DYNAMICAL FERMION MASS HIERARCHY AND FLAVOUR MIXING

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ABSTRACT

The chiral symmetry breaking of high colour representations produces dynamical breaking of the standard electroweak gauge symmetry. By enlarging the colour group and subsequently breaking it down to $SU(3)_C$ fermions acquire radiative masses from the chiral breaking. We present attempts to produce realistic fermion mass matrix in two classes of models depending on the way that the colour group is enlarged. A realistic example is found in one of these classes of models.

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1. INTRODUCTION

The recent discoveries of W and Z at CERN¹⁾ have established the standard electroweak model $SU(2)_L \otimes U(1)$ ²⁾ as a fundamental ingredient of our understanding of elementary particles. However, it is also widely recognized that the Higgs mechanism³⁾ which is used in the standard model in order to provide the spontaneous breakdown of the $SU(2)_L \otimes U(1)$ symmetry can serve only as a parametrization of some as yet unspecified dynamical mechanism. A few years ago some speculations were made about such a dynamical mechanism based on the existence of a new QCD-like interaction, the technicolour⁴⁾ interaction, whose fundamental scale $\Lambda_{T.C.}$ was of the order of 1 TeV. Then the gauge $SU(2)_L \otimes U(1)$ symmetry is dynamically broken by a system of massless fermions, the technifermions, which in addition to the usual $SU(3)_c \otimes SU(2)_L \otimes U(1)$ interactions feel also the technicolour interactions which force them to condense. However, the experimental bounds on the flavour changing neutral currents rule out this dynamical scenario in the full complexity which is required in order to produce fermion masses⁵⁾.

Here we shall follow another suggestion for dynamical symmetry breaking^{6,7,8,9)} which is much more attractive than the technicolour scenario, since it requires only ordinary QCD with additional fermions in higher than triplet representations of the $SU(3)_c$, gauge group. Chiral symmetry breaking in the high representation quark sector may occur at much larger mass scales than ordinary chiral symmetry breaking, and could produce dynamical breaking of the electroweak gauge symmetry. This might sound surprising, however with simple assumptions about chiral symmetry breaking such as: (i) chiral symmetry breaking depends entirely on the strength of the quark-antiquark binding potential which is based on the one gluon exchange approximation and (ii) perturbation theory holds in the region of chiral symmetry breaking, one obtains exponential enhancement for the scale of condensation of different representations. Of course one does not expect chiral symmetry breaking to have such a simple explanation especially one expects non-perturbative effects to be involved, however, in recent lattice calculations of the $SU(2)$ colour model, it is found that fermions in different colour representations do condense at exponentially separate mass scales.

The above dynamical scenario has been supplemented by a mechanism which provides also masses to quarks and leptons. Namely it was realized independently in Refs. (8) and (9) that if the colour group is extended to G_s and subsequently G_s is spontaneously broken to $SU(3)_c$, then the massive $G_s/SU(3)_c$ gauge bosons will mediate transitions between colour singlets or colour triplets and fermions in high colour representations which are sited in an appropriate irreducible representation of the strong group G_s . Then light fermions obtain radiatively masses as it is shown in Fig. 1. Of course, G_s can also be a subgroup of a larger group which breaks in successive steps to G_s . Depending on the choice of the group G_s and its breaking to $SU(3)_c$ we have constructed two classes of models. In one class the G_s is the semi-simple $(SU(3) \otimes SU(3))_s$ and it unifies ordinary quarks with quarks in high representations (echo-quarks) and similarly for leptons, but not quarks and leptons. In the second class G_s is the simple $SU(4)_s$ and it unifies quarks, leptons and echo-quarks. The latter class, since it unifies quarks and leptons without the induction of proton decay, would be a very interesting alternative to the usual grand unification schemes if protons appeared to be stable. However, within this new dynamical scenario, not only, as yet a realistic model explaining all the known elementary particle masses does not exist but even mechanisms for flavour mixing or mass splitting between fermions of the same charge or fermions which belong to the same weak $SU(2)$ doublet have not been suggested. It is the purpose of this paper to propose mechanisms that could solve these problems in the framework of the new dynamical scenario. As we shall see in the following, our proposals fail systematically to lead to a realistic model if we enlarge QCD to $SU(N)_s, N > 3$, while it seems that they are promising in the enlargement of QCD to $(SU(3) \otimes SU(3))_s$. Nevertheless, for instructive reasons to further model building towards a realistic model we present here our attempts in both directions.

The paper is organized as follows: in the second chapter we present an extensive introduction on the speculations about the dynamical symmetry breaking from fermions in high QCD representations and on the support they received from Monte Carlo lattice calculations. In the third chapter a model is presented which shows how fermions can acquire masses in these dynamical schemes. Some emphasis is placed on the attractive feature of

quark-lepton unification without proton decay, which naturally appears in a class of these models. In the fourth chapter we present attempts to produce realistic fermion masses and flavour mixing as well as the difficulties one faces in a class of models. In the fifth chapter, we present another class of models which naturally accommodates the various fermion scales and flavour mixing and gives promises to be the framework of a realistic model.

2. DYNAMICAL SYMMETRY BREAKING FROM QUARKS IN HIGH $SU(3)_C$ REPRESENTATIONS

Let us recall in this section the basis of the conjecture ^{6,7,8,9)} that ordinary QCD with new quarks in representations $R > 3$ can produce the dynamical symmetry breakdown of the electroweak group $SU(2)_L \otimes U(1)$. Neglecting current quark masses and electroweak interactions, there exists in the QCD Lagrangian for each of the quark sectors with quarks in the R -representation a separate global chiral symmetry

$$SU(N_R)_L \otimes SU(N_R)_R \otimes U(1) \quad (2.1)$$

where N_R is the number of quark flavours in the R representation ($R = 3, 6, 8, 10, 15$, etc.). At some value of the effective strength of colour forces it is assumed that quark-antiquark condensates are formed giving rise to a vacuum that breaks the axial-vector part of the global $SU(N_R)_L \otimes SU(N_R)_R$ symmetry and preserves the vector $SU(N_R)$. Failure of the conserved axial charges to annihilate the vacuum implies the existence of $N_R^2 - 1$ Goldstone bosons, which are massless quark-antiquark bound state excitations, and dynamical masses for the quarks. Switching on the standard electroweak theory with left-handed doublets and right-handed singlet quarks the local $SU(2)_L \otimes U(1)$ symmetry is also broken by the vacuum which is induced by the quark condensates, to the $U(1)$ of electromagnetism. The gauge bosons $W^\pm$ and Z acquire masses by absorbing three of the Goldstone bosons. Moreover, because of the multiplets assignment the relation $M_W = M_Z \cos \theta_w$, where θ_w is the Weinberg angle, is also preserved. This dynamical scenario is just a straightforward extension of Weinberg's and Susskind's scenario. Of course they referred only to the ordinary QCD triplet quark sector and they expected the chiral symmetry breaking to take place at $\mu \sim 1-2$ GeV. At this scale $\alpha_s(\mu)$ becomes strong enough to form quark-antiquark condensates. The would-be Goldstone bosons are pions and their coupling to weak current is:

$$\langle 0 | J_{5\mu}^W(0) | \pi(p) \rangle = -i f_\pi P_\mu \quad (2.2)$$

where f_π is the pion decay constant. In the absence of any source for quark masses these pions give masses to $W^\pm$ and Z while the photon remains massless. The $SU(2)_L \otimes U(1)$ vector boson masses are:

$$\begin{aligned} M_\gamma &= 0 \\ M_Z^2 &= \frac{1}{4} (g_1^2 + g_2^2) f_\pi^2 \\ M_W^2 &= \frac{1}{2} g_2^2 f_\pi^2 \end{aligned} \quad (2.3)$$

where g_2, g_1 are the coupling constants of $SU(2)$ and $U(1)$. Since isospin symmetry is unbroken each pion has the same f_π . Thus isospin conservation guarantees the successful relation $M_W = M_Z \cos \theta_W$. However, the resulting values for $W^\pm, Z$ are in the tens of MeV's region given the known values of g_1, g_2 and f_π . In order to obtain the measured values for the vector boson masses one needs a "pion" decay constant $F \sim 250$ GeV. Marciano⁶⁾ speculated that chiral symmetry breaking in the other quark sectors (where $R > 3$) may occur at mass scales much larger than the GeV region, maybe at $\mu \sim 500$ GeV giving rise probably to a Goldstone boson with the required $f_{\pi R} \sim 250$ GeV. In order to obtain this large scale, Marciano assumed that chiral symmetry breaking depends entirely on the strength of the effective quark-antiquark binding potential, based on the one gluon approximation, and takes place for a value of this strength which is common for all sectors. The quark-antiquark binding potential based on the one-gluon exchange approximation is proportional to

$$C_2(R) \propto_s(\mu) \quad (2.4)$$

where $C_2(R)$ is the quadratic $SU(3)$, Casimir invariant of the R -representation. The quantity in (2.4) is a measure of the effective colour flux linking a quark-antiquark pair at mass scale μ . Assuming a common value for all quark sectors and integrating the renormalization group equation one obtains

$$\frac{\mu_6}{\mu_3} = \exp \left(\int_{\alpha_s(\mu_3)}^{\alpha_s(\mu_6)} \frac{[C_2(3)/C_2(R)] \cdot \alpha_s(\mu)}{\beta(\alpha_s)} d\alpha_s \right) \quad (2.5)$$

where

$$\mu \frac{\partial}{\partial \mu} \alpha_s(\mu) \equiv \beta(\alpha_s) \simeq b \alpha_s^2 + O(\alpha_s^3) \quad (2.6)$$

Marciano has estimated this ratio assuming that α_s is small enough in the region of integration in eq. (2.5) so that it is enough to keep only the first term in the β -function of eq. (2.6)

$$b = -\frac{1}{2\pi} \left(11 - \frac{2}{3} N_3 - \frac{10}{3} N_6 - 4 N_8 + 10 N_{10} + \dots \right) \quad (2.7)$$

Then, taking $N_3=6$ and neglecting all the other quark sector contributions he obtained

$$\frac{\mu_6}{\mu_3} \simeq \exp \left[\frac{2\pi}{7\alpha_s(\mu_3)} \left(\frac{C_2(R)}{C_2(3)} - 1 \right) \right] \quad (2.8)$$

In Table 1, his results are listed too.

These results are based, as we have seen before, on assumptions about chiral symmetry breaking and oversimplification that one is not really ready to believe. However, in recent Monte Carlo studies¹⁰⁾ of the chiral symmetry breaking in the SU(2) lattice QCD confirmation of the "Casimir scaling", namely Marciano's assumptions for the higher (R=4 and R=5) SU(2)-representations has been found. They have carried out Monte

Carlo runs in the quenched approximation for several representations of SU(2) both at zero and at finite real physical temperature. They work with finite temperatures since, in this way, they have a relatively simple handle on the energy and length scales involved. The chiral symmetry restoration temperature in MeV is presumably related to the inverse separation of the fermions making up the condensate and directly comparable to the energy scale at which symmetry breaking becomes important in the zero temperature case. Thus by comparing the chiral symmetry restoration temperatures for different representations they are able to see whether a hierarchy of scales emerges. They found the following ratios for the chiral symmetry restoring temperatures T_χ

$$\frac{T_\chi(4)}{T_\chi(3)} \approx 10^{2.7 \pm 1.3} \quad ; \quad \frac{T_\chi(5)}{T_\chi(4)} \approx 10^{3.4 \pm .4} \quad (2.9)$$

Then in order to make contact with the "Casimir scaling" picture they tried to understand their results in terms of an effective Abelian coupling constant describing the strength of the fermion-fermion interaction with a coupling

$$e_c^2 \equiv C_2 g_c^2 \quad (2.10)$$

enough to trigger the spontaneous symmetry breaking. Of course there is no unique way to define the g_c^2 of eq. (2.10) and they chose to use the running coupling constant at the scale given by the chiral symmetry restoration temperature T_χ . Thus for a lattice definition of the coupling constant they took

$$g_c^2 \rightarrow g_c^2 = \frac{1}{2b_0 \ln \frac{T_\chi}{\Lambda_L}} \quad (2.11)$$

where $b_0 = 11/48\pi^2$. In terms of the continuum definition of the coupling constant used in the momentum-space subtraction scheme they took

$$g_c^2 \rightarrow g_{mom}^2 = \frac{1}{2b_0 \ln \frac{T_\chi}{\Lambda_{mom}}} = \frac{1}{2b \left[\ln \frac{T_\chi}{\Lambda_L} - \ln \frac{\Lambda_{mom}}{\Lambda_L} \right]} \quad (2.12)$$

For this scheme they found:

$$\begin{aligned} C_2 g_{\text{mom}}^2 / 4\pi &\simeq 0.34 \pm 0.02 & R = 4 \\ C_2 g_{\text{mom}}^2 / 4\pi &\simeq 0.30 \pm 0.02 & R = 5 \end{aligned} \quad (2.13)$$

These two numbers are close enough so that they interpreted their results as being consistent with the "Casimir scaling" hypothesis.

If the simple "Casimir scaling" really works, one sees from Table 1 that for $R=10$ under $SU(3)$ the spontaneous chiral breaking should occur almost at the grand unification scale while for the $R = 15'$ and $R = 21$ the predicted scale exceeds the Planck scale! We feel that one cannot really expect the "Casimir scaling" hypothesis to work with full strength for all the representations and it is natural to assume that some sort of "screening of the Casimir scaling" is taking place by increasing the dimensionality of the representations.

One could interpret the slight difference of the values found in eq. (2.13) as a weak indication that a screening is taking place. This difference is in fact very large for the lower representations ($R=2,3$) even we should admit that we are not really sure where it comes from. In any case, the same procedure described before gives

$$\begin{aligned} C_2 g_{\text{mom}}^2 / 4\pi &\simeq 1.19 & R = 2 \\ C_2 g_{\text{mom}}^2 / 4\pi &\simeq 0.59 & R = 3 \end{aligned} \quad (2.14)$$

As we shall see in the next sections, in order to have some chance to construct a realistic model one has to introduce various different representations which could account for the multiplicity of scales that we observe in fermions. We assume that there exists a screening factor $f(R)$ associated to every representation so that the strength of the fermion-antifermion force is proportional to

$$f(R) C_2(R) \propto s(\mu_R) \quad (2.15)$$

Furthermore we assume that this quantity takes a constant value when spontaneous symmetry breaking for a given representation R is taking place. In the absence of any solid information on the screening factor $f(R)$, we expect only intuitively that $f(R)$ decreases with R which is not in disagreement with the results (2.13), (2.14). We can only suggest that a study in a realistic $SU(3)_c$ model with many high representations should be achieved in order to justify or disprove our assumption.

3. DYNAMICAL MASS GENERATION AND QUARK-LEPTON UNIFICATION WITHOUT PROTON DECAY

In this chapter we are going to present an example showing dynamical breakdown of the electroweak group $SU(2)_L \otimes U(1)$ as well as dynamical generation of fermion masses. The basic mechanism as we have already mentioned is based on the enlargement of the colour group and its subsequent breaking to $SU(3)_c$. Examples of this mechanism have already been given in Refs (8) and (9). A characteristic feature of the models that one can construct by enlarging colour to $SU(N)_s$ gauge groups is that quarks and leptons appear unified in one representation. Within the context of the simplest of these models which involves enlargement to $SU(4)_s$ we are going to discuss also the proton stability. The feature of proton stability together with the feature of unification of quarks and leptons makes this class of models very attractive given the uncomfortable state of the usual unification schemes¹¹⁾.

Consider a model based on the gauge group $G = G_s \otimes SU(2)_L \otimes U(1)$ where $G_s = SU(4)_s$. The elementary fermions of the model have the following transformation properties:

$$\begin{aligned} G &= SU(4)_s \otimes SU(2)_L \otimes U(1) \\ \begin{pmatrix} H^u \\ H^d \end{pmatrix}_L &= \begin{pmatrix} 1 & 0 & , & 2 & , & 1/2 \end{pmatrix} \\ H^u_R &= \begin{pmatrix} 1 & 0 & , & 1 & , & 1 \end{pmatrix} \\ H^d_R &= \begin{pmatrix} 1 & 0 & , & 1 & , & 0 \end{pmatrix} \end{aligned} \quad (3.1)$$

Here we assume that the remaining families appear as replications of the first one. Next $SU(4)_s$ has to break to $SU(3)_c$ which can be done with a 4-plet Higgs in the absence of any dynamical mechanism for this breaking. We define the $U(1)_{e.m.}$ charge operator as

$$Q = U_4(1) + T_{3L} + U_Y(1) \quad (3.2)$$

where $U_4(1) = \sqrt{(2/3)} T_{15}$ with $\text{Tr} T_{15}^2 = 1/2$. Under the $SU(3)_c$ the 10-plets are decomposed as $10=1+3+6$. We naturally identify the triplet with a quark q and the singlet with a lepton l while the sextet is a new quark Q in high representation. Thus quarks and leptons appear to be unified in the same $SU(4)_s$ representation. According to our discussion in the previous chapter we expect the chiral symmetry breaking for the sextets to occur at a much higher scale than for triplets. Given the fermion assignment (3.1) under the gauge group G the sextet chiral breaking breaks also $SU(2)_L \otimes U(1)$ gauge symmetry to $U(1)_{e.m.}$ providing masses to $W^\pm$ and Z possibly of the right magnitude. The 15 gauge bosons of $SU(4)_s$ decompose under $SU(3)_c$ as $15=8+3+\bar{3}+1$. The 8-plet accommodates the usual $SU(3)_c$ gluons while all the other vector bosons obtain masses as $SU(4)_s$ breaks to $SU(3)_c$. The heavy vector bosons can now connect 1 to 3 and 3 to 6. Thus quarks and leptons^{F)} also obtain masses from the spontaneous chiral breaking of the sextets as is shown in Figs. 2 a,b. One obtains

$$m_q \simeq \frac{9\alpha_s (6)\mu_6^3}{4\pi M_4^2} \ln \left(\frac{M_4^2}{(3C_2(6))^{2/3}\mu_6^2} \right) \quad (3.3)$$

$$m_l \simeq \frac{\alpha_s}{\pi} m_q$$

where M_4 is the mass of the vector bosons which become heavy when $SU(4)_s$ breaks to $SU(3)_c$, μ_6 is the scale of the sextet chiral symmetry breaking and α_s is the strong coupling constant at the renormalization scale M_4 .

Let us look more generally at the effective Lagrangian which is due to the exchange of the heavy vector bosons $A_{4,1}, A_{4,3}$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = g_4^2 \sum_{\alpha, \beta = L, R} \sum_{i, j = 1, 2, 3} \left\{ \frac{1}{M_{4,1}^2} (\bar{q}_{\alpha, i} q_{\alpha, i} \bar{q}_{\beta, j} q_{\beta, j} + \bar{l}_{\alpha, i} l_{\alpha, i} \bar{l}_{\beta, j} l_{\beta, j} \right. \\ \left. + \bar{l}_{\alpha, i} l_{\alpha, i} \bar{Q}_{\beta, j} Q_{\beta, j} + \bar{q}_{\alpha, i} q_{\alpha, i} \bar{Q}_{\beta, j} Q_{\beta, j}) \right. \\ \left. + \frac{1}{M_{4,3}^2} (\bar{l}_{\alpha, i} q_{\alpha, i} \bar{q}_{\beta, j} l_{\beta, j} + \bar{l}_{\alpha, i} q_{\alpha, i} \bar{Q}_{\beta, j} q_{\beta, j} + \bar{q}_{\alpha, i} Q_{\alpha, i} \bar{Q}_{\beta, j} q_{\beta, j} \right. \\ \left. + \bar{q}_{\alpha, i} Q_{\alpha, i} \bar{q}_{\beta, j} l_{\beta, j}) \right\} \end{aligned} \quad (3.4)$$

From this effective Lagrangian one can see again that the terms $\bar{q}_{L,i} q_{L,i} \bar{q}_{R,j} q_{R,j}$ give rise to quark masses when the sextets form condensates $\langle 0 | \bar{Q}_L \bar{Q}_R | 0 \rangle$.

Moreover it is evident from the effective Lagrangian that there are no operators that could induce proton decay nor any baryon violating processes (the Q 's carry baryon number $2/3$ and lepton number -1).

The effective Lagrangian suggests also that the total lepton number is conserved while the family number is violated by the operators like $\bar{q}_{\alpha, i} q_{\alpha, i} \bar{q}_{\beta, j} q_{\beta, j}$, with $i \neq j$. Therefore, rare transitions as $K \rightarrow \mu e$, $\Delta M(K_L^0 - K_S^0)$, etc., can be induced. So the neutral singlet under $SU(3)_c$ heavy vector boson should be heavy enough not to violate the bounds on flavour changing neutral currents. The bounds on such a vector boson mass have been examined in the flavour changing horizontal interactions¹²⁾. They should be heavier than a few hundred TeV. This bound is safe also for the additional charged $SU(3)_c$ triplet vector boson of the present case.

4. FERMION MASS SPLITTING

In the previous chapter, we have seen a model which practically describes only one family of fermions. The other families were simply introduced as replications of the first one. There are several obvious problems linked to this model. First, even within the one family model there is not enough splitting between the up and down quarks. The reason is that both up and down quarks obtain their masses from the sextet condensation corresponding to each of them. The condensation for both sextets takes place practically at the same scale since electromagnetic and neutral weak corrections which differentiate them are very small. Secondly, even worse, there is nothing to make the rest of the families

heavier than the first one given that the sextet condensation is responsible for all fermion masses. Thirdly, there is no flavour mixing in the charged current. Here in order to cure these problems we propose to introduce fermions which generally belong to different representations of the enlarged strong group.

The introduction of higher representations in our framework forces us to assume some kind of "screening in the Casimir hypothesis" as we have already discussed in Chapter 2. Moreover, here we assume that the highest representation involved is responsible for the breaking of $SU(2)_L \otimes U(1)$ gauge symmetry. Thus we assume that spontaneous symmetry breaking takes place when the quantity

$$f(R_1) \cdot f(R_2) \cdot \lambda(1) \cdot \lambda(2) \quad (4.1)$$

obtains a value common to all the representations. $\lambda(1)$, $\lambda(2)$ denote the gauge charges and $f(R)$ are the screening factors. Then, (4.1) can be written as:

$$\frac{1}{2} f(R_1) f(R_2) \left\{ C_2(G) - C_2(2) - C_2(1) \right\} \quad (4.2)$$

where

$C_2(G)$, $C_2(1)$, $C_2(2)$ denote the quadratic Casimir operators for the combined and constituent systems, which take the form (2.15) for $R_1=R_2$, $R_1 \times \tilde{R}_2 = 1$.

Consider a model based on the gauge group $SU(4)_S \otimes SU(2)_L \otimes U(1)$, as before, but with the following assignments for fermions

$$G = SU(4)_S \otimes SU(2)_L \otimes U(1)$$

$$\begin{pmatrix} H^u \\ H^d \end{pmatrix}_L = \begin{pmatrix} \overline{20}'' & 2 & 1 \end{pmatrix} \quad (4.3)$$

$$H_R^u = \begin{pmatrix} 10 & 1 & 1 \end{pmatrix}$$

$$H_R^d = \begin{pmatrix} \overline{20}'' & 1 & 1/2 \end{pmatrix}$$

Again we expect $SU(4)_S$ to break to $SU(3)_C$ with a 4-plet Higgs. Under the $SU(3)_C$ the $\overline{20}$ and 10 have the following decomposition:

$$\overline{20}'' = 1 + 3 + 6 + 10$$

(4.4)

$$10 = 1 + 3 + 6$$

Next we let the quantity (4.2) to decide which is the most attractive channel where condensation is going to take place. Here we are once more confronted with the need for the screening factor $f(R)$ since if we use the formula (4.2) without any screening factor we find that the most attractive channel is $\langle 10_L^u \overline{6}_R^u \rangle$ which breaks $SU(3)_C$. However, for $f(6) > (6/5)f(10)$ the most attractive channel favours the $SU(3)_C$ singlets $\langle 10_L \overline{10}_R \rangle$ and $\langle 6_L \overline{6}_R \rangle$ to condense. The scale that the tenplet condenses in this model is expected to be appropriate to give to M_W, M_Z their measured values i.e. $\mu_{10} \approx 500 \text{ GeV}$. This corresponds to a screening factor $f(10) \approx 0.3$ for $\alpha_s \approx 0.15$. Then $f(6) \geq 0.4$ which in turn can be translated to $\mu_6 \geq 36 \text{ GeV}$. Then the fermion masses are going to be:

$$m_u \simeq \frac{9 \alpha_s C_2(6) \mu_6^3}{4\pi M_4^2} \ln \left(\frac{M_4^2}{(3C_2(6))^{2/3} \mu_6^2} \right)$$

$$m_\tau \simeq \frac{\alpha_s}{\pi} m_u \quad (4.5)$$

$$m_d \simeq \frac{9 \alpha_s}{4\pi M_4^2} \left\{ \mu_6^3 \ln \left(\frac{M_4^2}{(3C_2(6))^{2/3} \mu_6^2} \right) + \frac{\alpha_s}{\pi} \mu_{10}^3 \ln \left(\frac{M_4^2}{(3C_2(10))^{2/3} \mu_{10}^2} \right) \right\}$$

$$m_e \simeq \frac{\alpha_s}{\pi} m_d$$

for $\alpha_4 = 0.1$ and $M_4 = 38 \text{ TeV}$, $\mu_6 \approx 120 \text{ GeV}$ (i.e. with screening factor $f^2(6) = 0.53$) one obtains $m_u \approx 4 \text{ MeV}$; $m_d \approx 15 \text{ MeV}$; $m_\nu \approx 0.1 \text{ MeV}$; $m_e \approx 0.5 \text{ MeV}$.

Let us next consider what one can do for the rest of the families and for the mixing. We consider a two-family model based on the same group as before in order to make a few remarks and to demonstrate the problem involved:

$$\begin{aligned}
& - 13 - \\
G &= SU(4)_S \otimes SU(2)_L \otimes U(1) \\
\begin{pmatrix} H^u \\ H^d \end{pmatrix}_L &= \begin{pmatrix} 10 & ; & 2 & ; & 1/2 \end{pmatrix} \\
H_R^u &= \begin{pmatrix} 4 & ; & 1 & ; & 1/2 \end{pmatrix} \\
H_R^d &= \begin{pmatrix} 10 & ; & 1 & ; & 0 \end{pmatrix} \\
\begin{pmatrix} H^c \\ H^s \end{pmatrix}_L &= \begin{pmatrix} \overline{20}'' & ; & 2 & ; & 1 \end{pmatrix} \\
H_R^c &= \begin{pmatrix} \overline{20}'' & ; & 1 & ; & 3/2 \end{pmatrix} \\
H_R^s &= \begin{pmatrix} 10 & ; & 1 & ; & 0 \end{pmatrix}
\end{aligned} \tag{4.6}$$

Similar considerations as before on the most attractive channels lead us to the following assumptions on the screening factors:

$$\frac{6}{5} f(10) < f(6) < \frac{4}{5} f(3) \tag{4.7}$$

so that the produced condensates do not break $SU(3)_c$.

In this model the c-quark is going to obtain mass from the 10-plet condensation, while the u-quark remains massless, since it does not appear to be unified in a common representation under $SU(4)_S$ with another fermion of higher representation under $SU(3)_c$. For the quarks with charge $-1/3$ the situation is more complicated because it is not clear whether the sextets are going to condense in a way that conserves the family number $\langle 6_L^d \bar{6}_R^d \rangle$ and $\langle 6_L^s \bar{6}_R^s \rangle$ or in a way which violates the family number $\langle 6_L^d \bar{6}_R^s \rangle$ and $\langle 6_L^s \bar{6}_R^d \rangle$. In this case one must take into account also all the other forces which might distinguish the two cases. It turns out that also in the latter case, there is no preference for one of the two possible channels. Thus they are both going to appear with equal strength. We obtain the following quark mass matrices:

$$M^{2/3} = \begin{pmatrix} \bar{u}_L \\ \bar{c}_L \end{pmatrix} K_1 \begin{pmatrix} u_R & c_R \\ 0 & 0 \\ 0 & 1 \end{pmatrix} + h.c.$$

$$M^{-1/3} = \begin{pmatrix} \bar{d}_L \\ \bar{s}_L \end{pmatrix} K_2 \begin{pmatrix} d_R & s_R \\ 1 & 1 \\ 1 & 1 \end{pmatrix} + h.c. \quad (4.8)$$

$$K_1 \simeq (9\alpha_s C_2(10) \mu_{10}^3 / 4\pi M_4^2) \ln(M_4^2 / (3C_2(10))^{2/3} \mu_{10}^2)$$

$$K_2 \simeq (9\alpha_s C_2(6) \mu_6^3 / 4\pi M_4^2) \ln(M_4^2 / (3C_2(6))^{2/3} \mu_6^2)$$

The matrices (4.8) with $M_4 \simeq 980$ GeV, $\mu_{10} \simeq 500$ GeV, $\mu_6 \simeq 49$ GeV [corresponding to screening factor $f^2(6) \simeq 0.43$] lead to the following physical quark masses.

$$m_{u,d} = 0 ; m_s = 175 \text{ MeV} ; m_c = 1.4 \text{ GeV} \quad (4.9)$$

and no flavour mixing.

We have presented this model in order to demonstrate that the class of models in which the colour group is enlarged to $SU(N)_c$, admits a hierarchy of quark mass scales. Unfortunately, in this class of models there is a serious problem with flavour mixing as we have seen in the above particular example. In this model flavour mixing fails to appear in the charged weak current, in spite of the presence of off-diagonal terms in (4.8), because the produced mixing is trivial. It seems we have to complexify further the picture in order to obtain a realistic mass spectrum and flavour mixing. We have tried to complexify the picture in various ways none of which led to an attractive possibility. Particular attention was paid in enlarging further the strong gauge group. In this case there is a possibility to introduce an additional source of flavour mixing coming from the exchange of heavy vector bosons. Even if we succeed in finding a flavour mixing in the charged current, our models are confronted with problems the most serious of which are: (i) the presence of light particles with the quantum

numbers of (ν, e) sitting in a large representation with known particles which however could not be identified with known particles (ii) cancellation of the anomalies requires the introduction of extra representations but we did not find a satisfactory mechanism to make heavy the particles which are accommodated in these representations.

We think that more effort should be devoted in making this class of models a viable alternative to the usual unification schemes. In any case, the examples we present in this chapter appear to be useful to us in the construction of much more satisfactory models in the framework of enlarging the colour group to $SU(3) \otimes SU(3)$ which we describe in the next chapter.

5. TOWARDS A REALISTIC MODEL

With the experience we have obtained from the previous examples in which QCD was enlarged to $SU(N)_S$ we shall try in this chapter to apply the same mechanisms in the class of models that one can construct by enlarging QCD to $(SU(3) \otimes SU(3))_S$. It appears that it is much easier in the latter context to satisfy the requirement for a multiplicity of scales and on the other hand to cancel anomalies without leaving the theory with any unwanted light particles.

First, let us recall the model of Ref (8) which is based on the gauge group $G = (SU(3) \otimes SU(3))_S \otimes SU(2)_L \otimes U(1)$. A discrete symmetry interchanging the two $SU(3)$ subgroups was imposed. The elementary fermions (fermionic haplons) of the model have the following assignments:

$$\begin{aligned}
 G &= (SU(3) \otimes SU(3))_S \otimes SU(2)_L \otimes U(1) \\
 \begin{pmatrix} H^u \\ H^d \end{pmatrix}_L &= \left((\bar{3}, \bar{3}) \quad ; \quad 2 \quad ; \quad 1/3 \right) \\
 H^u_R &= \left((\bar{3}, \bar{3}) \quad ; \quad 1 \quad ; \quad 4/3 \right) \\
 H^d_R &= \left((\bar{3}, \bar{3}) \quad ; \quad 1 \quad ; \quad -2/3 \right) \\
 \begin{pmatrix} H^\nu \\ H^e \end{pmatrix}_L &= \left((3, \bar{3}) \quad ; \quad 2 \quad ; \quad 1 \right) \\
 H^e_R &= \left((3, \bar{3}) \quad ; \quad 1 \quad ; \quad -2 \right)
 \end{aligned} \tag{5.1}$$

Next the $(SU(3) \otimes SU(3))_S$ gauge group breaks down to the "diagonal" $SU(3)_C$ by introducing appropriate Higgs. After spontaneous symmetry breaking of G_S to $SU(3)_C$, a linear combination $V_\mu \equiv (A_\mu - B_\mu)/\sqrt{2}$ forms a colour octet of massive vector bosons [A_μ , B_μ are the vector bosons associated with the two $SU(3)$]. The orthogonal combination $V_\mu^+ \equiv (A_\mu + B_\mu)/\sqrt{2}$ emerges as the colour octet of massless gauge bosons of QCD. The quark-haplons decompose to a colour 3 and a colour 6 under the diagonal $SU(3)_C$. Naturally the 3 was identified with ordinary quarks q while the 6 was identified with a new species of quarks the echo-quarks Q that belong to high colour representations. Both ordinary and echo-quarks feel QCD forces but moreover the heavy vector bosons V_μ^- connect ordinary and echo-quarks so that when the sextets condense and break $SU(2)_L \otimes U(1)$ they give rise also to the known diagram of Fig. 1 which gives masses to ordinary quarks. Similarly lepton-haplons decompose under $SU(3)_C$ to $1 + 8$. Again the leptons obtain masses radiatively. Within this one family model, there is an "understanding" of the fact that leptons are lighter than quarks since the condensation scales $\mu_8 < \mu_6$ because $C_2(8) < C_2(6)$. For cancellation of the anomalies one needs to introduce quarks (ordinary and echo) three times more than leptons since the usual factor 3 in the contribution of the quark anomalies which in the standard model balance the contribution of lepton anomalies, does not enter here.

Of course we would like to construct a model in the framework described above which could explain the horizontal and vertical fermion mass hierarchies and flavour mixing. So let us next consider a two family-model

$$G = (SU(3) \otimes SU(3))_S \otimes SU(2)_L \otimes U(1)$$

$$\begin{pmatrix} H^u \\ H^d \end{pmatrix}_L = \left((8, 3) ; 2 ; 1/3 \right)$$

$$H^u_R = \left((\bar{3}, \bar{3}) ; 1 ; 4/3 \right)$$

$$H^d_R = \left((8, 3) ; 1 ; -2/3 \right)$$

$$\begin{pmatrix} H^V \\ H^e \end{pmatrix}_L = \begin{pmatrix} (\bar{3}, 3) & ; & 2 & ; & 1 \end{pmatrix}$$

$$H_R^e = \begin{pmatrix} (\bar{3}, 3) & ; & 1 & ; & -2 \end{pmatrix}$$

$$\begin{pmatrix} H^c \\ H^s \end{pmatrix}_L = \begin{pmatrix} (8, 15) & ; & 2 & ; & 1/3 \end{pmatrix}$$

$$H_R^c = \begin{pmatrix} (8, 15) & ; & 1 & ; & 4/3 \end{pmatrix} \quad (5.2)$$

$$H_R^s = \begin{pmatrix} (8, \bar{6}) & ; & 1 & ; & -2/3 \end{pmatrix}$$

$$\begin{pmatrix} H^V \\ H^\mu \end{pmatrix}_L = \begin{pmatrix} (8, 8) & ; & 2 & ; & 1 \end{pmatrix}$$

$$H_R^\mu = \begin{pmatrix} (8, 8) & ; & 1 & ; & -2 \end{pmatrix}$$

After spontaneous symmetry breaking of $(SU(3) \otimes SU(3))_s$ to $SU(3)_c$ the quark and lepton haplons are decomposed under QCD to ordinary quarks and leptons plus echo-quarks and leptons which belong to various high representations of $SU(3)_c$. It is expected that the highest representation which appears in the left-handed doublet and in the right handed singlet (here the 42-plet) is responsible for the breaking of $SU(2)_L \otimes U(1)$. Thus its condensation scale should be $O(500 \text{ GeV})$ and its screening factor should be $f(42) = 0.14$.

Let us first list the decompositions under $SU(3)_c$ of the various representations of $SU(3) \otimes SU(3)$

$$\begin{aligned}
 (SU(3) \times SU(3))_s & \quad SU(3)_c \\
 \bar{3} \times \bar{3} & = 3 + \bar{6} \\
 \bar{3} \times 3 & = 1 + 8 \\
 8 \times \bar{3} & = 3 + \bar{6} + 15 \\
 8 \times 6 & = 3 + \bar{6} + 15 + 24 \\
 8 \times 15 & = 3 + \bar{6} + 15_1 + 15_2 + 15' + 24 + 42 \\
 8 \times 8 & = 1 + 8 + 8_a + 10_a + 10'_a + 27
 \end{aligned} \tag{5.3}$$

From the assignments of the fermionic content (5.2) of the model and from the decompositions (5.3) we find the following lepton masses and quark mass matrices

$$\begin{aligned}
 m_e &= C(8) \quad ; \quad m_\mu = C(27) \quad ; \quad m_{\nu_e, \mu} = 0 \\
 m^{2/3} &= \begin{matrix} u_R & c_R \\ \bar{u}_L & \begin{pmatrix} C(6) & C(15) \\ C(6) & C'(42) \end{pmatrix} \end{matrix} + h.c. \\
 m^{-1/3} &= \begin{matrix} d_R & s_R \\ \bar{d}_L & \begin{pmatrix} C(15) & C(15) \\ \bar{s}_L & C(15) + C'(24) \end{pmatrix} \end{matrix} + h.c.
 \end{aligned} \tag{5.4}$$

where

$$C(Q) = \frac{9\alpha_s C_2(Q) \mu_R^3}{4\pi M_{V^-}^2} \ln\left(\frac{M_{V^-}^2}{(3C_2(Q))^{2/3} \mu_R^2}\right)$$

$$C'(Q) = \frac{9\alpha_s^2 C_2(Q) \mu_R^3}{4\pi^2 M_{V^-}} \ln\left(\frac{M_{V^-}^2}{(3C_2(Q))^{2/3} \mu_R^2}\right)$$

From the above mass matrices we observe that the present model leads to a very natural flavour mixing. This is due to the fact that there are channels of possible condensation which are flavour conserving and flavour violating but they appear with exactly the same strength. Take for example the sextet echo-quark coming from the decomposition of the haplon H_L^S . It has equal probability to condense with the echo-sextet coming from the decomposition of the haplon H_R^S and with the echo-sextet coming from the haplon H_R^d . This leads to a condensation in both channels with equal strength and the full quark mass matrices take the form given in (5.4). From (5.4) with

$$M_{V^-} \simeq 1.3 \text{ TeV} ; \mu_6 \simeq 10 \text{ GeV} \left(f(6)^2 \simeq 0.26\right)$$

$$\mu_8 \simeq 7 \text{ GeV} \left(f(8)^2 \simeq 0.26\right); \mu_{15} \simeq 12 \text{ GeV} \left(f(15)^2 \simeq 0.14\right) \quad (5.5)$$

$$\mu_{24} \simeq 170 \text{ GeV} \left(f(24)^2 \simeq 0.16\right); \mu_{27} \simeq 135 \text{ GeV} \left(f(27)^2 \simeq 0.16\right)$$

We obtain the following physical fermion masses and Cabibbo angle

$$m_u \simeq 4 \text{ MeV} ; m_d \simeq 7 \text{ MeV} ; m_e \simeq 0.5 \text{ MeV}$$

$$m_c \simeq 1.4 \text{ GeV} ; m_s \simeq 175 \text{ MeV} ; m_\mu \simeq 105 \text{ MeV} \quad (5.6)$$

$$\tan \theta_c = m_d/m_s$$

So far, we have not considered anomalies in this model. It appears that it is very easy to cancel anomalies by introducing new multiplets and the additional particles to make them heavy. In order, for example, to

cancel the strong sector anomalies, one has to introduce the following multiplets under $(SU(3) \otimes (SU(3)))_s$: $(1,15)_R$; $(1,15')_R$; $(1,42)_R$ with charge $-1/3$ which become 15_R ; $15'_R$; 42_R , respectively under $SU(3)_c$ and they are exactly the missing partners of left-handed fields necessary to make heavy condensates:

$$\langle 15_L \bar{15}_R \rangle; \langle 15'_L \bar{15}'_R \rangle; \langle 42_L \bar{42}_R \rangle.$$

In addition in order to cancel the $U(1)$ anomalies of the electroweak group one has to introduce multiplets that do not introduce strong anomalies in the model like $(1,27)$ for example and to give them the appropriate charge so that they satisfy the known condition $\Sigma Q = 0$. Again these particles obviously become very heavy. Needless to mention that with all the multiplets we have introduced, asymptotic freedom is lost. However, since echo-fermions decouple at lower energies, perturbative QCD is safe.

6. CONCLUSION

The recently found support for the "Casimir scaling" in Monte Carlo lattice calculations encourages us to look at a new scenario for dynamical symmetry breaking of the electro-weak group. This scenario involves new quarks in high QCD representations which according to "Casimir scaling" condense at exponentially different scales than ordinary quarks. The scenario suggests that electroweak symmetry breaking is due to the condensates of the new quarks. Fermions can also obtain masses radiatively from the condensates, if the strong group is enlarged and then broken to $SU(3)_c$. In this case a sufficiently high representation of the enlarged group decomposes under the $SU(3)_c$ to ordinary fermions plus new ones with exotic quantum numbers. These new quarks when they condense in addition to electroweak symmetry breaking also provide masses to fermions via the exchange of the heavy vector bosons.

Here, in order to accommodate in the above framework the realistic multiplicity of mass scales, we have proposed the introduction of different representations for the various fermions. The introduction of various representations led us to the reasonable assumption of some kind of "screening of the Casimir scaling" the exact form of which should be studied in Monte Carlo lattice investigations. In order to produce mass

splitting within the fermions belonging to the same $SU(2)_L$ weak doublet, as well as mass splitting between fermions of the same charge and flavour mixings, we have applied our proposal in two classes of models. In the first class the strong group was enlarged to $SU(N)_s$, $N > 3$, while the second was enlarged to $SU(3) \otimes SU(3)$. The first class even though appealing as it unifies quarks and leptons without the necessity to induce proton decay meets with problems, the most serious of which is the cancellation of anomalies. The anomalies can be cancelled with the introduction of new particles which however cannot become heavy. The second class of models we have examined is indeed much more promising. We have presented an example of a reasonably realistic model which does not suffer from unwanted light particles.

We think that more work should be done in this new scenario of dynamical symmetry breaking in both classes of models. In the first apparently a new mechanism is needed to make the unobserved particles heavy. In the second, the minimal realistic model should be constructed according to our prescriptions and then the assumptions about the screening of "Casimir scaling" should be tested in lattice calculations. We should also mention that we find unattractive the fact that we are forced to introduce Higgs field for the breaking of the unenlarged colour group. It would be very interesting to have a dynamical symmetry breaking also for this sector. We have tried extensively to construct a model of the present framework in which the enlarged colour group breaks down via the tumbling mechanism¹³⁾. Unfortunately we did not find a realistic model. Moreover, even if such a model exists the tumbling idea cannot be tested directly in lattice calculations. The reason is that one cannot put chiral fermions on the lattice, while for fermions in non-chiral representations the most attractive channel, according to the tumbling hypothesis, forms singlet condensates which leave the group unbroken.

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Footnote

Leptons obtain the main contribution to their masses from the 2-loop diagrams of Fig. 2 a,b and not from the 1-loop diagram where ordinary quarks condense. With our fermionic assignments we expect the relation given in (3.3) between masses of neutrino-up quarks and charged lepton-down-quarks. We found that the assignment suggested in Ref (9) which predicts $m_t \approx 70$ GeV is not possible.

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FIGURE CAPTIONS

Fig. 1 Diagram producing a radiative mass to an ordinary fermion f . A_μ denotes the massive gauge bosons of $G_s/SU(3)_c$. $Q(R)$ is a R-plet of exotic colour fermion and $\Sigma_R = \langle Q(R)_L \overline{Q(R)}_R \rangle$.

Fig. 2 Two-loop diagrams producing lepton masses.

$\alpha_3(\mu_3)$	$\mu_6(\text{GeV})$	$f^2(6)$	$\mu_8(\text{GeV})$	$f^2(8)$	$\mu_{10}(\text{GeV})$	$f^2(10)$	$\mu_{15}(\text{GeV})$	$f^2(15)$		
0.40	29	1.85	17	2.19	2580	0.80	840	0.93		
0.35	47	1.61	25	1.93	7910	0.70	2.2×10^3	0.81		
0.30	89	1.38	42	1.66	3.5×10^4	0.59	7.9×10^3	0.70		
0.25	218	1.15	89	1.38	2.9×10^5	0.49	4.8×10^4	0.58		
0.20	839	0.93	273	1.11	6.6×10^6	0.40	7×10^5	0.46		
0.15	7910	0.70	1772	0.81	1.2×10^9	0.29	6.3×10^7	0.35		
0.10	7×10^5	0.46	7.5×10^4	0.51	44×10^{13}	0.20	5×10^{11}	0.23		
$\alpha_3(\mu_3)$	$\mu_{15}^1(\text{GeV})$	$f^2(15^1)$	$\mu_{21}(\text{GeV})$	$f^2(21)$	$\mu_{24}(\text{GeV})$	$f^2(24)$	$\mu_{27}(\text{GeV})$	$f^2(27)$		
0.40	7×10^5	0.49	5.9×10^8	0.31	1.3×10^5	0.52	7.5×10^4	0.56		
0.35	4.8×10^6	0.40	1.1×10^{10}	0.27	7×10^5	0.46	3.7×10^5	0.48		
0.30	6.3×10^7	0.35	5×10^{11}	0.23	6.6×10^6	0.39	3.1×10^6	0.41		
0.25	2.3×10^9	0.28	1.1×10^{14}	0.20	1.5×10^8	0.33	6.3×10^7	0.35		
0.20	5.6×10^{11}	0.23	3.5×10^{17}	0.15	1.7×10^{10}	0.26	5.6×10^9	0.27		
0.15	3.9×10^{15}	0.17	2.5×10^{23}	0.12	4.4×10^{13}	0.20	9.9×10^{12}	0.21		
0.10	2.5×10^{23}	0.12	1.2×10^{35}	0.08	2.9×10^{20}	0.13	3.1×10^{19}	0.14		
$\alpha_3(\mu_3)$	$\mu_{35}(\text{GeV})$	$f^2(35)$	$\mu_{42}(\text{GeV})$	$f^2(42)$	$\mu_{48}(\text{GeV})$	$f^2(48)$	$\mu_{60}(\text{GeV})$	$f^2(60)$	$\mu_{64}(\text{GeV})$	$f^2(64)$
0.40	6.3×10^7	0.35	2×10^7	0.37	9.2×10^{10}	0.25	1.7×10^{10}	0.26	9.8×10^9	0.27
0.35	8.1×10^8	0.31	2.3×10^8	0.33	3.4×10^{12}	0.22	5×10^{11}	0.23	2.6×10^{11}	0.24
0.30	2.5×10^{10}	0.26	5.6×10^9	0.27	4.2×10^{14}	0.19	4.4×10^{13}	0.20	2.1×10^{13}	0.21
0.25	3×10^{12}	0.22	5×10^{11}	0.23	3.5×10^{17}	0.15	2.4×10^{16}	0.16	9.6×10^{15}	0.16
0.20	3.9×10^{15}	0.17	4.2×10^{14}	0.19	8.5×10^{21}	0.12	2.9×10^{20}	0.13	9.5×10^{19}	0.13
0.15	6.2×10^{20}	0.13	3.1×10^{19}	0.14	1.7×10^{29}	0.09	1.9×10^{27}	0.10	4.3×10^{26}	0.10
0.10	1.5×10^{31}	0.09	1.7×10^{29}	0.10	7.2×10^{43}	0.07	8.5×10^{40}	0.07	9.1×10^{39}	0.07

Table 1: μ_R is obtained from Eq. (2.5); $f^2(R)$ is defined in Eq. (2.15) and is given by $f^2(R) = \ln \mu_{R\text{screened}} / \ln \mu_R$ with $\mu_{R\text{screened}} \sim 500 \text{ GeV}$; $\alpha_3(\mu_3)$ is the $SU(3)_c$ coupling constant at $\mu_3 \sim 1 \text{ GeV}$.

Fig. 1

a)

b)

Fig. 2