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CHAOTIC INFLATION IN SUPERGRAVITY

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CHAOTIC INFLATION IN SUPERGRAVITY

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A new realization of the chaotic inflation scenario is suggested in the context of $N=1$ supergravity.

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One of the most interesting realizations of the inflationary universe scenario [1-3] is based on the investigation of the cosmological consequences of $N=1$ supergravity coupled to matter [4-8]. This realization became especially attractive after the primordial monopole problem in this version of the inflationary universe scenario was solved [6-8].

However, this scenario was to some extent incomplete. For realization of this scenario some supplementary chiral superfield Φ (different from the Polony field [9]) was introduced [4] and the effective potential $V(\bar{z}, z^*)$ of the first (scalar) component $\bar{z}$ of this superfield was investigated [10].

$$V(\bar{z}, z^*) = e^{\frac{2}{3} \bar{z} z^*} (2 |g_{\bar{z}} - \frac{2}{3} \bar{z}^* g|^2 - 3 |g|^2), \quad (1)$$

where $g_{\bar{z}} = \frac{dg}{d\bar{z}}$, $g(\bar{z})$ is the superpotential,

$$g(\bar{z}) = M^3 f(\bar{z}). \quad (2)$$

Here M is some mass parameter, $f(\bar{z})$ is some arbitrary dimensionless function of the field $\bar{z}$. The system of units in which $\frac{M_{Pl}}{\sqrt{8\pi}} = 1$ is used, where $M_{Pl} \sim 10^{19}$ GeV is the Planck mass [10].

In refs. [4,5] the superpotential was represented in the following form:

$$f(\bar{z}) = \sum_{n=1}^m \frac{\lambda_n}{n} \bar{z}^n, \quad (3)$$

and it was assumed that $\lambda_0 \geq 0$, $\lambda_1 > 0$. The effective potential $V(\varphi)$ with respect to the real part φ of the field $\bar{z}$ was written as follows,

$$V(\varphi) = M^6 (\alpha + \beta \varphi^2 - \gamma \varphi^3 + \delta \varphi^4 + \dots), \quad (4)$$

where $\alpha, \beta, \gamma, \delta$ are some functions of λ_n , which can be chosen in such a way that the minimum of $V(\varphi)$ is at

$\varphi = \varphi_0 = 1$, and $V(\varphi_0) = 0$. The last condition is necessary in order to have zero cosmological constant at the minimum of $V(\varphi)$.

In refs. [6, 8] another effective potential was used,

$$V(\varphi) = 3\mu^6 \left(1 - \alpha^2 \varphi^2 + \frac{\alpha^4}{4} \varphi^4 \right) . \quad (5)$$

At present there is no idea what would be a preferable choice for the superpotential $Q(\mathbf{z})$, and therefore the question which effective potential is more natural is to some extent scholastic. However, there exists a possible constraint on the function $Q(\mathbf{z})$. In the theories under consideration the mass of the gravitino $m_{\tilde{g}}$ is proportional to $Q(\varphi_0)$, and is assumed to be very small, $m_{\tilde{g}} \sim 10^2$ GeV [11], or $m_{\tilde{g}} \sim 10^{-16}$ in the units used above ($\frac{m_{\tilde{g}}}{M_{\text{Pl}}} = 1$) . This is necessary in order to obtain a comparatively simple solution to the gauge hierarchy problem in the context of supergravity [12] . In such theories the value of $Q(\mathbf{z})$ in the minimum of $V(\mathbf{z}, \bar{\mathbf{z}}^*)$ should be almost precisely zero.

In refs. [6, 8] it was not assumed that $Q(\varphi_0) = 0$, since the theories with $m_{\tilde{g}} \sim 10^2$ GeV lead to some cosmological difficulties [13] . One can overcome these difficulties in the inflationary universe scenario [14, 5] , but only in a rather restricted class of theories [15] . Nevertheless it would be important to understand whether it is possible to realize the inflationary universe scenario in supergravity under the condition $Q(\varphi_0) = 0$, as it was assumed in refs. [4, 5] .

To make our investigation as simple as possible we introduce the function

$$\Psi(\bar{z}) = e^{\frac{\bar{z}^2}{4}} f(\bar{z}) \quad (6)$$

and rewrite the effective potential $V(\bar{z}, \bar{z}^*)$ (1) as follows:

$$V(\bar{z}, \bar{z}^*) = \mu^6 e^{-\frac{(\bar{z} - \bar{z}^*)^2}{4}} \left(2|\Psi_{\bar{z}} + \frac{1}{2}(\bar{z}^* - \bar{z})\Psi|^2 - 3|\Psi|^2 \right), \quad (7)$$

where $\Psi_{\bar{z}} \equiv \frac{d\Psi}{d\bar{z}}$. At the real axis $\bar{z} = \varphi$ the effective potential has a very simple form,

$$V(\varphi) = \mu^6 [2\Psi_{\varphi}^2 - 3\Psi^2]. \quad (8)$$

Now let us note, that $\Psi(0) = f(0)$, $\Psi_{\varphi}(0) = f_{\varphi}(0)$, and that from the condition $f(\varphi_0) = 0$ it follows that $\Psi(\varphi_0) = 0$. Let us consider according to [4,5] the superpotential $f(\bar{z}) = \sum_{n=0}^m \frac{\lambda_n}{n!} \bar{z}^n$ (3) with $\lambda_0 \geq 0$, $\lambda_1 > 0$. In such theories $\Psi(0) = \lambda_0 \geq 0$, $\Psi_{\varphi}(0) = \lambda_1 > 0$. It can be easily understood, that in this case the condition $\Psi(\varphi_0) = 0$ can be satisfied only if Ψ_{φ} becomes zero at some point $\tilde{\varphi}$ between $\varphi = 0$ and $\varphi = \varphi_0$, in which $\Psi(\tilde{\varphi}) \neq 0$. From eq. (8) it follows that $V(\tilde{\varphi}) < 0$, which means that the point $\varphi = \varphi_0$ is not the absolute minimum of $V(\varphi)$, and that a more deep minimum with $V(\tilde{\varphi}) < 0$ lies somewhere between $\varphi = 0$ and $\varphi = \varphi_0$ [16].

This result is a strengthened version of the theorem, which has been proved independently in ref. [17]. The authors of ref. [17] have concluded, that the inflationary universe scenario in supergravity can be realized only in the theories, in which $V(\varphi_0) = 0$, $g_{\varphi}(\varphi_0) \neq 0$, just as in the model considered in [6,8]. This would mean that inflation in supergravity is incompatible with the above-mentioned possibility to solve the gauge hierarchy problem [12].

This general conclusion of the authors of ref. [17] (as distinct from the theories discussed above) was based on the investigation of the high-temperature corrections to $V(\varphi)$ (1). However, as it was shown by one of the present authors in [8], the only possibility to implement the inflationary universe scenario in the context of supergravity is connected with the chaotic inflation scenario [3], which is not based on the study of the high-temperature effects. The main aim of the present paper is to suggest a consistent realization of the chaotic inflation scenario in the context of supergravity in such a way as to have both $V(\varphi)=0$ and $G(\varphi)=0$ in the absolute minimum of the effective potential. We will not try to propose here a most natural superpotential, since at present we have no criterion for making a choice between different possible functions $G(\varphi)$. We would like just to show, that the superpotentials of the necessary type do actually exist, and to reveal some basic features of the chaotic inflation scenario in supergravity [8].

As an example of an appropriate superpotential let us consider the function

$$\Psi(\Sigma) = \text{th} \zeta \cdot \text{sh} \zeta, \quad \zeta = \xi + i\eta, \quad (9)$$

where

$$\zeta = \sqrt{\frac{3}{2}} (\Sigma - \varphi_0), \quad (10)$$

and φ_0 is some real number, see below.

It is seen, that $\Psi(\Sigma)=0$ at $\Sigma = \varphi_0$. The effective potential in the new variables is

$$V(\zeta, \zeta^*) = 3\mu^6 \exp\left(\frac{2(\text{Im}\zeta)^2}{3}\right) \left\{ |\psi - \frac{1}{3}(\zeta - \zeta^*)\psi|^2 - |\psi|^2 \right\}. \quad (11)$$

With the superpotential (9) the effective potential (11) at the real axis is given by

$$V(\zeta) = 3\mu^6 (\psi^2 - |\psi|^2) = 3\mu^6 \left(1 - \frac{2}{3\chi^4\zeta} - \frac{1}{3\chi^4\zeta} \right). \quad (12)$$

This potential has a minimum at $\zeta = 0$ ($\varphi = \varphi_0$) with $V(\varphi_0) = 0$. At large $|\zeta|$ the effective potential grows and asymptotically approaches $3\mu^6$. In the complex plane $V(\zeta, \zeta^*)$ is positive semidefinite. $V(\zeta, \zeta^*) = 0$ only at $\zeta = i\pi n$, $n=0, \pm 1, \pm 2, \dots$. Due to the presence of the factor $\exp\left(\frac{2(\text{Im}\zeta)^2}{3}\right)$ in (11), the potential $V(\zeta, \zeta^*)$ is exponentially large everywhere outside the narrow region near the real axis. The only exceptions are the exponentially narrow holes (of the width of the order of $\exp(-\frac{2\pi^2 n^2}{3})$) near the above-mentioned points $\zeta = i\pi n$, $n = \pm 1, \pm 2, \dots$

This potential is ideally fitted for the realization of the chaotic inflation scenario [3]. According to this scenario the universe initially was filled with some chaotic spatial distribution of the nonequilibrium field $\tilde{\chi}$. During the rolling of the field $\tilde{\chi}$ down to the minimum of the effective potential some of the domains of the universe, filled with a sufficiently large field $\tilde{\chi}$, exponentially grow and acquire a size exceeding the size of the observable part of the universe [1]. In a recent modification of this scenario [18] the large field $\tilde{\chi}$ appears after the quantum creation of the universe.

The chaotic inflation scenario can be implemented in the

theory with the effective potential (9)-(12) as follows. Almost independently of the initial value of the field Z in the universe, this field exponentially rapidly rolls down to the real axis, $\zeta = \bar{\zeta} = \sqrt{\frac{2}{3}} (\varphi - \varphi_0)$. After this process the main part of the universe becomes filled with some real field φ , which typically is very large, $|\varphi - \varphi_0| \gg 1$ (The probability to roll down to the holes near $\zeta = i\pi n$, $n = \pm 1, \pm 2, \dots$ or directly to the minimum at $\zeta = 0$ is negligibly small). Then the field φ exponentially slowly rolls along the real axis down to the minimum of $V(\varphi)$ at $\varphi = \varphi_0$. During this process the domains of the universe filled with a sufficiently homogeneous field φ (homogeneous at a scale $l \geq H^{-1}$ [3]) expand according to the Einstein equation

$$H = \frac{dZ}{dt} = \frac{1}{\sqrt{3}} \left(V + \frac{\dot{\varphi}^2}{2} \right)^{\frac{1}{2}} \quad (13)$$

where Z is the logarithm of the scale factor $a(t)$, $Z = \ln a$.

The evolution of the field φ is governed by equation

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV}{d\varphi} \quad (14)$$

In the region $|\varphi - \varphi_0| \gg 1$, which is of the main interest for us, the field φ rolls down very slowly, and the terms $\dot{\varphi}^2$ in (13) and $\ddot{\varphi}$ in (14) can be neglected [3]. In this region the potential $V(\varphi)$ is very flat and can be approximated by the asymptotic formula

$$V(\varphi) = gM^3 \left(1 - \frac{g}{3} e^{-\sqrt{\frac{2}{3}} |\varphi - \varphi_0|} \right) \quad (15)$$

During the rolling of the field φ to the minimum of $V(\varphi)$ the scale factor $a(t) = a(\varphi(t))$ grows as follows

$$\ln \frac{a(\varphi_0)}{a(\varphi)} = Z(\varphi_0) - Z(\varphi) \equiv \Delta Z_\varphi =$$

$$= \int_{\varphi}^{\varphi_0} \frac{dZ}{d\varphi} d\varphi = \int_{\varphi}^{\varphi_0} \left(\frac{d(\ln V)}{d\varphi} \right)^{-1} d\varphi \approx \frac{g}{16} e^{\sqrt{6}|\varphi - \varphi_0|}. \quad (16)$$

From eq. (16) it follows that the universe expands greater than e^{70} times (which is necessary for the realization of the inflationary universe scenario) if the value of $|\varphi - \varphi_0|$ initially was sufficiently large, $|\varphi - \varphi_0| \gg 2$.

In the chaotic inflation scenario in the theory with the effective potential (15) any initial value of φ can exist in a given domain of the universe with the probability, which is almost φ -independent, at least for sufficiently large φ . Therefore a considerable part of the universe (presumably, the main part of the universe) initially was in a state with $|\varphi - \varphi_0| \gg 2$. The regions of the universe with $|\varphi - \varphi_0| \gtrsim 2$ exponentially expand and acquire the size exceeding the size of the observable part of the universe $\{ \sim 10^{28} \text{ cm.} [3] \}$.

Now let us investigate the spectrum of inhomogeneities, which arise after the inflation. As it was shown in [19], these inhomogeneities arise from quantum fluctuations of the field φ , which at the beginning of the process of their amplification at the point $\varphi = \varphi_*$ have the momentum

$$k_*^2 \sim H^2 \approx 3\mu^6. \quad (17)$$

At the end of inflation this momentum decreases,

$$k^2 = k_*^2 e^{-2\Delta Z_*} \approx 3\mu^6 e^{-2\Delta Z_*}, \quad (18)$$

where $\Delta Z_* = Z(\varphi_0) - Z(\varphi_*)$. According to [19], the spectrum of density perturbations after inflation is given by

$$\frac{\delta \rho(\kappa)}{\rho} = \frac{1}{\sqrt{2\pi^3}} \frac{H^2}{|\dot{\Psi}|} \Big|_{\Psi=\Psi_*} = \frac{3\mu^2}{\pi\sqrt{\pi}} \Delta Z_* \approx 0.3 \mu^3 \ln \frac{3\mu^6}{\kappa^2}. \quad (19)$$

The galaxies arise due to the amplification of fluctuations with $\Delta Z_* \sim 50$, hence

$$\frac{\delta \rho}{\rho} \sim 30 \mu^3. \quad (20)$$

This means, that $\frac{\delta \rho}{\rho} \sim 10^{-4}$ if $\mu^3 \sim 3 \cdot 10^{-6}$, i.e. $\mu \sim 10^{-2}$, which seems quite reasonable. In these theories, unlike all other theories considered up to now, we do not need any other small parameters to obtain a sufficiently large inflation and the value $\frac{\delta \rho}{\rho} \sim 10^{-4}$. As for the parameter φ_0 , it determines the rate of decay of particles Ψ to other particles, $\Gamma_\Psi \sim \mu^3 \varphi_0^2$ [5,8], and with an appropriate choice of this parameter one can get an adequate temperature T_R of the universe after reheating, which is important for the solution of the gravitine problem in this scenario [15]. In particular, at $\varphi_0 = 1$ in our scenario $T_R \sim 10^{10} - 10^{11}$ GeV, and the decrease (increase) of φ_0 leads to the decrease (increase) of T_R proportional to φ_0 . A discussion of the gravitine problem and of the baryon asymmetry generation in our scenario is contained in ref. [15].

As it was shown above, the observable part of the universe in our scenario was formed when the field Ψ was in the interval $|\Psi - \varphi_0| \lesssim 2$. Therefore this scenario can be realized not only in the theory with the superpotential (9), but in any theory, in which $\Psi(\Xi)$ is approximately given by (9) in the interval $|\Xi - \varphi_0| \lesssim 2$. Thus, we have found a rather wide class of superpotentials, which have all properties necessary for the realization of the chaotic inflation scenario in super-

gravity [9]. An attractive feature of the proposed scenario is that it does not contain any small parameters except for the parameter $M \sim 10^{-2}$. In this respect our scenario differs from all previous versions of the inflationary universe scenario suggested so far.

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