \right)}{2 M_G^4 \frac{1}{6} e^{-M_G^2 \tau} + \left(\frac{1}{10\pi^2} \right) e^{-p_c} \tau^{-3} \left(1 + p_c + \frac{p_c^2}{2!} \right)} \quad (2.3a)$$

with

$$p_c \equiv t_c \tau \quad . \quad (2.3b)$$

We require the "asymptotic consistency behaviour" of the two sides of the sum rule [Eqs. (2.2) and (2.3)], in the small τ region. This leads to the constraint:

$$\left(\frac{1}{20\pi^2} \right) t_c^3 \simeq 6 M_G^4 \frac{1}{6} , \quad (2.4a)$$

which enables us to write a generic form of R which should be valid for any

two-point function having the asymptotic form like $(Q^2)^2 \log Q^2/v^2$ independently of the external normalization of the gluonic current:

$$R_{2+} = M_G^2 \frac{1 + 18 \frac{t_c}{M_G^2} (t_c \tau)^3 e^{-\rho_c} \left(1 + \rho_c + \frac{\rho_c^2}{2!} + \frac{\rho_c^3}{3!}\right)}{1 + 6 (t_c \tau)^3 e^{-\rho_c} \left(1 + \rho_c + \frac{\rho_c^2}{2!}\right)} . \quad (2.4b)$$

According to our conjecture in Section 1, one could consider the minimum of the R to be an upper bound for the lowest ground state mass provided that the non-perturbative contribution to the moments at the minimum is still a correction to the leading term. Here the corrections are about 33% and are reasonable. Then, the bound of the lowest ground state mass is:

$$M_G \leq \sqrt{R_{\min}} \simeq (2.4 \pm 0.3) \text{ GeV} , \quad (2.5)$$

where the error bar comes from the coefficient of the τ^4 term and from the assumption that the unknown non-perturbative terms induce an uncertainty of 20% to the moments. We organize our fitting procedure of Eqs. (2.2) and (2.3) as follows: we fix Λ , for example, to be 0.2 GeV. We do a preliminary step where we fix $\tau_{\min}$ about 10^{-3} GeV^{-2} and do the fit by letting $\tau_{\max}$ free. Then, we realize that the χ^2/NDF is minimum for $\tau_{\max}$ about 0.2 GeV^{-2} . Then, we fix $\tau_{\max}$ at such a value and move $\tau_{\min}$ (Fig. 2a). A simultaneous precise fit of (M_G^2, t_c) and a minimum χ^2/NDF can be obtained for $\tau_{\min}$ between 10^{-4} and 10^{-3} GeV^{-2} . Actually, because of the τ dependence of ΔR_{th} , as introduced by our parametrization in Eq. (1.8b) ($n = 3$ here), one can see that ΔR_{th} goes very fast to zero for τ going to zero. So, beyond some critical value of τ , the program can become invalid under the conditions of precision with which we ask it to determine the parameters to reduce $|R_{\text{th}}^j - R_{\text{exp}}^j|$ as small as ΔR_{th}^j . This fact is signalled by the jump of χ^2/NDF in Figs. 2a and 2c. For higher values of $\tau_{\min}$, the length of the interval $[\tau_{\min}, \tau_{\max}]$ becomes too short, so that the constraints on (M_G^2, t_c) become very weak. This is signalled by the big uncertainties in the results M_G^2, t_c .

For definiteness, we fix $\tau_{\min}$ to be $5 \cdot 10^{-4} \text{ GeV}^{-2}$ and we move again $\tau_{\max}^2$ (Fig. 2b). We see that the χ^2/NDF is small for $\tau_{\max}$ between 0.2 and 0.5 GeV^{-2} . We take the values of (M_G^2, t_c) obtained in this range of values of $\tau_{\max}$. We repeat the above procedure for $\Lambda = 0.1 \text{ GeV}$ (see Figs. 2c, 2d) where the value of χ^2/NDF is still stable in the above range of values of $\tau_{\max}$. At the second stage of the analysis, we fix for definiteness $\tau_{\min}$ to be $5 \cdot 10^{-4} \text{ GeV}^{-2}$ and $\tau_{\max}$ to be 0.6 GeV^{-2} . We do our fit for a given value of Λ and for two extremal values of the coefficient of the τ^4 term in Eq. (2.2), i.e., taking the coefficient between brackets to be 0.2 and 0.6 GeV^8 . We assume an uncertainty of about 0.02 GeV^8 in the coefficient in such a way that the constraint on (M_G^2, t_c) in doing the fit becomes strong. We obtain for $\Lambda \approx 0.2 \text{ GeV}$, the set of values of (M_G^2, t_c) to be (2.7, 3.3) GeV^2 and (3.3, 2.9) GeV^2 respectively for the values of the coefficient of τ^4 to be 0.2 and 0.6 GeV^8 . We repeat the above procedure for $\Lambda = 0.1 \text{ GeV}$. Our final results are:

$$\begin{aligned} 2.7 &\lesssim M_G^2 \lesssim 3.3 \text{ GeV}^2, \\ 2.7 &\lesssim t_c \lesssim 3.8 \text{ GeV}^2, \end{aligned} \tag{2.6}$$

for $0.1 \leq \Lambda \leq 0.2 \text{ GeV}$ and the value of the coefficient of the τ^4 term to be between 0.2 and 0.6 GeV^8 . The change of the coefficient of the τ^4 , as we have checked in the large range 0.2 and 0.8 GeV^8 , does not much affect the result in Eq. (2.6). This may be due to the fact that it is only important in a small interval ($0.4 \leq \tau \leq 0.6 \text{ GeV}^{-2}$) of the region where the fit is performed. Actually, we cannot also take the coefficient of the τ^4 term to be zero as we shall not have any criterion to control the maximal value of $\tau_{\max}$ at which we have to stop. In that respect, the presence of the condensate is necessary within our approximation in the OPE for our analysis. Our result shows that the continuum threshold is very close to the resonance mass, so that an isolation of the resonance parameters alone or a resonance dominance to the sum rule becomes unjustified here.

We compare our results in Table 2 with the bag model^{4a), b)}, the SU(3) lattice Monte Carlo³⁾ calculations and with the data of the $\phi\phi$ spectrum analysis^{2b)} coming from $\pi\text{-p} \rightarrow \phi\phi n$ and for ϕ radiative decays^{2a)}. Our result

is slightly lower than the $g_T(2.12)$, $g_{T'}(2.22)$ and $g_{T''}(2.36)$ reported earlier^{2b)}, but agrees with the $\Theta(1.64)^{2a)}$. Actually, the 2^{++} gluonium can have a small admixture of the 2^{++} tensor mesons which lie in the $1.3 \sim 1.5$ GeV region. This mixing mechanism could shift slightly the mass in Eq. (2.5) to the observed value. Inserting Eq. (2.5) into Eq. (2.4), one obtains as well:

$$41 \lesssim f_G \lesssim 74 \text{ MeV} , \quad (2.7)$$

which should be compared with $f_\pi \approx 93$ MeV! The finite width corrections to our results in Eq. (2.6) could be inserted using a Breit-Wigner form of the gluonium propagator. As we work here with a ratio of two Laplace transform sum rules, these corrections should be of the order of Γ_G/M_G which should be small^{*)} [$\Gamma_G \approx 300 \text{ MeV}^2$ and $M_G \approx 1.7$ GeV from Eq. (2.6)], and should not greatly affect our results [Eqs. (2.6) and (2.7)].

3. - THE 0^{-+} GLUONIUM

Here, we shall be concerned with the two-point function:

$$\Psi(q^2) = i \int d^4x e^{iqx} \langle 0 | T J_-(x) (\bar{J}_-(0))^{\dagger} | 0 \rangle , \quad (3.1)$$

with $J_-(x)$ as defined in Eq. (1.1). The above two-point function has been extensively studied within the QCD framework^{4c),5),15)}. The sum rule independent of the subtraction constants [Eq. (1.5b)] which we shall consider here is:

^{*)} One should remember that our analysis is done in the absence of quarks so that one can expect a very narrow gluonium state.

$$\begin{aligned}
 R_-(\tau) &\equiv -\frac{d}{d\tau} \log \int_{t_0}^{\infty} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \Psi_-(t) \\
 &\simeq \frac{3}{\tau} \left\{ 1 + 2\pi^2 g \langle F^3 \rangle \tau^3 \right\}
 \end{aligned} \tag{3.2}$$

$R_-(\tau)$ is given in Fig. 3 and has a nice feature; it presents a minimum at the approximation used here. Then, one could deduce the following upper bound to the lowest 0^{-+} ground state mass:

$$M_G \lesssim \sqrt{R_-^{\min}} \simeq (1.9 \pm 0.4) \text{ GeV}, \tag{3.3}$$

where the non-perturbative contributions to the sum rule are about 50% making the bound in Eq. (3.3) still quite reliable. The error bar in Eq. (3.3) is due to the coefficient of the τ^3 term and also due to the assumption that the contributions of the unknown non-perturbative terms to the moments are about 25%.

Now we do an analysis similar to that of the 2^{++} for our fitting procedure, where for convenience we define

$$\langle 0 | \bar{J}_- | G \rangle = \left(\frac{3\alpha_s}{4\pi} \right) \sqrt{2} f_G M_G^2 \equiv \sqrt{2} f_G^{\text{phys}} M_G^2. \tag{3.4a}$$

Using the small τ coincidence of the theoretical and experimental sides of the sum rule, we obtain the constraint:

$$\frac{1}{3\pi^2} t_c^3 \simeq M_G^4 f_G^2 \tag{3.4b}$$

with which the sum rule takes the generic form in Eq. (2.4b). Our discussion is done in the same way as for the 2^{++} channel and we illustrate it in Figs. 4 and 5. Our result, corresponding to the smallest χ^2/NDF which is about 0.42, is:

$$2.7 \lesssim M_G^2 \lesssim 3.1 \text{ GeV}^2, \quad (3.5a)$$

$$2.7 \lesssim t_c \lesssim 3 \text{ GeV}^2, \quad (3.5b)$$

for $0.1 \leq \Lambda \leq 0.2 \text{ GeV}$. It shows that the continuum plays an important role in the 0^{++} channel so that a resonance alone cannot dominate the spectral function. The corresponding value of f_G obtained from an analogy of Eq. (2.4), derived using the "asymptotic consistency condition", is between 0.26 and 0.35 GeV. This corresponds for $\alpha_s/\pi \approx 0.14$ ($\Lambda = 0.1 \text{ GeV}$, $M \approx 1 \text{ GeV}$), to:

$$23 \lesssim f_G^{\text{phys}} \lesssim 34 \text{ MeV}. \quad (3.6)$$

This result is much smaller than the η' decay amplitude $f_{\eta'} = 100 \text{ MeV}$ deduced from the branching ratio $\phi \rightarrow \eta' \gamma$ over $\phi \rightarrow \eta \gamma$ ¹⁶⁾. Actually, such a value of the decay amplitude agrees with the large N_c expectation, which states that the gluonium decay amplitude behaves like $1/\sqrt{N_c}$ while the meson one behaves like $\sqrt{N_c}$. The result in Eq. (3.5) agrees with the one of Ref. 5b) obtained from moments sum rules-like ratios. The results in Eq. (3.5) is much higher than the $\eta'(0.96)$ mass value. We compare in Table 2 various theoretical estimates of the lowest gluonium ground state mass. They are much more near the $\eta(1.44)$ than the $\eta'(0.96)$. Such results for the mass indicate that the quark component of the η' wave function should give an appreciable contribution to its mass. We should notice that the value of τ_{max} around 1 GeV^{-2} , corresponding to the best χ^2/NDF , is much larger than the one discussed in Ref. 5a). Actually, the value of τ_{max} in the latter analysis is governed by the contribution of $\phi_-(0)$ to their sum rule. This contribution is absent in our case. More speculatively, the authors in Ref. 5a) suggest that the instanton density-like contributions to the sum rule will spoil the series expansion for the values of τ_{max} about 1 GeV^{-2} . However, one should consider this restriction to be very qualitative, as the estimate of the real strength of such contributions is very uncertain due to the infra-red cut-off dependence of the analysis. Here we ignore such effects which are less well understood. The agreement of our result with the one obtained from independent methods may suggest a posteriori that the contribution of instanton-like density to the sum rule may be negligible at the value of τ smaller or equal to 1 GeV^{-2} .

4. - THE 0^{++} GLUONIUM

Here, we shall be concerned with the two-point function $\phi_+(q^2)$ associated with the J_+ current in Eq. (1.3). The lowest moment independent of $\phi_+(0)$ is:

$$R_+(\tau) \equiv -\frac{d}{d\tau} \log \int_{t_0}^{\infty} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \psi_+(t) \quad (4.1a)$$

$$\simeq \frac{3}{\tau} \left\{ 1 - 2\pi^2 g \langle F^3 \rangle \tau^3 \right\},$$

which we plot in Fig. 6.

The coefficient of the non-perturbative contribution in Eq. (4.1) is the opposite of the one in Eq. (3.2) and so Eq. (4.1) cannot have an extremum at this approximation. So, an upper bound to the lowest gluonium state based on our conjecture derived from the non-relativistic potential model case cannot be obtained at this level of approximation. However, if we use naively here the value of τ , where the 0^{-+} sum rule gets a minimum, we see that the corrections to the 0^{-+} and 0^{++} sum rules have the same amount, 50%, but are opposite in sign, making the approximation in Eq. (4.1a) still quite good. If we simply use the positivity of the continuum contribution to the moments R_+ , which can be verified in our model of parametrization of the spectral function, we can deduce the upper bound:

$$M_G \lesssim (1.1 \pm 0.1) \text{ GeV}. \quad (4.1b)$$

For our fitting procedure, we shall take for definiteness the value of $\tau_{\min} \simeq 4 \cdot 10^{-5} \text{ GeV}^{-2}$ as the one in the 0^{-+} channel, since $\phi_+(q^2)$ and $\phi_-(q^2)$ are identical up to an overall normalization for small τ . We analyze the influence of the values of $\tau_{\max}$ in the fitting procedure (see Figs. 7). The smallest χ^2/NDF is about 0.7 which is relatively high compared to the 0^{-+} one within the same conditions of analysis. For Λ between 0.1 and 0.2 GeV, we obtain:

$$0.9 \lesssim M_G^2 \lesssim 1.2 \text{ GeV}^2, \quad (4.2a)$$

$$2.3 \lesssim t_c \lesssim 2.7 \text{ GeV}^2, \quad (4.2b)$$

for τ_{max} about 1 and 1.2 GeV^{-2} . Consulting our Table 2, one can realize that various theoretical estimates are around 1 GeV, and seem to be far below the new $\eta\eta$ resonance recently found at GAMS^{2c)}. So, such a resonance could not be the lowest gluonium state but one of its excitations. The 0^{++} gluonium in Eq. (4.2) should mix with various 0^{++} meson states belonging to this 1 GeV region^{*)}. Using the $\tau = 0$ "asymptotic consistency condition", we deduce:

$$60 \lesssim g_G^{\text{phys}} \lesssim 100 \text{ MeV}, \quad (4.3)$$

which is comparable with the strength of the pion decay amplitude.

5. - CONCLUSIONS FROM THE MOMENTS ANALYSIS

The analysis of the gluonium parameters using some moments independent of the value of the gluonic two-point correlation function at zero momentum transfer, and using the χ^2 -minimization FUMILI program of the CERN library for the fitting procedure, leads to the conclusions summarized in Table 2, which suggest that various theoretical predictions agree with each other. However, one should notice that:

a) the predictions for the 0^{++} are far below the new GAMS data^{2c)}, finding a 0^{++} gluonium candidate at 1.59 GeV, but probably much nearer the $\sigma(700)$ or the $S^*(975)$ which have actually been speculatively interpreted as gluonium states^{1b), 5a), 18)}. The present theoretical predictions should then stimulate a more careful analysis of the coupling, production and decay of the σ and the S^* in order to clarify the less understood 0^{++} meson spectroscopy, which cannot be interpreted via a naïve Gell-Mann Okubo (GMO) mass formula including $SU(3)_F$ breaking effects¹⁸⁾. Actually, it is suggested that the

^{*)} Actually, it has been realized in Ref. 5f) that the 0^{++} gluonium might be a very broad resonance or many quasi-degenerate states populating the 1 GeV region!

$S^*(975)$ is an $\bar{s}s$ state because it couples strongly to the $K\bar{K}$ states. But a GMO mass formula analysis¹⁸⁾ shows that the $SU(3)_F$ partner of the $\delta(980)$ should be much more massive than the δ .

However, this is not the case as we know from the data. So, the $S^*(975)$, interpreted as a gluonium state, seems more reasonable¹⁷⁾. Such an interpretation seems to be supported by the preliminary $\gamma\text{-}\gamma$ scattering data which do not produce the $S^{*})$. Following the GMO line of reasoning, one should expect to have an isoscalar partner of the δ (maybe the σ) having approximately the same mass as the δ , like the case of a quasi-degeneracy of the ω and ρ masses, due to the good $SU(2)_F$ symmetry for the quark mass¹⁹⁾ and for the quark vacuum condensates^{7b)}.

b) None of the theoretical results agree with the naïve assumption that the $\eta'(0.96)$ is a pure gluonium candidate, as the lowest mass of the 0^{-+} gluonium is far above the η' mass. The results suggest a further analysis of the nature of the η' , in addition to the already known semi-quantitative large N_c result^{15b),d)} and to the current algebra-like approach¹⁹⁾. We shall discuss in Section 6 an alternative discussion for the determination of the η' mass.

c) In addition to the predictions of the gluonium masses, we also obtain a new estimate of the couplings of the gluonia to the currents having their quantum numbers [Eqs. (1.1)-(1.3)]. The results can be found in Eqs. (2.7), (3.6) and (4.3). One can notice that the couplings of the 2^{++} and 0^{++} gluonia are comparable to $f_\pi \approx 93$ MeV, but the one of the 0^{-+} is about 30 MeV which is very weak. Our estimates of the decay amplitudes might be useful for further analysis of the gluonium properties and decays.

6. - TOPOLOGICAL SUSCEPTIBILITY OF THE $U(1)_A$ SECTOR

We have seen previously that the nature of the η' cannot be explained either by a pure gluonium nor by a pure quark state. We also know from a $U(1)$ current algebra like-analysis¹⁵⁾ that the η' mass can be related to the subtraction constant $\psi_-(0)$ of the two-point function, the so-called

^{*)} I thank G. Mennessier for such information.

topological susceptibility, via an argument obtained from the independence of the QCD Lagrangian on the θ vacuum angle if one or more quark fields become massless²⁰⁾. Here, we shall estimate $\psi_-(0)$ and its first derivative $\psi'_-(0)$ using the sum rules method and taking into account the information on the 0^{++} gluonium parameters which we have obtained previously. The estimate of $\psi_-(0)$ can be done by applying the Laplace transform operator in Eq. (1.5c) to both sides of $(\psi(q^2) - \psi(0))/q^2$ given in Eq. (1.5c). This procedure leads to Eq. (25) in Ref. 5b). Combining such an equation with the $F^{(3)}$ sum rule [Eq. (10a) in Ref. 5b)], one can deduce in the world without quarks^{*)}:

$$\int_{t_0}^{\infty} \frac{dt}{t} e^{-t\tau} \left(1 - \frac{t}{2}\tau\right) \frac{1}{\pi} \text{Im} \psi_-(t) = \psi_-(0) \Big|_{\text{no-quarks}} +$$

$$+ \left(\frac{3\bar{\alpha}_s}{4\pi}\right)^2 \frac{2}{\pi^2} \tau^{-2} \left\{ \frac{1}{\log \tau \Lambda^2} + 2\pi^2 \langle F^2 \rangle \tau^2 + 6\pi^2 g \langle F^3 \rangle \tau^3 \right\}, \quad (6.1)$$

with $\bar{\alpha}_s = -4\pi/(g \log \tau \Lambda^2)$

and t_0 is the particle threshold.

We have isolated, on the left-hand side, the spectral function and on the right-hand side the QCD contribution. In this way, Eq. (6.1) is slightly different from Eq. (26) in Ref. 5b) [see also Ref. 4c), where a similar constraint has been derived]. We give in Fig. 8 the behaviour of the two sides of Eq. (6.1) for various values of τ . The left-hand side of Eq. (6.1) is parametrized using the gluonium obtained in Section 3 plus the QCD continuum (t_c is the continuum threshold) which is parametrized as:

$$\left(\frac{3\bar{\alpha}_s}{4\pi}\right)^2 \frac{2}{\pi^2} \tau^{-2} e^{-t_c \tau} (1 + t_c \tau) \left(\frac{1}{\log \tau \Lambda^2} \right). \quad (6.2)$$

One can see (dashed line) that the gluonium plus the continuum contributions cancel each other. The QCD part of the right-hand side of Eq. (6.1) is represented in Fig. 8 by the full line. This curve corresponds for

^{*)}The two-loop calculation of $\psi_-(q^2)$ has been done in Ref. 21).

definiteness to the values $\Lambda = 0.15 \text{ GeV}$, $\alpha_s \langle F^2 \rangle = 0.066 \text{ GeV}^4$ ³⁾ and $g^3 \langle F^3 \rangle = 1.1 \text{ GeV}^2 \alpha_s \langle F^2 \rangle$ ¹⁴⁾. One can realize that the curve reaches a small stability for the value of τ around 0.5 GeV^{-2} . For larger values of τ , the $\langle F^3 \rangle$ contribution to Eq. (6.1) becomes important and the stability of the curve in Fig. 8 is destroyed. However, at such higher values of τ , one can raise some doubts on the validity of the approximation in Eq. (6.1). The value of $\psi_-(0)$ at the stability point is:

$$\psi_-(0) /_{\text{no-quarks}} \simeq -(0.26 \text{ GeV})^4. \quad (6.3a)$$

It is easy to see that, ranging τ from 0.5 to 1 GeV^{-2} , the value of $\psi_-(0)$ moves by a factor 1.8, i.e., a perhaps more conservative attitude would be to consider Eq. (6.3a) within a factor two which leads to the result:

$$\psi_-(0) /_{\text{no-quarks}} \simeq -[(0.26 \sim 0.30) \text{ GeV}]^4. \quad (6.3b)$$

Now, let us compare our results with existing estimates coming from different sources. The first well-known estimate comes from a current $U(1)_A$ algebra low energy result ^{15), 4c)}. The derivation of such a result is based on the θ independence of the QCD Lagrangian when one (or more) massless quark fields are globally chiral rotated ²⁰⁾. In this way, the relation ^{*) 15)}:

$$M_{\eta'} \simeq \frac{1}{f_{\pi}} \left(6 \frac{\partial}{\partial \theta} \langle Q(x) \rangle_{\theta=0} \right)^{1/2} \quad (6.4)$$

has been obtained and the vanishing of the θ angle has been materialized by:

$$36 \frac{\partial}{\partial \theta} \langle Q(x) \rangle_{\theta=0} = -\psi_-(0) /_{\text{no-quarks}}. \quad (6.5)$$

*) Here we follow the notation in Ref. 15c):

$$\partial_{\mu} J_5^{\mu}(x) = \sum_{i,j} q_{ij} 2 m_j \bar{\psi}_j(i\gamma_5) \psi_j - (2 n_f Q(x) \equiv J_-),$$

$f = 93 \text{ MeV}$ and the quark model has been used in order to relate f to f_{π} .

Actually, the data for $\phi \rightarrow \eta' \gamma$ and $\phi \rightarrow \eta \gamma$ might indicate that π η'

$$f_{\eta'} /_{\text{data}} \simeq 0.6 f_{\eta'} /_{\text{quark model}}.$$

Using $M_\eta \approx 0.96$ GeV and solving Eqs. (6.4) and (6.5), we have the low-energy current algebra result:

$$-\psi(0)/_{\text{current algebra}} \approx (0.34 \sim 0.44 \text{ GeV})^4 \quad (6.6)$$

where the first number corresponds to the value of f_η , from the data^{*)} and the second from the value of f_η , from the quark model. Comparing the results in Eqs. (6.3) and (6.6), one can realize that our estimate in Eq. (6.3) can be lower by a factor 3 to 8 to the current algebra result. However, we might expect the current algebra result to be affected by $SU(3)_F$ breaking in the same way as the kaon PCAC estimate of an analogous subtraction constant for $SU(3)_L \times SU(3)_R$ has been strongly violated by a factor of about two^{7a)}. Before coming to the $SU(3)_F$ effects, let us also mention that there exist some Monte Carlo data simulations of the topological susceptibility. Previous analyses for $SU(2)_c$ ^{22a)} and $SU(3)_c$ ^{22b)} gauge theories have been reported.

They have given an estimate of $\psi_-(0)$ which is lower than the one in Eqs. (6.3) and (6.6) by a factor 10 to 20. A much more recent analysis^{22c)}, done for $SU(2)_c$, and which differs from the former two by a new definition of the topological charge of the lattice gauge field configuration, gives an estimate much closer to the one in Eqs. (6.3) and (6.6):

$$-\psi(0)/_{\text{lattice}}^{SU(2)} \approx [(0.41 \pm 0.06) \text{ GeV}]^4, \quad (6.7)$$

but it corresponds unfortunately to $SU(2)_c$ so that a clean comparison with the sum rules and current algebra estimates cannot be done^{*)}. Let us finally complete our discussion by an estimate of $\psi'_-(0)$ which has the sign convention fixed by Eq. (1.5b)^{**)} .

*) An improvement of the previous result for $SU(3)_c$ is in preparation^{22d)}.

**) I thank G. Veneziano and G.C. Rossi for suggesting the estimate of $\psi'_-(0)$ which should be useful for the current algebra approach.

The estimate of $\phi'_-(0)$ can be done by applying to both sides of

$$\frac{\psi_-(q^2) - \psi_-(0) - q^2 \psi'_-(0)}{(q^2)^2}$$

given in Eq. (1.5b) the Laplace transform operator [Eq. (1.5c)]. This procedure leads to:

$$\int_{t_0}^{\infty} \frac{dt}{t^2} e^{-t\tau} \frac{1}{\pi} \text{Im} \psi_-(t) = F^{(1)} \tau^{-1} - \psi_-(0) \tau + \psi'_-(0) \tau^2, \quad (6.8)$$

where $F^{(1)}$ is the right-hand side of Eq. (10c) in Ref. 5b). Eliminating $\psi_-(0)$ by using Eq. (6.1) gives:

$$\begin{aligned} \int_{t_0}^{\infty} \frac{dt}{t^2} e^{-t\tau} \left[1 + t\tau \left(1 - \frac{t\tau}{2} \right) \right] \frac{1}{\pi} \text{Im} \psi_-(t) \simeq \\ \psi'_-(0) + \left(\frac{3\bar{\alpha}_s}{4\pi} \right)^2 \frac{2}{\pi^2} \tau^{-1} \left\{ 1 - \frac{2}{9 \log \tau \Lambda^2} [10 - 2\gamma_E \beta_1 - \right. \quad (6.9) \\ \left. - 2 \frac{\beta_2}{\beta_1} \log(-\log \tau \Lambda^2)] + 4\pi^2 g \langle F^3 \rangle \tau^3 \right\}, \end{aligned}$$

with

$$\bar{\alpha}_s = -4\pi / (9 \log \tau \Lambda^2) \quad , \quad \beta_1 = -\frac{9}{2} \text{ and } \beta_2 = -8 \quad (6e,*)$$

and $\gamma_E = 0.5772$ is the Euler constant. We parametrize the spectral function appearing in Eq. (6.9) by the 0^{++} gluonium and the QCD continuum obtained in Section 3. The spectral function contribution to Eq. (6.9) is shown in Fig. 9 (dashed lines). The QCD plus the non-perturbative part of Eq. (6.9) is also shown in Fig. 9 (full line). The values of the parameters used here are the same as the one in Fig. 8. One can see the approximate cancellation of the resonance plus continuum contributions into the left-hand side of Eq. (6.9). So, the theoretical QCD contribution dominates the estimate of $\phi'_-(0)$. One sees in Fig. 9 that the resulting value of $\phi'_-(0)$ is quite insensitive to the

*) These values of the coefficient of the β -function correspond to three-flavours which are consistent with the value of $\Lambda_{\overline{MS}}$ obtained in the presence of quark matters. The uses of the β -function coefficient in the absence of quarks should correspond to another value of $\Lambda_{\overline{MS}}$ as mentioned previously. We notice that, in practice, the analysis is not crucially dependent on the value of Λ and β .

change of the τ values. The curve reaches an extremum for τ between 0.6 and 0.8 GeV^{-2} . The corresponding value of $\psi'_-(0)$ for $\Lambda \approx 150 \text{ MeV}$ is:

$$\psi'_-(0) \approx - (0.05 \text{ GeV})^2 \quad (6.10)$$

and is mainly due to the unit operator. Actually, for $\tau \approx 0.6 \text{ GeV}^{-2}$, the non-perturbative term is 10% of the lowest order one, while it becomes 30% at $\tau \approx 0.8 \text{ GeV}^{-2}$. Comparing Eq. (6.10) with Eqs. (6.3) and (6.6), one might expect that $\psi'_-(0)$ will not play a significant role in the current algebra analysis^{*)}.

7. - $\text{SU}(3)_F$ BREAKING EFFECTS AND THE η' PARAMETERS

Here, we discuss the effect of $\text{SU}(3)_F$ breaking on the current algebra relation quoted in Eq. (6.4). Actually, Eq. (6.4) has been derived by taking into account the effect of the term linear in the quark mass m_s (or equivalently in M_K^2) which enters via the condensate $m_s \langle \bar{\psi}\psi \rangle$. The effect of the quadratic term m_s^2 has been neglected. However, one knows that the strange quark invariant mass $\hat{m}_s$ (6c) is larger²³⁾ than the scale Λ of the $\overline{\text{MS}}$ renormalization scheme, so that its contribution can become very important as we have already encountered in the analysis of the $\text{SU}(3)_F$ breaking effects to the kaon PCAC ⁷⁾ and to the GMO mass formulae¹⁷⁾. In order to see the effect of $\text{SU}(3)_F$ breaking in this $\text{U}(1)_A$ sector, we shall consider the version of Eq. (6.1) in the world with massive quarks $\hat{m}_s \gg m_u = m_d = 0$, i.e., we shall derive an analogous sum rule starting from the two-point function built from the divergence of the $\text{U}(1)_A$ current:

$$\partial_\mu A^\mu(x) = 2m_s \bar{\psi}(i\gamma_5 \psi) - \left(\frac{3\alpha_s}{4\pi}\right) \vec{F} \cdot \vec{F}. \quad (7.1)$$

The fact that we have taken $m_u = m_d = 0$ allows us to respect the θ independence of the QCD Lagrangian so that we can consider $\psi_-(0)$ to be zero here. Using the two-loop expressions of the diagonal pieces of the two-point function which have been obtained in Refs. 23b), e), f) for the quark-quark

^{*)} We notice that similar estimate of $\psi'_-(0)$ has been done in Ref. 5e) but his result is based on his high value of the gluonium mass which should be interpreted as an upper bound, on the ad hoc value of the continuum threshold and on the important dependence of his result on the uncertain value of Λ . These weak points make his results unreliable.

component and in Ref. 21) for the gluon-gluon component^{*)}, we get the massive quark version of Eq. (6.1) for $SU(3)_C \times SU(3)_F$:

$$\int_{t_0}^{\infty} \frac{dt}{t} e^{-t\tau} \left(1 - \frac{t\tau}{2}\right) \frac{1}{\pi} \text{Im} \psi(t) \simeq \left(\frac{3\bar{a}_s}{4\pi}\right)^2 \frac{2}{\pi^2} \tau^{-2} \cdot \left\{ \frac{1}{\log \tau \Lambda^2} + \frac{27}{2} \hat{m}_s^2 \tau \left(-\frac{1}{2} \log \tau \Lambda^2\right)^{10/9} \left(1 - \frac{1}{\log \tau \Lambda^2} \cdot [4.66 - 0.7 \log(-\log \tau \Lambda^2)]\right) + 2\pi^2 \langle F^2 \rangle \tau^2 + 6\pi^2 g \langle F^3 \rangle \tau^3 + \dots \right\}. \quad (7.2)$$

If we saturate the spectral function by the η' plus the 0^{-+} gluonium and the continuum with the threshold obtained in Section 3, one can realize that the left-hand side of Eq. (7.2) is dominated by the η' as the gluonium plus the continuum contributions tend to cancel each other. In the right-hand side of Eq. (7.2), we can recognize terms which have been used in the estimate of $\psi_-(0) \Big|_{\text{no-quarks}}$ from Eq. (6.1). A straightforward algebraic manipulation from Eqs. (6.1) and (7.2) leads to:

$$2 M_{\eta'}^2 \frac{1}{g_{\eta'}} e^{-M_{\eta'}^2 \tau} \left(1 - \frac{M_{\eta'}^2 \tau}{2}\right) \simeq - \psi_-(0) \Big|_{\text{no-quarks}} + \frac{3}{4\pi^2} \hat{m}_s^2 \tau^{-1} \frac{1}{\left(-\frac{1}{2} \log \tau \Lambda^2\right)^{8/9}} \left\{ 1 - \frac{1}{\log \tau \Lambda^2} [4.66 - 0.7 \log(-\log \tau \Lambda^2)] \right\}, \quad (7.3)$$

which we may consider as the version of Eq. (6.4) including m_s^2 and $M_{\eta'}^2$ corrections. In order to see the consistency of our approach, let us use the value of $\psi_-(0)$ in Eq. (6.3) with the corresponding value of $\tau = 0.5 \text{ GeV}^{-2}$. Let us also use the value of $\Lambda \approx 150 \text{ MeV}$ and $\hat{m}_s \approx (250 \pm 50) \text{ MeV}^{20)}$. We obtain,

^{*)}We show in Ref. 24) that the off-diagonal contribution is relatively small compared to the diagonal one. We also use an alternative method for the analysis of the m_s^2 effect on the η' -mass in Ref. 24).

after using $M_{\eta'}^2 \approx 0.9 \text{ GeV}^2$ in the non-leading pieces of Eq. (7.3),

$$M_{\eta'}^2 f_{\eta'}^2 \approx (15 \pm 6) 10^{-3} \text{ GeV}^4, \quad (7.4)$$

where we notice that the $\hat{m}_s^2$ and the $\psi_-(0)$ contributions to Eq. (7.3) have been of the same order. Equation (7.4) gives only the by-product $(M_{\eta'}, f_{\eta'})$. We choose to deduce from Eq. (7.4) the decay amplitude of the η' which should be $\sqrt{3} f_\pi$ if the quark model has been used, or about $0.6\sqrt{3} f_\pi$ if the data from $\psi \rightarrow \eta' \gamma$ over $\psi \rightarrow \eta \gamma$ decays has been introduced¹⁶⁾. Using $M_{\eta'} \approx 0.96 \text{ GeV}$, we deduce:

$$f_{\eta'} \approx (0.8 \pm 0.2) \sqrt{3} f_\pi, \quad (7.5)$$

which is astonishingly between the previous numbers.

8. - SUBTRACTION CONSTANTS OF THE 0^{++} TWO-POINT FUNCTION

Sum rules similar to the one in Eqs. (6.1) and (6.9) can also be derived for the 0^{++} channel. The derivation of such sum rules becomes straightforward once one has derived the one for the 0^{-+} channel. We shall not repeat the analysis but we only quote the results:

$$\int_{t_0}^{\infty} \frac{dt}{t} e^{-t\tau} \left(1 - \frac{t\tau}{2}\right) \frac{1}{\pi} \text{Im } \psi_+(t) \approx \psi_+(0) +$$

$$+ \frac{2}{\pi^2} \bar{\alpha}_s^2 \tau^{-2} \left\{ \frac{1}{\log \tau \Lambda^2} - 2\pi^2 \langle F^2 \rangle \tau^2 - 6\pi^2 g \langle F^3 \rangle \tau^3 \right\} \quad (8.1)$$

and

$$\int_{t_0}^{\infty} \frac{dt}{t^2} e^{-t\tau} \left[1 + t\tau \left(1 - \frac{t\tau}{2} \right) \right] \frac{1}{\tau} \text{Im} \psi_+(t) \simeq \psi'_+(0) + \frac{2}{\pi^2} \cdot \bar{Q}_5^2 \cdot \tau^{-1} \left\{ 1 - \frac{2}{9 \log \tau \Lambda^2} \left[10 - 2\beta_1 - 2 \frac{\beta_2}{\beta_1} \log(-\log \tau \Lambda^2) - 4\pi^2 g \langle F^3 \rangle \tau^3 \right] \right\}. \quad (8.2)$$

We give in Fig. 10a the two sides of Eq. (8.1). One can realize that the QCD contribution also dominates the estimate of $\psi_+(0)$ like the case of $\psi_-(0)$. The QCD contribution reaches a maximum at $\tau \simeq 0.5 \text{ GeV}^{-2}$. This value of τ is also the one where $\psi_-(0)$ has a stability (Fig. 10b). The resulting value of $\psi_+(0)$ for τ ranging from 0.5 to 1 GeV^{-2} is:

$$\psi_+(0) \simeq [(0.48 \sim 0.54) \text{ GeV}]^4, \quad (8.3)$$

where we recall that the current has been normalized as in Eq. (1.3). One can notice that $\psi_+(0)$, due only to gluonium in Eq. (8.3), is positive, contrary to $\psi_-(0)$ in Eq. (6.3). If we take the same normalization for the 0^{-+} and 0^{++} currents [Eqs. (1.2) and (1.3)], one can remark that the absolute values of $\psi_-(0)$ and $\psi_+(0)$ are approximately the same. Equation (8.3) differs by appreciably to the low energy estimate derived using the functional integral formalism^{5a)}. It is important to distinguish the sum rules result and the low-energy one using independent methods such as, e.g., lattice Monte Carlo calculation.

We also give in Fig. 11a the behaviour of Eq. (8.2) for various values of τ . The QCD contribution still dominates the contribution to $\psi'_+(0)$. The optimal value of $\psi'_+(0)$ is obtained from τ about 0.5 GeV^{-2} (Fig. 11b) and is negative:

$$\psi'_+(0) \simeq - (0.1 \text{ GeV})^2 \quad (8.4)$$

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Typical examples of Feynman diagrams
contributing to the gluonic
two-point function

Gluonic current insertion
Lowest order
Next-to-leading
Dimension four vacuum condensates $\alpha_s \langle F^2 \rangle$
Dimension six vacuum condensates $f_{abc} g^3 \langle F_{abc}^3 \rangle$
Dimension eight vacuum condensates

Table 1

Table 2

FIGURE CAPTIONS

Fig. 1: Behaviour of the QCD expression of the moments of the R of the 2^{++} current for various values of τ . The uncertainties come from the determination of $\alpha_s F^2$.

Fig. 2: Fit of (M_G^2, t_c) in the 2^{++} channel:

- a) Fit of (M_G, t_c) fixing $\tau_{\max} \approx 0.2 \text{ GeV}^{-2}$, $\Lambda \approx 0.2 \text{ GeV}$ and varying $\tau_{\min}$.
- b) Fit of (M_G^2, t_c) fixing $\tau_{\min} = 5 \cdot 10^{-4} \text{ GeV}^{-2}$ and varying $\tau_{\max}$. $\Lambda \approx 0.2 \text{ GeV}$.
- c) Analogous to a) but $\Lambda \approx 0.1 \text{ GeV}$.
- d) Analogous to b) but $\Lambda \approx 0.1 \text{ GeV}$.

Fig. 3: Behaviour of the moment R in the 0^{-+} channel.

Fig. 4 to Fig. 5: Fit of (M_G^2, t_c) for the 0^{-+} channel.

Fig. 6: Behaviour of the moment R in the 0^{++} channel.

Fig. 7: Fit of (M_G^2, t_c) for the 0^{++} channel.

Fig. 8: Variation of the left-hand side and right-hand side of Eq. (6.1) versus τ for $\Lambda = 0.15 \text{ GeV}$, $f_G = 30 \text{ MeV}$ and $M_G = 1.7 \text{ GeV}$ and $\sqrt{t_c} = 1.8 \text{ GeV}$.

Fig. 9: Variation of various contributions to $\phi'(0)$ versus τ using the same parameters as in Fig. 8.

Fig. 10: a) Variation of various contributions to the left-hand side and right-hand side of Eq. (8.1) versus τ for $\Lambda = 0.15 \text{ GeV}$, $f_G = 80 \text{ MeV}$, $M_G = 1 \text{ GeV}$ and $\sqrt{t_c} = 1.58 \text{ GeV}$.
b) Behaviour of $\phi_+(0)$ versus τ .

Fig. 11: a) Contributions to the left-hand side and right-hand side of Eq. (8.2) versus τ for the parameters used in Fig. 10;
b) Behaviour of $\phi_+(0)$ versus τ .

Fig. 1

Fig. 2a

Fig. 2b

Fig. 2c

Fig. 2d

Fig. 3

Fig. 4a

Fig. 4b

Fig. 4c

Fig. 5a

Fig. 5b

Fig. 6

Fig. 7a

Fig. 7b

Fig. 8

Fig. 9

Fig. 10a

Fig. 10b

Fig. 11a

Fig. 11b