Ref.TH.3875-CERN

A POSSIBLE PROBLEM WITH THE RUBAKOV-CALLAN
CONDENSATE FORMATION FOR MASSIVE FERMIONS

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ABSTRACT

The effective s-wave fermion-monopole theory, with massive quarks and non-Abelian colour interactions near the monopole core taken into account, is solved in the $1/N$ approximation scheme. The zero energy four-point function vanishes strongly at the origin, hinting that the massive quarks may not reach the monopole core to trigger catalysis.

Ref.TH.3875-CERN
April 1984

Till recently the treatments of the monopole-catalysis of the fermion number violation phenomenon used drastic simplification, like the neglect of non-Abelian interactions and/or the fermion masses^{1),2)}. An interesting recent work³⁾ aims to reveal the role of the non-Abelian colour interactions of quarks near the SU(5) monopole core, neglecting, however, the fermion masses. They have shown that inclusion of the non-Abelian interactions does not change the condensate formation argument, thus reproducing the free quark result.

Mass effects were previously included in the bosonization framework, however, by replacing the non-Abelian interactions by an Abelian one²⁾. Still, the inclusion of the mass was not without drastic consequences. Indeed, Virasoro⁴⁾ has observed a non-perturbative growth of the mass which induces a repulsive barrier as one approaches the core.

In this work we follow closely Ref. 3), and study the same radially reduced monopole Lagrangian by adding a mass term:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} \left(\frac{4\pi r^2}{g^2} \right) F_{\mu\nu}^a F^{\mu\nu,a} + \bar{\chi} (\not{D} + m) \chi \\ D_\mu &= \partial_\mu - g a_\mu^a T^a \quad ; \quad x_\mu = (t, r) \end{aligned} \quad (1)$$

where χ are related to the 4-dimensional $J = 0$ fermion fields as

$$\begin{aligned} \chi &= \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} ; \quad \chi_1(r, t) = \frac{r}{\sqrt{8\pi}} \int d\Omega \bar{\psi}_{\alpha\ell}(\vec{x}, t) \varepsilon_{\alpha\ell} \\ \chi_2(r, t) &= i \frac{r}{\sqrt{8\pi}} \int d\Omega \bar{\psi}_{\alpha\ell}(\vec{x}, t) \varepsilon_{\ell\beta}(\hat{r}, \vec{\tau})_{\beta\alpha} \end{aligned} \quad (2)$$

and T are the generators of the colour group.

Our particular choice of the mass term perhaps needs some explanation. The fundamental SU(5) monopole⁵⁾ is a 't Hooft-Polyakov monopole⁶⁾ sitting in an SU(2) sub-group of SU(5). The fermions form doublets with respect to this SU(2). The monopole is generated in the first stage of the spontaneous breakdown:

$$SU(5) \longrightarrow SU(3)_c \times SU(2)_L \times U(1) \longrightarrow SU(3)_c \times U(1)_Q$$

For the fermions in the lowest partial wave, the only ones of physical relevance here, the gauge group breaks down to $SU(2)_c \times U(1)_{Y_c} \times U(1)_Q$ ⁷⁾. In the massless case, Ref. 3) manages to get a complete decoupling of the Abelian and

non-Abelian gauge interactions. However, the very transformations which decouple the Abelian interactions in the massless case, bring them back in a totally mixed-up way in the mass term. Our main aim is to study the role of the fermion mass, together with the non-Abelian interactions near the core, to come as close as possible to the realistic case. Therefore not to complicate the picture any further, we will neglect the Abelian interactions altogether. The inclusion of them may only worsen the case against the condensate formation as one could easily convince himself.

That this assumption physically is not very implausible can be seen in another way. The $U(1)_Q$ part introduced in the mass term by the decoupling transformation is a phase $\exp[-2i \square^{-1}(\partial_\mu a_\mu^Q)]$ which can be eliminated by a gauge choice $\partial_\mu a_\mu^Q = 0$. Furthermore $U(1)_Y$ interactions are inessential when calculating expectation values like $\langle \psi \bar{\psi} \dots \bar{\psi} \psi \dots \rangle$, because the sum of incoming and outgoing $U(1)_Y$ charges separately vanish. Thus the neglect of them should not affect the computation of the condensates of this type.

We will compute the two and four-point functions in the same $1/N$ approximation as in Ref. 3) by replacing $SU(2)_c$ with $SU(N)$, and letting $N \rightarrow \infty$; because only in this limit the theory is solvable. We will work in the Euclidian space, and make the gauge choice, $a_0 = 0$ (which seems to be a preferred gauge because of the lack of translation invariance in space).

A similar problem, QCD_2 , was first treated by 't Hooft⁸⁾ using the planar diagram method. We, instead, use the formulation developed in Refs. 9), 10) here.

First integrating over the gauge fields we get

$$\begin{aligned} Z[\eta, \bar{\eta}] = & \int \mathcal{D}\chi \mathcal{D}\bar{\chi} \exp \left\{ \int d^2x \left[\bar{\chi}_i (\not{\partial} + m) \chi_i + \bar{\eta} \chi + \bar{\chi} \eta \right] \right\} \times \\ & \times \exp \left\{ -g^2 \iint d^2x d^2y J_\mu^a(x) (\Delta_{\mu\nu}^{ab})^{-1} J_\nu^b(y) \right\} \end{aligned} \quad (3)$$

where

$$J_\mu^a(x) = \bar{\chi}_i(x) \gamma_\mu \tau_{ij}^a \chi_j(x) \quad (4)$$

and $\Delta^{-1}(x-y)$ is the solution of the equation

$$\frac{4\pi}{g^2} r^2 \frac{\partial^2}{\partial t^2} f(r, t; r', t') = \delta(r-r') \delta(t-t') \quad (5)$$

i.e., it is the gauge boson Green's function.

Because the action is not quadratic in $\bar{\chi}$ and χ , a direct path integral treatment of (3) is very difficult. It is therefore useful to introduce an action equivalent to (3), but quadratic in Fermi fields. When the interaction is local, one uses the Gross-Neveu trick to introduce an equivalent Lagrangian which is quadratic in the Fermi fields. Here we will use the non-local version of this trick^{9),10)}, to introduce bilocal fields depending on two-space and two-time coordinates.

Indeed using Fierz identities¹⁰⁾, and the identity

$$\tau_{ij}^a \tau_{kl}^a = N \left(\delta_{ik} \delta_{jl} - \frac{1}{N} \delta_{ij} \delta_{kl} \right) \quad (6)$$

one can write the current-current interaction term as

$$\begin{aligned} \bar{\chi}_i(x) \gamma^1 \chi_j(x) \bar{\chi}_k(y) \gamma^1 \chi_l(y) \tau_{ij}^a \tau_{kl}^a = \\ = \bar{\chi}_i(x) \gamma_5 \chi_i(y) \bar{\chi}_j(y) \gamma_5 \chi_j(x) - \bar{\chi}_i(x) \chi_i(y) \bar{\chi}_k(y) \chi_k(x) \\ + \bar{\chi}_i(x) \gamma^0 \chi_i(y) \bar{\chi}_k(y) \gamma^0 \chi_k(x) + \bar{\chi}_i(x) \gamma^1 \chi_i(y) \bar{\chi}_k(y) \gamma^1 \chi_k(x) \\ - \frac{1}{N} (\text{non-singlet part}) \end{aligned} \quad (7)$$

We thus see the emergence of the bilocals

$$\begin{aligned} \sigma^a(x, y) &= \bar{\chi}_i(x) \Gamma^a \chi_i(y) \\ \Gamma^a &= \{ 1, \gamma^0, \gamma^1, \gamma^3 \} \end{aligned} \quad (8)$$

One can now pass easily to the equivalent quadratic action by using the identity

$$\begin{aligned} \int \mathcal{D}\sigma \exp \{ (\sigma, \Delta^{-1} \sigma) + (\bar{\chi} \Gamma \chi, \sigma) \} \\ = C \exp \left\{ - \frac{1}{4} (\bar{\chi} \Gamma \chi, \Delta \bar{\chi} \Gamma \chi) \right\} \end{aligned} \quad (9)$$

After performing, the now possible, fermion integration, we get

$$Z = \int \prod_l d\sigma_l d\pi_l \exp \left\{ \bar{\eta} \bar{\sigma}^1 \eta + (\sigma_l, \Delta \sigma_l) + \frac{1}{N} (\pi_l, \Delta \pi_l) + N \text{tr} \ln (\not{\sigma} + m + \not{A}) \right\} \quad (10)$$

$l: 0, 1, 2, 5$

where

$$\not{\sigma} = \not{\sigma} + m + \not{A}$$

$$A = \sigma_0 + \gamma_1 \sigma_1 + \gamma_2 \sigma_2 + \gamma_5 \sigma_5 + \frac{1}{N} (\pi_0 + \gamma_1 \pi_1 + \gamma_2 \pi_2 + \gamma_5 \pi_5) \quad (11)$$

and π_1 are defined similarly to σ , by replacing Γ with $\gamma_5 \Gamma$.

In determining the large N limit⁹⁾, the first step is to absorb a factor $\sqrt{N}$ in the σ, π fields such that N appears an overall factor multiplying the entire action. Then, the integral (10) can be calculated by the stationary phase method; at $N = \infty$ the stationary phase approximation becomes exact. Thus, to evaluate Eq.(10) we first look for classical σ, π fields satisfying

$$\left. \frac{\delta S_{\text{eff}}}{\delta \sigma} \right|_{\sigma^{\text{cl}}} = 0, \quad \left. \frac{\delta S_{\text{eff}}}{\delta \pi} \right|_{\pi^{\text{cl}}} = 0 \quad (12)$$

After consistently setting $\pi = 0$, Eq. (12) yields the analogue of 't Hooft's equation⁸⁾ for the fermion self-energy

$$\tilde{A} = \Delta^{-1} \text{tr} \left(\frac{\tilde{A}}{\det(\tilde{A} + \not{\sigma} + m)} \right) \quad (13)$$

where

$$\tilde{A} = \sigma_0 + \gamma_1 \sigma_1 + \gamma_2 \sigma_2 + \gamma_5 \sigma_5 \quad (14)$$

In Eq. (13) the trace is only over space-time coordinates. Using this expression in the Dyson equation for the propagator, we get after performing a Fourier transformation in time

$$\left(\gamma_1 \partial_r - i\omega \gamma_2 + m + \frac{\alpha_s}{r^2} \int \frac{d\omega'}{2\pi} \frac{1}{\omega'^2} \tilde{S}_F(\omega - \omega', r, r) \right) \tilde{S}_F(\omega, r, r') = \delta(r - r') \quad (15)$$

The solution to this equation in the massless case is found in Ref. 3). For the massive case the solutions are

$$\begin{aligned} \tilde{S}_F(\omega, r, r') = & \frac{1}{2} (m\beta + \varepsilon(R)\gamma_1 + i\omega\beta\gamma_2) \times \\ & \times \exp\left\{ (m\gamma_1 - i\omega\gamma_5)R + \frac{\alpha_s}{2\pi} \beta P(\beta-\beta') (m + i\omega\gamma_2) \right\} \end{aligned} \quad (16)$$

for $\rho - \rho' < 0$, and

$$\begin{aligned} \tilde{S}_F(\omega, r, r') = & -\frac{1}{2} (m\beta - \varepsilon(R)\gamma_1 + i\omega\beta\gamma_2) \times \\ & \times \exp\left\{ (m\gamma_1 - i\omega\gamma_5)R - \frac{\alpha_s}{2\pi} P(\beta-\beta') (m + i\omega\gamma_2) \right\} \end{aligned} \quad (17)$$

for $\rho - \rho' > 0$; where

$$\begin{aligned} R &= r - r' \quad ; \quad \beta = -1/r \\ P &= \left(-2\beta + \frac{2}{\lambda}\right)\gamma_1 + im\alpha\gamma_5 \ln \frac{1-\omega\beta}{1+\omega\beta} \\ \beta &= 1/\sqrt{\omega^2 + m^2} \end{aligned} \quad (18)$$

and λ is the infra-red cut-off. As $\lambda \rightarrow 0$ we see that the two-point function vanishes as in the case of Ref. 3). Thus the fermions are confined as in the case of QCD₂. So one proceeds to the next question whether the colour singlet bound states form, a question which in the context of cluster argument is relevant for the formation of the condensate.

To answer this question we need to know the four-point function which is down by one power N . To compute this, one constructs a systematic expansion of S_{eff} in powers of $1/N$. Again consistently setting $\pi = 0$ as in the previous case, we write

$$\sigma = \sigma^{cl} + \frac{1}{\sqrt{N}} \delta\sigma \quad (19)$$

and expand S_{eff} in powers of $\delta\sigma$. The answer is

$$Z[0,0] = \exp[N S_{\text{eff}}(\sigma^{cl})] / \det \left[\delta^2 S_{\text{eff}} / \delta\sigma(x) \delta\sigma(y) \right]_{cl} \quad (20)$$

Because σ is a bilocal field with colour singlet quantum numbers, in inverting the operator $\delta^2 S_{\text{eff}} / \delta\sigma\delta\sigma'$ we are solving for the propagator of a field with the

quantum numbers to create or destroy a colour singlet bound state. Denoting the corresponding wave function by $\phi(r, \omega, E)$, the equation for inverting $\delta^2 S_{\text{eff}} / \delta c \delta \sigma'$ turns out to be equivalent to 't Hooft's Bethe-Salpeter equation for mesons (Fourier transformed in time)

$$\phi(r, \omega, E) = \int dr' \frac{\alpha_s}{r^2} \tilde{S}_F(r, r', \omega) \tilde{S}_F(r, r', E - \omega) \frac{1}{2\pi} \int d\omega' \frac{\phi(r', E, \omega')}{(\omega - \omega')^2} \quad (21)$$

We first note that the infra-red behaviour coming from the gluon exchange is

$$\int d\omega' \frac{\phi(r, \omega', E)}{(\omega - \omega')^2} = \frac{2}{\lambda} \phi(r, \omega, E) + P \int d\omega' \frac{\phi(r, \omega', E)}{(\omega - \omega')^2} \quad (22)$$

and expand both sides of the Eq. (21) in powers of λ , using

$$\begin{aligned} \lim_{\lambda \rightarrow 0} \exp\left(-\frac{2}{\lambda} |\varphi - \varphi'|\right) &\longrightarrow \lambda \delta(\varphi - \varphi') + O(\lambda^3) \\ \lim_{\lambda \rightarrow 0} \exp\left(-\frac{2}{\lambda} |\varphi - \varphi'|\right) \varepsilon(\varphi - \varphi') &\longrightarrow -\frac{\lambda^2}{2} \delta'(\varphi - \varphi') + O(\lambda^4) \\ \lim_{\lambda \rightarrow 0} \exp\left(-\frac{2}{\lambda} |\varphi - \varphi'|\right) \varepsilon^2(\varphi - \varphi') &\longrightarrow \frac{\lambda}{3} \delta(\varphi - \varphi') + \frac{\lambda^3}{6} \delta''(\varphi - \varphi') + O(\lambda^5) \end{aligned} \quad (23)$$

The equation obtained from Eq. (20) in the limit $\lambda \rightarrow 0$ is extremely complicated compared to the massless case of Ref. 3). However, using the ansatz

$$\phi(r, \omega, E) = \theta(E - \omega) \theta(\omega) f(r) ; \quad 0 < \omega < E \quad (24)$$

it simplifies considerably. By examining order-one and order- λ , we get, for $E = 0$, the following equation, respectively:

$$\begin{aligned} f(\varphi) &= \gamma_1 f(\varphi) \\ \gamma_1 \frac{df(\varphi)}{d\varphi} - \left(\frac{1}{\pi} + \frac{m\alpha_c}{4\pi\varphi^2} \right) f(\varphi) &= 0 \end{aligned} \quad (25)$$

The solution is easily found as

$$f(r) = c \exp \left[\left(\frac{1}{2} m r - \frac{\alpha_c}{2\pi m r} \right) \gamma_1 \right] \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (26)$$

This solution is not translationally invariant, and vanishes very strongly as $r \rightarrow 0$. Translated into the cluster argument of Rubakov¹⁾, this indicates that the massive fermions, as in Virasoro's case, may not reach the monopole core to trigger catalysis.

The translationally invariant solution for the massless case is known to survive the introduction of non-planar graphs⁸⁾; thus it is there to all orders in $1/N$. The question now is whether the same holds for our solution for the massive case. Given that physically $N_c = 2$, further work to settle the question of corrections (hopefully to all orders in $1/N$) is needed to decide whether the condensate $\langle u_1 u_2 d_3 \bar{e} \rangle$ forms or not. If the $1/N$ corrections work the same way as they do in the massless case, then our answer to this question will firmly be no. Perhaps a Monte-Carlo simulation as suggested by Ref. 4) could be an unambiguous way to decide whether massive quarks can really reach the core to trigger the catalysis.

Aside from the purely academic interest in solving an interesting two-dimensional field theoretical model, our solution at least illustrates that the final test to determine whether the Callan-Rubakov condensate forms or not should be done by a method which takes all the complications into account.

ACKNOWLEDGMENTS

We thank M. Arik and H. Beker for helpful conversations and G. Veneziano for reading the manuscript. This work is partially supported by TUBITAK, the Scientific and Technical Research Council of Turkey.

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