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DOES THE KOBAYASHI-MASKAWA MATRIX VIOLATE CP MAXIMALLY?

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ABSTRACT

Possible definitions of maximal CP violation in the Kobayashi-Maskawa model are explored.

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Analyses of the Kobayashi–Maskawa (KM) matrix based on recent experiments indicate that the complex phase in the matrix is large [1,2]. This suggests the possibility that in some sense the KM matrix violates CP maximally, an idea already proposed [3] before the recent results. However, it is unclear what maximal CP violation implies since different parametrizations and different conventions lead to different phase parameters. For example, a 90° phase in the Maiani form [4] is not equivalent to a 90° phase in the original KM form [5]. The purpose of this letter is to explore possible definitions of maximal CP violation within the context of the KM model.

Since the KM matrix U is close to the unit matrix we focus on the off-diagonal elements. It is instructive to consider first an example that does not conform to our present empirical information. We assume that all off-diagonal elements of U have the same small magnitude θ . Maximum CP violation could then correspond to the statement that there exists a phase convention in which, neglecting terms of order θ^2 , all off-diagonal elements are purely imaginary. Then to order θ

$$U = \begin{pmatrix} 1 & i\theta & i\theta \\ i\theta & 1 & i\theta \\ i\theta & i\theta & 1 \end{pmatrix} \quad (1)$$

By an alternate choice of phase convention two pairs of off-diagonal elements can be made real. Thus in this example maximal CP violation corresponds either to all off-diagonal elements purely imaginary or equivalently one pair purely imaginary.

It has recently been pointed out [2] that our present empirical information on the KM matrix allows an expansion in terms of a small parameter λ . Neglecting terms of order λ^4 , U is given by

$$U = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (2)$$

Experiments^{*)} not involving CP violation give $\lambda = 0.22$, $A = 1 \pm 0.2$, and

$$\rho^2 + \eta^2 < 0.3 \quad (3)$$

The phase convention is defined by setting the terms through λ^2 real since there is no CP violation to order λ^2 . Given this form we can see no obvious way to define maximal CP violation since even the λ^3 terms cannot be purely imaginary. Thus $\rho = 0$ yields V_{ub} purely imaginary but then the phase of V_{td} is at most about 30° .

^{*)} We use the Mark II result [6] for the B lifetime together with the branching ratio of 12% for $B \rightarrow e\nu X$ to determine A . We also use the recent result [7] that $\Gamma(b \rightarrow u e \nu) < 0.03 \Gamma(b \rightarrow c e \nu)$ to obtain eq. (3).

In order to enforce unitarity we can set

$$U = \exp(ih) \quad (3a)$$

and then expand h:

$$ih = \lambda h_1 + \lambda^2 h_2 + \lambda^3 h_3 \quad (3b)$$

We recover eq. (2) to order λ^3 by the choice

$$ih_1 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ih_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A \\ 0 & -A & 0 \end{pmatrix} \quad (4)$$

$$ih_3 = \begin{pmatrix} 0 & 0 & \rho - \frac{1}{2} - i\eta \\ 0 & 0 & 0 \\ \frac{1}{2} - \rho - i\eta & 0 & 0 \end{pmatrix}$$

We note in passing that the algebra of the matrices of eq. (4) assures that corrections to the approximate form of U given in eq. (2) correspond to multiplying terms by factors $(1 + \alpha\lambda^2 + \dots)$, where $\alpha \sim 1$. Since CP invariance is satisfied if $(ih) = (ih)^*$ maximal CP violation could be defined by $h = h^*$. The best we can do in this respect is to require $h_3 = h_3^*$, which corresponds to setting $\rho = 1/2$. This gives V_{ub} and V_{td} the same phase with neither being 90° . The constraint of eq. (3) together with the requirement that η be large enough to produce the experimental value of ϵ does not allow $\rho = 1/2$ if $m_t \leq 60$ GeV. Thus this definition of maximal CP violation is unlikely to be satisfied.

Instead of looking directly at U we may consider the mass matrix. We follow a proposal of Stech [8] to consider the down matrix M_d in the representation in which the up matrix M_u is diagonal and to assume that the unitary matrix U diagonalizes M_d rather than the more general possibility of a biunitary transformation. Stech makes the following ansatz as to the form of M_d : i) the diagonal elements are proportional to M_u ; ii) the off-diagonal elements form an antisymmetric Hermitian matrix, that is they are purely imaginary. This second assumption may be considered as a possible definition of maximal CP violation. Once again by a change of phase convention instead of having all off-diagonal elements imaginary only one pair need be.

We now use eq. (2) to reconstruct M_d . With the approximations $m_d \ll m_s$, $m_s \ll m_b$, $\lambda^4 m_b \ll m_s$, and setting $A = 1$

$$M_d \approx \begin{pmatrix} m_d + \lambda^2 m_s & \lambda m_s & \lambda^3 (\rho - i\eta) m_b \\ \lambda m_s & m_s & \lambda^2 m_b \\ \lambda^3 (\rho + i\eta) m_b & \lambda^2 m_b & m_b \end{pmatrix}$$

Setting $\rho = 0$ provides maximal CP violation in the sense of Stech. This corresponds to a 90° phase in the Maiani form but not in the original KM form. The value $\rho = 0$ is consistent with present experiments, although significant departures of the Maiani phase from 90° are allowed [1]. We may also ask in passing whether the first part of Stech's ansatz is satisfied; this requires in our approximation

$$m_s / m_b \approx m_c / m_t \quad (5a)$$

$$m_d + \lambda^2 m_s \approx m_s m_u / m_c \quad (5b)$$

Equation (5a) yields the prediction that m_t is between 30 and 40 GeV as discussed by Stech; eq. (5b) is a usual relation for the Cabibbo angle, generally believed to be reasonable.

In conclusion, we find that there is no reasonable definition by means of which it can be stated that the matrix U violates CP maximally. However, it is possible that the mass matrix violates CP maximally in the sense discussed by Stech.

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REFERENCES

1. L.L. Chau and W.Y. Keung, Phys. Rev. D **29** (1984) 592;
A.J. Buras, W. Slominski, and H. Steger, Max-Planck Institut, Munich, preprint MPI-PAE/PTU 77/83 (1983);
E.A. Paschos and U. Türke, Santa Barbara preprint NSF-ITP-83-168 (1983);
F. Gilman and J. Hagelin, Phys. Lett. **133B** (1983) 443.
2. L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.
3. David Hochberg and Robert G. Sachs, Phys. Rev. D **27** (1983) 606.
4. L. Maiani, Proc. 1977 Int. Symposium on Lepton and Photon Interaction at High Energies, Hamburg, 1977 (DESY, Hamburg, 1977) p. 867.
5. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
6. N.S. Lockyer et al., Phys. Rev. Lett. **51** (1983) 1316.
7. J. Lee-Franzini, talk given at Europhysics Conference on Flavour Mixing in Weak Interactions, Erice, Italy, March 1984.
8. B. Stech, Phys. Lett. **130B** (1983) 189.