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SLIDING SCALES IN MINIMAL SUPERGRAVITY

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ABSTRACT

Supergravity models obtained by minimal covariant extension from globally supersymmetric models automatically contain sliding scales if supersymmetry is spontaneously broken. The sliding gravitino mass is not affected by quadratic divergencies. Form invariance under $SU(n,1)$ transformations singles out cubic superpotentials. The supersymmetry breaking appears in the observed sector as soft F-type breaking.

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In the context of Grand Unified models global supersymmetry (Susy) has been used as a technical tool to maintain a once established hierarchy. Subsequently it was realized that the "accidental" occurrence of sliding scales in spontaneously broken Susy models might even generate the hierarchically different scales after radiative corrections via dimensional transmutation. Though no completely satisfying model was constructed, it was hoped that inclusion of gravity would improve the situation. Anyhow, the unification scale is not far away from the Planck scale $\tilde{M}_p$, and according to prevailing prejudice and experience, one should allow for a global symmetry to become gauged, thus implying supergravity as gauged Susy. Unfortunately one does not know how to deal with the gravitational radiative corrections and has to assume that ultimately the tree results together with quantum corrections for switched-off gravity ($\tilde{M}_p \rightarrow \infty$) will turn out to be a good approximation to the complete theory.

Inclusion of supergravity in the canonical case (a concise definition is given below), which has been nearly exclusively studied so far [1], did not shed new light on the origin of the Susy-breaking scale or the smallness of the weak scale.

If one attempts to explain these hierarchically small scales by dimensional transmutation from a sliding scale, one is naturally led to consider minimal coupling of supergravity, the subject of this note. The results are surprising: In minimal models all scales slide, the gravitino mass, the unification scale, and an ultraheavy scale. The multiplet

which carries the Susy-breaking couples gravitationally only, rendering a separation into observed and hidden sectors unnecessary. The gravitationally induced Susy-breaking splits the masses of the scalars and pseudoscalars¹ in contrast to canonical models, where both masses are raised .

To fix our notation we recall that a supergravity model is characterized by a real function $G(z, z^*)$ of the complex scalar matter fields z (and a possibly non-canonical kinetic energy of the gauge fields determined by analytic functions $f_{ab}(z)$). G is composed out of the real kinetic series $K(z, z^*)$ and the analytic superpotential $g(z)$ via² [2]

$$G(z, z^*) = -3 \ln K(z, z^*) + \ln |g(z)|^2 \quad (1)$$

The kinetic energy of the scalar fields

$$\partial z^i \partial z_j^* \left(\frac{\partial}{\partial z^i} \frac{\partial}{\partial z_j^*} G \right) \quad (2)$$

involves a (usually) field-dependent metric

$$G_{ij} = \frac{\partial}{\partial z^i} \frac{\partial}{\partial z_j^*} G \quad (3)$$

¹The breaking is analogous to an O'Raifeartaigh model in the limit where the Goldstone fermion decouples.

²We use the mass unit $1 = 2.4 \cdot 10^{18} \text{ GeV} = \frac{\tilde{M}_p}{\sqrt{8\pi}} = M_p$

With a few exceptions papers on $N = 1$ sugra-models dealt with the case of a canonical kinetic energy or flat metric

$$G_{i\bar{j}} = \delta_{i\bar{j}}$$

$$G = |z|^2 + \ln |g(z)|^2 \quad (4)$$

and obtained phenomenological results characteristic for this canonical choice. Note, however, that the canonical choice

$$K = e^{-\frac{|z|^2}{3}} \quad (5)$$

is by no means indicated as straightforward generalization from globally supersymmetric models. Minimally substituting covariant derivatives, densities, chiral projectors etc. in the globally supersymmetric Lagrangian leads to [3,4]

$$K = 1 - \frac{|z|^2}{3}$$

$$G = -3 \ln \left(1 - \frac{|z|^2}{3} \right) + \ln |g(z)|^2 \quad (6)$$

where the superpotential g is a polynomial of order 3. It is this choice of the superfunction G which we call minimal in contrast to canonical models. (The gauge fields retain their canonical kinetic energy $f_{ab}(z) = \delta_{ab}$ by minimal substitution).

We now investigate the prerequisites for models with sliding gravitino mass $m_{3/2}$ - i.e. sliding Susy breaking scale [5-9]

$$m_{3/2} = e^{\frac{\langle G \rangle}{2}} \quad (7)$$

Assume that the tree potential

$$V = e^G \left(\frac{\partial G}{\partial z_i^*} G_i^{-1} \frac{\partial G}{\partial z^i} - 3 \right) \quad (8)$$

(gauge interaction neglected)

does not determine this mass scale¹.

How do radiative corrections determine the gravitino mass? In 1-loop order there is a quadratic divergence of the potential energy multiplying the difference of the bosonic and fermionic squared masses

$$\text{Str } M^2 = \sum_{\text{Boson}} M^2 - \sum_{\text{Fermion}} M^2$$

We conclude that $\text{Str } M^2$ should vanish irrespective of the value of $m_{3/2}$ to keep $m_{3/2}$ sliding and to retain a vanishing cosmological constant².

¹This assumption holds for example in the canonical model with one scalar field

$$G = |z|^2 + \text{Re} \left(2\sqrt{3} z - z^2 \right)$$

which shows that sliding $m_{3/2}$ does not require a $U(1, 1)/U(1) \times U(1)$ coset geometry.

²This requirement is not met in the phenomenological no-scale models proposed so far [8,9]

General formulas for $\text{Str } M^2$ have been worked out [10]. For vanishing $\langle D \rangle$ -breaking (which is usual) and canonical kinetic energy of the gauge fields $\text{Str } M^2$ vanishes irrespective of the direction of the $\langle F \rangle$ -breaking, if the Ricci tensor R_i^j derived from the metric G_i^j fulfils

$$R_i^j = \frac{n+1}{3} G_i^j \quad (9)$$

where n is the number of scalar fields. This geometric condition is satisfied in the minimal models (6). [9]

They have a metric

$$G_i^j = \frac{\delta_i^j}{1 - \frac{|z|^2}{3}} + \frac{1}{3} \frac{z_i^* z^j}{\left(1 - \frac{|z|^2}{3}\right)^2} \quad (10)$$

One readily identifies this metric with the induced metric of a complex time-like mass shell $(U(n, 1)/U(n) \times U(1))$ [9]

$$\sum_{i=1}^n |z'^i|^2 - 3 |z'^{n+1}|^2 = -3 \quad z'^{n+1*} = z'^{n+1} \geq 1 \quad (11)$$

In fact if we parametrize the hyperboloid by

$$z'^i = \frac{z^i}{\sqrt{1 - \frac{|z|^2}{3}}} \quad z'^{n+1} = \frac{1}{\sqrt{1 - \frac{|z|^2}{3}}} \quad (12)$$

the metric (12) is the scalar product

$$G_{ij} = (e_i, e_j) = \sum_{\ell=1}^n e_i^{*\ell} e_j^{\ell} - 3 e_i^{*n+1} e_j^{n+1}$$

of the induced n-Bein

$$(e_i)^{\dagger} = \frac{\partial z'^j}{\partial z^i}$$

In particular the manifold $|z|^2 < 3$ is complete and we can consistently disregard field values which violate this bound. As a consequence, field configurations with negative energies [11] need not trouble us if they occur outside the manifold $|z|^2 < 3$.

Apart from U(n)-transformations, the metric (10) is also invariant under "boosts" (the transitive motions in the $U(n,1)/U(n) \times U(1)$ coset space), which in the coordinates z (12) take the form

$$\bar{y} = \frac{y + a\sqrt{3}}{1 + \frac{a^*y}{\sqrt{3}}} \quad \bar{z}^i = \frac{z^i \sqrt{1 - |a|^2}}{1 + \frac{a^*y}{\sqrt{3}}} \quad |a| < 1 \quad (13)$$

Here y denotes the direction of the boost and z are the transverse directions. The superfunction (6) keeps its form under a boost with a transformed superpotential

$$g(z, y) = \left(\frac{1 + \frac{a^*y}{\sqrt{3}}}{\sqrt{1 - |a|^2}} \right)^3 \bar{g}(\bar{z}(z, y), \bar{y}(y)) \quad (14)$$

This transformation law shows that a non-vanishing superpotential, even a constant one, breaks the $U(n, 1)$ -symmetry of the metric (10). If $g(z, y)$ is an arbitrary polynomial, it becomes a rational function (13) after a boost, but most notably, if $g(z, y)$ is a polynomial of degree 3, it remains so because the denominators cancel. The minimal coupling prescription that the superpotential be cubic (and that the kinetic energy of the gauge fields be canonical) is respected and indicated by the transformations of superpotentials under boosts!¹

We are now in a position to characterize the generic case of spontaneous (F-type) supersymmetry breaking in minimal models.² By help of the boosts (13) we can transform to fields with vanishing vacuum expectation values (vevs) without changing the form of the superfunction (6) and the potential. So we investigate minima of the potential (8) [12]

$$V = \left(1 - \frac{|z|^2}{3}\right)^{-2} \left(\sum_{i=1}^n \left| \frac{\partial g}{\partial z^i} \right|^2 - 3 \left| g - \frac{1}{3} z^i \frac{\partial g}{\partial z^i} \right|^2 \right) \quad (15)$$

with the assumption of vanishing vevs. Moreover, we assume the problem

¹We note in passing that the coefficients c^i , introduced by [3]

$$K = 1 - \frac{|z|^2}{3} + c_i z^i + c^{*i} z_i^*$$

to describe vevs of singlet field, can be absorbed by a redefinition of fields, similar to the boosts. These coefficients c^i can therefore be disregarded.

²We neglect gauge interactions.

of the cosmological constant to be solved, that is $V_{\min} = 0$.¹

In addition, by assumption supersymmetry is spontaneously broken, so

$$\left. \frac{\partial g}{\partial z^i} \right|_{z=0} \neq 0$$

Call the direction of this gradient y and the transverse direction z :

$$\left. \frac{\partial g}{\partial y} \right|_{y=z=0} \neq 0 \quad \left. \frac{\partial g}{\partial z^i} \right|_{y=z=0} = 0$$

expand the potential $V|_{y=z=0} = 0$ and require it to be a local minimum.

Then the form of the superpotential is fixed to be

$$g(y, z) = \mu \left(1 + \frac{y}{\sqrt{3}}\right)^3 + \mathcal{O}(z^2) \quad (16)$$

and the potential vanishes identically in y , if the transverse fields are put to their vevs.²

$$V(y, z=0) \equiv 0 \quad (17)$$

So all minimal sugra-models with spontaneous Susy-breaking contain a sliding field y entering the superpotential as

$$g(y, z=0) = \mu \left(1 + \frac{y}{\sqrt{3}}\right)^3 \quad (18)$$

¹One can easily show that for any local minimum one has $V_{\min} \leq 0$, if the gauge interactions are neglected.

²This is completely analogous to O'Raifeartaigh's result for the globally supersymmetric case [13]. $g(y, z) = \mu^2 y + \mathcal{O}(z^2)$

$V(y, z=0) \equiv \mu^4$ and y slides

By redefining the field y , it can be easily shown that this model [6] coincides with the no-scale model proposed in [7] and employed in [8,9].

Because y is not fixed by the potential, our assumption of vanishing vevs has to be reconsidered. If y has a vev $y = \sqrt{3}a$, then by a boost (13) we can reobtain the form (16) with fields with vanishing vevs. The coefficient μ , which is the gravitino mass for vanishing vevs, changes under the boost

$$m_{3/2} = \mu \left(\frac{1+a}{\sqrt{1-a^2}} \right)^3 \quad |a| < 1 \quad (19)$$

Thus, not only the field y slides, but also the gravitino mass.

The superpotential (16) suggests that y is always a gauge singlet and that 3 coefficients have to be finetuned. Both conclusions are disproved by the SU(5)-toy model with 24-multiplets Σ and Y

$$g(\Sigma, Y) = \frac{\kappa}{3} \Sigma^3 + \frac{m}{2} \Sigma^2 (1 - \sqrt{10} Y) \quad (20)$$

Apart from the supersymmetric minima at $\langle Y \rangle = 0$, the potential contains Susy breaking states with sliding $m_{3/2}$ and sliding SU(5)-scale and unique breaking direction $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$

$$\langle \Sigma \rangle = \frac{s}{\sqrt{30}} \text{diag} (2, 2, 2, -3, -3) \quad \langle Y \rangle = \left(\frac{\kappa}{\sqrt{10} m} - \frac{\sqrt{3}}{5} \right) \langle \Sigma \rangle \quad (21)$$

where s is arbitrary and only restricted by

$$\left| \frac{s}{r} - 1 \right|^2 < 1 \quad r = \frac{\sqrt{30} \kappa m}{\kappa^2 + 10 m^2}$$

The gravitino mass is

$$m_{3/2} = \left(\frac{10}{27} \right)^{1/4} \left(\frac{m}{r} \right)^{3/2} \frac{1}{\sqrt{\kappa}} \frac{\left| s^3 - \frac{3}{2} \left(1 + \sqrt{30} \frac{m}{\kappa} \right) s^2 + \frac{3}{2} \sqrt{30} \frac{m}{\kappa} s \right|}{\left(1 - \left| \frac{s}{r} - 1 \right|^2 \right)^{3/2}} \quad (22)$$

and the SU(5)-scale

$$M_{SU(5)}^2 = 10 \cdot 4\pi\alpha_{SU(5)} \cdot \frac{\left| \frac{s}{r} - 1 \right|^2 + 10 \frac{m^2}{\kappa^2}}{1 - \left| \frac{s}{r} - 1 \right|^2} \quad (23)$$

is always larger than

$$M_{\text{Lower Bound}} = 10 m \frac{\sqrt{4\pi\alpha}}{\kappa}$$

We see that not only the gravitino mass, but also the SU(5)-scale and other masses slide.

Using the form (16), we investigate the general case. If we restrict the superpotential to a cubic polynomial, then (16) implies

$$g(y, z) = \mu \left(1 + \frac{y}{13} \right)^3 + \left(1 + \frac{y}{13} \right) \frac{M}{2} z^2 + \frac{N}{2} z^2 + \frac{\kappa}{3} z^3 \quad (24)$$

with obvious index-contractions suppressed. Shifting back a non-vanishing vev of y , the superpotential changes to

$$\begin{aligned}
 g(y, z) = & M_p^2 \mu (1+a)^3 (1-|a|^2)^{-3/2} \left(1 + \frac{y}{\sqrt{3} M_p}\right)^3 + \\
 & + (1+a) (1-|a|^2)^{-1/2} \left(1 + \frac{y}{\sqrt{3} M_p}\right) \frac{M}{2} z^2 + \\
 & + \left(1 + \frac{1+a}{1+a^*} \frac{a^* y}{\sqrt{3} M_p}\right) (1-|a|^2)^{-1/2} \frac{N}{2} z^2 + \frac{\kappa}{3} z^3
 \end{aligned} \tag{25}$$

Here we made explicit all powers of M_p appropriate to obtain a non-trivial limit of vanishing gravitational interaction.

In this limit ($M_p \rightarrow \infty$) the field y decouples (apart from possible gauge interactions), leaving the fields z with a (sliding, F-type) supersymmetry breaking. The superpotential

$$h = \left(\frac{1+a}{\sqrt{1-|a|^2}} M + \frac{N}{\sqrt{1-|a|^2}} \right) \frac{z^2}{2} + \frac{\kappa}{3} z^3 \quad |a| < 1 \tag{26}$$

determines the surviving Yukawa interactions

$$\mathcal{L}_{Yuk} = \chi^i \chi^{\dagger j} \frac{\partial^2 h}{\partial z^i \partial z^{\dagger j}} + h.c.$$

and the supersymmetric part of the potential

$$V = \sum \left| \frac{\partial h}{\partial z^i} \right|^2 - \text{Re} \mu^* \frac{(1+a^*)^2}{1-|a|^2} N z^2 \tag{27}$$

Note that the masses of all fields slide, as does the Susy-breaking. Therefore the matrices M and N have to be chosen such that for all values of the slide parameter a one remains in a local minimum. The restriction of this parameter to the interior of the unit circle prevents one from applying immediately O'Raifeartaigh's investigation [13] to this case. One has to satisfy

$$| \mu N z^2 | \leq | (\frac{N}{1+a} + M) z |^2 \text{ for all } |a| < 1 \quad (28)$$

in all directions of the mass matrices N and M .

We are interested in the case of small Susy-breaking and therefore consider the slide parameter a near -1 . In this case all fields z which are not annihilated by the matrix N become superheavy. If, however, N has null-directions, the corresponding fields have the low scale $\frac{1+a}{\sqrt{1-|a|^2}} M$ and can participate in the Susy-breaking via mixing with the heavy sector. At the moment we do not have a completely satisfying $SU(5)$ -model, but we can demonstrate the occurrence of the different scales in the model

$$g(y, \Sigma, Y) = \mu \left(1 + \frac{y}{13} \right)^3 + \left(1 + \frac{y}{13} \right) M \Sigma Y + \frac{N}{2} Y^2 + \frac{\kappa}{2} Y \Sigma^2 \quad (29)$$

where Σ and Y are 24-dimensional representations of $SU(5)$.

The potential (in the limit $M_p \rightarrow \infty$ and with reshifted vev of y)

$$\begin{aligned}
 V = & \left| \frac{1+a}{\sqrt{1-|a|^2}} M \Sigma + \frac{N}{\sqrt{1-|a|^2}} Y + \frac{\kappa}{2} \left(\Sigma^2 - \frac{1}{5} \text{tr} \Sigma^2 \right) \right|^2 + \\
 & + \left| \frac{1+a}{\sqrt{1-|a|^2}} M Y + \frac{\kappa}{2} \left(\{ \Sigma Y \} - \frac{2}{5} \text{tr} \Sigma Y \right) \right|^2 - \text{Re} \mu^* N \frac{|1+a|^2}{1-|a|^2} \text{tr} Y^2 + \\
 & + D\text{-Terms} \quad |a| < 1
 \end{aligned}$$

has (besides others) a minimum at

$$\langle Y \rangle = 0 \quad \langle \Sigma \rangle = \frac{2M}{\kappa} \frac{1+a}{\sqrt{1-|a|^2}} w$$

where $w = \text{diag}(2, 2, 2, -3, -3)$ breaks $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. For small Susy-breaking ($a = -1 + \epsilon$, $\epsilon \ll 1$) the field Y is much more massive than the field Σ (far above M_p if the input mass parameters M and N are assumed to be of the Planck scale). Diagonalization of the mass matrix exhibits a "light" Σ' with fermion masses

$$m_{\text{Fermion}} = 25 \frac{M^2}{N} \frac{(1+a)^2}{\sqrt{1-|a|^2}}$$

for the $\Sigma^{(8, 1)}$ and $\Sigma^{(1, 3)}$ components. The masses of the real and imaginary parts of the scalar partners are no longer degenerate, but show F-splitting

$$m_{\text{Scalar}}^2 = \left(25 \frac{M^2}{N} \frac{|1+a|^2}{\sqrt{1-|a|^2}} \right)^2 \pm 25 \frac{\mu M^2}{N} \frac{|1+a|^4}{1-|a|^2}$$

For the $\Sigma^{(1, 1)}$ multiplet, the same formulas with 25 replaced by 1 are obtained. The $\Sigma^{(3, 2)}$, $\Sigma^{(\bar{3}, 2)}$ multiplets are fed to the super-symmetric heavy vector multiplet with (degenerate) mass

$$M_{SU(5)} = 10 e_{SU(5)} \frac{2M}{\kappa} \frac{|1+a|}{\sqrt{1-|a|^2}}$$

We note that for $a = a^* = -1 + \varepsilon$, $\varepsilon \ll 1$ the SU(5) scale and the mass of the chiral multiplets (proportional to $m_{3/2}$ (19) and the mass splitting) slide as

$$M_{\text{chiral}} \sim M_{\text{SU(5)}}^3$$

(modified by $\frac{M}{N}$ and K) and generate a hierarchy of scales. Due to the type of explicit Susy breaking (F-type splitting (27) in our case), the only logarithmic divergencies generated radiatively will be proportional to the F-component of gauge singlet fields [14]. Preliminary results show that in the presence of a suitable coupled gauge singlet field radiative corrections can indeed fix $a = -1 + \varepsilon$, $\varepsilon \ll 1$. The interplay of thermodynamic and radiative corrections has also to be investigated for its cosmological consequences.

A coupling to quintet Higgses is easily introduced as $H' \Sigma H$. Massive triplet Higgses together with light doublet Higgses could be generated by group-theoretical constructs. But there seems also a better chance in such models for the elegant sliding singlet mechanism [15] (an ansatz $H'(\Sigma + \alpha Z)H$ with Z a singlet field only appearing in this term) to work. The $\Sigma^{(8,1),(1,3)}$ may be rather light, not much above the weak scale. Then a breakdown of the mechanism through radiative corrections in global Susy does not follow like discussed in ref. [16]. Furthermore an $H^{(2)}$ -mass through light singlet exchange with the massive $H^{(3)}$ in a tadpole graph is not so dangerous since there are no direct soft Susy breakings on $H^{(3)}$.

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