**RAPIDITY RATIOS, FEYNMAN-X DISTRIBUTIONS AND FORWARD-BACKWARD
CORRELATIONS IN HADRON-NUCLEUS COLLISIONS IN A DUAL
MONTE-CARLO MULTI-CHAIN FRAGMENTATION MODEL**

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A B S T R A C T

Hadron-nucleus interactions are studied for different projectile hadrons and targets in the dual multi-chain fragmentation model. Using the Monte-Carlo method allows the study of inclusive as well as exclusive cross-sections. Energy momentum and all additive quantum numbers are conserved exactly in the model. Results are presented on the A-dependence of single particle distributions in the projectile fragmentation region, average multiplicities and forward-backward correlations. The inclusion of the Fermi momentum of the target nucleus makes it possible to study rapidity distributions as well as rapidity ratios in the target fragmentation region. Satisfactory agreement with recent data is obtained.

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1. INTRODUCTION

Multi-hadron production in high-energy hadron-nucleus collisions has been studied in detail over the last few years, both experimentally and theoretically. A recent review on all aspects of this problem was given by Fialkowski and Kittel [1].

In Ref. [2] we studied particle production in hadron-hadron and hadron-nucleus collisions using the dual multi-chain fragmentation model, see also Capella [3]). The model can be approximated by a simple two-chain fragmentation model for hadron-hadron interactions below ISR energies. This two-chain model describes reasonably well most of the present data for baryon, antibaryon and meson projectiles [2,4].

The generalization of such a model to a multi-chain model permits the study of hadron-hadron collisions at high energies [5-7] as well as hadron-nucleus interactions [8].

Here we continue to study our multi-chain Monte-Carlo fragmentation model presented in Ref. [2] with some improvements:

- i) We use a more realistic sampling of the actual number of collisions during a hadron-nucleus interaction [9].
- ii) We include the Fermi momentum of the nucleons in the target nuclei. This allows us to look for rapidity distributions as well as rapidity ratios in the whole y -region.

The paper is organized as follows: since a detailed description of the underlying model [2] already exists, we only give a brief summary of the main features of our model and discuss its new aspects in section 2. In sections 3, 4 and 5 we compare our Monte-Carlo results with recent data on average multiplicities, rapidity distributions and ratios, x -distributions in the fragmentation region and forwardbackward correlations.

2. THE MULTI-CHAIN FRAGMENTATION MODEL FOR PARTICLE PRODUCTION IN HADRON-NUCLEUS COLLISIONS

In hadron-scattering processes we assume that the interaction separates the valence quarks of the colliding hadrons into two colour-neutral systems, which give rise to two multi-particle chains (see Ref. [2]). Using only valence quarks is not correct for higher energies, i.e. at collider energies. In order to describe the particle production at these energies further sea quark chains have to be added according to the AGK rules [10]. Such an extension of the model has already been studied [6-8] and seems to describe all features of low P_t particle production known at the energies of the CERN SPS proton-antiproton collider [7].

In case of hadron-nucleus collisions many interactions occur. The average number of collisions $\bar{\nu}$ inside the nucleus is given by the mass number A and the inelastic cross-sections for h-h and hA interactions.

$$\bar{\nu} = A \sigma_{\text{inel}}^{\text{h-h}} / \sigma_{\text{inel}}^{\text{h-A}} \quad (1)$$

In our calculation we use inelastic cross-sections $\sigma_{\text{inel}}^{\text{h-h}}$ given in Ref. [11]. Each collision provides two chains like in hadron-hadron interactions. This gives rise on the average to $2 \bar{\nu}$ chains. In a Monte-Carlo model we have to sample the actual number ν of collisions for each event. Different from Ref. [2], where ν was sampled from a Poisson distribution, we use here the more realistic procedure described by Nielsson and Stenlund [9]. This method uses the nuclear density distribution in Wood-Saxon form. The probability of finding a nucleus at a distance r from the centre of the nucleus is

$$P(r) = \frac{K}{1 + \exp \left[\frac{r - r_0 A^{1/3}}{C_0} \right]} ; C_0 = 0.545 \text{ fm} \quad (2)$$

where K is a normalization constant such that

$$A = \int_0^{\infty} P(r) dr \quad (3)$$

and r_0 is an A -dependent number chosen to fit the inelastic cross-section σ_{inel}^{h-A} .

In Fig. 1 we show an example of a triple-scattering process in a proton-nucleus collision. As can be seen from this figure, both valence quarks and sea quarks of the projectile have to be included, to construct the chains. Two chains are initiated by the valence quarks of the projectile and the valence quarks of one of the target nucleons. All other chains are derived from sea quarks of the projectile and valence quarks of other target nucleons. In order to sample the x -fraction of the valence quarks we use

$$d_{baryon}^{val}(x) \sim \frac{(1-x)^2}{x^{1/2}} \quad (4)$$

for baryons and

$$d_{meson}^{val}(x) \sim \frac{(1-x)^{3/2}}{x^{1/2}} \quad (5)$$

for mesons. The x -fraction of the sea quarks of the target nucleons are sampled according to

$$d^{sea}(x) \sim \frac{(1-x)^5}{x} \quad (6)$$

The remaining diquark of a target nucleus gets the x -fraction $x_d = (1 - x_T^{val})$. In the case of the projectile we have $x_d = (1 - x_p^{val} - \int_{sea} x_p^{sea})$ for the diquark of the baryon. For meson projectiles we have two possibilities: either the quark or the antiquark can get this remaining x -fraction.

Now we have to fragment the chains into hadrons and hadron resonances. For this purpose we make a Lorentz boost of each chain into its centre of mass system and hadronize the chains via a Monte-Carlo chain decay fragmentation model [12,13].

The Monte-Carlo code BAMJET [13] for the fragmentation of quark and diquark jets into the observed hadrons used here was tested using data from e^+e^- annihilation [12] and from lepton-hadron interactions [14].

In order to describe the hadronization of quark and diquark jets produced in hadron-hadron or hadron-nucleus collisions we make the assumption that quark jets have universal properties in e^+e^- , lepton-hadron, hadron-hadron and hadron-nucleus collisions. There is no theoretical justification for such an assumption, but it seems to describe the data successfully (see Refs. [2,4]).

Now we return to the hadronization of our chains. The invariant masses of the chains are obtained from the x -fractions of the contributing quarks and diquarks. Since the chain decay into hadrons is only possible for sufficiently large jet masses we have to correct the kinematics for chains containing valence quarks with low invariant masses. Instead of the chains we create directly a stable hadron or hadron resonance with the appropriate quantum numbers. If this is not possible we sample a new event.

Finally all chains are transformed back into the lab system and a complete hadron-nucleus interaction is obtained conserving energy momentum and all additive quantum numbers.

The decay of the resonances into stable hadrons is done by means of the Monte-Carlo code DECAY [15].

The whole procedure as well as the underlying Monte-Carlo model is described in more detail in Refs. [2,16]. Here we discuss only some new features of our model:

- i) Contrary to Ref. [2] we now consider also sea quark chains with very low invariant masses as long as it is possible to replace such chains by hadrons or hadron resonances. This is more natural than the artificial cut-off introduced in [2].
- ii) We decrease the power b in the valence quark distribution [4]. Instead of $b = 3.5$ we use here $b = 2$. The reason for this change is to get a better agreement with experimental data in the projectile fragmentation region.

iii) In order to include the Fermi momentum of the target nucleons we approximate the nuclear density by

$$\rho = \frac{A}{V} = \frac{3}{4\pi r_0^3} \quad (7)$$

where $r_0 \approx 1.1 \dots 1.2$ Fermi represents the mean nucleon radius. A is the mass number and V the volume of the nucleus. This provides the following limit for the Fermi momentum:

$$P_{F,p,n} = 0.4 \left[\frac{N_{p,n}}{A} \right]^{1/3} \quad (8)$$

N_p is the number of protons and $N_n = A - N_p$ the number of neutrons inside the nucleus. The Fermi momentum distribution follows from

$$\int_0^{P_{F,p,n}} \frac{dn_{p,n}}{dp} dp = 4\pi \int f(p) p^2 dp = 1 \quad (9)$$

$$\text{with } f(p) = \begin{cases} \frac{3}{4\pi P_{F,p,n}^3} & 0 \leq p \leq P_{F,p,n} \\ 0 & p > P_{F,p,n} \end{cases}$$

and has the form

$$\frac{dn_{p,n}}{dp} = \frac{3p^2}{P_{F,p,n}^3} \quad (10)$$

Using an isotropic angular distribution we finally get a three-dimensional Fermi momentum for the nucleons in the nucleus rest frame. Handling each collision like before, we always have to make a Lorentz boost into the target nucleon rest frame. The inclusion of the Fermi momentum of the nucleons changes the particle production, especially in the target fragmentation region.

3. AVERAGE MULTIPLICITIES, RAPIDITY DISTRIBUTIONS AND RAPIDITY RATIOS

In Ref. [2] we discussed only the average charged multiplicity of proton nucleus collisions. In order to compare our model also with data [17-20] for other projectiles we give in Tables 1-3 average multiplicities for different kinds of charged projectiles. From the comparison we conclude that our Monte-Carlo multi-chain model provides average charged multiplicities that follow the essential trends of the data.

We have to take into account that in our model additional protons, caused by the soft intranuclear cascade, are not included but they are present in the experimental data. Although several cuts are used to subtract such additional cascade protons from the data, it seems to be more realistic to compare multiplicities of negative charged secondary particles only.

In Fig. 2 we plot rapidity distributions at $p_{\text{lab}} = 200 \text{ GeV/c}$ for projectile protons and antiprotons and different targets. In the case of nuclear targets the model provides a satisfactory description of the data [19] above $y \approx 1$. Below $y \approx 1$ the Monte-Carlo results are somewhat too low compared to the data. This might be due to additional nuclear effects, which are not in our present model, such as the cumulative effect [21] or the soft intranuclear cascade mentioned above.

Rapidity distributions for pp and $p\bar{p}$ collisions are also shown in Fig. 2. In contrast to the experiment, our two-chain model for hadron-hadron interactions gives forward-backward-symmetric distributions. This is what we expected. The difference between our model and the data [19] in the forward region could be caused by possible misidentifications of protons, antiprotons and kaons in the experiment.

Since the particle misidentification for positively charged particles is approximately the same in pp and pA interactions, it is useful to consider the ratio of rapidity distributions

$$R(y) = \frac{\frac{dN}{dy}(hA)}{\frac{dN}{dy}(hh)} . \quad (11)$$

The rapidity y is defined as

$$y = \frac{1}{2} \ln [E + P_{11}) / (E - P_{11})] . \quad (12)$$

In some experiments also the pseudorapidity η is used:

$$\eta = -\ln(\theta_{lab}/2) . \quad (13)$$

In Fig. 3a-d the ratio $R(y)$ for all charged particles is plotted for argon and xenon targets and incident protons and antiprotons. For $y > 1$ our Monte-Carlo calculations agree reasonably well with the data [19]. In the rapidity region $y < 1$ our results are clearly below the data, although we have included the Fermi motion of the target nucleons in the model. In order to obtain a better agreement in the backward region as well, one would have to include further nuclear effects into the model. Considering negatively charged particles only (see Fig. 4) the situation is somewhat better since additional cascade protons do not influence the result.

In Fig. 5a-b the ratio $R(\eta)$ as a function of the pseudorapidity η is presented for different projectiles and energies. The pseudorapidity distributions are calculated for $v = 1$ and $v = 3$; this corresponds to a hadron-hadron and to a hadron-lead interaction. Here, contrary to Figs. 3 and 4, the Monte-Carlo results are somewhat above the data [18] in the region $\eta > 2$, whereas we get a better agreement for $\eta < 1$.

We have to take into account that there might be some differences in determining the average number of collisions in the paper [18] and in our Monte-Carlo model. Furthermore there are no data below $\eta \approx 0$.

4. THE PROJECTILE FRAGMENTATION REGION

In order to study the projectile fragmentation region in hadron-nucleus collisions, we plot in Fig. 6 the invariant differential cross-section $E d^3\sigma/d^3p$ for inclusive proton production as a function of the Feynman-x variable for Pb, Cu, C and H targets.

The data [22] are measured at fixed transverse momentum $P_t = 0.3 \text{ GeV}/c$, whereas our Monte-Carlo results are integrated over all transverse momenta. Since the P_t and x -dependence should approximately factorize, a comparison seems to be possible at least in the region $0.2 \leq x \leq 0.8$. For this, only the normalization of our Monte-Carlo curves was corrected.

From Fig. 6 we see that the results of our model agree reasonably well with the data in this x -region. Since diffractive processes are not included in our Monte-Carlo model (see also Refs. [2,4], we are not able to describe the data at larger x -values, $x > 0.7$.

5. FORWARD-BACKWARD CORRELATIONS

Long-range correlations can be studied by considering the correlations between the number n_B of charged particles in the backward hemisphere (in the projectile-target nucleon c.m.s.) and the number n_F of charged particles in the forward hemisphere. In Ref. [7] it was found that these correlations are a sensitive test for the multistring nature of the model. Forward-backward correlations in hadron-nucleus collisions were measured by De Marzo et al. [19]. In Fig. 7 we compare the backward multiplicity $\langle n_B \rangle_{n_F}$ versus n_F and the forward multiplicity $\langle n_F \rangle_{n_B}$ versus n_B for p-Xe collisions. Strong correlations are seen. There is a good qualitative agreement of the model with the data, except for the fact, discussed already in section 3, that some particles in the region $y < 1$ of the backward hemisphere are missing in the model. To exclude the effects of short range correlations the data are presented in Fig. 7b also after the elimination of the particles in the central region $2.25 > y > 3.75$. With this cut the correlations almost disappear. This expresses the fact that all additional sea-quark (projectile)/valence-quark (target) chains are at small rapidities in the lab frame and do not extend beyond $y = 3.75$. In this respect the model for hadron-nucleus collisions differs from the multi-chain hadron-hadron model [7], where the additional chains are centred around c.m.s. rapidity $y = 0$. Also this drastic decrease in the long-range correlations after the rapidity cut is well described by the model.

6. SUMMARY

Several aspects of the multi-chain model for hadron-nucleus collisions presented in Ref. [2] have now been improved. Besides the original features of the model, like:

- i) the construction of multi-chain events and the subsequent transition of the chains into hadrons and hadron resonances using a Monte Carlo chain fragmentation model,
- ii) the decay of resonances into stable hadrons,
- iii) the inclusion of quantum number structure and correct kinematics leading to Monte Carlo events conserving exactly energy, momentum and all additive quantum numbers,

the model as described includes new features such as:

- iv) a more realistic sampling of the actual number of collisions in a hadron-nucleus interaction, and
- v) sea quark chains with low invariant masses down to typical hadron masses.

Since we have not taken into account diffractive events, our model is not able to describe data representing leading particle effects such as differential cross-sections at large x -values. Furthermore the model does not include nuclear effects like the cumulative effect or the soft intranuclear cascade, which influences the data especially in the backward region.

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TABLE 1

Average charged multiplicities obtained in hadron nucleus collisions with different projectile hadrons (π^+ , K^+ , p and $\bar{p}$) and target nuclei (C, Cu and Pb). The Monte Carlo results are compared to data from Ref. [18].

Project- ile	Target	P_{lab} (GeV/c)					
		50		100		150	
		Exp.[18] $\langle n_{ch} \rangle$	M.C. $\langle n_{ch} \rangle$	Exp.[18] $\langle n_{ch} \rangle$	M.C. $\langle n_{ch} \rangle$	Exp.[18] $\langle n_{ch} \rangle$	M.C. $\langle n_{ch} \rangle$
K^+	C	7.48 ± 0.14	7.63	-	9.36	10.37 ± 0.24	10.54
	Cu	8.71 ± 0.23	8.62	-	10.67	11.51 ± 0.34	12.05
	Pb	10.17 ± 0.39	9.64	-	11.96	13.46 ± 0.58	13.64
π^+	C	7.62 ± 0.14	7.79	9.19 ± 0.17	9.70	10.01 ± 0.18	10.08
	Cu	8.81 ± 0.23	9.25	10.41 ± 0.27	11.46	11.57 ± 0.30	13.07
	Pb	10.11 ± 0.38	10.90	11.54 ± 0.44	13.77	13.08 ± 0.50	15.6
p	C	7.88 ± 0.15	6.60	9.25 ± 0.18	8.20	10.58 ± 0.19	9.38
	Cu	9.52 ± 0.24	8.13	11.40 ± 0.29	10.75	12.93 ± 0.33	12.50
	Pb	11.31 ± 0.43	10.54	-	13.58	14.95 ± 0.57	15.58
$\bar{p}$	C	8.42 ± 0.15	7.89	9.87 ± 0.18	9.76	-	10.61
	Cu	10.30 ± 0.26	10.32	11.92 ± 0.32	12.85	-	14.36
	Pb	12.02 ± 0.45	12.81	14.16 ± 0.54	15.98	14.86 ± 0.57	18.0

TABLE 2

Average multiplicities for all charged particles and negatively charged particles measured in pXe, $\bar{p}$ Xe, pAr, $\bar{p}$ Ar, pp and $p\bar{p}$ collision at $P_{lab} = 200$ GeV/c.

The Monte Carlo results are compared with data from Ref. [19].

Target	Projectile	Experiment [19]		M.C. results (10'000 events)	
		$\langle n_{ch} \rangle$	$\langle n_- \rangle$	$\langle n_{ch} \rangle$	$\langle n_- \rangle$
Xe	p	17.33 ± 0.31	6.84 ± 0.13	15.53	6.34
	$\bar{p}$	18.85 ± 0.36	8.54 ± 0.15	17.56	8.40
Ar	p	13.31 ± 0.38	5.39 ± 0.17	12.30	4.98
	$\bar{p}$	14.23 ± 0.35	6.80 ± 0.16	14.08	6.92
p	p	7.67 ± 0.07	2.96 ± 0.03	7.83	2.85
	$\bar{p}$	7.53 ± 0.09	3.90 ± 0.05	8.20	3.46

TABLE 3

Average multiplicities for negatively charged particles produced in $\pi^+ \text{Al}$, $K^+ \text{Al}$, $\pi^+ \text{Au}$ and $K^+ \text{Au}$ interactions at $P_{\text{lab}} = 250 \text{ GeV}/c$.

The Monte Carlo results are compared to data from Ref. [20].

Target	Projectile	Experiment [20] $\langle n_- \rangle$	M.C. results (20'000 events) $\langle n_- \rangle$
Al	π^+	4.6 ± 0.2	5.20
	K^+	4.8 ± 0.3	5.03
Au	π^+	6.6 ± 0.3	7.10
	K^+	6.4 ± 0.6	6.60

FIGURE CAPTIONS

Fig. 1: Example of a triple-scattering diagram in a proton-nucleus interaction. P represents the incoming proton and T_1 , T_2 and T_3 are the three target nucleons taking part at the interaction. x^V and x^{Si} are valence and sea-quark momentum fractions.

Fig. 2: Rapidity distributions dN/dy at $P_{lab} = 200$ GeV/c for
 a) pXe , pAr and pp ,
 b) $\bar{p}Xe$, $\bar{p}Ar$ and $\bar{p}p$
 interactions, the Monte Carlo results (histograms) are compared to data from Ref. 19.

Fig. 3: Rapidity ratios $R(y) = [dN/dy(hA)]/[dN/dy(hp)]$ of all charged particles at $P_{lab} = 200$ GeV/c for
 a) $h = \text{proton}$, $A = \text{Argon target}$,
 b) $h = \text{proton}$, $A = \text{Xenon target}$,
 c) $h = \text{antiproton}$, $A = \text{Argon target}$,
 d) $h = \text{antiproton}$, $A = \text{Xenon target}$.
 The Monte Carlo results, represented by histograms, are compared to data from Ref. [19].

Fig. 4: Rapidity ratio $R^-(y)$ of negative charged particles at $P_{lab} = 200$ GeV/c. The data [19] are measured using incident protons and a Xenon target. The Monte Carlo results are represented by a histogram.

Fig. 5: Pseudorapidity ratios $R(\eta) = [dN/d\eta(v=3)]/[dN/d\eta(v=1)]$ of all charged particles for
 a) incident protons and antiprotons at $P_{lab} = 50$ GeV/c
 b) incident π^+ at $P_{lab} = 50$ GeV/c and 150 GeV/c.
 The data (18) measured at a target providing an average number of collisions $v=3$ are compared to Monte Carlo results (histograms) for a lead target ($v=2.8$).

Fig. 6: The invariant differential cross-section $E \, d\sigma/dp^3$ for $pA \rightarrow p$ plotted as a function of x for an incident momentum of 100 GeV/c. The data (22) are measured at a fixed transverse momentum of $p_t = 0.3$ GeV/c. The Monte Carlo results (40'000 events) are represented by histograms.

Fig. 7a: The average multiplicity $\langle n_B \rangle$ in the backward hemisphere versus the multiplicity n_F in the forward hemisphere for pXe reactions at $P_{lab} = 200$ GeV/c. All charged particles are considered.

7b: The average multiplicity $\langle n_F \rangle$ in the forward hemisphere versus the multiplicity n_B in the backward hemisphere. The Monte Carlo results are compared to data from Ref. [19].

Fig. 1

Fig. 2

$$y = \frac{1}{2} \ln \left(\frac{E+p_{\parallel}}{E-p_{\parallel}} \right)$$

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7a

Fig. 7 b