DETAILED INVESTIGATION OF THE CHANNELLING PHENOMENA INVOLVED
IN BENDING OF HIGH-ENERGY BEAMS BY MEANS OF CRYSTALS

J.F. Bak^{*)}, S.P. Møller^{*)} and E. Uggerhøj^{*)}
CERN, Geneva, Switzerland

P.R. Jensen, H. Madsbøll and H.E. Schiøtt
Institute of Physics, University of Aarhus, Aarhus, Denmark

J.J. Grob and P. Siffert
Centre de Recherches Nucléaires, Strasbourg, France

ABSTRACT

Strong bending effects for 12 GeV/c positive particles traversing 2-3 cm long silicon crystals are reported, and results for crystal bendings of 4, 20, and 52 mrad are presented. Bending efficiencies up to 20% are seen. A combination of axial and planar channelling is responsible for the effects observed. Computer simulations, based on binary collisions, can explain in detail the experimental findings, and also elucidate important channelling phenomena in the transition region between axial and planar channelling. For negative particles a weak bending effect, caused by doughnut scattering, is demonstrated. Interesting results for planar feed-in and dechannelling are found in an experiment where three detectors are placed along the crystal.

(Submitted to Nuclear Physics B)

^{*)} Permanent address: Institute of Physics, University of Aarhus, Aarhus, Denmark.

1. INTRODUCTION

A novel technique for bending beams with energies in the GeV region has recently been demonstrated experimentally^{1,2}. The technique is based on the channelling effect in perfect crystals^{3,4}. Particles entering a crystal nearly parallel to a planar direction will be trapped and oscillate between very steep potential walls. Even in a bent crystal the particles remain trapped between the planes unless the bending radius becomes very small or the beam momentum very high⁵. A radius of curvature of less than 1 m is easily obtained for GeV beams with an appreciable efficiency, but the inevitable loss due to dechannelling will increase as the radius of curvature is decreased.

We have been especially interested in studying the bending effect for axially-channelled particles. The range of angles for which axial-channelling effects are seen is much larger than for planes, i.e. the beam acceptance is larger, and dechannelling is less than for planes. The features we have observed can be explained by a model where the particles which follow the axis will be trapped in the planes, partially because of the centrifugal force due to the bending. In order to study this mechanism in detail, the trajectories of the particles were traced in a Monte Carlo simulation, based on binary collisions⁶. The result of these simulations will also be discussed.

In the present experiment, which was part of a wider investigation of the influence of channelling on the yield of various processes⁷⁻⁹, a large percentage of beams with positive polarity and a momentum of 12 GeV/c were bent up to 52 mrad through some centimetres of silicon. The bending effect was also studied for beams with negative polarity. Although an effect was seen, the bending efficiency was very low, because dechannelling is much

stronger for negative particles than for positive ones. A very pronounced feed-in from the doughnut region to the planes was observed. This compensates, to some extent, the loss due to dechannelling.

The presentation in the present paper has been divided into a theoretical and an experimental part. In section 2 we give the theoretical framework, starting with a short review of the basic channelling concepts. The bending by atomic planes is discussed, and the possibility of bending by strings is investigated. Special attention is paid to the combined effect of axial and planar bending occurring for axially-channelled beams. Finally the computer model based on binary collisions is introduced.

Details of the experimental set-up are described in section 3, and the experimental results are presented and discussed in section 4.

2. CHANNELLING IN A BENT CRYSTAL

2.1 Channelling description

Directional effects for non-relativistic particles penetrating single crystals were originally described theoretically by Lindhard³. In the theory, the concepts of atomic strings and atomic planes are introduced. When a particle enters a crystal at a small angle relative to a string or a plane, the correlation between successive collisions with the atoms in the string, or in the plane, causes the particle to be gently steered away from the string or plane.

In a first approximation the atomic string and the atomic plane may be described by continuum potentials

$$U(r) = \int_{-\infty}^{\infty} \frac{dz}{d} V \left(\sqrt{z^2 + r^2} \right) \quad (1)$$

and

$$Y(y) = Nd_p \int_0^{\infty} 2\pi \varrho \, d\varrho \, V \left(\sqrt{y^2 + \varrho^2} \right) \quad (2)$$

where, in eq. (1), r is the distance from the string, and d the inter-atomic spacing in the string; in eq. (2), y is the distance from the plane, Nd_p represents the number of atoms per unit area of the plane, d_p being the distance between planes. In both formulae $V(R)$ is the ion-atom potential.

The functions U and Y are the potential energies for the transverse motion of the channelled particles, i.e. the motion projected onto a plane perpendicular to the direction of string or plane. In the transverse plane energy is conserved in first approximation.

The basic quantity determining the conditions under which channelling effects occur is, for strings, the critical angle ψ_1

$$\psi_1 = \sqrt{\frac{4Z_1 Z_2 e^2}{pvd}} \quad (3)$$

where Z_1 and Z_2 are the atomic numbers of projectile and target, respectively, p is the (relativistic) momentum of the particle, and v its velocity. The corresponding planar critical angle is

$$\psi_p = \sqrt{\frac{4Z_1 Z_2 e^2 Nd_p}{pv}} \quad (4)$$

where C is a constant of the order of $\sqrt{3}$, and $a \approx a_0 Z_2^{-1/3}$ is a screening length in the ion-atom potential, a_0 being the Bohr radius. In deriving eq. (4) Lindhard's standard potential was used³. For particles incident on a crystal with an angle to a string or plane less than or of the order of ψ_1 , or ψ_p , respectively, channelling effects will be important. For 12 GeV/c particles along the $\langle 111 \rangle$ -direction in silicon, $\psi_1 = 0.12$ mrad, and, for the $\{110\}$ -planes, $\psi_p = 0.04$ mrad.

As shown in detail by Lindhard³, channelled relativistic particles move along well-defined orbits determined by the continuum potentials; further details may also be found in ref. 6.

In many experiments performed at GeV energies the crystals used are so thin that dechannelling is only of minor importance. For bending experiments, on the other hand, the crystals are long -- of the order of a few centimetres -- and dechannelling can no longer be neglected.

In an unbent crystal there are three ways in which a channelled particle can change its transverse energy: scattering by electrons, scattering by nuclei in the lattice, and scattering by impurities or other crystal defects. In the very perfect crystals used in high-energy channelling, the latter possibility may be neglected⁶. Scattering by nuclei is only effective for particles having a rather high transverse energy and therefore able to approach the atomic strings or planes closely. Scattering by electrons, on the other hand, affects all particles.

Dechannelling has been described as a kind of diffusion in transverse momentum space. Well-channelled particles are affected by electronic collisions and -- on the average -- gain transverse energy. Those particles which achieve a rather high transverse energy are also affected by nuclear collisions and have a good chance of gaining so much transverse energy that they cease to be channelled, i.e. they are dechannelled.

A characteristic length for dechannelling is the "electronic scattering length"¹⁰

$$Z_e = \frac{pv}{\pi Z_1 e^2 L_e N_d} \quad (5)$$

where L_e is the logarithmic term in Bethe-Bloch's stopping power formula

$$\frac{dE}{dx} = \frac{4\pi Z_1^2 e^4}{mv^2} N Z_2 L_e \quad (6)$$

Very roughly, the fraction of an originally channelled beam still channelled after traversing a thickness t of crystal is

$$\chi_c \approx e^{-t/t_d} \quad (7)$$

where t_d is proportional to Z_e , the proportionality constant being approximately unity. This formula is reasonably good in the case of sizeable dechannelling, whereas for thinner crystals one should consult the results obtained by means of the diffusion equation¹⁰. For positive particles in the most open direction in silicon we find $Z_e \sim 0.1 \text{ cm}/(\text{GeV}/c) \times p$ leading to a channelled fraction of $\sim 20\%$ after a 2 cm crystal. Nearly the same result is obtained by a simple extrapolation of experimental results at MeV energies¹¹.

In connection with dechannelling, the opposite phenomenon, feed-in, should also be mentioned, especially since it turns out to play an important part in the present paper. This process takes a particle from a momentum state outside the channelling region to a state inside the channelling region, as is known from blocking experiments⁴. According to the rule of reversibility³, the probability for a channelled particle to be

scattered from a certain channelled state to a certain unchannelled state equals the probability for the opposite process. For further details the reader is referred to ref. 3.

2.2 Motion along bent atomic planes

We shall now consider the motion along planes which have been bent by some external means. This is the case which was discussed originally by Tsyganov⁵ and is also the most easy one to understand.

Denote the curvature of the planes by κ , i.e. the radius of curvature is $R = 1/\kappa$. It is easily seen that if one looks at the particle motion in a coordinate system, in which the planar direction is fixed, the transverse potential must include a centrifugal term

$$Y_{\text{eff}}(r) = Y(r) + \frac{L^2}{2mr^2} . \quad (8)$$

Here, r is the distance of the particle from the centre of curvature, and L is the angular momentum with respect to this point. A series expansion in the quantity $x = r - R$ leads, in the relativistic case, to

$$Y_{\text{eff}}(x) = Y(x) - \kappa p v x . \quad (9)$$

Note that x is the distance from the midplane between two atomic planes. Clearly, this expression implies that the particles are subject to an additional constant force perpendicular to the bent planes

$$F_{\text{eff}}(x) = - \frac{dY}{dx} = F(x) + p v \kappa , \quad (10)$$

where $F(x)$ is the force from the atomic planes and the constant term is

the centrifugal force entering into the formula because the coordinate system is not a system of inertia. Using the standard potential and taking into account the contribution from the two planes forming the planar channel, the effective potential may be plotted for various values of the curvature κ . In fig. 1 the full-drawn curve corresponds to a flat crystal, and the horizontal line to the total transverse energy of a particle which crosses midway between the planes at an angle ψ_p . Clearly, for only slightly curved crystals, and for not too high transverse energies, the channelling effect is hardly affected by the bending.

For stronger bendings, corresponding to radii of curvature of ~ 10 cm to ~ 1 cm, the minimum of the potential is shifted considerably towards the outer plane, and few particles are able to follow the bend. It is also clear from fig. 1 that a particle which penetrates the outer plane will have an increased transverse energy between the neighbouring pairs of planes and therefore almost certainly ceases to follow the bend, i.e. it becomes dechannelled.

Tsyganov⁵ introduced a critical radius for bending, R_c , by demanding that the minimum in the effective potential be shifted to a distance y_c from the plane so small that a particle is almost certain to be dechannelled, i.e.

$$\kappa_c = \frac{1}{R_c} = \frac{1}{pv} \frac{dY}{dx} \bigg|_{y_c} ; \quad (11)$$

y_c should be of the order of the screening distance or, perhaps, ρ , the thermal vibrational amplitude. Using the standard potential and setting $y_c = Ca$ leads to the formula

$$R_c = \frac{pv}{\pi Z_1 Z_2 e^2 N_d p} \quad (12)$$

which in our case of 12 GeV/c particles in silicon is of the order of 1 cm. We were not able to test this limit, since it is far beyond the elastic limits of our crystals. In order to approach the critical bending it is necessary to go to higher momenta. However, as may be seen from fig. 1, only particles which have approximately zero transverse energy will be able to follow the bend, whereas for other particles nuclear scattering will be serious close to the outer plane, resulting in dechannelling. The bending efficiency is therefore expected to decrease drastically for bendings close to the critical bending.

More detailed calculations on dechannelling in curved crystals have been performed by Kudo¹² and by Ellison¹³, who calculated the fraction of channelled particles which become dechannelled by entering a region where the crystal is curved. Several other authors¹⁴⁻¹⁷ have performed calculations on various aspects of bending effects for planar channelled particles. Especially the calculations by Eriksen¹⁷ are of interest in the present work and will be discussed in connection with the binary collision programme. Lately, measurements at 200-400 GeV/c were performed at the Fermi National Laboratory¹⁸, where Ellison's results were tested.

2.3 Doughnut scattering

The present paper is chiefly concerned with the effects occurring when the beam is directed along an axis in a bent crystal. We shall now discuss a special scattering mechanism which turns out to play an important part in the bending.

For channelled particles the momentum is rapidly scattered around the axial direction because of collisions with atomic strings. This means

that a parallel beam which enters a thin crystal at a small angle ψ to an axis is turned, upon exit from the crystal, into a cone in physical space, with opening angle 2ψ . On a photographic plate the beam therefore shows up as a ring, which explains the name "doughnut scattering".

Lindhard³ argued that there will be a mean free path $\lambda_{\perp}$ for the equalization of transverse momentum, and gave an approximate formula for $\lambda_{\perp}$, valid for $\psi < \psi_1$

$$\lambda_{\perp} \approx \frac{4\psi}{\pi^2 N d a \psi_1^2}, \quad (13)$$

which, for angles larger than ψ_1 , was generalized to the rather crude estimate⁶

$$\lambda_{\perp} \approx \frac{4\psi}{\pi^2 N d a \psi_1^2} \left(4 \frac{\psi^2 q}{\psi_1^2 a} \right), \quad (14)$$

where q is the r.m.s. amplitude of the thermal vibrations of the atoms^{*}).

Equation (13) leads to values of $\lambda_{\perp}$ of less than 10 μm in the present case, whereas, for angles larger than ψ_1 , $\lambda_{\perp}$ becomes considerably longer, even of the order of millimetres.

We shall now estimate the contribution to dechannelling, when the crystal is bent. On a crystal length dx the bending is $d\psi_b = dx/R$. The corresponding change in transverse momentum becomes $d\vec{p}_{\perp} = \hat{\vec{p}} dx/R$, where $\hat{\vec{p}}$ is perpendicular to $\vec{p}$, with the same length, pointing in a direction opposite to the bending.

^{*}) A misprint in ref. 6 (a missing factor of 4) has been corrected.

The change per path length in average transverse momentum is, in the random string approximation, determined by the equation

$$\frac{d\langle \vec{p}_\perp \rangle}{dx} = -\frac{1}{\lambda_\perp} \langle \vec{p}_\perp \rangle + \frac{1}{R} \hat{z} \cdot \vec{p} . \quad (15)$$

Clearly, for a depth larger than a few $\lambda_\perp$ we have the stationary solution $\langle \vec{p}_\perp \rangle = \hat{z} \cdot \vec{p} \lambda_\perp / R$. The change in transverse energy is given by

$$\frac{d\langle E_\perp \rangle}{dx} = \frac{\langle (\vec{p}_\perp + d\vec{p}_\perp)^2 - p_\perp^2 \rangle}{2M_1 dx} ,$$

where we may neglect the contribution to $d\vec{p}_\perp$ from doughnut scattering since this does not lead to any change in transverse energy. The contribution from bending gives

$$\frac{d\langle E_\perp \rangle}{dx} = \frac{\langle 2\vec{p}_\perp d\vec{p}_\perp \rangle}{2M_1 dx} = 2 \frac{\lambda_\perp E}{R^2} \quad (16)$$

In order to obtain an upper limit for this expression for well-channelled particles we use the value of $\lambda_\perp$ for $\psi = \psi_1$. The increase in transverse energy on a path length l relative to $E\psi_1^2$ is then

$$\frac{d\langle E_\perp \rangle}{E\psi_1^2} \leq \frac{2l\lambda_\perp}{R^2\psi_1^2} . \quad (17)$$

This formula is valid relativistically, too, when E is replaced by $1/2pv$.

The distance l_d over which the particles are dechannelled, for a fixed radius of curvature and beam momentum, will be

$$L_d \approx \frac{R^2 \psi_1^2}{2\lambda_\perp}, \quad (18)$$

which is about 1/2 mm, for $R \approx 1$ m, $\psi_1 \approx 10^{-4}$ rad, $\lambda_\perp \approx 10^{-5}$ m. It is obviously necessary to demand

$$L_d \gg \lambda_\perp, \quad \text{or} \quad \frac{R\psi_1}{\lambda_\perp} > 1. \quad (19)$$

This condition is in our case clearly fulfilled: $R\psi_1/\lambda_\perp = 10$. However, $R\psi_1/\lambda_\perp \propto p^{-1}$ showing that an increase of p by a factor of 10 will give $L_d \approx \lambda_\perp$. In such a case doughnut scattering is not able to hold the particles in the axial direction, even for a short distance.

From eq. (18) it is also evident that for very modest bendings, $R \gtrsim 5$ m, the doughnut effect alone should be able to bend a particle beam in a 2-3 cm long crystal¹⁹. Would it be possible to measure directly such a bending effect for strings? For positive particles moving close to an axis there is a possibility of being captured by planes, in which case the bending effect would be composite. But for negative particles planar channelling is absent -- even in a flat crystal -- or at least very faint at these high energies^{6,8,9}, and there a contribution from strings is observed, to be shown in the experimental section. The computer simulations to be described in a later section show very nicely the bending by strings and the limitations of this bending.

2.4 Computer simulations of bending

In Eriksen's numerical calculations¹⁷ the particles collide with a continuous string as a whole, and thermal vibrations are neglected. He finds that doughnut scattering is able to force well-channelled particles

to follow a bent axis over a distance $\sim (10-50)\lambda_L$ for bendings comparable to ours, in good agreement with the simple estimates in section 2.3. A substantial part of the particles end up by being captured by the skew planes.

We have modified our program for investigating channelling phenomena at high energies, BINCOL, in order to be able to handle a bent crystal. The calculations follow a particle through a series of binary collisions with the crystal atoms. The two-body potential used is Lindhard's standard potential³, and the thermal vibrations of the crystal atoms are included. After each collision the direction of the axis is changed according to the bending desired. Interaction with the electrons in the crystal is neglected, and the lattice atom is assumed to be static during the collision.

Lindhard showed³ that the classical treatment of a collision with a perfect string becomes the more accurate the higher the velocity of the particle, in contrast with the case of a single collision, where classical mechanics is only valid for low particle velocities. For collisions with strings with vibrating atoms the classical treatment is not equally well justified. However, as demonstrated by Andersen and Feldman²⁰, also in this case a classical calculation based on binary collisions gives results in accord with simpler theoretical estimates, and, more important, with experimental results. Results from BINCOL are in complete agreement with Eriksen's results, for the case of a static lattice, i.e. without thermal vibrations. In the next sections we shall compare our experimental findings with BINCOL calculations.

3. EXPERIMENTAL SET-UP

The bending was investigated for 12 GeV/c protons, π^+ , and π^- , in the secondary beam p_{17} from the Proton Synchrotron at CERN. The beam divergence was ± 1 mrad and the beam intensity less than 3×10^5 /s. The structure of the actual set-up is shown in fig. 2. A set of scintillators was used to define a trigger condition, and the ingoing and outgoing tracks were recorded for each particle by means of drift chambers. Between the drift chambers were vacuum tubes in order to reduce the multiple scattering. Owing to the material in drift chamber DC-3, the angular resolution on the incident beam was 50 μ rad, but much better on the exit beam. The data from each event were written on magnetic tape, which was subsequently analysed off-line. A very detailed description of the detection system can be found elsewhere^{6,8}.

A schematic drawing of the bending device is given in fig. 3. It was mounted on a goniometer in the target chamber, where it could be cooled to 100 K. The crystal was bent by means of a fine-thread thumbscrew, which pushed the centre of the crystal down with respect to the fixed supports at the ends. The crystals were silicon, between 20 and 30 mm long, 10 mm wide, and 0.5 mm thick. The crystals were cut with a {110}-plane almost aligned with the horizontal, large-surface side of the crystal. The long crystal edges were parallel to a $\langle 111 \rangle$ -direction. The front end of the crystal was made into a solid-state surface barrier detector of 3×8 mm². One of the crystals was prepared with two extra detectors placed at the middle and at the end of the crystal.

Alignment of the beam with the crystal was accomplished by the low-energy-loss method⁶. Having located the axis, the final adjustment was done on the basis of a stereogram of the crystal obtained on-line. An

example of a stereogram for an aligned crystal is shown in fig. 4. The $\{110\}$ -planes are clearly seen, and the $\{112\}$ -planes may also be observed. After a change in bending, the crystal had to be realigned since the front end with the detector also changed its direction with respect to the incident beam.

The bending of a crystal was within its elastic limits. This was assured by a check after the bending with the central roller removed. The crystal was found to be straight within the angular resolution of the system ($\sim 50 \mu\text{rad}$).

Since we use a three-point bending device, the shape of the bent crystal is not cylindrical. In fact the curvature will increase nearly linearly from zero to the value $2\psi_b/l$ at the midpoint, and then decrease linearly to zero at the end point. Here ψ_b is the total bending angle of the crystal. These results can be found in any standard textbook on elasticity.

4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1 Structure of bent beam

Consider fig. 5, showing the stereogram close to the $\langle 111 \rangle$ -direction in silicon. One of the $\{110\}$ -planes is horizontal and perpendicular to the bending direction (this was the case in the experiment). The position of the stereogram at the back side of the crystal is also shown. The bending is "upwards". The orientation of planes will be measured with respect to this direction by the angle η . Planes for which $\eta \neq \pi/2$ are denoted "skew" planes.

In the following we shall use two equivalent ways to describe the motion of the particles with respect to the stereogram in fig. 5. One is to use a coordinate system fixed with respect to the crystal -- a laboratory frame [Fig. (5a)]. In this system the position of the stereogram is shifted upwards as one proceeds through the crystal. Particles will stay at the same position in the stereogram throughout unless they are i) multiply scattered, ii) doughnut scattered, i.e. scattered to another point on the circumference of a circle centred at the axis, or iii) trapped by a plane in which case they move perpendicular to that plane since the force from the plane is perpendicular to the plane.

In the alternative picture we use a non-inertial frame [Fig. (5b)] in which the axial direction is fixed. In this system particles will of course be subject to a centrifugal force directed downwards, and in addition to multiple scattering and doughnut scattering the particles slowly "fall" down. For particles bound to a skew plane only the component of the centrifugal force parallel to the plane will be effective and the particles therefore slowly slide down the plane.

At this point it seems worth while to make a remark concerning the relative magnitudes of various angles. Multiple scattering is characterized by the r.m.s. deviation, θ_G , of the Gaussian part of the scattering spectrum; in our case $\theta_G \simeq 0.7$ mrad. Doughnut scattering is very quick for $\psi \leq \psi_1$ but for the long crystals used here certainly important also for angles ψ up to $\sim 10 \psi_1 \simeq 1$ mrad. The bending angles span from 4 to 52 mrad and are by far the largest angles entering into the description.

From fig. 5a it is evident that particles entering the horizontal plane will be deviated the full bending angle ψ_b , whereas particles

entering a skew plane are deviated by an angle $\psi_b \sin \eta$. In both cases, of course, there is a possibility of them being dechannelled during the passage through the bent crystal.

As a preamble to detailed scattering-angle distributions for selected parts of the incident beam, to be discussed below, we start by showing, in figs. 6a, 6b, and 6c, the distribution of deflection angles in the laboratory frame for the whole incident proton and π^+ beam transmitted through crystals bent 4, 20, and 52 mrad, respectively. The lower, broad spots correspond to unbent parts of the beam, where the particles have undergone normal multiple scattering. A scan through such a spot (not shown here) gives the expected width for the Gaussian part of the multiple scattering distribution. The apparent difference in spot size is simply due to different selections of spot density.

The upper spots correspond to particles deflected 4, 20, and 52 mrad. They have been channelled along the horizontal $\{110\}$ -plane throughout the crystal. Particles in between lower and upper spots have been dechannelled during the passage through the bent crystal.

Strong side beams coming at 60° to the vertical are observed in all three distributions. These beams are caused by the two skew $\{110\}$ -planes ($\eta = \pm 30^\circ$). The maximum bending from these planes is half ($= \sin 30^\circ$) the bending of the horizontal plane. Wings from the $\{112\}$ -planes are also visible, but dechanneling is rather severe for these planes.

In these pictures the whole beam, with beam divergence ± 1 mrad, was used. As mentioned above, the axial and planar critical angles are much smaller, ~ 0.1 mrad, showing that only a small fraction of the beam becomes channelled upon entrance into the crystal. Obviously the bending is only expected to be efficient for particles which are -- or become -- planar channelled, or, perhaps, axial channelled. In the following we shall therefore look in more detail at the deflections for selected parts

of the incident beam. We have a possibility of selecting in the analysis particles with interesting incident angles and/or interesting (i.e. low) energy loss in the detector.

Figure 7a is equivalent to fig. 6, but here we only show deflection angles for particles incident along the horizontal $\{110\}$ -plane. This is equivalent to the type of bending investigated at Dubna¹. Projectiles participating in a possible doughnut bending were excluded by requiring incident angles to the axis to be larger than 0.5 mrad. A detailed scan through the angular distribution is shown in fig. 7b, where the intensity of particles, $Y(\theta)$, is plotted as a function of deflection angle θ . The peak at $\theta = 0$ is associated with particles which were either not channelled from the beginning (recall the poor angular resolution of the incident beam), or dechannelled during the first 5 mm crystal which was not bent. These particles are multiply scattered, $\theta_G \approx 0.7$ mrad.

In Fig. 7c we show the number $I(\theta)$ of channelled particles as a function of θ , i.e. $I(\theta) = \int_0^\infty Y(\theta) d\theta$. Since the curvature is linear in the depth x , as mentioned previously, we can easily convert the angle θ into crystal depth x . In fact, one finds, for $0 < x < L/2$, the relationship $x = L\sqrt{(\theta/2\psi_b)}$, and, because of the symmetry of the problem, the same formula applies for $L/2 < x < L$, provided one substitutes $(L - x)$ for x and $(\psi_b - \theta)$ for θ . In this way we have been able to extract a dechanneling length t_d , which turns out to be ~ 8 mm, close to the value of Z_e [eq. (5)].

The exact bending of the crystal can be read directly from fig. 7b. It appears that one can easily determine the bending with an accuracy better than ± 0.3 mrad. It is also apparent from fig. 7b that 10-15% of the

channelled beam is bent the full angle, which is also in fair agreement with simple dechannelling estimates [eq. (7)].

Figure 8 shows deflection angle distributions for particles incident along a skew $\{110\}$ -plane. In fig. 8a the particles have had incident angles corresponding to the lower part of the plane, as sketched by the insert. Obviously the particles have been deflected by that plane only. In fig. 8b the particles were incident in the upper part of the skew plane, but we see deflection effects for both skew planes! The explanation is straightforward. In the coordinate system where the axial direction is fixed (fig. 5b) the particles slide down the plane. These particles pass through the axial region, where the planar effects are absent inside $\sim \psi_1$. If such particles during their fall are retrapped in planes, they must be so with equal probability in two planes characterized by the angles $\pm\eta$; provided, of course, that the doughnut scattering is quick enough, see eq. (19).

We therefore should expect to see at least the prominent wings from the two skew $\{110\}$ -planes. The wings should not be exactly equal. The one corresponding to the original plane gives a deflection angle close to 10 mrad, whereas particles which "change plane" in the axial region experience a smaller net deflection: at first they are deflected to the left, and afterwards to the right. Since the total opening angle of our beam is ≤ 2 mrad, this effect is hardly noticeable on a qualitative picture such as fig. 8b, but can easily be made visible in a detailed scan.

In fig. 9 we show qualitative pictures of the bending effect for particles incident in various angular regions around the axial direction. In fig. 9a the beam is axially channelled, i.e. $\psi < 0.5\psi_1$. Only the wings from the skew $\{110\}$ -planes are seen. (Recall that particles which are not

affected by the bending appear in the central spot, which therefore contains either dechannelled particles or particles channelled in the vertical $\{112\}$ -plane which is unbent.) The results are identical with the case discussed above except that the wings are here equal in length.

Figure 9b shows results for particles with $\psi_1 < \psi < 1.5\psi_1$. For such a beam, in statistical equilibrium with respect to azimuthal angles, some particles are channelled along the horizontal $\{110\}$ -plane, and give rise to a distinct vertical wing in the picture. Considerably more particles become channelled along the skew planes.

In fig. 9c particles with $2.5\psi_1 < \psi < 3.5\psi_1$ are shown. Here we see more particles in the vertical wing, and even some in the wings from the skew planes. But still the number of particles channelled along one of the skew planes exceeds the number channelled along the horizontal plane. Finally, in figs. 9d and 9e, particles incident far from the axis in the upper and lower doughnut regions, respectively, are shown. The upper doughnut region populates all planes, whereas the lower one only populates the skew planes -- and rather poorly.

These results will now be compared with qualitative theoretical expectations as well as with detailed BINCOL simulations. Recall in this connection that the front end of the crystal (5 mm) is unbent.

Particles with $\psi < \psi_1$ reach the bent region in complete statistical equilibrium. They will try to follow the bent axis, but the radius of curvature is too small; the particles fall downwards thereby achieving larger transverse energy: doughnut scattering is still quick, but statistical equilibrium with respect to azimuthal angles cannot be reached. During the fall there is a chance of the particles being trapped

in the lower part of the skew planes, but hardly in the horizontal plane, which is perpendicular to the centrifugal force.

Next, consider particles incident on the crystal at angles ψ to the axis in some narrow interval outside ψ_1 . Assume the distribution of particles to be uniform with respect to azimuthal angle. This means that the beam is already in statistical equilibrium, and it will stay in equilibrium during its passage through the unbent part of the crystal, the only change being a radial broadening. The reason for this is that there is a detailed balance in the beam due to the rule of reversibility, as mentioned in section 2.1. When the bent part of the crystal is reached, this equilibrium is destroyed because the doughnut "falls" downwards as seen from the stereogram.

Look first at the particles in the three $\{110\}$ -planes. Except for dechannelling, the particles in the horizontal plane and in the lower parts of the skew planes stay in the planes and take part in the bending. Particles in the upper parts of the skew planes slide to the axial region and behave the same way as particles with $\psi < \psi_1$, i.e. some are retrapped in the lower parts of the skew planes and take part in the bending, and the rest are dechannelled and not bent.

Doughnut particles outside the planes but below the axis increase their transverse energy, thereby reducing the doughnut scattering. This means that most of these particles are not expected to become planar channelled but rather to be lost. Particles above the axis move closer to this, thereby increasing the doughnut scattering, which may lead to further planar feed-in. (An example of planar feed-in in the doughnut region for the present energies -- but in an unbent crystal -- may be seen

in ref. 6, fig. 17.) Some of the particles even reach the central axial region and then behave as mentioned above.

In total it seems as if -- although the arguments above are qualitative -- the bending by itself, provided it is not too strong, promotes the tendency to planar feed-in compared to the case of a flat crystal where the effect is modest. In the experiment we have tried to elucidate further this interesting effect of feed-in, which enhances the bending efficiency.

In fig. 10 examples of BINCOL simulations are presented. We use the coordinate system with fixed direction of the axis. The three $\{110\}$ -planes are shown in each figure. The program plots the angle to the axis every 100 atomic layers into the crystal as a continuous line through the points. The particles have been followed through 0.4 mm of crystal, with bending radius 10 cm, which is somewhat smaller than our strongest bending ($R = 50$ cm). The small bending radius forces the particles through the axial region in a shorter crystal length, and therefore a shorter running time in computing.

The centrifugal force pulls the particles downwards, and one can easily see axial channelling as well as planar channelling. Many interesting features -- also about normal channelling in unbent crystals -- are apparent from the figure.

In both plots four particles have been started above the axis, with the same angle, but at randomly chosen positions in the transverse plane. The starting angle corresponds to either a skew plane or the doughnut region outside major planes.

In fig. 10a the particles are well channelled in a skew plane (small angle to the plane) at the beginning. When they reach the axial region

they are doughnut scattered for a short while (the bending is not far from the critical bending), and they all happen to be caught by planes afterwards: two in the $\{110\}$ -planes, one in the vertical $\{112\}$ -plane, and one in the weak $\{123\}$ -plane.

As is well-known from channelling theory³ planar effects become weaker and gradually disappear when the angle to the axis is approaching ψ_1 . Also in this region between $\sim \psi_1$ and $\sim 2\psi_1$ the planar critical angles are smaller than they are far from the axis. The plane has been described in this region as a string of strings. The disappearance of the planar effects inside ψ_1 are clearly seen in the plots of fig. 10, and also the string-of-strings region is distinctly different from the region farther away. One sees directly that the particle motion is governed by a correlated series of single-string collisions close to the axis, whereas the motion is smoothed far from the axis.

Particles which become channelled in the lower parts of a skew plane may do so with a very small transverse energy (plot a1) or with rather high transverse energy (plots b1, b2). An example of a hard collision (a sudden shift of the trace) is seen in plot b1, the latter leading to dechannelling. In plot b2 a particle is channelled along the $\{112\}$ -plane and obviously very quickly will be dechannelled.

Doughnut scattering for non-channelled particles can also be seen: the path is distinctly concave towards the axis. Particles in the doughnut region do not normally become planar channelled without the help of the axis, but an attempt is seen in plot b3.

Although these BINCOL simulations give many more details than can be seen experimentally, we think that the experimental results are well explained by BINCOL. For example one will expect most of the particles,

which according to BINCOL are planar channelled along the weaker planes, to be dechannelled rather quickly so that only those channelled along the $\{110\}$ -planes will remain.

It is worth emphasizing that the reason why the various effects mentioned above can be seen in these BINCOL simulations is the bending of the crystal, which forces the particle to probe different regions in angle space around an axis, thereby revealing the various scattering mechanisms in channelling.

4.2 Bending efficiency

If one should contemplate the possibility of using the bending effect of a curved crystal in the handling of high-energy beams it is important to know the efficiency of the bending, i.e. how many particles are lost because of dechannelling, etc.

As mentioned above, the percentage of particles incident along the horizontal $\{110\}$ -plane, within the planar angle ψ_p , that follow the bent plane all the way through the crystal, is $\sim 15\%$ in the present case, i.e. that of 12 GeV/c particles in a 20 mm long crystal, bent 20 mrad. Since the beam divergence is ± 1 mrad, the number of channelled particles is small, i.e. $\sim 5\%$, and the bent beam has therefore only $\sim 0.7\%$.

The efficient way to increase this number is to make the beam more parallel, and at the same time keep the beam cross-section as small as possible (the crystal cross-section was 0.5 mm x 10 mm). More sophisticated bending devices can, of course, secure a uniform bending of the whole crystal, and somewhat shorter crystals, perhaps ~ 10 mm, can be used, which will increase the channelling yield -- in the present case up to $\sim 2\%$.

Probably a somewhat more efficient bending could be achieved at higher beam momenta, 200-300 GeV/c. Although the planar angles decrease as $p^{-1/2}$, the multiple scattering decreases even faster, as p^{-1} , and, very important, the dechannelling length increases proportionally to p [eq. (5)]. As mentioned above, an experiment has been performed at Fermilab¹⁸ where bending effects were also investigated.

The actual bending efficiencies measured in the present experiment are summarized in figs. 11 and 12. We select a ring in incident angle space centred around the axis. In fig. 11 we plot, as a function of azimuthal angle in this ring, the percentage of the beam bent in the vertical direction, i.e. by the horizontal plane. At 0° and at 180° we naturally observe peaks since here the beam is aligned with the plane. The peak values are $\sim 15\%$. In the rest of the scan we find a rather constant bending efficiency of a few per cent, showing that particles in the doughnut, during the 5 mm unbent crystal, are fed into the plane to a certain extent. For the larger doughnut "radii", $5-7\psi_1$, there is a distinct difference between the "upper" and the "lower" doughnut: equilibrium is not established in the front part of the crystal, but the particles "above" the horizontal plane have an increased chance of being caught in the plane.

A similar set of scans are shown in fig. 12. Here we look at the intensity in one of the side beams from a skew $\{110\}$ -plane. We have chosen the plane situated in the first and the third quadrant. For natural reasons there is a strong peak at 240° . The incident beam is in the lower part of the skew plane. The intensity at 60° is reduced by a factor of 2 compared to the peak at 240° , but now we observe a similar peak at 120° which stems from the mirror half-plane at 120° . Everything is in agreement

with expectations, both from the discussions in section 2, and from the results of fig. 10.

In figs. 11 and 12 particles are included if they have been deviated by more than 1 mrad by the plane. We then find from the figures that about 12% of the whole beam enters one of the three {110}-planes, 4% in each. Taking dechannelling into account through a 15 mm bent crystal we end up by having close to 1% of the particles in each of the three beams. This is an efficiency somewhat better than for the case of pure planar channelling (0.7%), and in addition we get here three beams instead of one!

4.3 Bending of negative beams

Planar effects for heavy negatively charged GeV particles have been observed to be very weak⁹. If planar channelling is weak the only possibility for steering a negative beam by means of a bent crystal is via the doughnut effect, which is known to be strong also for negative particles.

A negative particle is free to move everywhere in the transverse plane, except if it happens to be bound to one atomic string. In the latter case the probability of being scattered (because of thermal vibrations) is expected to be large. This means that, for channelled negative particles, the multiple scattering is of the same strength, or stronger, than for particles incident in a random direction, and much stronger than it is for channelled positive particles. This again means that a possible bending effect is expected to be weaker for negative particles than for positive particles.

In fig. 13 the results are shown for negative particles incident along the $\langle 111 \rangle$ -axis in a 2 cm silicon crystal bent 4 mrad, i.e. a bending radius of 5 m. We plot the intensity distribution along the bending

direction for particles with incident angles to the $\langle 111 \rangle$ -axis between $0.5\psi_1$ and ψ_1 (full curve) and for particles incident in a direction far from the axis. The bending effect is clearly present, although weak. No particles are observed to be deflected the full 4 mrad.

We have simulated the doughnut bending for negative particles by means of BINCOL. In fig. 14 we show an example of such a calculation, and, for comparison, the corresponding result for a positive particle. The crystal is 0.4 mm long with a bending radius of 5 m. In fig. 14a is shown the track in angle space (in the non-inertial frame) of a proton with incident angle $\psi = 0$. As in fig. 10, the radius of the circle is equal to ψ_1 . The particle is well channelled throughout the crystal. In fig. 14b we have used exactly the same starting conditions, i.e. the same direction and same position in transverse space, but changed the sign of the charge of the particle. The particle quickly acquires a rather high transverse energy, and is about to leave the axial region. Doughnut scattering is seen to be very prominent. Note that the tracks in angle space (which is equivalent to a transverse velocity space) are qualitatively different for positive and negative particles: the track of a positive particle is convex as seen from the axis, whereas the track of a negative particle is concave with respect to the axis, in accordance with the fact that a negative particle acquires its maximum (transverse) velocity at the closest distance of approach to the axis.

Some negative particle tracks show beautiful "rosettes", corresponding to particles which have for some distance been bound to one string. An example of this is shown in fig. 15. Figure 15a gives the track in the transverse angle space, and fig. 15b the corresponding track in the physical transverse space, i.e. the particle path itself. This particle is

unbound at the beginning but is captured by the string (recall that thermal vibrations of the atoms along the string are included in BINCOL, making capture processes possible). The particle makes some 50 revolutions around the axis, most of which are nearly circular and rather close to the axis. After a while it suffers some hard collisions and eventually escapes.

Qualitatively the BINCOL results explain the experimental findings of fig. 13. There is an effect of the doughnut scattering, and particles are seen to follow the axial direction for an appreciable distance.

Next we have looked into a possible planar channelling for negative particles. Again we compare directly positive and negative particles given exactly equal starting conditions. The particles start with a velocity parallel to a skew $\{111\}$ -plane. As an example, we show in fig. 16a the result for a positive particle in a crystal with a bending radius of 1 m. This picture is analogous to the ones shown in fig. 10.

Now consider fig. 16b where the particle has been given negative charge. We do see planar effects, but a plane only holds a particle for a very short time -- scattering is obviously too strong. Again we see strong doughnut scattering.

In a previous experiment we have seen another faint experimental trace of planar channelling for negative particles⁹. The number of δ electrons ejected from a thin (0.63 mm) silicon crystal by planar channelled negative pions of 12 GeV/c was a little bit larger than the number ejected by particles incident in a random direction.

The conclusion from both experiment and simulation is that planar channelling for negative particles does exist also for heavy GeV particles. Because of scattering the dechannelling is very quick, and planar effects will be weak, unless the crystal is very thin. The classical experiments

with MeV electrons, where planar channelling effects were very strong, were performed on crystals with a thickness of $\sim 1 \mu\text{m}$. In the present energy region one should probably not exceed $\sim 50 \mu\text{m}$ in order to observe strong channelling effects. There is no bending effect from planes in a 2 cm crystal!

4.4 Planar feed-in

As mentioned in section 4.1, we have tried to elucidate the feed-in mechanism for planes by means of a crystal equipped with three detectors along it; see fig. 3. Since a channelled particle suffers a lower than normal energy loss in the detector, we can with reasonable accuracy identify channelled particles by means of their energy loss.

In fig. 17 a number of energy spectra from the three detectors are shown. The crystal was $\langle 111 \rangle$ -Si, 3 cm, unbent, and the incident beam was 12 GeV/c, positive. Figure 17a gives the energy-loss spectra in the three detectors for the whole beam, of opening angle 2 mrad. The spectra are Landau-like except for the low-energy shoulder due to channelled particles. Only a few per cent of the beam is channelled, see fig. 5, and most channelled particles are planar channelled. The number of channelled (i.e. low-energy-loss) particles is roughly the same in the three detectors, but it is not the same particles which are channelled in the three spectra, as can be seen in fig. 17b-d.

Consider fig. 17b, where we have plotted the energy-loss spectra of those particles which had a low energy loss in detector 1. The results show a strong dechannelling through the crystal. In fig. 17d, we plot the energy-loss spectra of those particles which had a low energy loss in detector 3. These spectra look very much like the former, but are in

reverse order. This result must be interpreted as showing a strong feed-in. Actually, the feed-in appears to be of the same strength as dechannelling. Finally, in fig. 17c, we show the spectra for those particles which had a low energy loss in detector 2. The spectra in detectors 1 and 3 are much alike: more than 50% of the particles channelled in detector 2 have been fed in between detector 1 and 2, and more than 50% are dechannelled between detectors 2 and 3.

In order to explain these findings we again refer to the rule of reversibility, and to the fact that our incident beam is in statistical equilibrium with respect to azimuthal angles. The detailed balance means that the number of particles channelled will stay essentially the same throughout the crystal [except for the decrease due to the radial broadening (multiple scattering) of the beam]. Feed-in and dechannelling should be equal as is also seen in figs. 17b and 17d. In principle, it is possible to extract quantitative information on dechannelling and feed-in from these figures, but we have refrained from doing so at present.

5. CONCLUSION

There is a strong bending effect for positive particles with GeV energies. The bending is primarily caused by atomic planes, but the doughnut scattering close to an axial direction is so efficient in feeding particles into the planes that, for the present divergent beam, we end up with more particles being deviated by a plane than if we had used pure planar channelling. In addition we get three beams instead of one!

The effects observed are explained in detail by computer simulations based on binary collisions. These simulations also illustrate the

interesting transition region where axial and planar channelling merge -- the string-of-strings region. A bending process caused by doughnut scattering alone will only be effective for very modest bendings, and will be masked by planar bending effects. For energies much higher than used in the present experiment, pure doughnut bending will be even less important.

For negative particles a small bending effect is demonstrated, and from the computer simulations it is identified as due to doughnut scattering. The simulations also reveal the possibility of "rosette" motion for negative particles. The multi-detector crystal is a unique tool for studying planar feed-in and dechannelling.

Acknowledgements

We are much indebted to G. Melchart and G. Petersen, who participated in other parts of our experimental programme and who have helped us in the present context with valuable technical assistance. Fruitful discussions with J.U. Andersen, K.R. Eriksen and J.S. Forster are gratefully acknowledged. The Danish participation in the Collaboration was made possible through financial support from the Danish Committee for Accelerator Physics.

REFERENCES

- 1) A.F. Elishev et al., Phys. Lett. 88B, 387 (1979).
A.S. Vodopyanov et al., Pis'ma v Zh. Eksp. Teor. Fiz. 30, 474 (1979).
- 2) J. Bak et al., Phys. Lett. 93B, 387 (1980).
- 3) J. Lindhard, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 34, No. 14 (1965).
- 4) D.S. Gemmell, Rev. Mod. Phys. 46, 129 (1974).
- 5) E.N. Tsyganov, Fermilab TM-682, TM-684 (1976).
- 6) S.K. Andersen et al., Nucl. Phys. B167, 1 (1980).
- 7) S.K. Andersen et al., Proposal CERN/PSCC/78-13 (1978).
- 8) J.F. Bak et al., Phys. Rev. A 25, 1334 (1982).
- 9) J.F. Bak et al., Nucl. Phys. A389, 533 (1982).
- 10) E. Bonderup et al., Radiat. Eff. 12, 261 (1972).
H.E. Schiøtt et al., in Atomic collisions in solids (eds. S. Datz, B.R. Appleton and C.D. Moak) (Plenum, New York, 1975), Vol. 2.
- 11) L.C. Feldman and B.R. Appleton, Phys. Rev. B 8, 935 (1973).
- 12) H. Kudo, Nucl. Instrum. Methods 189, 609 (1981).
- 13) J.A. Ellison and S.T. Picraux, Phys. Lett. 83A, 271 (1981).
J.A. Ellison, Nucl. Phys. B206, 205 (1982).
- 14) V.V. Kaplin and S.A. Vorobiev, Phys. Lett. 67A, 135 (1978).
- 15) A.M. Taratin et al., Phys. Lett. 72A, 145 (1979).
A.M. Taratin et al., Phys. Status Solidi b 100, 273 (1980).
- 16) I.D. Feranchuk, Sov. Phys.-Tech. Phys. 26, 164 (1981).
- 17) K.R. Eriksen, Thesis, Aarhus University (1980), unpublished.

18) R.A. Carrigan, Jr, Fermilab-Pub-80/45-Exp (1980).

J.A. Ellison et al., Contribution to Int. Conf. on Atomic Collisions
in Solids, Bad Iburg, Germany, 1983, (to be published in Nucl.
Instrum. Methods).

S.I. Baker et al., Phys. Lett. 137B, 129 (1984).

19) J.U. Andersen (private communication).

20) J.U. Andersen and L.C. Feldman, Phys. Rev. B 1, 2063 (1970).

Figure captions

- Fig. 1 : Effective planar potential between two $\{110\}$ -planes in silicon, placed $1.92 \text{ \AA}$ apart. The full-drawn curve represents the potential in a flat crystal, the dash-dotted curve the potential in a crystal with a radius of curvature of 40 cm , and the dotted curve the potential for a radius of curvature of 10 cm . The potential is measured in units of the maximum potential from a single plane. The thermal vibrational amplitude q perpendicular to a plane is indicated (room temperature).
- Fig. 2 : Schematic drawing of experimental set-up. Beam enters from the left. DC designates drift chambers, SC scintillators, and VT vacuum tubes.
- Fig. 3 : Bending device. The crystal is placed between three rollers and bent by tightening the thumbscrew. Most crystals had one detector close to the front end, but in one case we used a crystal with three detectors as shown here.
- Fig. 4 : Stereogram of $\langle 111 \rangle$ -Si crystal. The relative number of 12 GeV/c positive particles in the two-dimensional incident angle space that have suffered lower than normal energy loss is plotted. The three $\{110\}$ -planes are very pronounced, and the $\{112\}$ -planes are just visible.
- Fig. 5 : a) Sketch of the stereogram with the three prominent $\{110\}$ -planes shown at both the front and rear ends of the crystal. From this picture one can easily see the bending from either the horizontal or a skew plane. b) The angular region around the $\langle 111 \rangle$ -axis. The circular region is the axial-channelling

region, of radius ψ_1 , where planar effects disappear. The planes are shown with a width corresponding to $2\psi_p$.

Fig. 6 : Distribution of deflection angles for 12 GeV/c positive particles transmitted through crystals bent a) 4 mrad, b) 20 mrad, and c) 52 mrad, respectively. The beam is aligned as accurately as possible with the $\langle 111 \rangle$ -axis, but the beam divergence is ~ 10 times the critical angle ψ_1 .

Fig. 7 : Bending along a horizontal plane. a) Similar to fig. 6a but, as shown in the small insert, the beam is directed along the plane, outside the axial region. b) A vertical scan through the distribution in (a). Many particles are dechannelled on the way, but $\sim 10\%$ survive as channelled particles throughout the crystal. c) The number of particles channelled as a function of deflection angle. All angles are measured in milliradians.

Fig. 8 : Bending along a skew $\{110\}$ -plane. When the particles are directed along the lower part of the skew plane (a) they stay there, and we see bending effects for that plane only. If the particles are directed along the upper part of the skew plane (b) they slide to the axial region and are redistributed between the skew planes. Angles in milliradians.

Fig. 9 : Bending of particles incident in various angular regions.

Fig. 10 : Particle tracks from BINCOL. The figure shows the direction of the particle path as a function of depth in the crystal, in a coordinate system with fixed axial direction. Particles start above the axis and are pulled "downwards" by the centrifugal force. The radius of the circle is equal to ψ_1 .

Fig. 11 : Bending efficiency as a function of position in incident-angle space. The percentage of the incident beam bent in the vertical direction.

Fig. 12 : Same as fig. 11 except that here we plot the percentage of the incident beam bent by one of the skew $\{110\}$ -planes. For a further discussion, see text.

Fig. 13 : Scattering-angle distributions for 12 GeV/c π^- incident along the $\langle 111 \rangle$ -axis in silicon (full curve) and in a random direction (dashed curve). Abscissa in milliradians.

Fig. 14 : Axial bending for (a) a positive and (b) a negative particle. Analogous to fig. 10. We show the tracks in angle space for two particles which differ only in charge. Bending radius is 5 m and crystal thickness is 0.4 mm. The positive particle is well-channelled, with low transverse energy, whereas the negative particle quickly acquires a rather high transverse energy and is about to be dechannelled. Doughnut scattering is very prominent.

Fig. 15 : A track similar to the one in fig. 14b, shown both (a) in angle space and (b) in physical space. Both in the frame with fixed axial direction.

Fig. 16 : Comparison between (a) positive and (b) negative particles incident along a skew plane. Analogous to fig. 14 except that the bending radius is 1 m. The planar channelling for the positive particle changes rather abruptly into axial channelling around ψ_1 . The negative particle stays in the skew plane only for a very short interval, is dechannelled, and is recaptured for a few oscillations. Doughnut scattering is strong.

Fig. 17 : Energy-loss spectra for 12 GeV/c protons transmitted through the three-detector silicon crystal. a) The spectra for all particles. b)-d) Spectra for those particles which have had a low energy loss in detectors 1, 2, and 3, respectively.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

a)

b)

Fig. 16