Ref.TH.3850-CERN

HEAVY FLAVOUR CONTRIBUTIONS TO MASSIVE LEPTON PAIR PRODUCTION

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A B S T R A C T

Massive lepton pair production in hadronic collisions, $A+B \rightarrow \mu^+\mu^-X$, is usually assumed to proceed via an intermediate virtual photon, $\gamma^* \rightarrow \mu^+\mu^-$, the Drell-Yan process. We examine other contributions arising from heavy flavour production, $A+B \rightarrow Q\bar{Q}X$; $Q \rightarrow \mu\nu X$. We find that this background is comparable to the Drell-Yan signal at small values of $M_{\mu\mu}^2/s$ but negligible at large values. Characteristic signatures are described that distinguish the background from the signal.

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Ref.TH.3850-CERN

March 1984

The Drell-Yan process, in which a massive lepton pair is produced through an intermediate virtual photon, is recognized as perhaps the best hadronic process for tests of parton dynamics¹⁾. In this paper we address the question of backgrounds to the Drell-Yan process arising from the production and subsequent weak decay of heavy quarks, e.g., $pp \rightarrow \bar{Q}QX$; $(\bar{Q}) \rightarrow (\bar{\mu})X$, at energies typical of current experiments.

A crude estimate of the signal to background ratio can be obtained by replacing α_{em}^2 in the Drell-Yan cross-section by α_s^2 (to produce $\bar{Q}Q$) and then multiplying by the semi-leptonic branching ratio (~ 0.1) for $Q \rightarrow \mu X$:

$$\sigma_{dy}/\sigma_{background} \approx \alpha_{em}^2 / (0.1 \alpha_s)^2 \sim 1.$$

If the background were this large, and not removed from the data sample, it would seriously affect not only the absolute yield but also the invariant mass and rapidity distributions, the transverse momentum distribution, and the angular distribution of the lepton pair in its rest frame --- all of which are of substantial theoretical interest.

In principle, the background from heavy flavour decay could be subtracted, at least on a statistical basis. Most direct would be measurements of both flavour alike neutral pairs and flavour unlike pairs, e.g., $\mu\bar{\mu}$ and μe . The backgrounds contribute equally to both, but there is no Drell-Yan contribution to μe . In practice, however, technical limitations have precluded good measurements of both μ 's and e 's in the same experiment. Other methods for separating production mechanisms, such as requiring that a lepton be unaccompanied by associated hadrons in the same region of phase space, are also inapplicable in closed geometry beam dump experiments. Finally, measurements of same sign dilepton spectra (e.g., $\mu^+\mu^+$) do not permit an adequate estimate of the desired opposite sign background. Contributions to same sign spectra arise from sequential decay of more massive quarks (e.g., $b \rightarrow c \rightarrow \bar{\mu}X$; $\bar{b} \rightarrow \bar{c} \rightarrow \mu X$), whereas opposite sign spectra originate with much greater probability ($c \rightarrow \bar{\mu}X$; $\bar{c} \rightarrow \mu X$).

We are motivated therefore to consider careful theoretical estimates of the possible heavy flavour backgrounds. Earlier studies have suggested that these backgrounds could be alarmingly large^{2),3)}. Our investigation is based on an evaluation of first-order perturbative QCD contributions to heavy

flavour production. An obvious criticism of this method is that QCD estimates for heavy flavour yields are generally neither unambiguous nor entirely successful⁴⁾. However, we argue that if our attention is restricted to a specific region of phase space, defined below, in which perturbation theory can be justified, we should be able to rely upon the QCD estimates to within a factor of two or so in accuracy of over-all normalization. This region of phase space, corresponding to small centre-of-mass rapidity y , and polar decay angle $\theta^* \approx \pi/2$ in the massive lepton pair rest system, is precisely the region in which most Drell-Yan experiments have concentrated coverage.

Our aim, therefore, is to derive an estimate of the background which is as reliable as possible in a well defined and interesting region of phase space. We specialize to the contributions from charm quark production and decay. We define a new set of variables not used previously in studies of massive lepton pair production, and we propose a methodology for the analysis of data which should facilitate the separation of background from the signal. The approach may be generalized in a straightforward way to include contributions from bottom and top quark decays at energies well above their respective production thresholds.

KINEMATICS

In $A+B \rightarrow \mu^+ + \mu^- + X$, we let λ_μ and $\bar{\lambda}_\mu$ be the four-momenta of the μ^+ and μ^- respectively in the hadron c.m. frame with the z-axis along the beam direction. Five variables, which we denote as M , Q_T^{in} , Q_T^{out} , Y , and θ , suffice to determine λ_μ and $\bar{\lambda}_\mu$ up to a rotation about the z-axis. We choose these variables as follows.

We let $M^2 = (\lambda_\mu + \bar{\lambda}_\mu)^2$, as usual, and let $Q_T = \lambda_T + \bar{\lambda}_T$ be the projection of $\lambda_\mu + \bar{\lambda}_\mu$ on the transverse plane. In a typical charm background event, λ_T and $\bar{\lambda}_T$ have nearly opposite directions but quite unequal magnitudes. That is, Q_T will typically have a large component along the direction of $\hat{\Delta} = \lambda_T - \bar{\lambda}_T$ but a small component perpendicular to $\hat{\Delta}$. We therefore define unit vectors $u_{\text{in}} = \hat{\Delta} / |\hat{\Delta}|$,

$\underline{u}_{out} = z^x \underline{u}_{in}$ and corresponding components of Q_T : $Q_T^{in} = Q_T \cdot \underline{u}_{in}$, $Q_T^{out} = Q_T \cdot \underline{u}_{out}$. We let Y be the average of the rapidities of the μ^+ and the μ^- : $Y = \frac{1}{2}(y+\bar{y})$. Finally, from the rapidity difference we construct an angle θ , with $\cot(\theta/2) = \exp[\frac{1}{2}(y-\bar{y})]$.

In a typical true Drell-Yan event, one has $Q_T \approx 0$. In this special case, the usual massive lepton pair rapidity equals Y , and the angle between the μ^+ momentum and the z -axis in the dimuon c.m. frame equals θ . In a typical charm background event, Q_T^{in} is not negligible. We will see the significance of Y and θ in the parton dynamics for this case later, in Eqs. (3) and (4).

We now define a set of nine variables x_A , x_B , θ_c , $\bar{\theta}_c$, ξ , $\bar{\xi}$, z and $\bar{z}$ that appear naturally in the description of the parton dynamics. In the parton dynamics, we assume that the energies are high enough so that we can neglect all particle masses, the transverse momenta of the incoming gluons or quarks, and the components of the final muons' momenta transverse to the momenta of their parent charmed quarks. We then define $x_A(x_B)$ to be the momentum fraction carried by the incoming gluon or quark from hadron A(B). We let $\theta_c(\bar{\theta}_c)$ be the scattering angle of the outgoing charmed quark (antiquark) relative to the beam direction in the c.m. frame of the incoming gluons or quarks. Because of the collinear kinematics assumed, this is also the angle of the outgoing $\mu^+(\mu^-)$. We let ϕ be the azimuthal angle between the momenta of the c and $\bar{c}$ quarks, and thus between the μ^+ and μ^- momenta. Finally, $\xi(\bar{\xi})$ is the momentum fraction carried by the outgoing $c(\bar{c})$ and $z(\bar{z})$ is the momentum fraction carried by the observed $\mu^+(\mu^-)$: $\xi = 2E_c/\hat{s}^{\frac{1}{2}}$, $z = 2E_\mu/\hat{s}^{\frac{1}{2}}$. Here and in the following, $\hat{s}$ denotes the c.m. energy squared of the incoming partons: $\hat{s} = x_A x_B s$.

If the parton variables x_A , x_B , ξ , and $\bar{\xi}$ are given, then the remaining parton variables θ_c , $\bar{\theta}_c$, ϕ , z and $\bar{z}$ are determined by the observed muon variables M , Q_T^{in} , Q_T^{out} , Y , and θ . We record these relationships in the Appendix. In general, these kinematic relationships are fairly complicated, but simpler expressions apply in the region of phase space of interest to us.

DYNAMICS

We now suppose that s is large and that the observed muons are restricted to regions of phase space in which perturbation theory is applicable. Qualitatively, this implies that, as viewed in the hadron c.m. system, the observed muons (and thus the parent quarks) should be energetic, with momentum vectors that are not nearly parallel to the beam or target momenta, or to each other. Specifically, M should be large, θ should be bounded away from 0 and π , and $|Y|$ should be small.

In this region of phase space, we may use the standard factorized form for the cross-section:

$$\begin{aligned}
 & \frac{d\sigma}{dM dQ_T^{\text{in}} dQ_T^{\text{out}} d\cos\theta dY} \\
 & \approx \sum_{a,b} \int_0^1 dx_A dx_B \frac{d\xi}{\xi} \frac{d\bar{\xi}}{\bar{\xi}} f_{a/A}(x_A) f_{b/B}(x_B) \\
 & \times \frac{d\hat{\sigma}(a+b \rightarrow c+\bar{c}+X)}{d\cos\theta_c d\cos\bar{\theta}_c d\phi d\xi d\bar{\xi}} d_{\mu/c}\left(\frac{z}{\xi}\right) d_{\mu/\bar{c}}\left(\frac{\bar{z}}{\bar{\xi}}\right) \\
 & \times \frac{\partial(\cos\theta_c, \cos\bar{\theta}_c, \phi, z, \bar{z})}{\partial(M, Q_T^{\text{in}}, Q_T^{\text{out}}, \cos\theta, Y)} .
 \end{aligned}
 \tag{1}$$

Here, $f_{a/A}(x_A)dx_A$ is the probability to find a parton of type a in hadron A carrying a fraction x_A of the hadron's momentum, and $d_{\mu/c}(z/\xi)dz/\xi$ is the probability to find a muon as a decay product of a charmed quark carrying a fraction z/ξ of the muon's momentum. The functions $f(x)$ and $d(z/\xi)$ depend (rather weakly) on the momentum scale of the hard process, which we take to be $\hat{s}$. [We neglect this dependence in the case of the rather poorly known functions $d(z/\xi)$.]

The hard scattering cross-section $\hat{\sigma}$ in Eq. (1) can be calculated perturbatively. Working at the factor of two level in presumed accuracy, we will be content to evaluate $\hat{\sigma}$ in first-order perturbation theory. At this order, the contributing hard processes are $g+g \rightarrow c+\bar{c}$ and $q+\bar{q} \rightarrow c+\bar{c}$, where g and q denote respectively a gluon and a quark from the initial hadrons. We

will neglect the contributions from charmed and heavier quarks in the initial hadrons because the corresponding probabilities $f_{a/A}, f_{b/B}$ are small.

There are important kinematic simplifications associated with the first-order graphs because all these graphs have only two partons in the final state. One has

$$\xi = \bar{\xi} = 1, \quad \bar{\theta}_c = \pi - \theta_c, \quad \phi = \pi. \quad (2)$$

The conditions $\xi = \bar{\xi} = 1$ eliminate the ξ and $\bar{\xi}$ integrations in Eq. (1). The condition $\bar{\theta}_c = \pi - \theta_c$ implies

$$e^{2Y} = \frac{x_A}{x_B}. \quad (3)$$

That is, the average muon rapidity Y gives the boost angle from the hadron-hadron c.m. frame to the parton-parton c.m. frame. As a consequence of Eq. (3), the integrations over the two variables x_A and x_B in Eq. (1) are replaced by an integration over only one variable, which we take to be $x_A x_B s = \hat{s}$.

Another consequence of Eq. (2) is

$$\theta = \theta_c = \pi - \bar{\theta}_c. \quad (4)$$

Correspondingly, the angle $\theta = 2 \tan^{-1}(\exp[\frac{1}{2}(\bar{y}-y)])$, determined from the rapidities of the muons, is the charmed quark scattering angle in the parton-parton c.m. frame. Thus for the present purpose of studying the charm background to the Drell-Yan process, the angle θ which we define is directly useful, although other definitions are more useful for studying the Drell-Yan process itself. (One may note that our definition is useful for the study of any process - for example $A+B \rightarrow \text{jet}+\text{jet}+X$ - that is dominated by an underlying two-parton $\rightarrow$ two-parton hard scattering.)

Returning to Eq. (2), we see that the final relation, $\phi = \pi$, implies

$$Q_T^{\text{out}} = 0. \quad (5)$$

This relation is only approximate since the intrinsic transverse momentum of the incoming partons, and the transverse momentum imparted to the incoming and outgoing particles by nearly collinear or soft gluon bremsstrahlung, will yield non-zero Q_T^{out} , just as such effects lead to $Q_T \neq 0$ in the Drell-Yan process.

Using first-order perturbation theory for the hard scattering cross-section in Eq. (1), we obtain

$$\begin{aligned} & \frac{d\sigma}{dM dQ_T^{\text{in}} dQ_T^{\text{out}} d\cos\theta dY} \\ & \approx \delta(Q_T^{\text{out}}) \frac{4M^2}{s [Q_T^2 + M^2 \sin^2\theta]^{1/2}} \int \frac{d\hat{s}}{\hat{s}} \sum_{a,b} \\ & \times f_{a/A}(x_A) f_{b/B}(x_B) \frac{d\hat{\sigma}^{(1)}(a+b \rightarrow c+\bar{c})}{d\cos\theta} d_{\mu c}(\bar{z}) d_{\mu \bar{c}}(\bar{\bar{z}}), \quad (6) \end{aligned}$$

where

$$\begin{aligned} x_A &= [\hat{s}/s]^{1/2} e^Y \\ x_B &= [\hat{s}/s]^{1/2} e^{-Y} \\ \bar{z} &= \{[M^2 \sin^2\theta + (Q_T^{\text{in}})^2]^{1/2} + Q_T^{\text{in}}\} / \{\hat{s}^{1/2} \sin\theta\} \\ \bar{\bar{z}} &= \{[M^2 \sin^2\theta + (Q_T^{\text{in}})^2]^{1/2} - Q_T^{\text{in}}\} / \{\hat{s}^{1/2} \sin\theta\} \end{aligned} \quad (7)$$

For the first-order cross-sections $d\hat{\sigma}^{(1)}/d\cos\theta$ we use the standard results⁸⁾

$$\begin{aligned} \frac{d\hat{\sigma}^{(1)}(g+g \rightarrow c+\bar{c})}{d\cos\theta} &= \frac{\pi \alpha_s^2}{48 \hat{s}} \left\{ 18 \sin^2\left(\frac{\theta}{2}\right) \cos^2\left(\frac{\theta}{2}\right) - 9 \right. \\ & \quad \left. + 4 \tan\left(\frac{\theta}{2}\right) + 4 \cot\left(\frac{\theta}{2}\right) \right\}; \quad (8) \end{aligned}$$

$$\frac{d\hat{\sigma}^{(1)}(q+\bar{q} \rightarrow c+\bar{c})}{d\cos\theta} = \frac{\pi\alpha_s^2}{9\hat{s}} \{1 + \cos^2\theta\} . \quad (9)$$

We set $\alpha_s(\hat{s}) = 12\pi/25\ln(\hat{s}/\Lambda^2)$ with $\Lambda = 200$ MeV.

For the light quark distributions in the nucleon we use the $\hat{s}$ dependent CDHS⁵⁾ parametrizations. For the quark distribution functions in the pion, we use the $\hat{s}$ dependent parametrizations of Ref. 6). We find that the $gg \rightarrow c\bar{c}$ subprocess is more important than the $q\bar{q} \rightarrow c\bar{c}$ subprocess, while at the same time the gluon structure functions are not precisely known. Therefore, we provide an estimate of the uncertainty of the results arising from the uncertainty of the structure function by considering two different gluon structure functions. For most of our computations, we use

$$xg(x) = 3(1-x)^5 \quad (10)$$

for both the pion and the nucleon. Alternatively, we use the softer CDHS form for $g(x)$ ⁷⁾.

The final factor in Eq. (6) is the decay probability $d_{\mu/c}(z)$ for finding a $\bar{\mu}$ in the decay of a charmed quark. We resort to some simple models in agreement with available data^{9),10)}. We imagine that most charmed quark decays proceed through the chain $c \rightarrow DX$; $D \rightarrow \mu X$. To describe $c \rightarrow DX$, we employ the normalized fragmentation function $f(z_D) = 42z_D^5(1-z_D)$, whose average is $\langle z_D \rangle = 0.75$. Both the shape of this function and its mean are consistent with data⁹⁾. For the subsequent semi-leptonic weak decay $D \rightarrow \mu\nu X$, we begin with an acceptable fit to data¹⁰⁾,

$$\frac{d\Gamma}{dy} \propto y^2 (1-y)^2 , \quad (11)$$

where $y = 2E_\mu/M_D$. Converting to z , we derive

$$D(z) = \int_z^1 \frac{dy}{y} \frac{d\Gamma}{dy} = \frac{5}{2}(1+3z)(1-z)^3 , \quad (12)$$

with $\langle z \rangle = 1/4$. Finally, the desired fragmentation function for $c \rightarrow \mu X$ is

$$d(z) = B \int_z^1 \frac{d\tau}{\tau} f(\tau) D(z/\tau) , \quad (13)$$

where B denotes the semi-leptonic branching fraction, which we take^{9),10)} to be $B = 0.08$. We note that the factor B enters Eq. (6) as the square. The mean value of z computed from Eq. (13) is $\langle z \rangle = 3/16 \approx 0.19$, consistent with data reported by the TASSO group⁹⁾. To examine the sensitivity of results to our choices, we also made computations with a somewhat harder weak decay formula for $D \rightarrow \mu \nu X$, viz., that provided by the elementary four-fermion process $c \rightarrow \mu \nu s$:

$$\frac{d\Gamma}{dy} \propto y^2 (1-y) . \quad (14)$$

This alternative expression yields

$$D(z) = 2(1-z)^2 (1+2z) , \quad (15)$$

instead of Eq. (12), and results in an average $\langle z \rangle = 9/40 \approx 0.23$ obtained from Eq. (13).

RESULTS

In Figs. 1 and 2, we present cross-sections $d\sigma/dM dY d\cos\theta$ as functions of M for $Y = 0$ and $\theta = \pi$. The cross-sections are for $pp \rightarrow \mu\bar{\mu}X$ and $\pi^- p \rightarrow \mu\mu X$ at energies typical of current Drell-Yan experiments. We show the contributions to the charm background from both the $gg \rightarrow c\bar{c}$ and $q\bar{q} \rightarrow c\bar{c}$ subprocesses and also the Drell-Yan signal, obtained from standard theoretical expressions¹⁾ (with no K factor). The fact that the heavy flavour backgrounds grow less important as $\tau^{\frac{1}{2}} = M/\sqrt{s}$ increases is easily understood. At rapidity $Y = 0$, the Drell-Yan process probes the quark and antiquark densities at values of $x_A = x_B = M/\sqrt{s}$. On the other hand, since $\langle z \rangle \approx 3/16$ in the decay $c \rightarrow \mu$, a given value of M requires $\langle \hat{s}^{\frac{1}{2}} \rangle \approx 16M/3$, or typically $\langle x_A \rangle \approx \langle x_B \rangle \approx 16M/3\sqrt{s}$. If $M/\sqrt{s}$ is not small, the charm background is suppressed because the gluon and quark distribution functions fall rapidly as $x \rightarrow 1$.

In Fig. 1b, we provide a shaded band of values for the $gg \rightarrow c\bar{c}$ contribution. The central line through this band is obtained from the gluon distribution in Eq. (10) and the $D \rightarrow \mu X$ decay function in Eq. (12). Changing the decay function to the harder form, Eq. (15), we find that the gg contribution increases to the upper edge of the shaded band. Using the softer CDHS form⁷⁾ for $xg(x)$ and our standard choice for $D \rightarrow \mu X$, Eq. (12), we compute the lower edge of the shaded band. The spread in possible values of the $gg \rightarrow \mu\bar{\mu}X$ contribution is obviously considerable. The uncertainty could be reduced if one had a direct measurement of the c quark decay function (e.g., measurements of the muon yield opposite to charmed mesons in e^+e^- annihilation events).

The curves in Fig. 1 show that the central value of the possible charm contribution to the $\mu\bar{\mu}$ yield at $Y = 0$ and $\cos\theta = 0$ is below the 25% level for $M \gtrsim 4$ GeV in pp interactions at $\sqrt{s} = 30$ GeV but only below this level for $M \gtrsim 6$ GeV at $\sqrt{s} = 60$ GeV. In π^-p collisions, as shown in Fig. 2, the background is less important, primarily because the Drell-Yan signal is greater. We have also computed charm backgrounds for $\bar{p}p \rightarrow \mu\bar{\mu}X$ at $\sqrt{s} = 540$ GeV. We find that the central value of our estimated background exceeds the signal at $Y = 0$ and $\cos\theta = 0$ for all $M \lesssim 20$ GeV. We expect to present these extrapolations to collider energies elsewhere, including contributions from bottom quark decay. In this paper, we prefer to limit our attention to CERN-ISR and to fixed-target energies, and to an exposition of the methodology.

In Fig. 3, we show the angular dependence of $d\sigma/dM dY d\cos\theta$ for $pp \rightarrow \mu\bar{\mu}X$ at $M = 4$ GeV, $Y = 0$, and $\sqrt{s} = 60$ GeV. The angular dependence at other values of M and Y is similar. The Drell-Yan signal shows the expected behaviour $\propto (1+\cos^2\theta)$, as does the background from $\bar{q}q \rightarrow c\bar{c}$ (not shown). However, the gluon induced process $gg \rightarrow c\bar{c} \rightarrow \mu\bar{\mu}X$ grows steeply in potential importance as $|\cos\theta| \rightarrow 1$. This growth as $|\cos\theta| \rightarrow 1$ reflects the $\hat{t}^{-1}$ and $\hat{u}^{-1}$ behaviour of Eq. (8). When either $\hat{t}$ or $\hat{u}$ is small, perturbation theory is unreliable. Therefore we do not trust the $gg \rightarrow c\bar{c}$ curve in Fig. 3 for $|\cos\theta| \gtrsim 0.6$. Nevertheless, one may draw the qualitative conclusion that the heavy flavour background grows in importance relative to the Drell-Yan signal as $|\cos\theta|$ increases.

Because the background and the Drell-Yan signal have such different dependences on $\cos\theta$, one must pay careful attention to possible contamination

from background at large $|\cos\theta|$ when attempting to compare data with the standard prediction $d\sigma/d\cos\theta \propto (1+\cos^2\theta)$, valid when Q_T is small and Y is not large, and with predicted deviations associated with gluon emission¹¹⁾ at large Q_T , and high-twist terms¹²⁾ at large Y .

In Fig. 4, we provide computations of the $|Q_T^{\text{in}}|$ distribution at $M = 4$ and 8 GeV for $pp \rightarrow \mu\bar{\mu}X$ at $\sqrt{s} = 60$ GeV. We note that the background distribution becomes broader as M increases. To obtain the Drell-Yan signal in Fig. 4, we assumed that the total Drell-Yan contribution is distributed in Gaussian fashion

$$\frac{d\sigma_{DY}}{dQ_T^{\text{in}} dQ_T^{\text{out}}} \propto \exp\left[-\frac{(Q_T^{\text{in}})^2 + (Q_T^{\text{out}})^2}{\sigma^2}\right]$$

with $\sigma^2 = \langle Q_T^2 \rangle = 1 \text{ GeV}^2$; $\langle (Q_T^{\text{in}})^2 \rangle = 0.5 \text{ GeV}^2$. Thus, Fig. 4 shows that the charm background increases in importance relative to the signal as Q_T^{in} grows.

These results offer also the possibility for an internal experimental check on the relative importance of background at different values of M , $\cos\theta$, and Y . We suggest that comparisons be made of the distributions $d\sigma/dQ_T^{\text{in}}$ and $d\sigma/dQ_T^{\text{out}}$. They should be essentially identical if the heavy flavour background is negligible. However, an important heavy flavour background should result in a distribution $d\sigma/dQ_T^{\text{in}}$ which is much broader than $d\sigma/dQ_T^{\text{out}}$.

DISCUSSION AND CONCLUSIONS

The results of this paper apply to massive lepton pairs produced not too far from $y = 0$, $\bar{y} = 0$ [or Y not far from 0 and θ not far from $\pi/2$, where, we recall, $Y = \frac{1}{2}(y+\bar{y})$ and $\cot(\theta/2) = \exp[\frac{1}{2}(y-\bar{y})]$]. We expect the first order QCD formulas used here to be reliable at the factor of two level in this region.

If either of the muon rapidities y , $\bar{y}$ becomes large, physical processes other than those studied here may become important. These additional processes include i) intrinsic charm components of the hadron wave function¹³⁾, which

will produce charmed hadrons with small transverse momenta and thus leptons with large y and $\bar{y}$ (unless, of course, the charm quarks undergo a hard QCD scattering); ii) whatever processes, perturbative or otherwise, are responsible for diffractive production of open charm¹⁴⁾; and iii) higher order perturbative QCD diagrams¹⁵⁾ in which at least one of the final charm quarks emerges at a small angle relative to the incident hadron direction. A study of these additional processes is beyond the scope of this paper.

We are, of course, aware that total charm yields are observed to be roughly a factor of ten larger than predicted by first-order QCD. We have argued that first-order QCD is reliable for, say, $|y| < 1$, $|\bar{y}| < 1$. It follows that the "extra" charm production must arise in the large y , or $\bar{y}$ region, where first-order perturbation theory is not reliable. Confirmation of this prediction must await the accumulation of sufficient charm production data to permit detailed comparisons with theory as a function of y and q_T .

Our principal conclusion is that prompt leptons from charm decay should not be a significant source of background for studies of the Drell-Yan process in π^-p and pp collisions for $M/\sqrt{s} \gtrsim 0.1$, so long as $|y|$, $|\bar{y}| < 1$ and $|\cos\theta| < 0.6$. If these conditions are satisfied, our computations show that the background to signal ratio is $\lesssim 25\%$, decreasing as $M/\sqrt{s}$ increases. We have little to offer in terms of estimates for $|\cos\theta| > 0.6$ and/or $|y| > 1$, other than to repeat that a comparison of the Q_T^{in} and Q_T^{out} distributions may provide a purely experimental check on the possible role of heavy flavour backgrounds. We suggest that these comparisons be made.

ACKNOWLEDGEMENTS

We have benefited from conversations with Guido Altarelli and Walter Geist.

APPENDIX - KINEMATICS IN THE GENERAL CASE

In this Appendix we record the kinematic relations needed in the general QCD formula (1) for the cross-section. For any given values of the integration variables x_A and x_B , one needs the relation between the observed variables M , Q_T^{in} , Q_T^{out} , θ , and Y and the subprocess variables θ_c , $\bar{\theta}_c$, z , $\bar{z}$ and ϕ . This relation is

$$M = [z \bar{z} x_A x_B S]^{1/2} \quad (\text{A.1})$$

$$Q_T^{\text{in}} = \frac{1}{2} [x_A x_B S]^{1/2} \frac{z^2 \sin^2 \theta_c - \bar{z}^2 \sin^2 \bar{\theta}_c}{[z^2 \sin^2(\theta_c) + \bar{z}^2 \sin^2(\bar{\theta}_c) - 2z\bar{z} \sin \theta_c \sin \bar{\theta}_c \cos \phi]^{1/2}} \quad (\text{A.2})$$

$$Q_T^{\text{out}} = [x_A x_B S]^{1/2} \frac{z \bar{z} \sin \theta_c \sin \bar{\theta}_c \sin \phi}{[z^2 \sin^2(\theta_c) + \bar{z}^2 \sin^2(\bar{\theta}_c) - 2z\bar{z} \sin \theta_c \sin \bar{\theta}_c \cos \phi]^{1/2}} \quad (\text{A.3})$$

$$\tan(\frac{1}{2}\theta) = [\tan(\frac{1}{2}\theta_c) / \tan(\frac{1}{2}\bar{\theta}_c)]^{1/2} \quad (\text{A.4})$$

$$e^{2Y} = (x_A/x_B) \cot(\frac{1}{2}\theta_c) \cot(\frac{1}{2}\bar{\theta}_c) \quad (\text{A.5})$$

The inverse relation is

$$\cot(\frac{1}{2}\theta_c) = [x_B/x_A]^{1/2} e^Y \cot(\frac{1}{2}\theta) \quad (\text{A.6})$$

$$\cot(\frac{1}{2}\theta_c) = [x_B/x_A]^{1/2} e^Y \tan(\frac{1}{2}\theta) \quad (\text{A.7})$$

$$\bar{z} = \frac{1}{\sin \theta_c} \left[\frac{(\Delta + Q_T^{\text{in}})^2 + (Q_T^{\text{out}})^2}{x_A x_B S} \right]^{1/2} \quad (\text{A.8})$$

$$\bar{z} = \frac{1}{\sin \bar{\theta}_c} \left[\frac{(\Delta - Q_T^{\text{in}})^2 + (Q_T^{\text{out}})^2}{x_A x_B s} \right]^{1/2} \quad (\text{A.9})$$

$$\cos \phi = -(1 + 2 \cot^2 \theta) \frac{\Delta^2 - (Q_T^{\text{in}})^2 - (Q_T^{\text{out}})^2}{2M^2 + (Q_T^{\text{in}})^2 + (Q_T^{\text{out}})^2 - \Delta^2}, \quad (\text{A.10})$$

where

$$\Delta^2 = \left[\sin^2 \theta + \frac{(Q_T^{\text{in}})^2}{M^2 + (Q_T^{\text{out}})^2} \right] M^2 - 2 \cot^2 \theta (Q_T^{\text{out}})^2 \zeta \quad (\text{A.11})$$

and ζ is the solution of $A\zeta^2 + B\zeta + C = 0$ with

$$A = 4 \cot^4 \theta \quad (\text{A.12})$$

$$\begin{aligned} B = & 2(2 - \sin^2 \theta) [M^2 + (Q_T^{\text{out}})^2] \\ & + 4 \cot^2 \theta [\cos^2 \theta M^2 + (2 - \sin^2 \theta) (Q_T^{\text{out}})^2] \\ & \times \frac{(Q_T^{\text{out}})^2}{M^2 + (Q_T^{\text{out}})^2} \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} C = & -\sin^2 \theta (2 - \sin^2 \theta) M^2 - 2(2 - \sin^2 \theta) (Q_T^{\text{in}})^2 - (Q_T^{\text{out}})^2 \\ & + 2 \cos^2 \theta \frac{(Q_T^{\text{in}} Q_T^{\text{out}})^2}{M^2 + (Q_T^{\text{out}})^2} - \frac{(Q_T^{\text{in}})^4 (Q_T^{\text{out}})^2}{[M^2 + (Q_T^{\text{out}})^2]^2} \end{aligned} \quad (\text{A.14})$$

Notice that if $Q_T^{\text{out}} \approx 0$ or if $\theta \approx \pi/2$, one can neglect the ζ term in Eq. (A.11).

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FIGURE CAPTIONS

- Figure 1 Contributions to $pp \rightarrow \mu\bar{\mu}X$ from three sources are shown as a function of lepton pair mass M for (a) $\sqrt{s} = 30$ GeV, and (b) $\sqrt{s} = 60$ GeV. For the $gg \rightarrow \mu\bar{\mu}X$ contribution in (b), a range of values is provided corresponding to the different choices of gluon structure functions and charm fragmentation functions, discussed in the text.
- Figure 2 Contributions to $\pi^-p \rightarrow \mu\bar{\mu}X$ as a function of M for (a) $p_{\text{lab}} = 200$ GeV/c and (b) $p_{\text{lab}} = 600$ GeV/c.
- Figure 3 The angular distribution $d\sigma/dM dY d\cos\theta$ is provided for $pp \rightarrow \mu\bar{\mu}X$ at $M = 4$ GeV, $Y = 0$, and $\sqrt{s} = 60$ GeV. Shown are the contributions from the Drell-Yan process, $\gamma^* \rightarrow \mu\bar{\mu}$, and from the charm background produced via $gg \rightarrow c\bar{c}X$.
- Figure 4 The $|Q_T^{\text{in}}|$ distribution $d\sigma/dM dY d\cos\theta d|Q_T^{\text{in}}|$ is displayed at $M = 4$ GeV and at $M = 8$ GeV with $Y = 0$, $\cos\theta = 0$. Here Q_T^{in} is the component of Q_T along the direction $\hat{l}_T - \hat{\bar{l}}_T$, as defined in the text.

- Figure 1 -

- Figure 2 -

- Figure 4 -

- Figure 3 -