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TRIVIALITY OF THE S-MATRIX
IN THE QUANTUM LIOUVILLE FIELD THEORY

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ABSTRACT

Within the framework of the translation non-invariant quantization suggested recently by D'Hoker, Freedman and Jackiw, it is established that the exact S-matrix of the quantum Liouville field theory is trivial. In connection with this result, we clarify some geometrical aspects of the theory as interpreted as a model for quantum gravity in two dimensions.

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Quantization of the Liouville field theory has attracted much attention. Most of the recent works have mainly been intended for application to string theories [1-3]. However, viewed as an ordinary field theory model in two space-time dimensions, the Liouville system has several peculiar properties [4]. For instance, it seems that a normalizable translation-invariant ground state does not exist. Concerning this problem, D'Hoker, Freedman and Jackiw [5] have made a proposal that the quantum theory be defined on background fields which are translation non-invariant^{*)}. They showed that there exists a well-defined perturbation theory, without infrared divergence, around such a background field, and that the tree level S-matrix is trivial. The question whether the S-matrix is trivial in the full-fledged quantum theory is important for understanding the quantum-theoretical significance of the geometrical nature of the Liouville system. This question, however, remained unanswered in their work. In the present note we shall re-examine this question, and establish that the exact S-matrix is indeed trivial if a suitable revision is made in the formulation.

As is well known, the Liouville equation

$$\square\phi + \frac{m^2}{\beta} e^{\beta\phi} = 0 \quad (1)$$

determines a two-dimensional space-time with constant (negative) curvature, in the conformal gauge in which the metric tensor takes the form $g_{\mu\nu} = \eta_{\mu\nu} \exp \beta\phi$, with $\eta_{\mu\nu}$ being the flat metric^{**)}. The conformal

*) This reminds us of an earlier proposal of Fubini [6] for general conformally invariant field theories.

***) We use the Pauli metric. $\square \equiv -\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$

invariance of (1) merely reflects the freedom of coordinate transformation remaining in the conformal gauge. The classical Hamiltonian structure corresponding to this geometrical property has been elucidated by Teitelboim [7]. If we decompose the general metric tensor in the form

$$g_{\mu\nu} = e^{\beta\phi} \begin{bmatrix} (\gamma^1)^2 - (\gamma^\perp)^2 & \gamma^1 \\ \gamma^\perp & 1 \end{bmatrix}, \quad (2)$$

the generator which describes the deformation of the Cauchy curve at "time" x^0 to the curve at "time" $x^0 + \delta x^0$ is given by $H\delta x^0$:

$$H = \int dx^1 \left[\gamma^\perp(x) \mathcal{H}_\perp(x) + \gamma^1(x) \mathcal{H}_1(x) \right] \quad (3)$$

$$\mathcal{H}_\perp = \frac{1}{2} (\pi^2 + \phi'^2) - \frac{2}{\beta} \phi'' + \frac{m^2}{\beta^2} e^{\beta\phi} \quad (4a)$$

$$\mathcal{H}_1 = \pi\phi' - \frac{2}{\beta} \pi' \quad (4b)$$

where $\pi(x)$ is the conjugate momentum for $\phi(x)$. The $H_\perp$ and H_1 satisfy the following Poisson bracket algebra:

$$[\mathcal{H}_\perp(x), \mathcal{H}_\perp(y)]_{PB} = \partial_x \delta(x, y) (\mathcal{H}_1(x) + \mathcal{H}_1(y)) \quad (5a)$$

$$[\mathcal{H}_1(x), \mathcal{H}_\perp(y)]_{PB} = \partial_x \delta(x, y) (\mathcal{H}_\perp(x) + \mathcal{H}_\perp(y)) - x \partial_x^3 \delta(x, y) \quad (5b)$$

$$[\mathcal{H}_1(x), \mathcal{H}_1(y)]_{PB} = \partial_x \delta(x, y) (\mathcal{H}_1(x) + \mathcal{H}_1(y)) \quad (5c)$$

with a non-vanishing central charge $Z = 4/\beta^2$. The algebra (5) expresses the fact that the evolution from a given initial curve, embedded in a two-dimensional space-time, to a given final curve is independent of the paths of deformation. By choosing $\eta^\perp = 1$ and $\eta^1 = 0$, we find (3) to be the Liouville Hamiltonian. A consequence of this and the Hamilton equation of motion is that $H_\perp$ and H_1 satisfy $\dot{H}_\perp = H'_1$ and $\dot{H}_1 = H'_\perp$ and hence are free massless fields. This also reflects that $H_\perp$ and H_1 coincide with the diagonal and off-diagonal components, respectively, of the improved energy momentum tensor of the Liouville system. Because of these properties, the Liouville model is also attractive as a possible model for quantum gravity in two dimensions [7,8].

Now the observation that $H_\perp$ and H_1 satisfy the free field equation will immediately lead to the triviality of the classical (or tree level) S-matrix [5] provided that the asymptotic field is massless and couples with $H_\perp$ or H_1 . It is thus reasonable to expect that the preservation of (5) at the quantum level will result in the triviality of the exact S-matrix. It is in fact already known that, if one naïvely defines a quantum theory by performing the normal ordering [2,4] with respect to a free translation-invariant Fock vacuum, (5) can be preserved. Unfortunately, however, this does not solve our problem, since i) as mentioned above it is highly plausible that there exists no normalizable and translation-invariant state in an exact theory, and ii) it is not known how to define the asymptotic states without knowing the nature of the ground state. In the naïve translation-invariant quantization, any perturbative attempts toward checking the properties of would-be asymptotic states are marred by the infrared divergences in the infinite volume limit.

Therefore, let us follow the suggestion made in ref. [5] and examine^{*)} whether it is possible to preserve the geometrical structure expressed in (5) within the new scheme. This task is not trivial because there is, at least apparently, no canonical transformation which could connect both schemes.

The quantization starts with the splitting $\varphi = \varphi_s + \hat{\varphi}$, where φ_s is a classical static solution of (1). In general, φ_s is left invariant under $SO(2,1)$ subalgebra of the conformal algebra. Hence Poincaré symmetry in ordinary quantization is now replaced by the $SO(2,1)$ symmetry.

Following [5], we choose the simplest solution

$$\phi_s(x^\mu) = -\beta^{-1} \ln \frac{m^2 r^2}{2}, \quad (6)$$

where $r = x^1 - x_0^1$, and x_0^1 is an arbitrary integration constant associated with the translation invariance of (1). The key feature of this quantization is that there is no normalizable zero mode, and the complete set of the linearized oscillation modes about (6) are given by $e^{\pm i\omega t} \psi_\omega(r)$, with $\psi_\omega(r) = (|\omega|r)^{1/2} J_{3/2}(|\omega|r)$, which satisfy

$$\begin{aligned} \int_0^\infty d\omega \psi_\omega(r) \psi_\omega(r') &= \delta(r-r') \\ \int_0^\infty dr \psi_\omega(r) \psi_{\omega'}(r) &= \delta(\omega-\omega') \end{aligned} \quad (7)$$

Thus, unlike the quantization around solitons, Poincaré invariance is never restored. This should be expected because the Poincaré group is not

*) Another possible approach might be to try to define a finite-volume counterpart of the S-matrix and finally to take the limit of infinite volume. We do not know how to carry out this.

a symmetry of space-times with constant (non-zero) curvature. Equations (7) further show that the representation of the canonical commutation relation (CCR) should be built on a half (infinite) space, e.g. $r \geq 0$. Since $\psi_\omega(0) = \psi'_\omega(0) = 0$, there is no communication to the other half space. We therefore postulate the following expansion as a representation of CCR:

$$\begin{aligned}\hat{\phi}(r) &= \int_0^\infty \frac{d\omega}{\sqrt{2\omega}} (a_\omega + a_\omega^\dagger) \psi_\omega(r) \\ \hat{\pi}(r) &= -i \int_0^\infty d\omega \sqrt{\frac{\omega}{2}} (a_\omega - a_\omega^\dagger) \psi_\omega(r) \\ [a_\omega, a_{\omega'}^\dagger] &= \hbar \delta(\omega - \omega')\end{aligned}\tag{8}$$

and define the normal ordering with respect to the creation and annihilation operators $a_\omega^\dagger$ and a_ω which diagonalize the free Hamiltonian $H^{(0)} = \int_0^\infty 1/2 [\hat{\pi}^2 + \hat{\phi}'^2 + 2\hat{\phi}^2/r^2] dr$. This prescription removes all ultraviolet divergences in the loop expansion. Also, there is no infrared divergence because $\psi_\omega(r) \sim O(\omega^2)$ as $\omega \rightarrow 0$.

We now come to the main question: What are the deformation generators H_μ satisfying (5)? We may first try to see whether the simple substitution $\varphi \rightarrow \varphi_S + \hat{\phi}$ in the normal ordered expression of ref. [4] meets our requirement. The answer is no. Thus there is no canonical transformation, even formally, which connects both quantization schemes. Fortunately, however, a slight modification can be shown to work. Let us first define

$$\begin{aligned}L_\pm(r) &\equiv : \frac{1}{2} A_\pm(r)^2 : \mp \frac{2}{\zeta} \left(\frac{d}{dr} + \frac{1}{r} \right) A_\pm(r) \\ A_\pm(r) &\equiv \hat{\pi}(r) \pm \hat{\phi}'(r)\end{aligned}\tag{9}$$

Formulas (9) satisfy the commutation relations

$$\begin{aligned}
 [L_{\pm}(r), L_{\pm}(r')] = & \pm 2i\hbar \partial_r \delta(r-r') (L_{\pm}(r) + L_{\pm}(r')) \\
 & \pm i\hbar \frac{8}{\beta^2} \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \left(\frac{\partial}{\partial r'} + \frac{1}{r'} \right) \partial_r \delta(r-r') \\
 & + \frac{1}{2} \left([A_{\pm}^{(+)}(r), A_{\pm}^{(-)}(r')]^2 - [A_{\pm}^{(+)}(r'), A_{\pm}^{(-)}(r)]^2 \right) \quad (10a)
 \end{aligned}$$

$$[L_{+}(r), L_{-}(r')] = 0 \quad (10b)$$

$$\begin{aligned}
 [L_{\pm}(r), :e^{\hat{\phi}(r')}:] = & -i\hbar \delta(r-r') :e^{\hat{\phi}(r')} p A_{\pm}(r') : \\
 & \pm i\hbar \frac{2\hbar}{\beta} \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \delta(r-r') :e^{\hat{\phi}(r')} : \\
 & + \frac{\hbar^2}{2} \left([A_{\pm}^{(+)}(r), \hat{\phi}^{(-)}(r')]^2 - [A_{\pm}^{(-)}(r), \hat{\phi}^{(+)}(r')]^2 \right) :e^{\hat{\phi}(r')} : \quad (10c)
 \end{aligned}$$

where the superscripts (+) and (-) denote the annihilation and creation part, respectively. In terms of $\psi_{\omega}(r)$, the commutators in the r.h.s. of (10a) and (10c) are given by

$$\begin{aligned}
 [A_{\pm}^{(+)}(r), A_{\pm}^{(-)}(r')] = & \frac{\hbar}{2} \int_0^{\infty} d\omega \left[\pm i \psi'_{\omega}(r) \psi_{\omega}(r') \mp i \psi_{\omega}(r) \psi'_{\omega}(r') \right. \\
 & \left. + \omega \psi_{\omega}(r) \psi_{\omega}(r') + \frac{1}{\omega} \psi'_{\omega}(r) \psi'_{\omega}(r') \right] \quad (11a)
 \end{aligned}$$

$$[A_{\pm}^{(+)}(r), \hat{\phi}^{(-)}(r')] = \frac{\hbar}{2} \int_0^{\infty} d\omega \left[-i \psi_{\omega}(r) \psi_{\omega}(r') \pm \frac{1}{\omega} \psi'_{\omega}(r) \psi_{\omega}(r') \right] \quad (11b)$$

It is not difficult to see that (11) have the following short-distance behaviours

$$[A_{\pm}^{(+)}(r), A_{\pm}^{(-)}(r')] = \frac{\hbar}{\pi} \left\{ -\frac{1}{(r-r' \mp i\epsilon)^2} + \frac{2}{(r+r')^2} + \overline{\mathcal{S}}(r, r') \right\} \quad (12a)$$

$$[A_{\pm}^{(n)}(r), \hat{\phi}^{(\pm)}(r')] = \frac{\tau_1}{2\pi} \left\{ \mp \frac{1}{r-r' \mp i\epsilon} \pm \frac{1}{r+r'} \pm \bar{S}(r, r') \right\}, \quad (12b)$$

where the remainder parts $\bar{S}(r, r')$ and $\bar{L}(r, r')$ are real and satisfy $\bar{S}(r, r') = \bar{S}(r', r) = O[(r-r')^2, (r-r')^2 \log(r-r')]$, $\bar{L}(r, r') = O[r-r', (r-r') \log(r-r')]$. Then, using the formulas

$$(x-y \mp i\epsilon)^n - (x-y \pm i\epsilon)^n = \pm 2\pi i (-)^{n-1} \frac{1}{(n-1)!} \delta^{(n-1)}(x-y),$$

we find that the last terms of the r.h.s. of (10a) and (10c) take the form $\mp i\hbar^2 \partial_r^3 \delta(r-r')/6\pi \pm 4i\hbar^2 \partial_r \delta(r-r')/\pi(r+r')^2$ and $\mp i\hbar^2 (\partial_r + r^{-1}) \delta(r-r') \times :e^{\hat{p}\hat{\phi}(r')}:/4\pi$, respectively. Putting all these results together, we establish that

$$\begin{aligned} \mathcal{L}_{\perp}^{\hat{\phi}}(r) \equiv & : \frac{1}{2} (\hat{\pi}(r)^2 + \hat{\phi}(r)^2) : - \frac{2}{\beta} \hat{\phi}''(r) - \frac{2}{\beta r} \hat{\phi}'(r) \\ & + \frac{2}{\beta^2 r^2} \frac{m^2}{\mu^2} : e^{\hat{p}\hat{\phi}(r)} : + \frac{\sigma}{r^2} \end{aligned} \quad (13a)$$

$$\mathcal{L}_{\parallel}^{\hat{\phi}}(r) \equiv : \hat{\pi}(r) \hat{\phi}'(r) : - \frac{2}{\beta} \left(\hat{\pi}'(r) + \frac{1}{r} \hat{\pi}(r) \right) \quad (13b)$$

satisfy the required commutation relations corresponding to (5), provided that

$$\frac{p}{\beta} - \frac{p^2 \tau_1}{8\pi} = 1 \quad (14a)$$

$$z = \frac{4}{\beta^2} + \frac{\tau_1}{12\pi} \quad (14b)$$

$$\sigma = -\frac{2}{\beta^2} + \frac{\tau_1}{4\pi} \quad (14c)$$

It should be noted that both the condition (14a) and the quantum contribution to the central charge coincide with those obtained in the translation-invariant quantization in a finite (periodic) box [2], but not with the results of reference [4]. The undetermined constant μ^2 in (13a), corresponding to the arbitrariness of constant field translation, is fixed by demanding that the vacuum expectation value of $\hat{\phi}$ be zero. Then, $\mu^2/m^2 = 1 - \hbar\beta^2\delta/4\pi$ with $\delta \sim O(1)$ because of the existence of the $\hbar$ -dependent linear term in the Hamiltonian.

In the classical limit, formulas (13) are identical with (4). For non-zero $\hbar$, however, the Hamiltonian constructed from (13a) does not coincide with the classical one or with the one given in the literature [2-5]. The expansion in $\hbar$ of (13a) gives an infinite number of $\hbar$ -dependent terms in the interaction Hamiltonian. These terms, which are absent in ref. [5], should be interpreted as the (finite) counter terms which must exist in order to preserve the full geometrical significance of the classical Liouville theory at the quantum level. As shown before, establishing the existence of the deformation generators H_μ satisfying the algebra (5) implies that they obey the free (massless) field equations in the conformal gauge. Hence, defining

$$\hat{\partial}_\mu^{\hat{\phi}}(r) \equiv \left(\frac{\partial}{\partial r} - \frac{1}{r}\right) \hat{\partial}_r^{\hat{\phi}}(r) \quad , \quad (\mu = \pm, 1)$$

we now have

$$\left(\square + \frac{2}{r^2}\right) \hat{\partial}_\mu^{\hat{\phi}}(r, \tau) = 0 \quad . \quad (15)$$

Furthermore, it is easy to see that the operators $\hat{H}_\mu^{\hat{\phi}}$ connect the vacuum to one-particle states, and hence can serve as interpolating fields [5]. Consequences of this and the LSZ reduction formula are that, to all orders

in $\hbar$, i) there is no quantum correction to the eigenvalue of the $SO(2.1)$ Casimir operator r^2 [5] for the asymptotic one-particle states, and ii) the S-matrix is an identity.

As a check, we compute the self-energy correction to the first non-trivial order in $\hbar$. The one-loop graph and the contributions coming from the $\hbar$ -dependent counter terms are shown in Fig. 1. We find that, on the energy shell, the sum of all the contributions exactly cancels:

a) $= -\hbar\beta^2\omega/12\pi$; b) $= \hbar\beta^2\omega(1+\delta)/6\pi$; c) $= -\hbar\beta^2\omega(1+2\delta)/12\pi$. This explicitly demonstrates the crucial role of the $\hbar$ -dependent terms in the Hamiltonian, without which the Casimir eigenvalue is subject to quantum correction^{*)}.

We note that although the language of the manifestly $SO(2.1)$ invariant formulation [5] has not been used, the validity of the algebra (5) assures us that the $SO(2.1)$ symmetry of the space-time with constant curvature can be exactly preserved since the central charge vanishes for the $SO(2.1)$ subalgebra.

Let us now return again to the geometrical aspect of the theory and discuss about the general coordinate transformation in the theory as interpreted as a model for quantum gravity in two dimensions. A characteristic feature of the Hamiltonian dynamics for systems with general coordinate invariance is the existence of the constraints associated with the arbitrariness of the coordinate choice, i.e. arbitrariness of η^μ . In quantum theory, the constraints are usually interpreted as the conditions on permissible state vectors, in analogy with the Klein-Gordon equation

*) In ref. [5], it is asserted that the Casimir eigenvalue is unchanged by the quantum correction without the $\hbar$ -dependent-terms. There seems no reason for expecting this, and the explicit calculation indeed shows the contrary: a) $\neq 0$. The $SO(2.1)$ symmetry demands only the vanishing of the imaginary part of the one-loop graph.

for the case of a relativistic point particle. The resulting system of equations replaces the Schrödinger equation of ordinary quantum mechanics, and is called the Wheeler-DeWitt equation [9]. In our case, however, the presence of the non-zero central charge in (5) prevents us from doing this. Although, from the point of view of the integrability of the deformation of the Cauchy curves in two dimensions, the central charge is allowed to exist [7], the resulting non-invariance of the action under general coordinate transformation would, in general, imply that the physical amplitudes change when the space-time parametrization is changed.

There is a natural way of remedying this situation. Given an auxiliary free scalar field $\chi(r)$ and its conjugate momentum $P(r)$, the operators

$$\mathcal{H}_\perp^\chi = -:\frac{1}{2}(P^2 + \chi'^2): + \frac{2}{\beta}\chi'' \quad (16a)$$

$$\mathcal{H}_\perp^\chi = :P\chi': - \frac{2}{\beta}P' \quad (16b)$$

satisfy the algebra (5), which is the same as for (13) but the opposite sign for the central charge because of the "wrong" sign in (16a). Here, we assumed the following CCR representation, which diagonalizes the "Hamiltonian" $\int H_\perp^\chi dr$

$$\begin{aligned} \chi(r) &= \int_0^\infty \frac{d\omega}{\sqrt{2\omega}} \sqrt{\frac{2}{\pi}} \cos \omega r (b_\omega + b_\omega^\dagger) \\ P(r) &= -i \int_0^\infty d\omega \sqrt{\frac{\omega}{2}} \sqrt{\frac{2}{\pi}} \cos \omega r (b_\omega - b_\omega^\dagger) \\ [b_\omega, b_{\omega'}^\dagger] &= \pi \delta(\omega - \omega') \end{aligned} \quad (17)$$

The sum $H_\mu \equiv H_\mu^\phi + H_\mu^\chi$ therefore satisfies the algebra (5) without the central charge. That this is a natural (and almost compulsory) way of restoring

the general coordinate invariance can be seen by taking the classical limit. The action $A = \int d^2x [\dot{\phi}\pi + \dot{\chi}P - \eta^\mu H_\mu]$ is then rewritten as

$$A = \int \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \tilde{\chi} \partial_\nu \tilde{\chi} - \Lambda + R \tilde{\chi} \right) d^2x \quad (18)$$

($\Lambda = m^2$) with $\tilde{\chi} = \chi - \phi$. Formula (18) is just the local version of the non-local invariant action, written down by Polyakov [1], which is reduced to the Liouville action in the conformal gauge. The Hamiltonian formulation of the non-local action necessitates the introduction of the auxiliary field $\tilde{\chi}$, which is dynamical. Formula (18) then leads to the constraints $H_\perp = H_1 = 0$ in the standard way^{*}). We emphasize that the "wrong" sign in (16a) does not do any immediate harm, since $H_\perp$ is a constraint rather than the energy density. It is amusing to note that the classical general solution of the Liouville equation in terms of the free field is, if expressed in the canonical language, nothing but the solution of these constraints. In this sense, the Wheeler-DeWitt equation itself is a natural quantum generalization of that classical solution. It is interesting to study the solution of the Wheeler-DeWitt equation in the light of this observation. However, it is beyond the scope of the present note and is left to a future publication.

Finally, we add a few remarks related to our generally covariant interpretation: i) The standard counting rule of the dynamical degrees of freedom for the constrained system tells us that there is no true dynamics in the system. It is therefore natural that the S-matrix of the Liouville system is trivial. ii) The energy of a generally covariant system must be

^{*}) The action (18) has previously appeared in the literature [10] where, however, it was only used as a formal device for deriving the improved energy momentum tensor.

expressed by a surface term, following the standard method [11]. It is easily seen that the only possible (finite) value of the energy of our system is then zero, at least classically. A closely related (but not identical) phenomenon will be discussed, in the context of a different model, in another paper [12]. iii) In terms of $\tilde{\chi}$ and $\tilde{\pi} \equiv \pi + P$, the constraints in the classical theory take the form $H_{\perp} = 2^{-1}\tilde{\pi}^2 - \tilde{\pi}P + 2\beta^{-1}\tilde{\chi}'^2 - 2^{-1}\tilde{\chi}'^2 - \varphi'\tilde{\chi}' + \Lambda \exp(\beta/\varphi)$ and $H_1 = \tilde{\chi}'P + \varphi'\tilde{\pi} - 2\beta^{-1}\tilde{\pi}'$. From these expressions, we observe that in a recent work [13] by Banks and Susskind only the special case where $P = \tilde{\chi}' = 0$ was treated. From our point of view, this is not, however, a gauge choice. This explains why these authors had only a collapsed solution for $\Lambda > 0$ (negative scalar curvature). It should also be remarked that with the above restriction we have now $[H_{\perp}(x), H_{\perp}(y)] = 0$, which may be interpreted as an indication that the space-times swept out by the deformation generators with this restriction are in fact something different [14] from the ordinary (pseudo)-Riemannian space-times.

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REFERENCES

- [1] A. Polyakov, Phys. Lett. 103B (1981) 207.
- [2] T. Curtright and C. Thorn, Phys. Rev. Lett. 48 (1982) 1309.
E. Braaten, T. Curtright and C. Thorn, Phys. Lett. 118 (1982) 115.
- [3] J.L. Gervais and A. Neveu, Nucl. Phys. B209 (1982) 125 and B224
(1983) 329; CERN preprints TH.3731 and 3732.
- [4] E. D'Hoker and R. Jackiw, Phys. Rev. D26 (1982) 3517.
- [5] E. D'Hoker, D. Freedman and R. Jackiw, Phys. Rev. D28 (1984) 2583.
E. D'Hoker and R. Jackiw, Phys. Rev. Lett. 50 (1983) 1719.
- [6] S. Fubini, Nuovo Cimento 34A (1976) 521.
- [7] C. Teitelboim, Phys. Lett. 126B (1983) 41.
- [8] R. Jackiw, in Ahead of his time: Bryce S. DeWitt -- a collection of essays in honor of his 60th birthday, ed. S. Christensen (Adam Hilger, Bristol, 1983).
- [9] See, for example, B.S. DeWitt, Phys. Rev. 160 (1967) 1113.
- [10] R. Marnelius, Nucl. Phys. B211 (1983) 14.
- [11] T. Regge and C. Teitelboim, Ann. Phys. (NY) 88 (1974) 286.
- [12] T. Yoneya, Higher derivative quantum gravity in two dimensions, CERN preprint TH.3867, submitted to Phys. Lett. B.
- [13] T. Banks and L. Susskind, Stanford Univ. preprint ITP-743 (1983).
- [14] C. Teitelboim, in General relativity and gravitation: one hundred years after the birth of Albert Einstein, ed. A. Held (Plenum Press, New York, 1980).

Fig. 1 : The Feynman diagrams contributing to the self-energy to order $\hbar$.