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NO-SCALE SUPERGRAVITY MODELS WITH A PLANCK MASS GRAVITINO

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A B S T R A C T

We construct phenomenological $N = 1$ supergravity models in which the gravitino may be fixed to have a mass $O(10^{19}$ GeV), whilst the weak interaction mass scale is determined by radiative corrections to be hierarchically smaller. Weak gauge symmetry breaking is driven by dynamically determined gaugino masses, and all the SUSY partners of standard model particles have masses $O(100)$ GeV.

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Supersymmetry (SUSY) is currently of phenomenological interest because of its ability to stabilize the hierarchical ratio $m_W/m_P \ll 1$ ¹⁾. Some time ago, it was pointed out in the context of global SUSY models^{2,3)} that the hierarchy could be stabilized even if the SUSY-breaking scale $m_S \gg m_W$. It was, however, soon realized^{4,5)} that supergravity effects were no longer negligible if $m_S \approx \sqrt{m_W m_P}$. For example, minimally coupled Higgses and scalar partners of quarks and leptons in general⁵⁾ acquire masses $m_0 = O(m_{3/2}) = O(m_S^2/m_P) = O(m_W)$. Models could be constructed in which $m_0 \ll m_{3/2}$ at the tree level⁶⁾, but radiative corrections nevertheless tended to generate non-zero scalar masses at the loop level⁶⁾. For this reason, all previous phenomenological $N = 1$ supergravity model-building has incorporated a gravitino with mass $m_{3/2} = O(m_W)$ ¹⁾. In many models m_W has been put by hand into the theory, but recently it has been realized how one may determine m_W and $m_{3/2}$ dynamically to be $m_P \exp[-O(1)/\alpha_t]$, where $\alpha_t \equiv g_{\text{H}\bar{t}t}^2/4\pi$ -- the so-called no-scale models⁷⁻⁹⁾.

A gravitino mass of order m_W may be a cosmological embarrassment in all these phenomenological $N = 1$ supergravity models¹⁰⁾. Such a light gravitino is sufficiently stable to live beyond the epoch of primordial nucleosynthesis, but its mass density invalidates conventional calculations of light element abundances, and the gravitinos' subsequent decays disrupt light nuclei and/or distort the microwave background radiation. To avoid these problems, it was proposed¹¹⁾ that one suppress the primordial gravitino abundance by an early inflationary epoch, and it was suggested¹¹⁾ that the gravitinos subsequently regenerated would be insufficient to perturb nucleosynthesis. Unfortunately, initial estimates of the number of regenerated gravitinos were too low¹²⁾, and now it seems¹³⁾ that sufficiently few gravitinos are produced only if the reheating temperature after inflation is less than about 10^{10} GeV. Such a low reheating temperature makes it difficult to achieve cosmological baryosynthesis¹⁴⁾.

In this paper we show how these difficulties can be avoided by constructing a phenomenological $N = 1$ supergravity model in which the gravitino mass can be fixed to be $O(m_P)$, whilst the weak interaction mass scale and global SUSY breaking

spin-zero and gaugino masses are determined dynamically to be hierarchically smaller: $O(100 \text{ GeV})$. This scenario exploits the non-compact $SU(n,1)/SU(n) \times U(1)$ coset structure introduced in our previous paper⁹⁾, which gives masses and couplings for chiral superfields that are globally supersymmetric. The SUSY breaking in the observable sector emanates from gaugino masses, which are generated by non-minimal kinetic terms for the gauge superfields when local SUSY is broken^{9,15)}. In the model we presented here, the gaugino to gravitino mass ratio is dynamically determined by radiative corrections, and the gravitino mass itself can be fixed to be $O(m_P)$. Despite this large gravitino mass, the sparticle masses are all $O(m_W)$ as in conventional phenomenological supergravity models¹⁾.

Our starting point is the class of $SU(n,1)/SU(n) \times U(1)$ non-compact coset space models introduced in Ref. 9. These models have a Kähler potential

$$G = -3 \log \left(c + z + z^\dagger - \frac{y^i y_i^\dagger}{3} \right) + (F(y^i) + \text{h.c.}) \quad (1)$$

$i=1, \dots, n-1$

where z is a complex gauge singlet "hidden sector" field, and the y^i : $i = 1, \dots, n-1$ are the observable complex chiral fields (squarks, sleptons, and Higgses: $n = 50$ for the SUSY standard model). It was shown in Ref. 9 that the supertrace of the mass squared matrix for the gravitino, the gauge fields and the chiral fields in (1) has the suggestive value

$$\text{Str } M^2 = -4m_{3/2}^2 - 16M_{\tilde{g}}^2 - 6M_{\tilde{W}}^2 - 2M_{\tilde{B}}^2 \quad (2)$$

We verified explicitly that indeed all the Standard Model particles and their chiral spartners have equal masses even in the presence of a non-trivial superpotential $F(y^i)$ for the observable sector, that the real u and imaginary v components of the spin-zero z field have zero mass, and that all the dimension ≤ 4 terms in the Lagrangian generated by the Kähler potential (1) are explicitly globally supersymmetric⁹⁾. The masslessness of the u and v components of z tells us that at the tree level the effective potential is flat in the complex z plane, and hence that the gravitino mass $m_{3/2} = e^{G/2}$ is not determined at the tree level. This flatness

is guaranteed by invariance under an $SU(1,1)$ subgroup¹⁶⁾ of $SU(n,1)$ which is broken down to $U(1)$ by the gravitino mass⁸⁾. This breaking of $SU(1,1)$ means that the effective potential is no longer flat when radiative corrections are taken into account. It is just this unflatness of the effective potential which has been exploited in previous models⁷⁻⁹⁾ to generate a dynamically preferred value for the gravitino mass $m_{3/2} = e^{G/2}$.

We have said that the Kähler potential (1) yields a globally supersymmetric low-energy theory for the chiral fields. So where does the necessary global SUSY breaking ($m_{\tilde{q}} > m_q$, $m_{\tilde{\ell}} > m_\ell$, etc.) originate? The answer is in the gauge sector. It is possible¹⁵⁾ in supergravity models to have a non-minimal kinetic term for the vector superfields V_a :

$$h_{ab}(z, y^i) V_a V_b \quad (3)$$

where h_{ab} is an analytic function of the chiral superfields. Assuming for simplicity that $h_{ab} = \delta_{ab} h(z, y^i)$, the non-minimal kinetic term (3) gives a gaugino mass

$$|m_{\tilde{V}}| = \left| e^{G/2} \frac{G_{,z}}{G_{,zz^*}} (\log h)_{,z} \right| \quad (4)$$

We see that while the over-all scale for $m_{\tilde{V}}$ is set by $m_{3/2} = e^{G/2}$, the combination of field variables which determines the gaugino mass (3) is different, and it could well be that $m_{\tilde{V}} \ll m_{3/2}$. As an example, let us consider the most economical case where the brunt of the gaugino and gravitino mass determination is borne by the singlet complex field z . Using

$$\hat{G} = -3 \log(c + z + z^*) \quad , \quad \hat{h}_{ab} = \delta_{ab} (1 + \phi(z)) \quad (5)$$

we get:

$$\left. \begin{aligned} m_{3/2} &= e^{\hat{G}/2} = |c + z + z^*|^{-3/2} \\ m_{\tilde{V}} &= |e^{\hat{G}/2} \frac{\hat{G}_{,z}}{\hat{G}_{,zz^*}} \frac{\phi_{,z}}{1+\phi}| \end{aligned} \right\} \quad (6)$$

To obtain an explicit expression for $m_{\tilde{V}}$, let us take

$$\phi(z) = k e^{i\alpha z} \quad (7)$$

and suppose that some as yet unspecified dynamics fixes $\text{Re } z = 0$. This fixing of u may be blamed on the theory of everything (TOE) which comes into play at the Planck scale. Later we will provide a toy model which fixes $u = 0$ in this way.

In order for the QCD vacuum angle θ to vanish, we must require k real in Eq. (7).

We obtain from Eqs. (6) and (7)

$$m_{3/2} = c^{-3/2} = O(m_P) \quad (8a)$$

$$m_{\tilde{V}} = \sqrt{3} i m_{3/2} k d \left[\frac{e^{-d\phi}}{1 + k e^{-d\phi}} \right] \quad (8b)$$

The relative phase between the gravitino and gaugino masses is irrelevant, since all the soft SUSY-breaking terms induced at low energies $\ll m_P$ are proportional to $m_{\tilde{V}}$. Therefore, we can rotate away the imaginary phase in $m_{\tilde{V}}$ (8b), and thus the dipole moment of the neutron is given by the SUSY Kobayashi-Maskawa model¹⁷⁾. The value of $m_{3/2}$ by itself plays no role in the low-energy spectrum. The massive gravitino decouples from physics at energy scales $\ll m_P$, its primordial abundance is suppressed by inflation, and it is not subsequently regenerated because the reheating temperature $T_R \ll m_{3/2} = O(m_P)$.

To have a hierarchically low weak interaction scale, we follow the philosophy of Refs. 7-9, using radiative corrections to fix the scale of soft SUSY-breaking and m_W to be $O(\mu_0)$ where μ_0 is the dimensional transmutation scale at which $\det m_H^2$

evolves through zero to negative values, thereby triggering weak gauge-symmetry breaking¹⁸⁾. The calculations are identical to those in Ref. 9: all that changes slightly is the interpretation. Whereas previously we assumed

$$m_{\tilde{V}} = \xi m_{3/2} \quad (9)$$

and assumed ξ fixed^{*)} while $m_{3/2}$ has to be dynamically determined, now we assume $m_{3/2}$ to be fixed and regard the equations as determining dynamically ξ :

$$\xi = \sqrt{3} k d \left[\frac{e^{-d\tau}}{1 + k e^{-d\tau}} \right] \quad (10)$$

One possible difference with the analysis of Ref. 9 is that whereas there in constructing a no-scale GUT we required zero SUSY-breaking masses for the heavy fields, here, if one does not try to go beyond constructing a SUSY $SU(3) \times SU(2) \times U(1)$ model, we do not need to require $m_{\tilde{Q}} = m_{\tilde{X}} = m_H = 0$ at the high energy renormalization scale $\mu = O(m_P)$. Instead, we can allow ourselves to start with $m_{\tilde{Q}}, m_{\tilde{X}}, m_H = O(m_{\tilde{V}})$, in which case the low-energy spectrum can resemble more closely that of Ref. 7 than that of Ref. 9.

We now flesh out the ideas introduced above with some explicit calculations in a model with u fixed to be $u_0 = 0$. We take a Kähler potential which generalizes slightly our starting point (1):

$$G = -3 \log \left(f(z, z^*) - \frac{y^i y_i^*}{3} \right) + [F(y^i) + h.c.] \equiv \tilde{G} + F + F^\dagger \quad (11)$$

whose corresponding tree-level effective scalar potential may be written as

$$V = 9 e^{\frac{2}{3} G} (G^{-1})^i_j \left[\partial_i \partial^j e^{-\frac{1}{3} G} + \frac{n-1}{3n} G_i^j e^{-\frac{1}{3} G} \right] + D\text{-terms} \quad (12)$$

$i, j = 0, 1, 2, \dots, n-1$

where n is the total number of complex chiral fields. This can be written as:

*) In general, the ξ for the $SU(3)$, $SU(2)$, and $U(1)$ gauginos could be different as in Ref. 9, but here we assume for simplicity (5) that the ξ 's are equal.

$$\begin{aligned}
 V = & e^{\bar{G}+F+F^\dagger} \times \\
 & \left\{ \frac{3 e^{-\frac{1}{3}\bar{G}} f_{,zz^\dagger}}{f_{,z} f_{,z^\dagger} - f f_{,zz^\dagger}} + e^{-\frac{1}{3}\bar{G}} \left(\frac{f_{,z} f_{,z^\dagger} - e^{-\frac{1}{3}\bar{G}} f_{,zz^\dagger}}{f_{,z} f_{,z^\dagger} - f f_{,zz^\dagger}} \right) F_{,y^i} F_{,y^i}^\dagger \right. \\
 & \left. - \left(\frac{e^{-\frac{1}{3}\bar{G}} f_{,zz^\dagger}}{f_{,z} f_{,z^\dagger} - f f_{,zz^\dagger}} \right) (y_i^\dagger F_{,y_i} + y^i F_{,y^i}) \right\} + \text{D-terms}
 \end{aligned} \tag{13}$$

which collapses to

$$V = F_{,y^i} F_{,y^i}^\dagger e^{F+F^\dagger} + \text{D-terms} \tag{14}$$

if

$$f_{,zz^\dagger} = 0 \tag{15}$$

and we transform to the normalized⁹⁾ field variables

$$\hat{y}^i \equiv e^{\frac{1}{6}\bar{G}} y^i \tag{16}$$

Thus, we recover a globally SUSY low-energy theory for the chiral fields, but with a Planck mass gravitino, if we ensure that condition (15) holds at $u = u_0$. This is easily arranged by choosing

$$f(z, z^\dagger) = c + z + z^\dagger + b(z + z^\dagger)^m : m > 2 \tag{17}$$

As far as the low-energy spectrum is concerned, if we choose $m_{\tilde{q}} = m_{\tilde{\ell}} = m_H = 0$ at some renormalization scale $\mu \simeq O(m_p)$, then we simply reproduce the results of Ref. 9. There we allowed for the possibility of different initial masses for the SU(3), SU(2), and U(1) gauginos; here we have assumed (4), for simplicity, equal masses $h_{ab} \propto \delta_{ab}$. With this simplifying assumption we find, using the standard renormalization group analysis, that:

$$m_t = 44 \text{ GeV} \quad (18)$$

In general, the values of m_t and of other physical mass parameters depend on the values of the quantities $a_{L,Q,U,A,D}$ denoting the initial magnitudes of effective SUSY-breaking parameters in units of $m_{\tilde{V}}$. Thus we take $m_L^2 = a_L m_{\tilde{V}}^2$, $m_Q^2 = a_Q m_{\tilde{V}}^2$, etc., and we allow for a SUSY-breaking term

$$AF_3(y^i) : A = a_A m_{\tilde{V}} \quad (19)$$

in the effective scalar potential. Using the standard renormalization group analysis, we find

$$m_t \simeq 460 \text{ GeV} \times \left\{ \frac{0.48 + a_L}{3(17.7 + (a_Q + a_U - a_L) + 0.77a_A^2 + 6.15a_A)} \right\}^{1/2} \quad (20a)$$

$$m_{\tilde{g}} \simeq 440 \text{ GeV} \times \frac{1}{\sqrt{1+2.2a_L}}, \quad m_{\tilde{W}} \simeq \frac{1}{3} m_{\tilde{g}}, \quad m_{\tilde{B}} \simeq \frac{1}{6} m_{\tilde{g}} \quad (20b)$$

$$m_{\tilde{e}, \tilde{\mu}, \tilde{\tau}_L}^2 = (a_L + 0.46) m_{\tilde{g}}^2 + \dots; \quad m_{\tilde{e}, \tilde{\mu}, \tilde{\tau}_R}^2 = (a_U + 0.16) m_{\tilde{g}}^2 + \dots \quad (20c)$$

$$m_{\tilde{d}_L}^2 = (a_Q + 3.6) m_{\tilde{g}}^2 + \dots, \quad m_{\tilde{u}_R}^2 = (a_U + 3.3) m_{\tilde{g}}^2 + \dots, \quad m_{\tilde{d}_R}^2 = (a_D + 3.2) m_{\tilde{g}}^2 + \dots \quad (20d)$$

$$m_{\tilde{t}_{1,2}} = \text{eigenvalues of} \begin{pmatrix} (a_Q + 3.0) m_{\tilde{g}}^2 + m_t^2 + \dots & m_{\tilde{g}} m_t A \\ m_{\tilde{g}} m_t A & (a_U + 2.9) m_{\tilde{g}}^2 + m_t^2 + \dots \end{pmatrix} \quad (20e)$$

$$A = 0.88 (a_Q + 4.0) \quad (20f)$$

where the dots in Eqs. (20c-f) represent the D-term contributions to the physical spectrum. In order to avoid colour-breaking minima, we must satisfy the bound¹⁹⁾:

$$A^2 \leq 3 \left(\frac{m_{Q_3}^2 + m_{U_3}^2}{m_{\tilde{g}}^2} \right)_{\mu_0} = 3(5.5 + a_Q + a_U - a_L) \quad (21)$$

In the case where $a_Q = a_V = a_L = 1$, this bound implies $A \leq 4.42$ or $a_A \leq 1.02$.

If we take, for example, $a_L = a_Q = a_U = a_A = a_D = 1$, then we find

$$\left. \begin{aligned} m_{\tilde{t}} &= 634 \text{ GeV}, \quad m_{\tilde{g}} = 240 \text{ GeV}, \quad m_{\tilde{\delta}} = 49 \text{ GeV} \\ m_{\tilde{e}, \tilde{\mu}, \tilde{\tau}_{L,R}} &= 290, 260 \text{ GeV} \\ m_{\tilde{\tau}_L, \tilde{u}_R, \tilde{d}_R} &= 520, 500, 490 \text{ GeV} \\ m_{\tilde{e}_1, \tilde{e}_2} &= 560, 420 \text{ GeV} \end{aligned} \right\} \quad (22)$$

The masses in this spectrum would be reduced if one included mixing between the two light Higgs doublets, which we have neglected in the above example.

We therefore conclude that it is easy to construct a variety of satisfactory phenomenological models with a gravitino of fixed mass $O(m_P)$, and in which the low-energy soft SUSY breaking and weak gauge-symmetry breaking are driven by a gaugino mass dynamically determined to be

$$m_{\tilde{V}} = m_P \exp \left[-\frac{O(1)}{\alpha_t} \right] \quad (23)$$

In addition to their interest for particle physics, these models have certain cosmological advantages which were mentioned earlier in this paper. We plan to return, in another paper¹⁴⁾, to these aspects of no-scale models with a Planck mass gravitino.

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