A. Vector and Tensor Designations

The following tensor designations are used in the book:

- $a$ tensor of zero rank (scalar),
- $a (a_k)$ tensor of first rank (vector),
- $A (A_{kj})$ tensor of second rank,
- $U (\delta_{kj})$ unit tensor ($\delta_{kj}$ — Kronecker symbol),
- $J (J_{ijk})$ tensor of third rank.

Simmetric and Antisymmetric Tensors

Simmetric and antisymmetric tensors are defined as follows:

Simmetric:

$$A = A^{\text{transp}} \quad (A_{kj} = A_{jk}) \quad (A.1)$$

Antisymmetric:

$$A = -A^{\text{transp}} \quad (A_{kj} = -A_{jk}) \quad (A.2)$$

Trace of tensor is defined as the sum of its diagonal elements:

$$\text{Sp}A = \sum_{k} A_{kk} \quad (A.3)$$
Scalar and Tensor (Internal) Product

Scalar product:

- of two vectors \( \mathbf{a} \cdot \mathbf{b} = \sum k a_k b_k \) (scalar)
- of vector and tensor \( \mathbf{A} \cdot \mathbf{b} = \sum k A_k b_k \) (vector)
- of tensor and vector \( \mathbf{b} \cdot \mathbf{A} = \sum k b_k A_k \) (vector)
- of two tensors \( \mathbf{A} \cdot \mathbf{B} = \sum k A_k B_k \) (tensor)

Dual scalar product of tensors:

\[ \mathbf{A} : \mathbf{B} = \sum_{k,j} A_{jk} B_{kj} \] (scalar) \( \tag{A.5} \)

Internal (dyad) tensor product:

- of two vectors \( \mathbf{ab} \) \( (ab)_{jk} = a_j b_k \) (tensor of second rank)
- of vector and tensor \( \mathbf{aB} \) \( (aB)_{ijk} = a_i B_{jk} \) (tensor of third rank)
- of tensor and vector \( \mathbf{Ba} \) \( (Ba)_{ijk} = B_{ijk} a_k \) (tensor of third rank)
- of two tensors \( \mathbf{AB} \) \( (AB)_{ijkl} = A_{ij} B_{kl} \) (tensor of fourth rank)

Vector product of two vectors and tensor and vector:

\[ (\mathbf{a} \times \mathbf{b})_k = \sum_{i,j} \varepsilon_{ijk} a_i b_j \] (vector), \( (\mathbf{B} \times \mathbf{a})_{ijk} = \sum_{j,l} \varepsilon_{ijk} B_{jl} a_j \) (tensor) \( \tag{A.7} \)

where symbol of permutation \( \varepsilon_{ijk} \) takes the values:

\[ \varepsilon_{ijk} = \begin{cases} 
+1 \text{ under even permutation of indexes (i.e. 123, 231, 312)} \\
-1 \text{ under odd permutation of indexes (i.e. 321, 132, 213)} \\
0 \text{ under recurring indexes.} 
\end{cases} \] \( \tag{A.8} \)

B. Cylindrical Coordinates

Expressions for the different operators used in the equations of heterogeneous mechanics are represented here, for the convenience, in the cylindrical coordinate system \( r, \varphi, z \) (in axisymmetric case, \( \partial / \partial \varphi = 0 \)). Here are the expressions of operators acting on:
(1) scalars:

\[ \frac{d \mathbf{B}}{dt} \equiv \frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \frac{\partial \mathbf{B}}{\partial t} + u_r \frac{\partial \mathbf{B}}{\partial r} + u_z \frac{\partial \mathbf{B}}{\partial z}, \quad \nabla \mathbf{B} = i_r \frac{\partial \mathbf{B}}{\partial r} + i_z \frac{\partial \mathbf{B}}{\partial z} \]  

(B.1)

\[ \nabla^2 \mathbf{B} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \mathbf{B}}{\partial r} \right) + \frac{\partial^2 \mathbf{B}}{\partial z^2}, \]  

(B.2)

(2) vectors:

\[ \nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{\partial A_z}{\partial z}, \]  

(B.3)

\[ \nabla \mathbf{A} = i_r i_r \frac{\partial A_r}{\partial r} + i_r i_\varphi \frac{\partial A_\varphi}{\partial r} + i_r i_z \frac{\partial A_z}{\partial r} + i_\varphi i_r A_\varphi \frac{\partial A_r}{\partial r} + i_\varphi i_z A_\varphi \frac{\partial A_z}{\partial r} + i_z i_r A_z \frac{\partial A_r}{\partial z} + i_z i_\varphi A_z \frac{\partial A_\varphi}{\partial z} + i_z i_z A_z \frac{\partial A_z}{\partial z}, \]  

(B.4)

(3) dyads:

\[ \mathbf{P} = i_r i_r P_{rr} + i_r i_\varphi P_{r\varphi} + i_r i_z P_{rz} + i_\varphi i_r P_{r\varphi} + i_\varphi i_\varphi P_{\varphi \varphi} + i_\varphi i_z P_{\varphi z} + i_z i_r P_{rz} + i_z i_\varphi P_{\varphi z} + i_z i_z P_{zz}, \]  

(B.5)

\[ \nabla \cdot \mathbf{P} = i_r \left[ \frac{1}{r} \frac{\partial}{\partial r} (r P_{rr}) - \frac{\partial P_{rz}}{\partial z} - \frac{P_{\varphi \varphi}}{r} \right] + i_\varphi \left[ \frac{1}{r} \frac{\partial}{\partial r} (r P_{r\varphi}) + \frac{\partial P_{\varphi z}}{\partial z} + \frac{P_{\varphi \varphi}}{r} \right] + i_z \left[ \frac{1}{r} \frac{\partial}{\partial r} (r P_{rz}) + \frac{\partial P_{\varphi z}}{\partial z} \right]. \]  

(B.6)

Then for strain tensor and strain velocity tensor we obtain, respectively:

\[ \mathbf{D} \equiv \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^{\text{transp}}) \]

\[ = i_r i_r \frac{\partial u_r}{\partial r} + i_r i_\varphi \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right) \right) + i_r i_z \left( \frac{1}{2} \left( \frac{\partial u_z}{\partial r} + \frac{u_z}{r} \right) \right) + i_\varphi i_r \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right) \right) + i_\varphi i_z \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r} \right) \right) + i_z i_r \left( \frac{1}{2} \left( \frac{\partial u_z}{\partial r} + \frac{u_z}{r} \right) \right) + i_z i_\varphi \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r} \right) \right) + i_z i_z \frac{\partial u_z}{\partial z}, \]  

(B.7)

\[ \mathbf{\dot{D}} \equiv \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^{\text{transp}}) - \frac{1}{3} \nabla \cdot \mathbf{u} \]

\[ = i_r i_r \left( \frac{\partial u_r}{\partial r} - \frac{1}{3} \nabla \cdot \mathbf{u} \right) + i_r i_\varphi \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right) \right) + i_r i_z \left( \frac{1}{2} \left( \frac{\partial u_z}{\partial r} + \frac{u_z}{r} \right) \right) + i_\varphi i_r \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r} \right) \right) + i_\varphi i_z \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right) \right) + i_z i_r \left( \frac{1}{2} \left( \frac{\partial u_z}{\partial r} - \frac{1}{3} \nabla \cdot \mathbf{u} \right) \right) + i_z i_\varphi \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} - \frac{1}{3} \nabla \cdot \mathbf{u} \right) \right) + i_z i_z \left( \frac{1}{2} \left( \frac{\partial u_\varphi}{\partial r} + \frac{1}{3} \nabla \cdot \mathbf{u} \right) \right). \]  

(B.8)
Double-point Gibbs multiplication serves as an operator widely used in hydrodynamics. If, following the Gibbs notations, \( \mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \) are arbitrary vectors, then \( \mathbf{a} \cdot \mathbf{c} = (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{d} \). In particular, for unit vectors we have:

\[
\hat{i}_j \hat{i}_k : \hat{i}_l \hat{i}_m = \hat{i}_j \cdot \hat{i}_l (\hat{i}_k \cdot \hat{i}_m) = \delta_{jk} \delta_{lm}, \quad (B.9)
\]

and for two dyads we have:

\[
\mathbf{D}^{(1)} : \mathbf{D}^{(2)} = \left( \sum_j \sum_k \hat{i}_j \hat{i}_k \mathbf{D}^{(1)}_{jk} \right) : \left( \sum_l \sum_m \hat{i}_l \hat{i}_m \mathbf{D}^{(2)}_{lm} \right) = \sum_j \sum_k \sum_l \sum_m \delta_{jl} \delta_{km} \mathbf{D}^{(1)}_{jk} \mathbf{D}^{(2)}_{lm} = \sum_j \sum_k \mathbf{D}^{(1)}_{jk} \mathbf{D}^{(2)}_{jk}, \quad (B.10)
\]

or

\[
2 \mathbf{D} \cdot \hat{\mathbf{D}} = 2D_{rr}^2 + 2D_{\varphi\varphi}^2 + D_{zz}^2 + 4D_{r\varphi}^2 + 4D_{rz}^2 + 4D_{z\varphi}^2 - \frac{2}{3} (\nabla \cdot \mathbf{u})^2. \quad (B.11)
\]
Conclusion

The natural dynamical systems and their physical properties, specifically those observed in outer space, draw much attention, and constant progress is made in studying them. The fundamental patterns of the formation of macromolecular structures from a homogeneous non-differentiable (chaotic) medium toward ordering structures becomes a paradigm of how modern science broadens the horizons of knowledge. The goal of our mathematical models, in particular those that model cosmic objects that are inaccessible to direct investigation, is understanding the genesis and evolution of the key mechanisms that underlie natural processes in our environment and in the Universe at large.

We focused on the theoretical approach of developed turbulence in multicomponent mixtures of reacting gases and heterogeneous gas-dust media as well as on the construction of continuum models for turbulized hydrodynamic systems, with application to space objects. Because these systems, as a rule, have complicated physical and chemical characteristics, their mathematical modeling presents serious problems. Generally it requires taking into account the compressibility of flow, the variability of thermophysical properties, heat and mass exchange, chemical reactions, phase transitions and radiation transfer, and the processes occurring under the influence of gravitational and electromagnetic forces.

We considered a number of complex astrophysical problems based on the methods of continuum mechanics using the proposed stochastic-thermodynamic approach to semi-empirical modeling of the developed turbulence in reacting multicomponent gases and gas-dust media, incorporating structured turbulence properties in a homogeneous fluid. This theoretical advancement serves the purpose of reconstructing space object evolution based on the refined numerical models.

The topical theory of self-organization in space environment was thoroughly discussed in support of the general concept of stochastic dynamics related to the emerging ordered structures in the case of significant deviation from equilibrium. Great attention was paid to the theory of self-organization in the developed hydrodynamic turbulence that involves cooperative vortex structure formation. We considered the origin and evolution of coherent vortex structures in turbulent flows by analyzing the relationship between order and chaos in open dissipative
systems from the standpoint of stochastic nonlinear thermodynamics of irreversible processes. This study focuses on applications to specific phenomena, first of all, the origin and evolution of protoplanetary accretion gas-dust disks and the emerging primary dust clusters on the background of turbulent transport and self-organization.

Basically, examples of self-organizing dynamical systems can be consciously expanded from accretion disks to the observed peculiar features of numerous objects in the Universe that underlie cosmology and cosmogony, but we are aware that this approach to the structure formation and evolution of processes in nature may not be shared by other researchers. This problem was specifically addressed in the first chapter, where different dynamical systems (nature of stars and planets, stellar-galactic and planetary evolution, protoplanetary accretion disk formation and evolution, structure and evolution of the Universe) were discussed, though this problem extends far beyond the narrower problems of modeling structured turbulence. Essentially, the subsequent chapters supported the general basic concept of natural ordering and, in particular, the formation of highly organized nonequilibrium structures in cosmic media.
References

Chapter 1


Dirac House, Temple Back, Bristol


Delsemme, A.H.: The deuterium enrichment observed in recent comet is consistent with the cometary origin of sea water. Planet. Space Sci. 47, 125–131 (1999)


Eneev, T.M.: On the possible structure of the outer (Trans-Neptunian) regions of the solar system. Pis’ma Astron. Zh. 6, 295 (1981)


Ipatov, S.I., Mather, J.C., Marov, M.Ya.: Migration of Icy Bodies to the Terrestrial Planets, Joint Assembly AGU, GS, MAS, MSA, SEG, and UGM, 23–26 May, Baltimore, Maryland, USA, Eos Trans.AGU, 87(36) (2006a)
Kaplan, J., Yorke, J.A.: Springer lecture notes in mathematics. 730, 228 (1978)


Klein, T., Touboul, M., Burkhardt, C., Bourdon, B.: Dating the first ~100 Ma of the solar system: From the formation of CAIs to the origin of the Moon. Goldschmidt Conference Abstracts, Vancouver, Canada, A480, 13–18 July (2008a)


Makalkin, A.B.: Radial compaction of the dust subdisk in a protoplanetary disk as possible way to gravitational instability. Lunar Planet. Sci. 25, 827 (1994)


References


References


Paige, D.A., 26 co-authors: Diviner lunar radiometer observations of cold traps in the Moon’s south polar region. Science. 330, 479 (2010)


References


References


Vinogradov, A.P. (ed.): Cosmochemistry of the Moon and Planets, Nauka, Moscow (in Russian)
References 631


Chapter 2

Stefan, J.: Wien Sitzungsber 63, 63 (1871)

Chapter 3

References


References

Nikuradze (1936)

Chapter 4

Keller, L.V.: Theory of Convection and Turbulence, Trudy GGO, 4 (1930)
References


Kuznetsov, V.P.: Influence of Temperature and Concentration Fluctuations on the Mean Chemical Reaction Rate in a Turbulent Flow in the Second USSR Symposium on Combustion and Detonation, Institute of Chemical Physics, Academy of Sciences of USSR, Moscow, 99 (in Russian) (1969)


Rotta, J.: Statistische theorie nichthomogener turbulenz teil 1. Physik 129, 547 (1951)


Chapter 5


Chapter 6

References


Chapter 7

Barenblatt, G.I., Golitsyn, G.S.: Local Structure of Developed Dust Storms. Moscow State University, Moscow (in Russian) (1973)


Derevich, I.V.: Influence of an admixture of large particles on turbulent characteristics of a gas suspension in channels. PMTF 2, 70 (1994)

Dominik, C., Blum, J., Cuzzi, J., Wurm, G.: Growth of dust as the initial step toward planet formation in protostars and planets V. Arizona, AZ (2007)


References


Loginov, V.I.: Dehydration and Desalinization of Oil. Khimiya, Moscow (in Russian) (1979)


Makalkin, A.B.: Radial compaction of the dust subdisk in a protoplanetary disk as possible way to gravitational instability. Lunar Planet. Sci. 25, 827 (1994)


Mazin, I.P.: Theoretical estimation of the coagulation coefficient of droplets in clouds. Trudy TsAO 95, 12 (1971)


Smoluchowski, M.: Three Lectures on Diffusion, Brownian Molecular Motion, and Coagulation of Colloidal Particles. Brownian Motion in Colloid Coagulation. ONTI, Moscow (in Russian) (1936)


Stokes, G.G.: On the effect of the internal friction of fluids on the motion of pendulums. Trans. Camb. Phil. Soc. 9, 8 (1851)


Chapter 8


Kolesnichenko, A.V.: On the possibility of synergetic birth of mesoscale coherent structures in the macroscopic theory of developed turbulence. Mat. Mod. 17, 47 (2005)
Kraichnan, R.H.: Diffusion of passive scalar and magnetic fields by helical turbulence. J. Fluid Mech. 77, 753 (1976a)
Kraichnan, R.H.: Eddy viscosity in two and three dimensions. J. Atmos. Sci. 33, 1521 (1976b)
Steenbeck, M., Krause, F., Radler, K.-H.: A calculation of the mean electromotive force in an electrically conducting fluid in turbulent motion under the influence of coriolis forces. Z. Naturforsch. 21a, 369 (1966)
References

Von Weizsacker, C.F.: Rotation Kosmischer Gasmassen. Z. Naturforsch. 3a, 524 (1948)

Chapter 9


## Index

### A

Accretion disks, 1, 2, 104, 107, 116–130, 425–553, 555–606
protoplanetary, 116–130
thin, 472, 486, 598–606
Aerodynamic drag coefficient, 441–447, 475
subsystems of, 223, 304, 357, 374, 530, 584, 587
Averaging operator, 191–195, 199, 208, 212, 471, 485

### B


### C

Chemical reactions source
  correlation moments of, 266–268
  fluctuation of, 266–268
Closure problem, 23, 189, 196, 212–215, 257, 268, 282, 473
Coagulation kinetics equation, 519–524
Conducting medium, 555, 556, 558, 560, 562–597
  averaged entropy of, 583–584
  with magnetic field, 567–597
  of variable density, 594–597
Conservation laws of mass and momentum, 435
equations of, 435
in locally isotropic turbulence, 533–534
Continuous media, 146–149, 161, 165, 188, 191
  balance equation of, 583–584
  mathematical modeling of, 146–149
Corona, 555–567, 598–600, 603
Cosmic media, 2, 39
Cosmic objects, 2, 39–144
evolution of, 100–109
Critical stationary states, 349–356

### D

Differential Kolmogorov–Prandtl model, 248–252
Diffusion, 16, 148, 189, 255, 304, 374, 425, 557
defining relations, 444
Disk dust particles, 441–447
Disk matter, 92, 96, 98, 459–460, 486–494, 527, 529, 532
  energy balance equations of, 486–494
  thermodynamic state equation of, 459–463
Dust grains, 85, 460–464
  optical properties of, 460–464
Dust multifractional composition approach, 447–451
Dynamical astronomy, 39–42
Dynamical systems, 2, 21, 23–39, 333, 334, 356, 359, 375, 403, 406, 422, 532
  chaos of, 23–39
  self-organization of, 23–39
E
Electrically conducting matter, 576–577
mean motion energy equations of, 576–577
Electrically conducting medium, 556, 558, 562–566, 568, 574, 579, 580, 582–597
energy of, 562–566
heat influx equations of, 562–566
Energy cascade, 9, 15, 311, 528–537, 539–542
balance equation of, 166–168
weighted mean of, 216–220

F
Fokker–Planck–Kolmogorov equations, 23
fractional-order, 360–371
Fractional derivative, 367–369
Fractional integral, 366–369

G
Gas-dust disk, 125, 129
averaged equation of motion of, 478–481
rotating, 515–519
Gas-dust matter, 469–476, 492, 524, 606
momentum conservation of, 451–454
monodisperse, 437–440
Gas-dust subdisk, 125, 432, 503–524
conservation laws of, 146–160
mass balance equations of, 151–153
regular motion of, 146–160
viscous heat-conducting, 160–161
averaged internal energy of, 481–486
balance equation of, 481–486
Gaussian process, 335–342, 351, 361
Gradient hypothesis, 214, 239–244, 255, 473, 582
H
defining relations, 169–188
Helicity, 405, 525–553, 594
cascades, 537–545
influence on energy cascade, 525–553
Heterogeneous gas-dust medium, 454–459
heat influx equation of, 454–459
Heterogeneous media, 425–524
equations of, 436–466
mechanics of, 436–466
Hydrodynamic helicity, 525–553, 594
modeling of, 525–529
in rotating disk, 542–545
theoretical prerequisites of, 525–529
I
Isotropic fluid, 170–171, 560
cross effects of, 171–172
diffusion of, 171–172
heat conduction of, 171–172
viscous flow of, 170–171
K
Kinetic coagulation equation, 447–451
Kolmogorov’s theory, 9–13
L
Limiting saturation approach, 515–519
Linear kinematic constitutive equations, 169–170
Magnetic induction equation, 556–559, 569–571
for mean fields, 569–571
Mean flow background, 531–533
Mesoscale coherent structures, 34, 308, 414, 530, 531, 548
formation of, 411–414
Method of moments, 426, 451, 519–524
Mirror symmetry, 533, 537–539, 542, 551, 552, 593
breaking in protoplanetary disk, 537–539
averaged equations of, 212–215
Modeling disk structure, 556–565
basic equations of, 556–565
magnetohydrodynamic approach to, 556–565
Momentum transfer theory, 545–547
Motion, averaged equation of, 571–572
Multicomponent diffusion, 171, 173, 176, 177, 180, 184, 186–188, 214, 216, 230
Multicomponent gas mixtures, 157, 159, 173, 176, 196, 211, 230–290, 435
energy balance equations of, 157–159
equation of motion of, 153–154
model transfer equations of, 268–285
Multicomponent medium, 139, 146, 151, 159, 190, 193, 208, 212, 215, 216, 241, 258, 272–280
with variable density, 272–280

N
Negative viscosity, 318, 405, 527–529, 537, 538, 542, 545–553
in rotating disk, 545–553
thermodynamic approach to, 547–549
Non-equilibrium Arrhenius kinetics, 257–268
averaging chemical reaction rates, 261–266
in turbulized flow, 257–268
Non-equilibrium phase transitions, 375–402
Non-equilibrium stationary states, 36, 298–300, 313, 334, 343–349, 352, 359
turbulent chaos of, 343–349
Non-Markovian processes, 362–366
causality principle for, 362–366

O
Onsager principle, 161–165
Order
relationship to turbulent chaos, 32–37
turbulent flows, 37–39

P
Periodic self-oscillations, 406–411
phase dynamics approximation of, 406–411
related synchronization of, 406–411
Planetary atmospheres, 16, 56, 78, 91, 93
Prigogine’s principle, 322–324
Protoplanetary gas-dust cloud, 51, 125, 425, 426, 436–466

R
Radiative transfer equation, 460–464
Reacting gas mixtures, 145–188, 281
entropy production of, 166–168
turbulence of, 190–215
Rotational viscosity, 549–553

S
Scalar second moments, 285–290
dissipation of, 285–290
transfer equations of, 285–290
Small bodies, 44, 45, 47, 107–116, 125
dynamics of, 107–116
Smoluchowski coagulation equation, 476–478
Solar system
dynamical properties of, 42–46
giant planets in, 45, 51, 67, 78–91
terrestrial planets in, 44, 48
Stable limit cycles, 406–410
Stationary non-equilibrium regime, 588–593
H-theorem of, 384–388
Stefan-Maxwell relations, 176–186
Stochastic dynamics, elements of, 24–31
Stochastic Langevin equations, 298, 323, 333–343
in internal coordinates space, 341–343
Stochastic thermodynamic model, 37–39, 295–371
Structured turbulence, 5, 23, 32, 295–423
generation of, 402–423
phase synchronization of, 402–423
thermodynamics of, 303–317
Structured turbulent chaos, 303–317
Synchronized clusters, 416, 418–423
oscillations of, 418–423

T
Thermal equation of state, 149, 159–160, 212
Thermodynamic derivation, 177–183
Thermodynamics, second law of, 160–169
Thermodynamic stability, 349–356
Thin Keplerian disk, 600–603
viscosity of, 600–603
Turbulence
developed, 9–13
spectrum of, 13–16
disk reflection-invariant, 537–545
geophysical, 18–20
hydrodynamic, 1, 2, 6, 29, 32, 37, 215, 296, 304, 359, 373, 375–388
isotropic, 10–12, 530–545
with reflection symmetry, 530–545
as manifestation of helicity cascade, 545–553
modeling methods of, 21–23
stationary-nonequilibrium state of, 319–322
Turbulence model, small-scale, 388–393
phenomenology of, 388–393
Turbulence scale, equations of, 252–254
Turbulent accretion disks, 426–436
evolution of, 426–436
modeling of, 426–436
theoretical prerequisites of, 426–436
equations of, 220–223, 324–328
evolution of, 335–341
with memory, 360–371
multiplicative noise of, 388
phase transitions of, 395–399
self-organization of, 333–360
subsystem of, 220–223
in conducting medium, 584–586
entropy balance equations of, 220–223
entropy production of, 220–223
internal fluctuating coordinates of, 303–310
phase dynamics equations of, 418–423
rheological relations of, 215–238
transfer equations of, 281–285
dissipation rate of, 393–395, 399–402
stationary-nonequilibrium state of, 319–322
defining relations of, 582–597
energy of, 201–211
Turbulent fluid motion, 2–23, 33, 241, 301, 304, 378
Turbulent gas-dust disk, 466–503
  averaged mass balance equations of, 469–471
two-phase mechanics averaged equations of, 466–503
rheological relations of, 215–254, 593
transfer equations of, 281–283
Turbulent motion, 1–4, 6, 9, 12, 15, 19, 33, 34, 36, 37, 89, 130, 145, 149, 189–254, 280, 290, 296, 301, 311, 314, 320, 322, 324, 333, 337, 360, 361, 390, 405, 425, 430, 432, 465, 467, 469, 486, 495, 499, 528, 529, 531, 534, 556, 567–582, 588, 589, 593, 602
  equations of, 189–254, 567–582
Turbulent stress tensor, 189, 200, 228, 268, 272–280, 467, 495, 550, 555, 572, 580, 590, 601
  transfer equations of, 272–280
Turbulent subdisk, 503–524
  steady motions in, 503–524
Turbulent transport coefficients, 229, 238–254, 291, 331, 360, 555, 598–606
  modeling of, 238–254, 598–606
Turbulent viscosity coefficient, 228, 239, 245, 248, 249, 251, 280, 494–503, 512–515, 597, 601–606
correction function to, 594–597
  modeling of, 494–503
total entropy of, 223–226
  balance equation of, 223–226
Turbulized gas-dust disk, 466–503
  two-phase mechanics averaged equations of, 466–503
  rheological relations of, 215–254, 593
  transfer equations of, 281–283
  equation of state, 211–212
  one-point second moments of, 269–272
  transfer equations of, 269–272
Turbulized multicomponent gas mixtures, 196, 226–232
  linear closing relations of, 226–232
Turbulized plasma, 577–582, 587
  magnetic energy equations of, 577–582
Two-dimensional turbulence, 528, 529, 536–537, 540
  two transfer equations, 252–254
  model of, 252–254

V

Verhulst’s mathematical model, 399–401
Viscous stress tensor, 21, 149, 153, 160, 167–188, 199, 200, 213, 226, 228, 278, 442, 452, 454, 479, 491, 560, 571
Vorticity dynamics, 534–536