SUPERSYMMETRIC GAUGE THEORIES COUPLED TO NON-MINIMAL
AND NEW MINIMAL SUPERGRAVITY

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A B S T R A C T

The coupling of the various supergravity multiplets to gauge invariant theories is constructed using a chiral basis for curved superspace. It is shown that R invariance is not required for such a construction.

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1. - INTRODUCTION

In a previous paper¹⁾ we have analysed superspace constraints in a systematic way. In this paper we are going to show that a chiral formulation of matter fields, coupled to supergravity can be done for the non minimal²⁾ and new minimal³⁾ supergravity multiplets in very much the same way as it has been done for minimal supergravity⁴⁾.

Our method for constructing these Lagrangians rests very much on a superfield $\bar{\Omega}$, which has already been defined in Ref. 1), chapter 5:

$$\bar{\Omega} = 2(3n+1) \frac{\bar{T}_{\dot{\alpha}} \bar{T}^{\dot{\alpha}}}{\bar{S}} \quad (1.1)$$

Its essential property is

$$D^{\dot{\alpha}} \bar{\Omega} = -2(3n+1) \bar{T}^{\dot{\alpha}} \quad (1.2)$$

$\bar{\Omega}$ is singular in the field $\bar{S}$. Therefore the action constructed with our method will look singular in $\bar{S}$ as well. There is however a way to make this singularity acceptable. We can construct potentials such that S gets a vacuum expectation value. If we expand S around this value, S^{-1} will be meaningful. In a recent paper⁵⁾, Galperin, Ogievetsky and Sokatchev have made a similar observation. There is however a special class of models - these are the R-invariant models - where the singularity of S^{-1} can be absorbed in a redefinition of the fields. This will be carefully discussed in this paper.

Chapter 2 deals with the concept of chiral superfields and the chiral density. In chapter 3 we construct the chiral Lagrangian for

the non minimal and the new minimal supergravity multiplets. Chapter 4 describes the coupling of gauge theories to these supergravity multiplets. Here we analyse the S^{-1} singularities in great detail. The last chapter 5 is devoted to Weyl invariance and the Fayet-Iliopoulos term.

2. - CHIRAL SUPERFIELDS AND CHIRAL DENSITY

The concept of chiral superfields, chiral density and new Θ variables has been very useful for the construction of Lagrangians coupled to minimal supergravity⁶⁾. The same methods can be used for non minimal and new minimal supergravity as well.

We start with chiral superfields, which satisfy the covariant constraint condition

$$\bar{D}^{\dot{\alpha}} \phi = 0 \quad (2.1)$$

Such a superfield contains three component fields which we define as follows:

$$A(x) = \phi \Big|_{\theta=\bar{\theta}=0}$$

$$\chi_{\alpha}(x) = \frac{1}{\sqrt{2}} D_{\alpha} \phi \Big|_{\theta=\bar{\theta}=0} \quad (2.2)$$

$$F(x) = -\frac{1}{4} D^{\alpha} D_{\alpha} \phi \Big|_{\theta=\bar{\theta}=0}$$

The transformation laws of these component fields are found from the transformation law of the superfield Φ :

$$\delta\Phi = - \zeta^A \mathcal{D}_A \Phi \quad (2.3)$$

For gauged supersymmetry transformations, the parameters ζ^A have been given in Ref. 1), equ. (3.14). For what follows we use the $\theta = \bar{\theta} = 0$ components of ζ^A only:

$$\zeta^A \Big| = \left(0, \zeta^\alpha(x), \bar{\zeta}_{\dot{\alpha}}(x) \right) \quad (2.4)$$

To compute the transformation laws of the component fields (2.2) we have to use the solution of the Bianchi identities for the various constraints, and the identification with component fields of the supergravity multiplet. All this has been given in Ref. 1). Note that the supergravity component field $\chi_\alpha = \mathcal{D}_\alpha R \Big|$ of tab. 2 in Ref. 1) does not appear in the following formulas. Therefore it cannot be confused with the component field χ_α of the chiral multiplet.

A consequent application of the relation

$$\begin{aligned} & \left(\mathcal{D}_c \mathcal{D}_B - (-1)^{b \cdot c} \mathcal{D}_B \mathcal{D}_c \right) V^A = \\ & = -T_{cB}{}^D \mathcal{D}_D V^A + (-1)^{d(b+c)} V^D R_{cB D}{}^A \end{aligned} \quad (2.5)$$

leads to the desired result:

$$\delta A = -\sqrt{2} \zeta^\alpha \chi_\alpha$$

$$\begin{aligned} \delta \chi_\alpha = & -\sqrt{2} \zeta_\alpha F - \sqrt{2} i (\delta^a \bar{\zeta})_\alpha \hat{D}_a A \\ & + \frac{1}{2} (n+1) \zeta^\beta (T_\beta \chi_\alpha + T_\alpha \chi_\beta) \\ & + (n-1) \bar{\zeta}_\beta \dot{T}^\beta \chi_\alpha \end{aligned}$$

$$\begin{aligned} \delta F = & \left[\frac{3}{2} (n+1) \zeta^\alpha T_\alpha + 2(n-1) \bar{\zeta}_{\dot{\alpha}} \dot{T}^{\dot{\alpha}} \right] F \\ & + \frac{\sqrt{2}}{8} (n+1) \zeta^\alpha \left[S + \frac{3}{2} (n+1) T^\beta T_\beta \right] \chi_\alpha \quad (2.6) \\ & - \frac{\sqrt{2}}{4} \bar{\zeta}_{\dot{\alpha}} \left[-\frac{2}{3} b^{\alpha\dot{\alpha}} + (5n-\frac{1}{3})(c+id)^{\alpha\dot{\alpha}} \right. \\ & \left. - 4(n+\frac{1}{3})(c-id)^{\alpha\dot{\alpha}} + (n-1)(n-\frac{5}{3}) \bar{T}^{\dot{\alpha}} T^\alpha \right] \chi_\alpha \\ & - i\sqrt{2} (\bar{\zeta} \bar{\delta}^a)^\alpha \hat{D}_a \chi_\alpha \\ & + \frac{i}{2} (n+1) (\bar{\zeta} \bar{\delta}^a)^\alpha T_\alpha \hat{D}_a A \end{aligned}$$

The supercovariant derivatives $\hat{D}_a$ are defined as follows:

$$\begin{aligned}\hat{D}_a A &= e_a^m \left(\partial_m A - \frac{1}{\sqrt{2}} \Psi_m^\beta \chi_\beta \right) \\ \hat{D}_a \chi_\alpha &= e_a^m \left[D_m \chi_\alpha - \frac{1}{\sqrt{2}} \Psi_{m\alpha} F \right. \\ &\quad + \frac{1}{4} (n+1) \Psi_m^\beta (T_\beta \chi_\alpha + T_\alpha \chi_\beta) \\ &\quad \left. + \frac{1}{2} (n-1) \bar{\Psi}_{m\dot{\beta}} \bar{T}^{\dot{\beta}} \chi_\alpha - \frac{i}{\sqrt{2}} (\sigma^b \bar{\Psi}_m)_\alpha \hat{D}_b A \right]\end{aligned}\tag{2.7}$$

The new Θ variables are again introduced such that the expansion coefficients of chiral superfields are the covariant components (2.2):

$$\phi = A(x) + \sqrt{2} \Theta^\alpha \chi_\alpha(x) + \Theta^\alpha \Theta_\alpha F(x)\tag{2.8}$$

The transformation law (2.6) can be reproduced in the following form:

$$\delta\phi = -\eta^M(x, \Theta) \partial_M \phi\tag{2.9}$$

The new transformation parameters

$$\eta^M(x, \Theta) = \eta_{(0)}^M(x) + \Theta^\alpha \eta_{(1)\alpha}^M(x) + \Theta^\alpha \Theta_\alpha \eta_{(2)}^M(x) \quad (2.10)$$

are found to be:

$$\eta_{(0)}^m = 0, \quad \eta_{(0)}^\mu = \zeta^\mu,$$

$$\eta_{(1)}^m{}_\alpha = 2i(\sigma^m \bar{\zeta})_\alpha,$$

$$\begin{aligned} \eta_{(1)}^\mu{}_\alpha = & -\frac{1}{2}(n+1)\zeta^\beta (\delta_\alpha^\mu T_\beta + \delta_\beta^\mu T_\alpha) \\ & - (n-1)\delta_\alpha^\mu \bar{\zeta}_{\dot{\alpha}} \bar{T}^{\dot{\alpha}} - i(\sigma^m \bar{\zeta})_\alpha \psi_m^\mu, \end{aligned}$$

$$\eta_{(2)}^m = -\frac{i}{2}(n+1)(\bar{\zeta} \sigma^m T) + (\bar{\zeta} \sigma^n \sigma^m \bar{\psi}_n),$$

$$\begin{aligned} \eta_{(2)}^\mu = & -\frac{1}{8}(n+1)\zeta^\mu \left[S + \frac{3}{2}(n+1)T^\alpha T_\alpha \right] \\ & + \frac{1}{4}\bar{\zeta}_{\dot{\alpha}} \left[-\frac{2}{3}b^{\mu\dot{\alpha}} + (5n-\frac{1}{3})(c+id)^{\mu\dot{\alpha}} - 4(n+\frac{1}{3})(c-id)^{\mu\dot{\alpha}} \right. \\ & \left. + (n-1)(n-\frac{5}{3})\bar{T}^{\dot{\alpha}} T^\mu \right] + \frac{i}{4}(\bar{\zeta} \sigma^m)^\mu \left[(n+1)\psi_m^\alpha T_\alpha \right. \\ & \left. + 2(n-1)\bar{\psi}_{m\dot{\alpha}} \bar{T}^{\dot{\alpha}} \right] - \frac{1}{2}(\bar{\zeta} \sigma^m \sigma^n \bar{\psi}_m) \psi_n^\mu \\ & - i(\bar{\zeta} \sigma^m)^\alpha \omega_{m\alpha}^\mu. \end{aligned} \quad (2.11)$$

The quantity $\omega_m{}^\mu{}_\alpha$ can be computed from Ref. 1), equ. (3.13).

We generalize the concept of a chiral field to the concept of a chiral density: chiral densities are functions of x and Θ and, by definition, their transformation law is:

$$\begin{aligned}\delta \Delta &= -(-1)^m \partial_M (\eta^M \Delta) \\ \Delta &= a + \sqrt{2} \Theta^\alpha \rho_\alpha + \Theta^\alpha \Theta_\alpha f\end{aligned}\tag{2.12}$$

It is straightforward to compute the transformation law of a , ρ_α and f from (2.12) and (2.11). It is not straightforward however to find functions of the components of the supergravity multiplet that transform in accord with (2.12). The transformation law of the supergravity multiplet has been derived in Ref. 1). We find that the following combinations transform like density components:

$$\begin{aligned}a &= e \left(1 + \bar{W}^{(0)}(x) \right) \\ \sqrt{2} \rho_\alpha &= a \left[\bar{W}^{(1)}_\alpha + \frac{1}{2} (13n-3) T_\alpha + i (\delta^m \bar{\Psi}_m)_\alpha \right] \\ f &= -\frac{a}{4} \left[\bar{W}^{(2)} + \bar{W}^{(1)\alpha} \bar{W}^{(1)}_\alpha + (13n-3) T^\alpha \bar{W}_\alpha^{(1)} \right. \\ &\quad \left. - 2i (\bar{\Psi}_m \delta^m)^\alpha W_\alpha^{(1)} + 2(3n-1) (S + 8n T^\alpha T_\alpha \right. \\ &\quad \left. - 2i (\bar{\Psi}_m \delta^m)^\alpha T_\alpha) + 4 (\bar{\Psi}_m \delta^{mn} \bar{\Psi}_n) \right].\end{aligned}\tag{2.13}$$

Here we have used the following symbols:

$$e = \det e_m^a$$

$$\bar{W}^{(0)} = \bar{\Omega} \Big|_{\theta=\bar{\theta}=0} = 2(3n+1) \frac{\bar{T}_{\dot{\alpha}} \bar{T}^{\dot{\alpha}}}{\bar{S}} \quad (2.14)$$

$$W^{(1)}_{\alpha} = \mathcal{D}_{\alpha} \bar{\Omega} \Big|_{\theta=\bar{\theta}=0}$$

$$W^{(2)} = \mathcal{D}^{\alpha} \mathcal{D}_{\alpha} \bar{\Omega} \Big|_{\theta=\bar{\theta}=0}$$

The corresponding chiral density will be denoted by $\mathcal{E}$. Note that the density multiplet is singular in the field S . In the next section we shall show that it can be used nevertheless to construct meaningful Lagrangians.

3. - CHIRAL SUPERGRAVITY LAGRANGIAN

We first try to formulate an invariant action for the supergravity multiplet. This should be done in terms of the chiral density $\mathcal{E}$ (2.13) and a chiral superfield of dimension one. It is natural and it will turn out to be correct, to try the following Lagrangian:

$$\mathcal{L}_{S.G.} = -\frac{1}{2} \int d^2\Theta \mathcal{E} \bar{S} + \text{h.c.} \quad (3.1)$$

We show that the singular contributions from $\mathcal{E}$ cancel in (3.1). This

is facilitated by splitting ξ into two parts, a singular one and a nonsingular one:

$$\xi = \xi^\circ \gamma \quad (3.2)$$

The nonsingular one, ξ° is supposed to have the determinant of the ordinary vierbein, e as lowest component. This determines the splitting:

$$\begin{aligned} \xi^\circ &= e + \Theta^\alpha e \varrho_\alpha^\circ + \Theta^\alpha \Theta_\alpha e f^\circ \\ \gamma &= e^{\bar{W}^{(0)}} \left[1 + \Theta^\alpha \bar{W}_\alpha^{(1)} - \frac{1}{4} \Theta^\alpha \Theta_\alpha \left(\bar{W}^{(2)} + \bar{W}^{(1)\beta} \bar{W}_\beta^{(1)} \right) \right] = \\ &= e^{\bar{\Omega}} \left| + \Theta^\alpha \partial_\alpha e^{\bar{\Omega}} \right| - \frac{1}{4} \Theta^\alpha \Theta_\alpha \partial^\rho \partial_\rho e^{\bar{\Omega}} \end{aligned} \quad (3.3)$$

$$\varrho_\alpha^\circ = \frac{3n-3}{2} T_\alpha + i (\delta^m \bar{\Psi}_m)_\alpha$$

$$\begin{aligned} f^\circ &= -\frac{1}{2} (3n-1) S - 4n (3n-1) T^\alpha T_\alpha \\ &\quad + i (3n-1) (\bar{\Psi}_m \delta^m)^\alpha T_\alpha - (\bar{\Psi}_m \delta^{mn} \bar{\Psi}_n) \end{aligned}$$

Warning: ξ° and γ are not chiral densities or chiral superfields. We

merely observe that their product reproduces ξ and that all the singular terms are contained in Y .

From (3.3) follows that the product of Y and a chiral superfield ϕ , $\bar{D}_\alpha \phi = 0$ has the following expansion in Θ :

$$\begin{aligned}
 Y^a \phi &= e^{a\bar{\Omega}} \phi \Big|_{\theta=\bar{\theta}=0} + \Theta^\alpha (\mathcal{D}_\alpha e^{a\bar{\Omega}} \phi) \Big|_{\theta=\bar{\theta}=0} \\
 &\quad - \frac{1}{4} \Theta^2 (\mathcal{D}^2 e^{a\bar{\Omega}} \phi) \Big|_{\theta=\bar{\theta}=0}
 \end{aligned} \tag{3.4}$$

(a is an arbitrary number). Equ. (3.4) is an identity in the new variables but again $Y\phi$ is not a superfield, $e^{\bar{\Omega}}\phi$ is not chiral.

However we can use this identity to show that $Y\bar{S}$ is not singular. Replace ϕ by $\bar{S}$, which is chiral. The product $e^{\bar{\Omega}}\bar{S}$ is not singular and $Y\bar{S}$ is not singular either. This shows that the action (3.1) is free of $\bar{S}^{-1}$ singularities.

To prove that (3.1) represents a proper action for the supergravity multiplet, we compute it:

$$\begin{aligned}
\frac{1}{e} \mathcal{L}_{S.G.} = & -\frac{1}{2} \mathcal{R} + \frac{1}{4} \varepsilon^{klmn} (\bar{\Psi}_k \bar{\sigma}_l \Psi_m - \Psi_k \sigma_l \bar{\Psi}_m) \\
& + n S \bar{S} + \frac{1}{3} b_a b^a - \frac{4}{3} (3n+1) (c_a c^a + 3n d_a d^a) \\
& + 2n (T^\alpha \lambda_\alpha + \bar{T}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}) + 4in \sigma_{\alpha\dot{\alpha}}^m (T^\alpha \partial_m \bar{T}^{\dot{\alpha}} + \bar{T}^{\dot{\alpha}} \partial_m T^\alpha) \\
& + 2n (5n-1) (T^\alpha T_\alpha \bar{S} + \bar{T}_{\dot{\alpha}} \bar{T}^{\dot{\alpha}} S) + \frac{4n}{3} T^\alpha T^{\dot{\alpha}} b_{\alpha\dot{\alpha}} \\
& - \frac{4}{3} (3n+1) (4n-1) T^\alpha T^{\dot{\alpha}} c_{\alpha\dot{\alpha}} + \frac{2}{3} (3n+1) (5n-1) (7n-1) T^\alpha T_\alpha \bar{T}_{\dot{\alpha}} \bar{T}^{\dot{\alpha}} \\
& - 2in (\Psi_m \sigma^m \bar{T}) S - 2in (\bar{\Psi}_m \bar{\sigma}^m T) \bar{S} \\
& - \frac{2i}{3} (\Psi_m T - \bar{\Psi}_m \bar{T}) b^m + 4n (3n+1) (\Psi_m T + \bar{\Psi}_m \bar{T}) d^m \\
& + \frac{4i}{3} (3n+1) (\Psi_m T - \bar{\Psi}_m \bar{T}) c^m \\
& - \frac{i}{3} (27n^2 + 4n - 1) (\Psi_m \sigma^m \bar{T} T^2 + \bar{\Psi}_m \bar{\sigma}^m T \bar{T}^2) \\
& + 4n (\Psi_n \sigma^{mn} T) (\bar{T} \bar{\Psi}_n) + 4n (\Psi_m T) (\bar{T} \bar{\sigma}^{mn} \bar{\Psi}_n) \\
& + \frac{2}{e} \partial_m \left[-(3n+1) e d^m - e \Psi_n \sigma^{mn} T + e \bar{\Psi}_n \bar{\sigma}^{mn} \bar{T} \right].
\end{aligned} \tag{3.5}$$

It is interesting to note that this is a proper Lagrangian for

the case $n=0$ as well. In this case it is the Lagrangian for the new minimal supergravity multiplet. As was explained in Ref. 1), we have to define the auxiliary fields as follows:

$$\begin{aligned} X_{\alpha i} &= -\frac{1}{3} b_{\alpha i} + \frac{2}{3} C_{\alpha i} + \frac{2}{3} T_{\alpha} T_i \\ A_m &= e_m^a b_a - i (\psi_m^{\alpha} T_{\alpha} - \bar{\psi}_m^i T_i) \end{aligned} \quad (3.6)$$

The Lagrangian (3.6) reduces to:

$$\begin{aligned} \frac{1}{e} \mathcal{L}_{s.g.} &= -\frac{1}{2} \mathcal{R} + \frac{1}{4} \varepsilon^{klmn} (\bar{\psi}_k \delta_l \psi_{mn} - \psi_k \delta_l \bar{\psi}_{mn}) \\ &\quad - 3 X_a X^a - 2 e_a^m A_m X^a . \\ &\quad (n=0) \end{aligned} \quad (3.7)$$

For the construction of Lagrangians with matter fields coupled to the supergravity multiplet we shall frequently use the chiral projection operator $\bar{\Delta}$, which was introduced in Ref. 1), equ. (2.23):

$$\bar{\Delta} = \bar{\partial}_{\dot{\alpha}} \bar{\partial}^{\dot{\alpha}} - 3(n+1) \bar{T}_{\dot{\alpha}} \bar{\partial}^{\dot{\alpha}} - 2(n+1) \bar{S} \quad (3.8)$$

This operator projects on a chiral field, independent of the tensorial structure of the superfield to which it is applied:

$$\bar{D}_i \bar{\Delta} = 0 \quad (3.9)$$

This operator has another interesting property. We commute the projection operator with $e^{-a\bar{\Omega}}$, a is an arbitrary number:

$$\begin{aligned} [\bar{\Delta}, e^{-a\bar{\Omega}}] &= \\ &= 2a(3n+1) \left(\bar{S} + 2\bar{T}_i \bar{D}^i - 2(n+1)\bar{T}_i \bar{T}^i \right) \end{aligned} \quad (3.10)$$

The right side of this equation is not singular in S . We can rewrite equ. (3.10):

$$\begin{aligned} e^{a\bar{\Omega}} \bar{\Delta} e^{-a\bar{\Omega}} &= \bar{\Delta} \\ &+ e^{a\bar{\Omega}} 2a(3n+1) \left(\bar{S} + 2\bar{T}_i \bar{D}^i - 2(n+1)\bar{T}_i \bar{T}^i \right) \end{aligned} \quad (3.11)$$

The right hand side of this equation is not singular either because $e^{a\bar{\Omega}}\bar{S}$ and $e^{a\bar{\Omega}}\bar{T}_i$ are not singular. We evaluate the right hand side and find:

$$\begin{aligned}
 e^{a\bar{\Omega}} \bar{\Delta} e^{-a\bar{\Omega}} &= \\
 &= \bar{\partial}_i \bar{\partial}^i + (4a(3n+1) - 3(n+1)) \bar{T}_i \delta^i \\
 &\quad + (2a(3n+1) - 2(n+1)) (\bar{S} + 2a(3n+1) \bar{T}_i \bar{T}^i) \equiv \\
 &\equiv X .
 \end{aligned} \tag{3.12}$$

In the next chapter we shall use this equation in an expansion in the new Θ variables. Then it can be written as follows:

$$\begin{aligned}
 \gamma^a \bar{\Delta} e^{-a\bar{\Omega}} U &= X U \Big|_{\Theta=\bar{\Theta}=0} \\
 &+ \Theta^a (\partial_a X U) \Big|_{\Theta=\bar{\Theta}=0} - \frac{1}{4} \Theta^2 (\partial^2 X U) \Big|_{\Theta=\bar{\Theta}=0}
 \end{aligned} \tag{3.13}$$

U is an arbitrary superfield, however, neither the right side of (3.13) nor the left side are superfields. The Θ terms in the expansion have been dropped.

4. - CHIRAL LAGRANGIANS

In this chapter we extend the Lagrangian

$$\begin{aligned} \mathcal{L} = & \int d^2\theta d^2\bar{\theta} \phi^\dagger e^V \phi \\ & + \left[\int d^2\theta \left(\frac{1}{4} \text{Tr} W^\alpha W_\alpha + V(\phi) \right) + h.c. \right] \end{aligned} \quad (4.1)$$

to curved superspace. Let us first consider a model with chiral fields only ($V=0$). We replace $d^2\bar{\theta}$ by the chiral projection operator $\bar{\Delta}$ and $d^2\theta$ by $d^2\Theta \mathcal{E}$:

$$\begin{aligned} \mathcal{L}_\phi = & \int d^2\Theta \mathcal{E} \left[-\frac{1}{8} \bar{\Delta} \phi^\dagger \phi \right. \\ & \left. + V(\phi) + \text{const.} + h.c. \right] \end{aligned} \quad (4.2)$$

The chiral superfield ϕ has been defined in (2.8). In general this Lagrangian will be singular. However, the constant in (4.2) can be chosen such that S starts with a constant as well. This is the case because f of (2.13) contains a part linear in S whereas the supergravity Lagrangian of equ. (3.5) has a part quadratic in S . In such a situation, S^{-1} would be meaningful as well.

There is, however a class of Lagrangians where the S^{-1} part can be eliminated by a redefinition of the fields. These are exactly the R-invariant Lagrangians. Let us demonstrate this on a simple example

when there is one chiral field only. For the potential, R-invariance allows a monomial only:

$$V(\phi) = \lambda \phi^m \quad (4.3)$$

In curved space we obtain from (4.2):

$$\lambda \int d^2 \Theta \xi \phi^m = \lambda \int d^2 \Theta \xi^0 \gamma \phi^m \quad (4.4)$$

We can reparametrise the component fields of ϕ :

$$\phi^{(0)} = \gamma^{\frac{1}{m}} \phi, \quad \phi = \gamma^{-\frac{1}{m}} \phi^{(0)} \quad (4.5)$$

This is a singular reparametrisation. We can use one set or the other as our non singular dynamical variables. If we choose $\phi^{(0)}$, we obtain from (4.4) a non singular potential:

$$\lambda \int d^2 \Theta \xi^0 \gamma \phi^m = \lambda \int d^2 \Theta \xi^0 \phi^{(0)m} \quad (4.6)$$

Now we are left to show that the kinematic part $\phi^\dagger \phi$ can be made nonsingular in the fields as well. There is a more general way to

extend this part to curved superspace:

$$\begin{aligned}\mathcal{L}_{kin} &= -\frac{1}{8} \int d^2\Theta \, \xi \bar{\Delta} \left(e^{-a\bar{\Omega}} e^{b\Omega} \phi^+ \right) \phi \\ &= -\frac{1}{8} \int d^2\Theta \, \xi^0 \gamma^{1-\frac{1}{m}} \bar{\Delta} \left(e^{-a\bar{\Omega}} e^{b\Omega} \phi^+ \right) \phi^{(0)} \quad (4.7)\end{aligned}$$

The constants a and b are arbitrary. If we choose $a=1-\frac{1}{m}$ and use equ. (3.13), we obtain:

$$\mathcal{L}_{kin} = -\frac{1}{8} \int d^2\Theta \, \xi^0 \phi^{(0)} \left\| X(e^{b\Omega} \phi^+) \right\| \Big|_{\bar{\Theta}=0} \quad (4.8)$$

We shall show that for $b=\frac{1}{m}$, the Lagrangian (4.8) is not singular in the $\phi^{+(0)}$ variables. We start from the conjugate equation of (3.4):

$$\begin{aligned}\phi^{(0)+} &= \gamma^{+\frac{1}{m}} \phi^+ = e^{\frac{1}{m}\Omega} \phi^+ \Big|_{\Theta=\bar{\Theta}=0} \\ &+ \bar{\Theta}_i \left(\bar{\mathcal{D}}^i e^{\frac{1}{m}\Omega} \phi^+ \right) \Big|_{\Theta=\bar{\Theta}=0} - \frac{1}{4} \bar{\Theta}^2 \left(\bar{\mathcal{D}}^2 e^{\frac{1}{m}\Omega} \phi^+ \right) \Big|_{\Theta=\bar{\Theta}=0} \quad (4.9)\end{aligned}$$

This is an identity in $\overline{\mathcal{M}}$. The left side is not singular by definition, the same has to be true for all the $\overline{\theta}$ coefficients on the right side. This proves that

$$X e^{\frac{1}{m} \Omega} \phi^+ \Big|_{\theta=\overline{\theta}=0} \quad (4.10)$$

is not singular. Next we have to compute

$$\mathcal{D}_\alpha \left(X e^{\frac{1}{m} \Omega} \phi^+ \right) \Big|_{\theta=\overline{\theta}=0} \quad (4.11)$$

The operator X is not singular, nor are its derivatives. Singularities could only arise from derivatives acting on $e^{\frac{1}{m} \Omega} \phi^+$:

$$\begin{aligned} \mathcal{D}_\alpha e^{\frac{1}{m} \Omega} \phi^+ &= \frac{1}{m} (\mathcal{D}_\alpha \Omega) e^{\frac{1}{m} \Omega} \phi^+ = \\ &= - \frac{2(3n+1)}{m} T_\alpha e^{\frac{1}{m} \Omega} \phi^+ \end{aligned} \quad (4.12)$$

This shows that the expression is not singular. It is clear that the

same is true for

$$\mathcal{D}^2 \left(X e^{\frac{1}{m} \Omega} \phi^\dagger \right) \Big|_{\theta = \bar{\theta} = 0} \quad (4.13)$$

We conclude that the Lagrangian (4.7) has no singular parts in S for $a = 1 - \frac{1}{m}$, $b = \frac{1}{m}$, when it is expressed in terms of the component fields of $\phi^{(6)}$ and their complex conjugates. A similar analysis can be done for an arbitrary R -invariant Lagrangian.

Now we study the extension to gauge theories. Let us first consider an Abelian group. We introduce a vector superfield V with its gauge transformations

$$V \rightarrow V + i (\Lambda - \bar{\Lambda})$$

$$\mathcal{D}_\alpha \bar{\Lambda} = 0, \quad \bar{\mathcal{D}}_\alpha \Lambda = 0 \quad (4.14)$$

It is not as straightforward to construct a gauge invariant field strength. The flat space definition of W_α is:

$$W_\alpha = -\frac{1}{4} \bar{\mathcal{D}}^2 \mathcal{D}_\alpha V \quad (4.15)$$

Replacing $\bar{\mathcal{D}}^2$ by the chiral projection operator $\bar{\Delta}$ does not lead

to a gauge invariant object. It needs some algebra to prove that

$$W_\alpha = -\frac{1}{4} \bar{\Delta} e^{-\frac{\bar{\Delta}}{2}} \mathcal{D}_\alpha V \quad (4.16)$$

is gauge invariant. An extensive use of (2.5), the solution of the Bianchi identities as they were given in Ref. 1) and the chirality property of Λ yield the desired result. The same is true for the non Abelian case:

$$e^V \rightarrow e^{-i\bar{\Lambda}} e^V e^{i\Lambda}$$

$$W_\alpha = -\frac{1}{4} \bar{\Delta} e^{-\frac{\bar{\Delta}}{2}} e^{-V} \mathcal{D}_\alpha e^V \quad (4.17)$$

The factor $e^{-\frac{\bar{\Delta}}{2}}$ is needed for gauge invariance. The gauge invariant Lagrangian is not singular in S. This is due to the factors $e^{-\frac{\bar{\Delta}}{2}}$ which we have introduced for gauge invariance:

$$\begin{aligned} \mathcal{L} &= \frac{1}{4} \int d^2 \Theta \sum \text{Tr} W^\alpha W_\alpha = \\ &= \frac{1}{4} \int d^2 \Theta \sum \left(-\frac{1}{16} \right) \text{Tr} \left(\gamma^{\frac{1}{2}} \bar{\Delta} e^{-\frac{\bar{\Delta}}{2}} e^{-V} \mathcal{D}_\alpha e^V \right) \cdot \\ &\quad \cdot \left(\gamma^{\frac{1}{2}} \bar{\Delta} e^{-\frac{\bar{\Delta}}{2}} e^{-V} \mathcal{D}_\alpha e^V \right) \} \end{aligned} \quad (4.18)$$

It is again a consequence of equ. (3.13) that (4.18) is not singular.

For the coupling term $\phi^\dagger e^V \phi$ we have to go through a similar analysis as for $\phi^\dagger \phi$. Depending on the choice of non singular fields we find a gauge invariant, non singular Lagrangian:

$$\mathcal{L} = -\frac{1}{8} \left(d^2 \Theta \right) \xi \bar{\Delta} \left(e^{(-1+\frac{1}{m})\bar{\Omega}} e^{\frac{1}{m}\Omega} \phi^\dagger e^V \right) \phi \quad (4.19)$$

5. - WEYL INVARIANCE AND THE FAYET ILIOPOULOS TERM

Weyl transformations are transformations of the vielbein that leave the torsion constraints invariant. In Ref. 1) they have been characterized by a parameter Σ subject to the condition:

$$\left[\mathcal{D}^\alpha \mathcal{D}_\alpha + (n+1) T^\alpha \mathcal{D}_\alpha \right] \left[3n(\bar{\Sigma} - \Sigma) - (\bar{\Sigma} + \Sigma) \right] = 0 \quad (5.1)$$

This condition arises from the constraints for non minimal supergravity, $R=0$.

In appendix A of Ref. 1) we computed the transformation law of the determinant of the vielbein E , and of the torsion components $\bar{S}, S, T_\alpha, \bar{T}^\alpha$. This allows us to compute the transformation law of the superfields $\bar{\Omega}$ and Ω as well:

$$\begin{aligned} \delta E &= 2(\Sigma + \bar{\Sigma}) E \\ \delta \bar{\Omega} &= -6\Sigma + W \\ \delta \Omega &= -6\bar{\Sigma} + \bar{W} \end{aligned} \quad (5.2)$$

Here we have introduced the symbol W for the chiral superfield:

$$W = -\frac{3}{(3n+1)\bar{S}} \bar{\Delta} \bar{\Omega} \Sigma \quad (5.3)$$

The concept of Weyl transformations can be extended to include chiral superfields and vector superfields as well. A chiral superfield should remain chiral; it is therefore natural to postulate

$$\delta\phi = x W\phi \quad (5.4)$$

For the vector superfield we shall postulate:

$$\delta V = 0 \quad (5.5)$$

We shall show that the kinetic part of the chiral Lagrangian can be made invariant under Weyl transformations for $x = \frac{1}{3}$. To construct this Lagrangian we shall use the fact that $\Omega, \bar{\Omega}$ can be used to change the $\bar{\Sigma}, \Sigma$ parameters to $\bar{W}, W$ parameters, respectively:

$$\delta \left(E e^{\frac{1}{3}(\Omega + \bar{\Omega})} \right) = \frac{1}{3} (W + \bar{W}) \left(E e^{\frac{1}{3}(\Omega + \bar{\Omega})} \right) \quad (5.6)$$

The invariant Lagrangian for the kinetic part of the chiral fields therefore is:

$$\int d^4x \, e^{\frac{1}{3}(\Omega + \bar{\Omega})} \phi^\dagger \phi \quad (5.7)$$

The Weyl transformation can be combined with a gauge transformation to construct an invariant Fayet-Iliopoulos term. Under a gauge transformation, V transforms as in (4.14):

$$\delta V = i(\Lambda - \bar{\Lambda}) \quad (5.8)$$

Λ is a chiral superfield. The Lagrangian

$$\mathcal{L}_{PI} = \int d^4x \, e^{\frac{1}{3}(\Omega + \bar{\Omega})} e^V \quad (5.9)$$

is invariant under a combined Weyl and gauge transformation:

$$\Lambda = \frac{i}{3} W \quad (5.10)$$

The Lagrangian (5.9) is known as the Fayet-Iliopoulos term. Expanding the exponential e^V , we obtain the action for the gravity multiplet. In flat space, the linear term reduces to the well known Fayet Iliopoulos term.

We are now going to translate the concept of Weyl transformations to our new $\oplus$ variables.

The transformation law of the vielbein is known. Therefore we can compute the transformation law of the chiral density $\mathcal{E}$. The result is, after a componentwise calculation, that it can be written in the following form:

$$\begin{aligned}\delta \mathcal{E} &= W \mathcal{E} + \frac{\partial}{\partial \oplus^\alpha} \left(\tau^\alpha \mathcal{E} \right) \\ \tau^\alpha &= \oplus^\alpha \left(2 \bar{\Sigma} - \Sigma \right) \\ &- \frac{1}{4(3h+1)} \oplus^2 \mathcal{D}^\alpha \left[-4(3h+1) \Sigma + (15h-1) \bar{\Sigma} \right]\end{aligned}\tag{5.11}$$

The transformation law (5.4) of a chiral superfield, when expressed in the new $\oplus$ variables is:

$$\delta \phi = x W \phi - \tau^\alpha \frac{\partial}{\partial \oplus^\alpha} \phi\tag{5.12}$$

To investigate the transformation law of the chiral Lagrangians we have to know the transformation law of the chiral projection operator $\bar{\Delta}$. If we apply $\bar{\Delta}$ to an arbitrary superfield U , whose Weyl transformation is δU , we obtain:

$$\delta \bar{\Delta} U = \bar{\Delta} (\delta U + 2(\bar{\Sigma} - 2\Sigma)U) \quad (5.13)$$

If δU is such that:

$$\delta U + 2(\bar{\Sigma} - 2\Sigma)U = \chi W U \quad (5.14)$$

then (5.13) reduces to the transformation law of a chiral superfield (5.4) because $\bar{\Delta}$ commutes with the chiral field W . For the new $\textcircled{W}$ variables, (5.12) becomes applicable.

From (5.4) and (5.2) follows that

$$e^{-\frac{2}{3}\bar{\Omega}} e^{\frac{1}{3}\Omega} \phi^+ \phi$$

has exactly this property and, as a consequence,

$$\mathcal{L}_\phi = \xi \bar{\Delta} \left(e^{-\frac{2}{3}\bar{\Omega}} e^{\frac{1}{3}\Omega} \phi^+ \phi \right) \quad (5.15)$$

is Weyl invariant. So is the chiral potential:

$$V(\phi) = \xi \phi^3 \quad (5.16)$$

Note that the Ω factors in (5.15) are exactly of the type that make (5.15) and (5.16) non singular when the component fields are taken from

$$\phi^{(0)} = \gamma^{1/3} \phi$$

Next we study the Yang Mills case. From the transformation law (5.5) follows:

$$\delta \mathcal{D}_\alpha V = (\Sigma - 2\bar{\Sigma}) \mathcal{D}_\alpha V \quad (5.17)$$

Miraculously enough, the phenomenon described at (5.14) happens again for W_α

$$\delta \left(\bar{\Delta} e^{-V - \frac{1}{2}\bar{\Omega}} \mathcal{D}_\alpha e^V \right) = -\frac{1}{2} W \left(\bar{\Delta} e^{-V - \frac{1}{2}\bar{\Omega}} \mathcal{D}_\alpha e^V \right) \quad (5.18)$$

The factor $e^{-\frac{1}{2}\bar{\Omega}}$ is needed to make W_α gauge invariant (4.16) and to have it transform like a chiral field under Weyl transformations:

$$\delta W_\alpha = -\frac{1}{2} W W_\alpha - \tau^\beta \frac{\partial}{\partial \Theta^\beta} W_\alpha \quad (5.19)$$

This transformation law leads to a Weyl invariant action (4.18) for the gauge fields.

The Fayet Iliopoulos term can also be formulated in the new Θ basis:

$$\mathcal{L}_{FI} = \xi \left(\bar{\Delta} e^{\frac{2}{3}\Omega + \frac{1}{3}\bar{\Omega}} e^V \right) \quad (5.20)$$

This action is invariant under the combined Weyl and gauge transformations (5.10).

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