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VIA THE VACUUM BIREFRINGENCE INDUCED BY A MAGNETIC FIELD:
STATUS OF THE EXPERIMENT

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1. INTRODUCTION

The light-by-light scattering is a process which cannot be described by classical electrodynamics, because of the linear structure of the Maxwell equations. The linearity of the equations is a consequence of the fact that electromagnetic waves do not possess any electromagnetic structure, such as electric charge, magnetic moment, etc. This is not so in the case of gravity waves, for example, where the "charge role" is played by the energy-momentum density tensor, which is carried also by the wave itself; and this is why Einstein's equations are intrinsically non-linear.

In the framework of the theory of quantum electrodynamics (QED), the photon remains without electromagnetic structure, which means that photons cannot scatter directly. However, they can scatter via the interaction with virtual charged pairs, i.e. because of the vacuum polarization process.

In terms of Feynman graphs, the diagrams describing the lowest order contribution to the photon-photon scattering are shown in Fig. 1, for virtual electron-positron pairs. The amplitude corresponding to such Feynman diagrams is the so-called vacuum polarization tensor of fourth rank $G_{\mu\nu\rho\sigma}(1234) = G_{\mu\nu\rho\sigma}(k_1, k_2, k_3, k_4)$.

Fig. 1. Three of the lowest order Feynman graphs contributing to the photon-photon interaction (the other three differ only in the arrow direction in the closed loop).

The general structure of the tensor $G_{\mu\nu\rho\sigma}$ has been studied by Karplus and Neuman¹ and, more recently, also by De Tollis and collaborators² using Mandelstam variables and double-dispersion techniques. On a general basis, by using only relativistic invariance, gauge invariance, and symmetry properties, Karplus and Neuman showed that the tensor $G_{\mu\nu\rho\sigma}$ can be written in terms of only five independent, scalar amplitudes A^{2143} , A^{2341} , A^{2111} , A^{2121} , and A^{2311}

$$G_{\mu\nu\rho\sigma}(1234) = \sum \{ A^{2143}(1234) I_{\mu\nu\rho\sigma}^{(1)}(1234) + A^{2341}(1234) I_{\mu\nu\rho\sigma}^{(2)}(1234) + A^{2111}(1234) I_{\mu\nu\rho\sigma}^{(3)}(1234) + A^{2121}(1234) I_{\mu\nu\rho\sigma}^{(4)}(1234) + A^{2311}(1234) I_{\mu\nu\rho\sigma}^{(5)}(1234) \} , \quad (1)$$

where the sum Σ refers to simultaneous permutations of the four arguments k_1, k_2, k_3, k_4 and of the tensor indices, and we have put

$$I_{\mu\nu\rho\sigma}^{(1)} = (1/32) T_{\mu\alpha\beta}(k_1) T_{\nu\beta\alpha}(k_2) T_{\rho\gamma\delta}(k_3) T_{\sigma\delta\gamma}(k_4) ,$$

$$I_{\mu\nu\rho\sigma}^{(2)} = (1/8) T_{\mu\alpha\beta}(k_1) T_{\nu\beta\gamma}(k_2) T_{\rho\gamma\delta}(k_3) T_{\sigma\delta\alpha}(k_4) ,$$

$$I_{\mu\nu\rho\sigma}^{(3)} = - [1/4(k_3 \cdot k_4)] T_{\mu\alpha\beta}(k_1) T_{\nu\beta\alpha}(k_2) k_{1\gamma} T_{\rho\gamma\delta}(k_3) T_{\sigma\delta\epsilon}(k_4) k_{1\epsilon} ,$$

$$I_{\mu\nu\rho\sigma}^{(4)} = - [1/4(k_3 \cdot k_4)] T_{\mu\alpha\beta}(k_1) T_{\nu\beta\alpha}(k_2) k_{2\gamma} T_{\rho\gamma\delta}(k_3) T_{\sigma\delta\epsilon}(k_4) k_{1\epsilon} ,$$

$$I_{\mu\nu\rho\sigma}^{(5)} = [1/3(k_2 \cdot k_4)] k_{2\alpha} T_{\sigma\alpha\beta}(k_4) T_{\mu\beta\gamma}(k_1) \\ [T_{\nu\gamma\delta}(k_2) T_{\rho\delta\epsilon}(k_3) - T_{\rho\gamma\delta}(k_3) T_{\nu\delta\epsilon}(k_2)] k_{1\epsilon} ,$$

the tensor $T_{\alpha\beta\gamma}(k)$ being defined as

$$T_{\alpha\beta\gamma}(k) = \delta_{\alpha\beta} k_\gamma - \delta_{\alpha\gamma} k_\beta . \quad (2)$$

The dynamics of the process is only contained in the five amplitudes A^{2143} , A^{2341} , A^{2111} , A^{2121} , and A^{2311} . Since in the low-energy limit the last three amplitudes vanish, only A^{2143} and A^{2341} contribute to the photon-photon scattering process far from the pair-production threshold. In this limit ($\omega \ll mc^2$) the photon-photon cross-section due to spinor (σ_{sp}) and scalar (σ_{sc}) virtual charged pairs reads

$$\sigma_{sp} = (973/10125\pi)\alpha^4 (\hbar/mc)^2 (\omega/mc^2)^6 , \\ \sigma_{sc} = (119/10125\pi)\alpha^4 (\hbar/mc)^2 (\omega/mc^2)^6 , \quad (3)$$

where ω is the photon energy in the centre-of-mass reference frame and m is the mass of the virtual charged particle. For visible light ($\omega = 2.5$ eV) we obtain

$$\sigma_e \approx 2.0 \times 10^{-63} \text{ cm}^2 , \quad \sigma_\pi \approx 6.4 \times 10^{-84} \text{ cm}^2 . \quad (4)$$

Below pair-production threshold, the photon-photon scattering can be described without using the virtual pair field by only modifying the linear structure of Maxwell equations. Euler, Heisenberg and Weisskopf^{3,4} have shown that, for slowly varying fields*

$$(\hbar/mc) |\text{grad}(\vec{A})| \ll |\vec{A}| , \quad (\hbar/mc^2) |\partial\vec{A}/\partial t| \ll |\vec{A}| , \quad (5)$$

*A stands for the electric (E) or magnetic (B) field amplitude.

and far from the critical value

$$|\vec{A}_{cr}| = m_e^2 c^3 / e \hbar , \quad (6)$$

i.e.

$$|\vec{E}_{cr}| \approx 1.3 \times 10^{18} \text{ V/m} , \quad |\vec{B}_{cr}| \approx 4.5 \times 10^{13} \text{ G} , \quad (7)$$

the real part of the photon-photon scattering amplitude can be accounted for by introducing in the free electromagnetic Lagrangian suitable higher order terms in the two relativistic invariants

$$F = E^2 - B^2 , \quad G = \vec{E} \cdot \vec{B} . \quad (8)$$

If the only contribution comes from electron-positron virtual pairs, then the e.m. field is described (to the lowest order) by the Lagrangian

$$8\pi \mathcal{L} = F + 2R [F^2 + 7G^2] , \quad (9)$$

where

$$R = (\alpha^2 / 90\pi) \hbar^3 / mc^2 . \quad (10)$$

According to Schwinger⁵, if the virtual pair is made of spinless particles (e.g. pions), the Lagrangian becomes

$$8\pi \mathcal{L} = F + (R/8) [7F^2 + 4G^2] . \quad (11)$$

The constant R, in the two above-mentioned cases, takes the values

$$R_e \approx 1.3 \times 10^{-32} \text{ cm}^3/\text{erg} , \quad R_\pi \approx 2.2 \times 10^{-42} \text{ cm}^3/\text{erg} . \quad (12)$$

The highest continuous electromagnetic field amplitudes which can be produced in vacuum are

$$E_{max} \approx 3 \times 10^2 \text{ sV/cm} , \quad B_{max} \approx 10^5 \text{ G} . \quad (13)$$

They correspond to e.m. energy densities of the order of

$$u_{E_{max}} \approx 10^4 \text{ erg/cm}^3 , \quad u_{B_{max}} \approx 10^9 \text{ erg/cm}^3 . \quad (14)$$

They are to be multiplied by R to obtain the order of magnitude of the linearity violation of Maxwell's equations that one can get in the laboratory*. It is then obvious why Jauch and Rohrlich⁷ affirm:

"The non-linear corrections to the linear classical Maxwell theory are extremely small, so that the basic principle of superposition is not violated to an observable degree".

2. INDIRECT OBSERVATIONS

The photon-photon interaction contributes, in general, to the higher order radiative corrections of any electromagnetic process, but it turns out that these corrections generally give rise to small contributions to the main process, which would be present also in the absence of γ - γ scattering.

In some sense, the situation is similar to the one found in the detection of gravitational waves. The observation of the period time-derivative in the binary pulsar PSW1913+16 shows⁸ a power loss of the system which fits very well with the one due to the emission of gravity waves. However this finding, although of primary relevance, constitutes effectively an indirect observation of gravity wave emission.

The two more relevant experiments that I will discuss in connection with the indirect observation of the γ - γ scattering are the g_{μ} -2 experiment^{9,10} and the Delbruck scattering¹¹.

2.1 g_{μ} -2 Experiment

The g_{μ} -2 experiment is one of the most stringent and precise tests of QED. It is, therefore, not surprising that it is able to say something also concerning the photon-photon interaction. Accor-

*Near neutron stars, where magnetic fields up to 10^{12} G could be present, the QED non-linear corrections to Maxwell's equations become very important⁶ and they explain the depolarization observed in the X-rays emitted near the surface of some pulsars.

ding to Ref. 9, the value of the muon anomaly $a = (g_\mu - 2)/2$ experimentally found so far is

$$a = (1165924 \pm 8.5) \times 10^{-9} . \quad (15)$$

By assuming that the muon is a structureless particle described by a Dirac field, in the framework of QED, the radiative corrections¹⁰ to the fundamental Feynman graph shown in Fig. 2a are responsible for an anomaly

$$\begin{aligned} a_{\text{QED}} &= 0.5(\alpha/\pi) + 0.76578223(\alpha/\pi)^2 + (24.452 \pm 0.056)(\alpha/\pi)^3 \\ &\quad + (135 \pm 63)(\alpha/\pi)^4 + (420 \pm 30)(\alpha/\pi)^5 + \dots \\ &= (1165852.0 \pm 1.9) \times 10^{-9} . \end{aligned} \quad (16)$$

Vacuum polarization terms involving hadrons add a contribution of

$$a_{\text{hadr}} = (66.7 \pm 8.1) \times 10^{-9} , \quad (17)$$

and the virtual emission of Weinberg-Salam weak bosons gives

$$a_{\text{weak}} = (2.1 \pm 0.2) \times 10^{-9} . \quad (18)$$

Therefore, the theoretical value for the muon anomaly amounts to

$$a_{\text{th}} = (1165921 \pm 8.3) \times 10^{-9} , \quad (19)$$

Fig. 2. Feynman graphs concerning the $g_\mu - 2$; a) fundamental diagram; b) three of the lowest order corrections¹¹ to the muon anomaly, due to the photon-photon interaction (the other three are obtained by reversing the arrow direction in the closed loop).

the uncertainty being essentially due to the calculations in the hadron sector. The agreement between values (15) and (19), i.e. between QED and experiment is very striking and is the reason why the experiment is one of the most important confirmations of QED.

The photon-photon interaction contributes (to the lowest order) to the anomaly a_{QED} via the Feynman graphs shown in Fig. 2b. According to Ref. 10 the photon-photon contribution is

$$a_{\gamma\gamma} = 21.69 \pm 0.05 (\alpha/\pi)^3 = (271.8 \pm 0.6) \times 10^{-9} . \quad (20)$$

As a consequence, given the errors quoted in Eqs. (15) and (19), the $g_{\mu}-2$ experiment tests the photon-photon interaction at the 5% level, besides the fact that its contribution is only 2.3×10^{-4} of the whole anomaly.

This test must, however, be considered "an indirect observation" of $\gamma-\gamma$ interaction since it is based on an interpretation of the experimental result which requires the use of QED and some hypothesis on the muon interactions, and on the absence of any other mechanism or particle able to influence the anomaly calculations.

2.2 Delbruck Scattering

The Delbruck scattering¹¹⁻¹³ is a radiative correction to the Compton scattering of a photon from a nucleus (See Feynman diagrams shown in Fig. 3a). In the Thomson region, the nuclear field can be assumed to be a static Coulomb field. The scattering is, in this case, elastic and is sometimes referred to as "potential scattering of light" (see Fig. 3b).

According to Refs. 2 and 12, the two differential cross-sections for no photon spin-flip (σ_{++}) and photon spin-flip (σ_{+-}) read, respectively,

$$\begin{aligned} d\sigma_{++}/d\Omega &= (Z\alpha)^4 r_0^2 |M_{++}|^2 \\ &= (Z\alpha)^4 r_0^2 \{ (73/72) (1/32) (\omega/m_e c^2)^2 \cos^2 \theta/2 \}^2 \\ d\sigma_{+-}/d\Omega &= (Z\alpha)^4 r_0^2 |M_{+-}|^2 \\ &= (Z\alpha)^4 r_0^2 \{ (5/8) (1/32) (\omega/m_e c^2)^2 \sin^2 \theta/2 \}^2 , \end{aligned} \quad (21)$$

Fig. 3. Delbruck scattering Feynman graphs; a) one of the diagrams describing the Delbruck scattering (the others are obtained by permutations of the photon legs); b) Feynman diagram concerning the electric potential scattering of light.

in both cases a factor $Z^4\alpha^2$ higher than in the pure $\gamma\text{-}\gamma$ case.

In the forward direction, the scattering amplitude is proportional to the second power of the photon energy ω ($\omega \ll m_e c^2$). As a consequence, in the presence of a macroscopic electric field in vacuum, one should expect¹⁴ an index of refraction of vacuum such that

$$n - 1 \propto 1/\omega^2 M(\theta=0) , \quad (22)$$

i.e. independent from the photon energy. This result is in fact what the Lagrangian (9) predicts for an electric field. This is also what does occur for the magnetically induced vacuum birefringence (see Section 3), because of the symmetry in E and B of Lagrangian (9).

Above pair-production threshold there is, of course, a non-zero imaginary part in the scattering amplitude, which is related, via the optical theorem, to the total cross-section for real-pair creation. However, only the real part of the Delbruck scattering is connected to virtual-pair creation, i.e. to the vacuum polarization phenomenon. It is, therefore, the experimental observation of this real part that should be proved in order to confirm in this way the existence of a photon-photon interaction.

In general, when a photon scatters on a nucleus, four coherent processes take place:

- i) Thomson scattering;
- ii) Delbruck scattering;
- iii) Rayleigh scattering;
- iv) Nuclear resonance scattering.

The four amplitudes add together coherently, and this is why it is quite difficult (apart from when $\omega \gg m_e c^2$) to prove the presence of a Delbruck real amplitude.

The best evidence of the phenomenon, to my knowledge, has been obtained by Kahane et al.¹⁵ by using ^{181}Ta ($Z = 73$) and photons having an energy of 7.9 and 9.0 MeV. Figure 4 is taken from Ref. 15 and shows the comparison between theoretical calculations and experimental data. On this basis, one can conclude that the real amplitude of the photon scattering from a Coulomb potential has been tested at the 20% level.

Fig. 4. Experimental results (taken from Ref. 15) concerning the Delbruck scattering on tantalum: a) 7.9 MeV of photon energy; b) 9.0 MeV of photon energy.

3. DIRECT OBSERVATION BY VACUUM BIREFRINGENCE DETECTION

The idea of the experiment^{16,17} is to observe, by optical means, the birefringence induced in vacuum by a magnetic field via the electron-positron quantized field. As a matter of fact, from the Lagrangian (9), one obtains that the vacuum, in the presence of a magnetic (electric) field, behaves, in some ways, as a material medium and becomes polarized.

The electric and magnetic susceptibility tensors ϵ_{ij} and μ_{ij} defined, in terms of the auxiliary fields

$$\vec{D} = 4\pi \partial \mathcal{L} / \partial \vec{E} \quad \text{and} \quad \vec{H} = -4\pi \partial \mathcal{L} / \partial \vec{B}, \quad (23)$$

as

$$D_i = \epsilon_{ij}(E, B) E_j \quad \text{and} \quad B_i = \mu_{ij}(E, B) H_j, \quad (24)$$

read

$$\epsilon_{ij} = \delta_{ij} + 2R[2(E^2 - B^2)\delta_{ij} + B_i B_j], \quad (26)$$

$$\mu_{ij} = \delta_{ij} - 2R[2(E^2 - B^2)\delta_{ij} - E_i E_j]. \quad (27)$$

The structure of ϵ_{ij} and μ_{ij} shows that in the presence of a static magnetic field, the vacuum is no longer isotropic for the propagation of light. It can be shown¹⁸ that the indices of refraction n_L and n_T of vacuum in a magnetic field B_0 , for light linearly polarized respectively parallel and transverse to the plane defined by $\vec{B}_0$ and the light propagation vector $\vec{k}$, are

$$n_L = 1 + 7R_e B_0^2 \sin^2 \theta, \quad (28)$$

$$n_T = 1 + 4R_e B_0^2 \sin^2 \theta, \quad (29)$$

where θ is the angle between $\vec{B}$ and $\vec{k}$.

Clearly, a measurement of a non-zero vacuum birefringence induced by a magnetic field is a direct, genuine observation of the violation of the linearity of Maxwell's equations, i.e. of the existence of a photon-photon interaction. A measurement of the value

$$\Delta n_{\text{QED}} = n_L - n_T = 3R_e B_0^2 \sin^2 \theta = (\alpha^2/30\pi)(\lambda_e^3/m_e c^2)B_0^2 \sin^2 \theta \quad (30)$$

would show that such a violation is really due to the presence of virtual electron-positron pairs, as affirmed by QED.

It should be noted that if the electron were a spinless particle, according to the Lagrangian (11), one would have obtained

$$n_L = 1 + (R/4)B_0^2 \sin^2 \theta , \quad (31)$$

$$n_T = 1 + (7R/4)B_0^2 \sin^2 \theta ,$$

i.e. a value for the magnetically induced vacuum birefringence of

$$\Delta n_{\text{spinless}} = -(3R/2)B_0^2 \sin^2 \theta , \quad (32)$$

of opposite sign with respect to value (30).

Finally, it may be interesting to note that Born-Infeld's non-linear theory of electrodynamics¹⁹ predicts no vacuum birefringence induced by a magnetic (electric) field, as is pointed out in an earlier paper²⁰.

The first tentative of observing a violation of the linearity of Maxwell's equations by studying the propagation of light in a magnetic field (10 kG) in vacuum is, to my knowledge, that of Watson²¹, performed in 1929. He wanted to measure a possible magnetic moment of the photon which, in a magnetic field, would change its energy and, therefore, its frequency. He tried to observe changes in the fringes pattern of a Fabry-Perot interferometer placed in a transverse magnetic field, and he established an upper limit for the photon magnetic moment of

$$|\mu| < 0.015 \mu_B . \quad (33)$$

More recently, another experimental proposal²² has been published to observe the departure from unity of the index of refraction given by Eq. (28) with a Michelson interferometer having in the two arms modulated transverse magnetic fields. From Eqs. (28) and (29) the signal is 7/3 higher in this case than for the vacuum birefringence expressed by Eq. (30). There are, however, many experimental difficulties that we have considered and that have convinced us that the "birefringent method" is experimentally easier.

3.1 The Method

When light propagates through a transverse magnetic field $\vec{B}_0$ in vacuum, the two linear polarization states, parallel and perpendicular to the $\vec{B}_0$ direction, propagate with different phase velocities. As a consequence, light originally linearly polarized at an angle β with respect to $\vec{B}_0$ emerges from the magnetic field region (of length L) elliptically polarized, with an ellipticity ψ_{QED} such that

$$\psi_{\text{QED}} = \pi \Delta n_{\text{QED}} (L/\lambda) \sin(2\beta) = 3\pi R_e B_0^2 (L/\lambda) \sin(2\beta), \quad (34)$$

where λ is the light wavelength.

In our case, by considering $B_0 = 80$ kG, from Eq. (30) we get

$$\Delta n_{\text{QED}} = 0.25 \times 10^{-21} \quad (35)$$

and therefore we shall have to measure very small ellipticities.

The intensity transmitted by an analyser prism nearly crossed with respect to the original light polarization vector is quadratic in the total light ellipticity ψ (to the lowest order), and reads

$$W = W_0 (\sigma^2 + \psi^2 + \phi^2), \quad (36)$$

where W_0 is the light power prior to the analyser, ϕ is the residual (misalignment) angle between the prism axis and the light polarization vector, and σ^2 is the extinction factor* of the system. In order to obtain a linear dependence of the signal on ψ_{qed} we have introduced an external modulation $\Phi_F(t)$ in the light polarization state. We use a glass Faraday modulator which maintains an extinction factor of the order of 10^{-7} , necessary to approach the ultimate sensitivity represented by the shot-noise limit** (see Section 3.3)

*It measures the quality of the light polarization state of the light beam. Using good crystal polarizer prisms, one can reach $\sigma^2 \approx 10^{-8}$.

**For interferometer experiments, the quantity equivalent to the extinction factor is the fringe contrast, which for Michelson interferometers is of the order of 10^{-2} .

with a light power in the watt region. The Faraday modulator requires an ellipticity-to-rotation conversion which is obtained by reflection on a gold mirror. A very schematic diagram of the apparatus is shown in Fig. 5.

The light intensity after the analyser prism is then

$$W(t) = W_0 \{ \sigma^2(t) + [\psi_{\text{QED}}(t) + \Phi_F(t) + \phi(t)]^2 + \chi(t)^2 \} , \quad (37)$$

where χ is the light ellipticity after reflection on the gold mirror.

The current from a photodiode detector placed after the analyser prism A (see Fig. 5), to the lowest order in ψ , ϕ , and χ , reads

$$i(t) = W_0 \eta \left[\sigma^2 + (\Phi_{0F}^2/2) - (\Phi_{0F}^2/2) \cos 2\omega_F t \right. \\ \left. + 2\Phi_{0F}\psi_0 \sin \omega_F t \sin 2\omega_M t + 2\Phi_{0F}\phi(t) \sin \omega_F t \right] , \quad (38)$$

where η is the photodetector efficiency and

$$\Phi_F(t) = \Phi_{0F} \sin \omega_F t , \quad (39)$$

$$\psi_{\text{QED}}(t) = \psi_0 \sin 2\omega_M t . \quad (40)$$

The useful signal, proportional to $W_0 \eta \Phi_{0F} \psi_0$ is present, buried in noise, at the two frequencies $\omega_F \pm 2\omega_M$, in the presence of a d.c. level proportional to $W_0 \eta (\sigma^2 + \Phi_{0F}^2/2)$ and a pure modulation signal at $2\omega_F$ proportional to $W_0 \eta \Phi_{0F}^2/2$. We have found it extremely important to have the possibility of adjusting the frequency ω_F in order to place the signal in a frequency region with a minimum of disturbances. Moreover, in order to overcome the low-frequency noise on the polarization state described by $\phi(t)$, the signal frequency needs to be as high as possible ($f_M = \omega_M/2\pi > 25$ mHz). According to Eq. (34), the modulation of the signal can be obtained by pulsating the current in the magnet or by rotating the magnetic field itself.

The signal is recovered by Fourier analysis of the photodetector current sampled during the period T .

In test experiments we have proved²³ the possibility of detecting ellipticities smaller than 10^{-10} by integrating the signal over sampling periods of one day.

Fig. 5. Schematic diagram of the apparatus.

The test measurements have also been used to study possible sources of systematic errors. For instance, the stray field from the magnet on the cavity mirrors (see Section 3.2) must be smaller than 1 μG , otherwise the Faraday effect in the dielectric multilayers²⁴ would generate systematic errors of the order of the effect to be measured. Also, the residual pressure in the vacuum pipe inside the magnetic field must be kept below 10^{-8} Torr in order to eliminate the effect due to the magnetically induced gas birefringence^{25,26} (Cotton-Mouton effect).

However, to get an ellipticity ψ_0 of the order of 10^{-11} , the optical path L required in the magnetic field is of the order of several kilometres. The method to achieve this condition is explained in the next section.

3.2 Optical Cavity

To obtain $L \approx$ some kilometres we intend to use an optical delay line^{27,28}, consisting of two concave spherical mirrors, placed at a distance of about 7 m from one another. The central cavity region (6 m) is traversed by a magnetic field with an intensity of $B \approx 80$ kG. In this way, with 500 reflections, i.e. $L = 3$ km, we will get

$$\psi_{\text{QED}} = 0.46 \times 10^{-11} . \quad (41)$$

An optical delay line having these characteristics and a diameter of 7 cm has been realized without magnetic field using interferential spherical mirrors having a reflectivity of 99.5%. The problems studied experimentally were the depolarization of the beam produced by the reflections and the cavity stability, which influences the noise in the polarization state.

The light source used was the same as planned for the final experiment, i.e. a CW argon-ion laser ($\lambda = 514.5$ nm) used at an output power of up to 2 W. The use of laser light is only dictated by the necessity of having the maximum source brilliance, i.e. the max-

imum power light in the minimum beam phase space (no coherence properties are used).

It is easy to show²⁷ that, in the Gauss approximations, a light beam in the cavity hits the mirror surfaces in points disposed on an ellipse, which is centred at the intersection point between the mirror surface and the optical axis defined by the two mirror surface centres. The beam spot positions are given by

$$\begin{aligned}x_n &= A \cos (n\Theta + \xi) , \\y_n &= B \cos (n\Theta + \zeta) ,\end{aligned}\tag{42}$$

with Θ such that

$$\cos \Theta = 1 - d(f_1 + f_2 - d/2)/f_1 f_2 ,\tag{43}$$

where d is the mirror distance, f_1 and f_2 are the two mirror focal lengths, and A , B , ξ , and ζ are constants depending upon the initial conditions. To make better use of the mirror surface, we have introduced an astigmatism in the optical delay line by mechanically deforming one of the two mirrors. Instead of ellipses one then obtains Lissajous patterns which cover almost all the mirror surface, as shown in Fig. 6. The beam enters and comes out from the delay line by the same hole drilled in one of the two mirrors, which guarantees a minimum of ellipticity produced by the reflections.

We have matched the TEM_{00} laser beam to the fundamental mode of the cavity, using a suitable telescope. The mode-matching eliminates spot-size oscillations on the mirrors and, since the constant-phase beam surfaces have the same radius of curvature as the mirrors when the beam is reflected, the spread in the incidence angle is minimized, which reduces the beam depolarization produced by the delay line.

Another important consideration has been to orient properly the mirrors with respect to the light polarization direction. In agreement with what was also found by Bouchiat²⁹, it turns out in

Fig. 6. Photograph of a Lissajous pattern on the cavity entrance mirror.

fact that interferential mirrors present a birefringence axis ($\psi \approx 10^{-4}$ per reflection).

With all these precautions, we have been able to maintain the extinction factor in the 10^{-7} region after up to 600 reflections.

3.3 Sensitivity

Several sources of noise and instabilities determine the sensitivity of the apparatus. We have performed various test measurements to establish the sensitivity we are able to reach. It has been determined by generating a known Faraday rotation (0.67×10^{-8} rad) with an air Faraday cell, and by comparing this with the noise during the measurements. The experimental conditions chosen are

summarized in Table 1 and are essentially those planned for the final experiment.

Table 1. Experimental Conditions

Laser power	~ 2 W
Laser wavelength	514.5 nm
Number of reflections	~ 500
Cavity attenuation	~ 12
Optical path	$\sim 3.5 \times 10^5$ cm
Extinction factor σ^2	$\sim 10^{-7}$
Faraday modulation amplitude ϕ_F	3.0×10^{-4} rad
Experimental sensitivity	6.8×10^{-11} $\sqrt{\text{day}}$

The shot-noise in the photocurrent of the photodiode is given, from Eq. (40), by

$$\langle i_{\text{shot}} \rangle = \sqrt{2e\eta I_0 (\sigma^2 + \phi_{0F}^2/2) \Delta\nu} \approx 3 \times 10^{-14} \sqrt{\Delta\nu} \text{ A.} \quad (44)$$

and would correspond to a sensitivity of

$$\psi_{\text{shot}} = 3.5 \times 10^{-9} / \sqrt{\text{Hz}} = 1.2 \times 10^{-11} \sqrt{\text{day}}. \quad (45)$$

The Johnson noise in the 400 M Ω feedback resistor R on the current-to-voltage photodiode preamplifier is

$$\langle i_{\text{thermal}} \rangle = \sqrt{(4kT/R) \Delta\nu} \approx 0.7 \times 10^{-14} \sqrt{\Delta\nu} \text{ A.} \quad (46)$$

The equivalent input noise of the preamplifiers has been measured to be

$$\langle i_{\text{electr}} \rangle \approx 1.0 \times 10^{-14} \sqrt{\Delta\nu} \text{ A.} \quad (47)$$

Whilst all these noise sources are "white", the laser intensity fluctuations show a spectral density increasing towards 0 Hz. Figure 7a shows the Fourier spectrum of the laser intensity between 0 and 1 Hz taken with a low-frequency spectrum analyser. At 0.050 mHz the laser intensity noise is 35 dB higher than the statistical limit for the same value of d.c. voltage output. The Faraday modulation, acting as a carrier at $f_F = 370$ Hz, shifts the signal to a region in which the excess noise of the laser is only 4 dB higher than the statistical limit. Figure 7b shows the laser light intensity spectrum between 0 and 500 Hz. Above 250 Hz, in agreement with Refs. 23 and 30, we observe

$$\langle i_{\text{laser}} \rangle \approx 0.7 \times 10^{-5} i_{\text{dc}} \sqrt{\Delta v} . \quad (48)$$

Fig. 7. Laser intensity noise, measured after the prism analyser A, with a light power ~ 15 nW. The dashed line represents the shot-noise limit. a) Fourier noise spectrum taken between 0 and 1 Hz. b) Fourier noise spectrum taken between 0 and 500 Hz. The peaks correspond to harmonics of the 50 Hz mains.

The excellent extinction factor of 10^{-7} reached in the apparatus reduces the light power on the detector to such values that the laser noise becomes comparable with the shot noise; in fact we have

$$\langle i_{\text{laser}} \rangle \approx 2.1 \times 10^{-14} \sqrt{\Delta\nu} \text{ A} . \quad (49)$$

The sum of the above-mentioned noise contributions gives

$$\langle i_n \rangle \approx 3.8 \times 10^{-14} \sqrt{\Delta\nu} \text{ A} , \quad (50)$$

only a factor of 1.3 higher than the shot-noise limit.

However, in the test measurements so far performed, the sensitivity obtained above 50 mHz has been

$$\psi_{\text{sens}} = 6.8 \times 10^{-11} \sqrt{\text{day}} , \quad (51)$$

a factor 4.5 worse than the shot-noise limit. This is because the Faraday modulation Φ_F , according to Eq. (40), also acts as a carrier for the fluctuations of the misalignment angle $\phi(t)$ which contribute to the low-frequency noise in the signal. These fluctuations are mainly due to stress birefringence induced by thermal gradients in the optical elements and by mechanical instabilities of the multi-pass optical cavity. To reduce this noise, we are at present mounting another optical cavity, in which much more care has been given to the mechanical mountings. Moreover no windows (see Fig. 8) are used in the polarized "zone", in order to reduce the ellipticity due to thermal gradients in the optics. With this improved cavity we will also study the possibility of reducing its cross-section to 4 cm, in order to provide for the possible future use of a bending magnet developed for 10 TeV proton accelerators. This would possibly lessen the cost problem concerning the magnet.

With the improved apparatus, we hope to reach a sensitivity of

$$\psi_{\text{sens}} \approx 2.5 \times 10^{-11} \sqrt{\text{day}} . \quad (52)$$

In this case, the magnetically induced vacuum birefringence due to the photon-photon interaction could be seen, with a signal-to-noise

Fig. 8. Layout of the experimental apparatus proposed in order to observe the vacuum birefringence induced by a magnetic field.

ratio of one, by taking data for one month, on the hypothesis of a 6 m long, 80 kG magnet, of 500 reflections and of a signal modulation frequency $\omega_M > 25$ mHz.

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