IMPACT-PICTURE EXPECTATIONS FOR VERY HIGH ENERGY
ELASTIC pp AND $\bar{p}p$ SCATTERING

C. Bourrely
 CERN - Geneva

J. Soffer
 Centre de Physique théorique, CNRS
 Marseille

T.T. Wu
 Harvard University,
 Cambridge, MA

A B S T R A C T

We review the predictions of the impact-picture for the many features which characterize elastic nucleon-nucleon scattering at very high energy. The comparison with recent CERN data up to SPS collider energy shows the consistency of this theoretical approach and we give some predictions for the next generation of collider machine with the highest energy $\sqrt{s} = 40$ TeV.

*) Permanent address: Centre de Physique théorique, CNRS, Marseille.

IMPACT-PICTURE EXPECTATIONS

1. INTRODUCTION.

In the limit of extremely high energies, the general properties of relativistic quantum field theory, make possible to deduce some essential features of high energy hadron scattering physics [1]. Among the simplest processes one can think of, hadron-hadron elastic scattering has been extensively studied and one interesting result obtained from this theoretical analysis is the prediction of rising total cross section [2] which was first observed experimentally for pp at CERN-ISR in 1973 [3]. Since then, in more than one decade of operation, a fair amount of data has been accumulated at the ISR for pp total cross section, real part of the forward hadronic amplitude and differential cross section. More recently, systematic measurements of the same quantities for $\bar{p}p$ scattering [4-6] have been performed at ISR energies and at the CERN collider [7,8]. Even at that collider energy $\sqrt{s} = 550$ GeV, which is at the moment the highest one obtained in a laboratory, s is still finite and to describe the data we have to use phenomenology compatible with the $s \rightarrow \infty$ behaviour of quantum field theory.

After a first attempt of this impact-picture phenomenology [9], we have proposed several improvements a few years ago [10] and the purpose of this paper is to compare it with the new data. We will see that we can explain, in a simple way, the energy dependence of the elastic differential cross section both for the forward peak and for large t values, from ISR energies up to collider energy. More specifically, we understand the experimental fact that the cross section at ISR energies drops by seven orders of magnitude from the optical point to the first structure and only to five orders of magnitude at the $\bar{p}p$ collider. We predict only four orders of magnitude at the energy $\sqrt{s} = 40$ TeV.

The paper is organized as follow. In section 2 we shall recall the main features of the model we use to describe diffraction scattering and we will explain our parametrization. Section 3 is devoted to the presentation of the numerical results which will be compared with the present experimental situation together with some predictions for future collider machines. Section 4 covers a discussion of the implications of other models based on different physical considerations and we give our concluding remarks in section 5.

2. DESCRIPTION OF THE MODEL.

In the impact parameter representation, the spin independent amplitude ¹ for $\bar{p}p$ and pp elastic scattering reads as

$$a_0(s, t) = \frac{is}{2\pi} \int e^{-i\vec{q}\vec{b}} (1 - e^{-\Omega_0(s, \vec{b})}) d\vec{b} \quad (1)$$

where $\vec{q}$ is the momentum transfer ($t = -\vec{q}^2$) and $\Omega_0(s, \vec{b})$ is defined to be the opaqueness at impact parameter b and at a given energy s . We define Ω_0 as a sum of two terms

$$\Omega_0(s, \vec{b}) = S_0(s) F(\vec{b}^2) + R_0(s, \vec{b}) \quad (2)$$

the first term associated to the "Pomeron" exchange generates the diffractive component of the scattering, the second term is the Regge background, whose exact expression can be found in ref. [10]; it is needed to separate pp and $\bar{p}p$ but it dies out rapidly as s increases. The function $S_0(s)$ comes from the high energy behaviour of quantum field theory [1] and we shall use the crossing symmetric expression

$$S_0(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}} \quad (3)$$

where u is the third Mandelstam variable. Of course the fact that the diffractive contribution to the opaqueness is assumed to be factorized into the product of a function of energy and a function of the impact parameter is a simplifying assumption. Let us emphasize that the factorization property is valid for the Born term only, the amplitude $a_0(s, t)$ obtained by eikonalization does not have this property. The choice one makes for $F(b^2)$ is crucial; we will not use the arbitrary form as in [9], where it was chosen on

¹ Here we will neglect the spin-dependent part of the amplitude which has been considered in refs. [10-12] for the description of polarizations and spin correlation parameters.

IMPACT-PICTURE EXPECTATIONS

the basis of computational convenience. Instead, as explained in ref. [10] we will take it, such as its Fourier transform $F(t)$ is proportional to the square of the electromagnetic proton form factor $G(t)$ [13,14] up to a slowly varying function, namely

$$\tilde{F}(t) = f [G(t)]^2 \left(\frac{a^2 + t}{a^2 - t} \right) \quad (4)$$

This ansatz is based on the approximate proportionality between the charge density and the internal hadronic matter distribution inside a proton [15]. A reasonable parametrization of $G(t)$ is

$$G(t) = \frac{1}{\left(1 - \frac{t}{m_1^2}\right) \left(1 - \frac{t}{m_2^2}\right)} \quad (5)$$

and the last factor in eq.(4) stands for the remaining t dependence which is unknown². To summarize, the scattering amplitude reduce to a Bessel transform

$$a_0(s, t) = i s \int_0^\infty J_0(b\sqrt{-t}) (1 - e^{-\Omega_0(s, b)}) b db \quad (6)$$

where all quantities are defined by eqs.(2-5). This completes³ the description of the model and we now turn to the numerical results.

² This factor seems also to be required in an analysis of hadronic matter from factor [16].

³ The normalization is such that $d\sigma/dt = \pi |a_0(s, t)|^2/s^2$, so that the amplitude is dimensionless.

3. NUMERICAL RESULTS.

The diffractive part of the scattering amplitude $a_0(s,t)$ is parametrized according to eqs.(2,3 and 4) by means of six parameters that we have fitted using the available pp and $\bar{p}p$ data for the total cross section σ_{tot} , the ratio of the real to imaginary parts of the forward hadronic amplitude $\rho = \text{Re } a_0 / \text{Im } a_0$ [3,5-8,17-19,20], and the differential cross section $d\sigma/dt$ from ISR up to SPS collider energy [4,7,8,17,21]. We have obtained the following set of values

$$\begin{aligned} c &= 0.167 & c' &= 0.748 \\ m_1 &= 0.586 \text{ GeV} & m_2 &= 1.704 \text{ GeV} \\ a &= 1.953 \text{ GeV} & f &= 7.115 \text{ GeV}^{-2} \end{aligned} \quad (7)$$

The asymptotic energy behaviour of the hadronic interactions is determined by c and c' , and the above values are not too different from those we used in ref.[10]. It is also the case for the values of the other four parameters ⁴, so this means that our impact-parameter phenomenology is fairly reliable when extrapolated to higher energy.

We have also investigated a solution with $c' \geq 1$ which leads to a higher value of σ_{tot} at SPS collider energy, and gives large deviations for $d\sigma/dt$ in pp scattering at ISR energies with an underestimate of the data by a factor 1.5 to 2 for $|t|$ above 0.8 GeV². This second solution is certainly not favoured by present ISR data. So in this model the energy dependence is controlled by the forward $d\sigma/dt$ and σ_{tot} , but as well as by the large t behaviour. Concerning the Regge background contribution R_0 (see eq. 2) of the scattering amplitude, it is mainly needed to separate σ_{tot} for pp and $\bar{p}p$ and its determination was found to be very similar to that of ref.[10]. It also leads to a small difference between pp and $\bar{p}p$ for $d\sigma/dt$ which is noticeable below $\sqrt{s} = 30.5$ GeV (see fig. 3).

The correct energy behaviour of the forward amplitude is shown in figs. 1 and 2 where we have plotted σ_{tot} and ρ versus $\sqrt{s}$, respectively. The value of ρ which was rising in the ISR range is not expected to go above 0.13 at SPS collider energy and for higher energy it decreases slowly to zero. In fig. 3 we compare our predictions with pp data for $d\sigma/dt$ at

⁴ In our previous work [10] eq.(7) contains a misprint, and it should read $\tilde{\Omega}_0(s,t) = i s S_0(s) \tilde{F}(t) + \tilde{R}_0(s,t)$. As a result f has dimension of GeV⁻².

different ISR energies and we also show the theoretical curves for $\bar{p}p$ elastic scattering.

For $\bar{p}p$ scattering we found in $d\sigma/dt$ a crossover with pp around $|t| = 0.2 \text{ GeV}^2$, away from the forward direction, the difference between the two processes becomes visible for $\sqrt{s} \leq 30.5 \text{ GeV}$, in that case the dip is slightly shifted towards smaller $|t|$ value and less pronounced than in pp . Now if we increase the energy $\bar{p}p$ and pp differential cross sections tend to become equal as it results from the vanishing of the Regge background and from the crossing symmetric diffractive part (see eq. 3). It will be interesting to confront them with data away from the forward peak which are soon expected from the last ISR measurements [4] ⁵. We have a fair description of the pp data, in particular, the dip-bump structures at the right position and the correct slope of the cross section out to large t values with no second dip.

The recent data from the CERN $\bar{p}p$ collider at $\sqrt{s} = 550 \text{ GeV}$ for $d\sigma/dt$ out to $|t| = 1.6 \text{ GeV}^2$ is shown in fig. 4 with our model prediction ⁶, again the agreement is quite satisfactory, in particular, we are able to produce a shoulder at $|t| = 0.8 \text{ GeV}^2$ instead of a pronounced dip. The analysis of the forward peak, which is narrower at the collider than at ISR, is also well described as shown in fig. 5, where we plot the expected logarithmic slope of $d\sigma/dt$ versus t in the interval $0. - 0.8 \text{ GeV}^2$.

The variation of the logarithmic slope as a function of t looks like a parabola, because the imaginary part of the diffractive amplitude begins with a curvature oriented upward, then it vanishes and goes downward due to the presence of a zero around 0.9 GeV^2 (we notice that there is a small influence of the real part which vanishes around 0.3 GeV^2 , see fig. 6). This sort of behaviour for the slope is already observed at lower energy [20,23]. The present analysis of the UA4 experiment [7] fitted the slope with a straight line in the interval $0.03 \leq |t| \leq 0.5 \text{ GeV}^2$, the inclusion of more data up to the shoulder will probably induce a deviation from a straight line.

We have also represented in fig. 6 the magnitude of the imaginary part of the amplitude at $\sqrt{s} = 550 \text{ GeV}$ and we see on this graph that at the

⁵ It is unfortunate that precocious the shutdown of the ISR has not allowed a data taking for $\bar{p}p$ comparable to pp .

⁶ A direct extrapolation of ref.[10] shows already the main features presented here (see ref.[22]).

position of the zero, the real part is substantial and this is the reason for the shoulder in $d\sigma/dt$. A summary of some observables is collected in table 1. In particular the ratio σ_{el}/σ_{tot} is predicted to be 0.21, in good agreement with the data value 0.213 ± 0.008 [7].

We find interesting to display in fig. 7 one particular feature of our model, that is, the first zero position $|t_0|$ of the imaginary part of the diffractive amplitude as a function of $\sqrt{s}$, in the same figure full circles represent the geometrical relation $|t_0| = \text{const}/\sigma_{tot}$. As a consequence, we expect the corresponding structure in $d\sigma/dt$ to move in to lower t values as $\sqrt{s}$ increases.

In view of the new energy domain accessible to future collider machines (SSC, LHC) we show in fig. 8 our predictions for $d\sigma/dt$ at $\sqrt{s} = 2, 10, 20$ and 40 TeV. We note that the dip-bump structure is gradually restored as s increases and we find a new shoulder moving in from $|t| = 4.2 \text{ GeV}^2$ to around $|t| = 1.7 \text{ GeV}^2$, this shoulder is due to the appearance of a second zero in the diffractive imaginary part which is filled in by the real part. We predict that at the highest energy the cross section drops only by four orders of magnitude from the optical point to the dip-bump structure, in clear contrast with the situation at ISR energies. The observation of such a pattern at these very high energies will be an important step which might stimulate new theoretical developments. In table 1, we summarize some characteristic features associated with fig. 8. We observe that the slope near the forward direction increases with energy, and that σ_{el}/σ_{tot} is also increasing, its limiting value is $1/2$ for $s = \infty$.

4. COMPARISON WITH OTHER MODELS.

Several models have proposed to describe high energy pp and $\bar{p}p$ elastic scattering and we would like now to discuss them briefly and compare their predictions with ours.

The model proposed by Donnachie and Landshoff [24] is essentially based on three main contributions to the scattering amplitude at high energy : the single Pomeron exchange for low t region, the double Pomeron exchange for medium t region and the triple gluon exchange which dominates at large t . The dip-bump structure seen in pp at the ISR is produced by a cancellation in the imaginary part of the Pomeron between single and double exchanges, while the real part of the Pomeron is balanced by the triple gluon term. This last contribution which is crossing odd, changes sign when one goes from pp to $\bar{p}p$, therefore the real part survives for $\bar{p}p$

and is expected to exhibit only shoulders both at ISR energies and at collider energy.

Clearly the predictions of this model, which gives a fair description of pp at ISR out to large t values, are very different from ours for $\bar{p}p$ at ISR energies and it will be interesting, in the near future, to compare them with data. In the case where the measurement of the cross section cannot provide a definite conclusion, a measurement of the $\bar{p}p$ polarization will give another check. We know from experiment that the pp polarization at 150 and 300 GeV/c [30] shows a marked minimum at the dip position of the cross section, the absence of a minimum for $\bar{p}p$ polarization will imply no dip in the cross section. Unfortunately at collider energy, the predicted differential cross section from this model is definitely too low, for $|t| > 0.8 \text{ GeV}^2$, by one order of magnitude compared to the UA4 data [7].

In the geometrical scaling model proposed by Dias de Deus and Kroll [25], the elastic cross section is expressed in terms of a scaling function $\Phi(\tau)$ where $\tau = -t/\sigma_{\text{tot}}$ is the scaling variable. The following form has been suggested [26]

$$\frac{d\sigma}{dt} = \frac{(\sigma_{\text{tot}})^2}{16\pi} \left\{ \phi^2(\tau) + \rho \left(\frac{d}{d\tau} \phi(\tau) \right)^2 \right\} \quad (8)$$

given the fact that $\sigma_{\text{el}}/\sigma_{\text{tot}}$ for pp is almost constant in the ISR range, they assume that the asymptotic regime is clearly reached there, so they can determine numerically Φ by fitting the pp data at $\sqrt{s} = 53 \text{ GeV}$. The model can then predict the cross section at any other energy by inserting the relevant values of σ_{tot} and ρ in eq.(8). The dip-bump structure in pp at ISR requires a zero in the scaling function Φ , but since $\sigma_{\text{tot}}(\bar{p}p) > \sigma_{\text{tot}}(pp)$ one anticipates a $\bar{p}p$ dip at a smaller $|t|$ than for pp in the ISR energy range. At collider energy, from the expected large value of ρ , the model predicts correctly a shoulder instead of a dip, but in this t region it fails badly because, as the previous model, the cross section is clearly [27] below the data.

In the geometrical picture of Chou and Yang [28], as the energy increases the colliding hadrons become more opaque with a black disk behaviour as a limit that is $\sigma_{\text{el}}/\sigma_{\text{tot}} = 1/2$. Therefore we are still far from asymptopia and the energy dependence is entirely given by the value of σ_{tot} . This is the only parameter which fixes the value of $\sigma_{\text{el}}/\sigma_{\text{tot}}$, the slope and the position of the structures in the elastic cross section. Many dips are predicted and in particular a second dip around $|t| = 4.5 \text{ GeV}^2$ at

ISR energy which is absent in the pp data. Of course, real part effects may fill the dips and this is what they have proposed at collider energy to turn the first dip into a shoulder [29]. Unfortunately the resulting cross section is still too high in the shoulder region by a factor three to four compared to the UA4 data. The fact that the determination of the real part requires to adjust the free parameter ρ at each energy is a weakness of this model and of the previous one. In our case the real part is built in and we predict both its t dependence and its energy behaviour. A prediction for $d\sigma/dt$ at collider energy from different models discussed above is shown in fig. 9.

Finally let us mention the critical Reggeon Field Theory (RFT) which has been proposed to describe diffraction scattering at asymptotic energy [31]. However the predictions for $d\sigma/dt$ at SPS collider energy fails [32] and σ_{el}/σ_{tot} decreases when one goes from ISR to SPS collider, contrarily to the trend of the data. Since RFT is an asymptotic theory, the pending question remains " where asymptopia begins to set in ? ".

5. CONCLUDING REMARKS.

We have presented a review of the predictions of the impact-picture for pp and $\bar{p}p$ elastic scattering from ISR up to the CERN collider energies. This theoretical approach appears to be rather unique to give a fair description of both the pp data at ISR up to large t values and of the $\bar{p}p$ data at the collider. Further comparison of our predictions with new data will be possible in the near future and in particular for $\bar{p}p$ at ISR and at FNAL collider. We have also calculated all the relevant parameters which characterize elastic scattering up to $\sqrt{s} = 40$ TeV and we found an interesting pattern emerging. Let us hope that this superhigh energy domain where hadrons become larger and larger will not be left unexplored before long.

We thank G. Matthiae and M. Haguenaue for their comments on the UA4 experiments. Work supported in part by the U.S. Department of Energy Contract n° DE-ASO2-76ERO3227.

REFERENCES

- [1] H. Cheng and T.T. Wu, Phys. Rev. Lett. 22(1969)666 ; 23(1969) 1311; 24(1970)759; Phys. Rev. 182(1969)1852; D1(1970)2775.
- [2] H. Cheng and T.T. Wu, Phys. Rev. Lett. 24(1970)1456.
- [3] U. Amaldi et al, Phys. Lett. 44B(1973)112; S.R. Amendolia et al ,Phys. Lett. 44B(1973)119.
- [4] A. Breakstone et al., Contribution to the International Europhysics Conference on High Energy Physics, Brighton (UK) 1983
- [5] N. Amos et al., Phys. Lett. 120B(1983)460.
- [6] M. Ambrosio et al., Phys. Lett. 115B(1982)495.
- [7] UA4 Collaboration, R. Battiston et al., Phys. Lett. 115B(1982) 333; 117B(1982)126; 127B(1983)472.
G. Matthiae et al., Contribution to the International Europhysics Conference on High Energy Physics, Brighton (UK) 1983.
- [8] UA1 Collaboration, G. Arnison et al., Phys. Lett. 121B(1983)472.
- [9] H. Cheng, J.K. Walker and T.T. Wu, Phys. Lett. 44B(1973)97.
- [10] C. Bourrely, J. Soffer and T.T. Wu, Phys. Rev. D19(1979)3249.
- [11] C. Bourrely, J. Soffer and T.T. Wu, Phys. Lett. 76B(1978)481.
- [12] C. Bourrely, H.A. Neal, H.A. Ogren, J. Soffer and T.T. Wu, Phys. Rev. D26(1982)1781.
- [13] T.T. Wu and C.N. Yang, Phys. Rev. 137(1965)B708.
- [14] T.T. Chou and C.N. Yang, Phys Rev. 170(1968)1591.
- [15] H. Cheng and T.T. Wu, Phys. Rev. 184(1969)1868.
- [16] S. Sanielevici and P. Valin, Phys. Rev. D29(1984)52.
- [17] For a complete list of references see : A.N. Diddens and K. Schubert, in Landolt-Borstein, Elastic and charge exchange scattering of elementary particles, vol 7 (1973), vol 9 (1980), Edit. H. Schopper.
- [18] S.P. Denisov et al., Nucl. Phys. B65(1973)1.
A.S. Carroll et al., Phys. Lett. 80B(1979)423.
J. Allaby et al., Phys. Lett. 62B(1976)460.
- [19] K.J. Foley et al., Phys. Rev. Lett. 19(1967)857.
G.G. Beznogikh et al., Phys. Lett. 39B(1972)411.
V. Bartenev et al., Phys. Rev. Lett. 31(1973)1367.
U. Amaldi et al., Phys. Lett. 66B(1977)390.
- [20] L. Fajardo et al., Phys. Rev. D24(1981)46.

- [21] G. Barbiellini et al., Phys. Lett. 39B(1972)663.
A. Bohm et al., Phys. Lett. 49B(1974)491.
N. Kwak et al., Phys. Lett. 58B(1975)233.
H. de Kerret et al., Phys. Lett. 62B(1976)363; 68B(1977)374.
U. Amaldi et al., Phys. Lett. 66B(1977)390.
L. Baksay et al., Nucl. Phys. B141(1978)1.
E. Nagy et al., Nucl. Phys. B150(1979)221.
- [22] J. Soffer, Invited talk at the 3rd International Conference on "Physics in collision : high energy ee/ep/pp interactions" Como (1983), Preprint CPT-83/p.1548.
- [23] J. Burq et al., Nucl. Phys. B217(1983)285.
- [24] A. Donnachie and P.V. Landshoff, Phys. Lett. 123B(1983)345, Nucl. Phys. B231(1983)189, Preprint DAMTP 84/6 (1984).
- [25] J. Dias de Deus and P. Kroll, Acta Phys. Pol. B9(1978)159.
- [26] A. Martin, Nuovo Cimento Lett. 7(1978)811.
- [27] P. Kroll, J. Phys. G9(1983)L81.
- [28] T.T. Chou and C.N. Yang, Phys. Rev. D19(1979)3268.
- [29] T.T. Chou and C.N. Yang, Phys. Lett. 128B(1983)457.
- [30] G. Fidecaro et al., Nucl. Phys. B173(1980)513.
J.H. Snyder et al., Phys. Rev. Lett. 41(1978)781.
- [31] M. Moshe, Phys. Rep. 37C(1978)255.
- [32] J. Baumel, M. Feingold and M. Moshe, Nucl. Phys. B198(1982)13.
M. Baig, J. Bartels and J. Dash, preprint CPT-83/PE.1514 (June 1983).

TABLE 1.

Summary of some observables in the TeV domain.

$\sqrt{s}$ (TeV)	0.55	10.	20.	40.
σ_{tot} (mb)	62.	98.4	109.4	121.2
$\rho(s, t=0)$	0.13	0.122	0.118	0.114
$\langle \text{slope} \rangle$ (GeV^{-2}) $0 \leq t \leq 0.15$ (GeV^2)	15.6	18.6	19.6	20.7
$\sigma_{\text{el}}/\sigma_{\text{tot}}$	0.21	0.272	0.287	0.3
Imag. Part Pomeron. First zero (GeV^2)	0.85	0.47	0.41	0.37
Second zero		2.35	1.97	1.67

FIGURE CAPTIONS

- Fig. 1 σ_{tot} for pp (black points) and $\bar{p}p$ (open points), (data from refs. [3,5-8,17,18]) and our predictions up to future collider machines.
- Fig. 2 $\rho = \text{Re}a_0/\text{Im}a_0$ for pp (black points) and $\bar{p}p$ (open points) (data from refs. [5,6,17,19,20]) as a function of $\sqrt{s}$.
- Fig. 3 $d\sigma/dt$ for pp at $\sqrt{s} = 30.5, 53$ and 62 GeV (data are from refs. [17,21]) compared to our model fit (solid curve). Our prediction for $\bar{p}p$ is the dashed curve.
- Fig. 4 $d\sigma/dt$ for $\bar{p}p$ at $\sqrt{s} = 550$ GeV (preliminary data are from ref. [7]) and our model fit.
- Fig. 5 Logarithmic slope parameter of $d\sigma/dt$ for $\bar{p}p$ at $\sqrt{s} = 550$ GeV, (straight lines are from ref. [7]).
- Fig. 6 The real part (dashed curve) and the imaginary part (solid curve) of the elastic $\bar{p}p$ amplitude at $\sqrt{s} = 550$ GeV.
- Fig. 7 First zero position of the imaginary part of the diffractive amplitude versus $\sqrt{s}$. Full circles are the geometrical scaling relation $|\text{t}_0| = \text{const}/\sigma_{\text{tot}}$.
- Fig. 8 Predictions for $d\sigma/dt$ at $\sqrt{s} = 2, 10, 20$ and 40 TeV.
- Fig. 9 Comparison between different models for $\bar{p}p$ $d\sigma/dt$ at collider energy. Solid curve: our model, dotted curve: ref.[29] for $\rho = 0.2$, dashed curve: ref.[24], dashed-dotted: curve ref.[27] for $\sigma_{\text{tot}} = 66$ mb.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 6

Fig. 5

Fig. 7

Fig. 8

Fig. 9