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QCD AND THE STRANGE QUARK PARAMETERS

L.J. Reinders and H.R. Rubinstein
CERN-Geneva

ABSTRACT

We determine the mass and the condensate of the strange quark from QCD sum rules. The resulting values are slightly unexpected.

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At the present time it is not possible to determine the QCD parameters from first principles. Some like the quark and gluon condensates should be expressible in terms of Λ_{QCD} . Others, like quark masses, lie outside the theory.

A more modest approach, given that QCD is the correct theory, is trying to determine the parameters from constraints derived from the details of the mass spectrum and the couplings. These parameters are not totally free. Some combinations depend on general properties. For example, current algebra and the Goldstone realization of chiral symmetry breaking imply

$$(m_u + m_d) \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle = - f_\pi^2 m_\pi^2 \quad (1)$$

and a similar relation for $f_K^2 m_K^2$.

To go further and determine the quark masses or the value of the condensates one must use dynamical information. In the case of charmonium and bottomium the quark masses m_c and m_b can be accurately determined from a moment analysis of the vector channel [1]. In this case the task is facilitated by the fact that $\langle \bar{c}c \rangle = \langle \bar{b}b \rangle = 0$. For light quarks, the problem is more delicate. Narison and de Rafael [2], and Gasser and Leutwyler [3] have used QCD sum rules to determine the quark masses. Their work is based on saturating QCD sum rules for the divergence of the axial current by pseudoscalar mesons. They obtain

$$m_s(1 \text{ GeV}) = (180 \pm 50) \text{ MeV} \quad (\Lambda_{\text{QCD}} = 140 \text{ MeV}). \quad (2)$$

The purpose of this letter is to obtain a more accurate determination of this quantity as well as of the strange quark condensate. It will also show that the sum rules are more accurate than what has been the general impression so far.

Our strategy is to take advantage of the constraints coming from heavy quark physics and then analyze the light quark meson channels with strange quarks. Heavy quark systems (in particular charmonium [1]) require

$$\begin{aligned} \langle 0 | \alpha_s / \pi G_{\mu\nu} G_{\mu\nu} | 0 \rangle &= (360 \pm 20 \text{ MeV})^4 \\ \Lambda_{\text{QCD}} &= 140 \pm 40 \text{ MeV}. \end{aligned} \quad (3)$$

Given these constraints one can analyze the vector, scalar, and tensor mesons as a function of m_s and $\langle \bar{s}s \rangle$. A similar analysis for u and d quarks is not easy.

The light quark vector mesons have first been analyzed by SVZ [4] in their original paper. The corresponding polarization operators are known up to and including dimension six operators. The results for the ρ and ω

$$\begin{aligned} m_\rho &= m_\omega = 770 \text{ MeV} & (\text{exp } 770 \pm 10 \text{ MeV}) \\ g_\rho^2/4\pi &= g_\omega^2/36\pi = 2\pi/e & (\text{exp } 2.26 \pm 0.1) \end{aligned} \quad (4)$$

are in excellent agreement with experiment. The degeneracy is reproduced and the values for the masses and couplings are predicted within a few percent. We note that the theory can predict the symmetry breaking and does so correctly to high accuracy. The original analysis of the strange quark systems was more qualitative and shows a systematic overestimate of the masses and couplings. We are therefore interested in a more detailed analysis of these systems. For the ϕ meson current we choose

$$j_\mu(x) = \bar{s}(x)\gamma_\mu s(x). \quad (5)$$

The polarization operator is defined as

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(Q^2) = i \int e^{iqx} dx \langle 0 | T(j_\mu(x) j_\nu(0)) | 0 \rangle, \quad Q^2 = -q^2. \quad (6)$$

Including power corrections up to order Q^6 and the first quadratic mass correction to the bare loop we have [4]

$$\begin{aligned} \Pi(Q^2) &= -1/4\pi^2([1+\alpha_s/\pi]\ln(Q^2/\mu^2) + 6m_s^2/Q^2) + 2\langle m_s \bar{s}s \rangle/Q^4 \\ &+ 1/12\langle \alpha_s/\pi G_{\mu\nu} G_{\mu\nu} \rangle/Q^4 - 2\pi\alpha_s \langle \bar{s}\gamma_\mu \gamma_5 t^a s \bar{s}\gamma_\mu \gamma_5 t^a s \rangle/Q^6 \\ &- 4\pi\alpha_s/9 \langle \bar{s}\gamma_\mu t^a s \Sigma \bar{q}\gamma_\mu t^a q \rangle/Q^6. \end{aligned} \quad (7)$$

The summation in the last term is over the light flavours u , d , and s . The matrix elements of the four-fermion operators can be evaluated by vacuum saturation. Large N limit arguments suggest that the approximation is good to 10% [5]. The contribution of the dimension six operators is enhanced since their Wilson coefficients do not have loop suppression factors. The dimension five operator $\bar{q}\sigma_{\mu\nu}qG_{\mu\nu}$ is suppressed since its Wilson coefficient has an extra factor m_q^2/Q^2 which makes it negligible. Writing a dispersion relation and performing a Borel transform one obtains

$$\begin{aligned} \int e^{-s/M^2} \text{Im}\Pi(s) ds &= \\ &= M^2/4\pi\{1+\alpha_s(M)/\pi - 6m_s^2/M^2 + 8\pi^2\langle m_s \bar{s}s \rangle/M^4 \end{aligned}$$

$$+ \pi^2/3 \langle \alpha_s / \pi G_{\mu\nu} G_{\mu\nu} \rangle / M^4 - (448/81) \pi^3 \alpha_s(M) \langle \bar{s}s \rangle^2 / M^6 \}. \quad (8)$$

As usual the left-hand side is saturated by a single resonance plus a continuum contribution (with threshold s_0)

$$\text{Im}\Pi(s) = 9\pi m_\phi^2 / g_\phi^2 \delta(s - m_\phi^2) + 1/4\pi(1 + \alpha_s/\pi) \theta(s - s_0). \quad (9)$$

The location of the threshold is determined by the first radial excitation on the ϕ trajectory. Its precise value is fixed by the stability of the sum rule (see below). This requirement allows a very narrow range for s_0 and in agreement with the position of the radial excitation.

A second sum rule can be derived by differentiating (8) with respect to $1/M^2$. Transferring the continuum contribution to the right-hand side and denoting the result from eq.(8) by N and its derivative by D we obtain

$$m_\phi^2 = M^2 D / N \quad (10)$$

where

$$N = (1 + \alpha_s/\pi)(1 - e^{-s_0/M^2}) - 6m_s^2/M^2 + 8\pi^2 \langle m_s \bar{s}s \rangle / M^4 + \pi^2/3 \langle \alpha_s / \pi G_{\mu\nu} G_{\mu\nu} \rangle / M^4 - (448/81) \pi^3 \alpha_s(M) \langle \bar{s}s \rangle^2 / M^6 \quad (11)$$

and

$$D = (1 + \alpha_s/\pi)(1 - \{1 + s_0/M^2\} e^{-s_0/M^2}) - 8\pi^2 \langle m_s \bar{s}s \rangle / M^4 - \pi^2/3 \langle \alpha_s / \pi G_{\mu\nu} G_{\mu\nu} \rangle / M^4 + (896/81) \pi^3 \alpha_s(M) \langle \bar{s}s \rangle^2 / M^6 \quad (12)$$

And for the coupling we have

$$g_\phi^2/4\pi = (9\pi/M^2) m_\phi^2 e^{-(m_\phi/M)^2} N^{-1} \quad (13)$$

with m_ϕ given by (10).

Similar formulae hold for the K^* meson. For that case the mass correction to the bare loop is a factor four smaller and the quark condensate a factor two.

The structure of the sum rules for P-wave mesons is not very different. We refer the reader to the original papers [6].

We first analyze the structure of the formulae. Since the ρ , K^* , and φ mesons are well predicted by the sum rules, one can ask which effect is responsible for the symmetry breaking. The leading term is the $1/M^4$ contribution from the quark and gluon condensates. For the mesons with strange quarks (K^* , φ) the gluon condensate that dominates the correction in the ρ is overwhelmed by the chiral condensate which is now proportional to m_s . This term is pure octet and responsible for the equal spacing, in first approximation, between ρ - K^* - φ . It is in fact the mass correction to the bare loop that goes beyond octet. As we shall see it amounts to about 20 MeV, and therefore is about 10% of the dominant Gell-Mann-Okubo term.

To analyze the sum rules numerically we first point out that it is worthwhile to introduce, whenever possible, renormalization group invariant parameters. The operators that enter into (11) and (12) are all renormalized at the scale M . This does not affect $\alpha_s GG$ or $m_s \bar{s}s$, which are renormalization group invariant. In the $1/M^6$ terms we substitute the quantity $\langle m_s \bar{s}s \rangle / m_s(M)$ for $\langle \bar{s}s \rangle$. This only leaves m_s and α_s as scale dependent parameters in the sum rules. The value for Λ_{QCD} (eq.(3)), determined from charmonium, implies $\alpha_s(1 \text{ GeV}) = 0.30 \pm 0.06$. The analysis is done in the following way: we look at all available sum rules and establish the masses and couplings as a function of the three parameters m_s , $\langle m_s \bar{s}s \rangle$, and s_0 , the threshold of the continuum contribution. The pattern in all cases K^* , φ , E , and f' is similar.

There is a steep dependence on $\langle m_s \bar{s}s \rangle$. This dependence affects the value of the mass and coupling as well as the stability of the sum rule.

There is some dependence on the threshold s_0 . However, even if used to adjust the mass and coupling, this only allows variation in a narrow range for s_0 . It is satisfying that its value is always very near the expected or known position of the first radial recurrence.

The dependence on m_s is more channel dependent and goes in different directions in different channels. The reliability of the sum rules and consequently of the values of the parameters we are determining must be carefully analyzed. The interesting result, which could not be foretold, is that different

systems require different ranges of $\langle m_s \bar{s} s \rangle$ irrespective of s_0 , if one requires the mass of the state to be accurate within 20 MeV and the stability condition to be fulfilled. This situation is best seen comparing the K^* and ϕ mesons. As can be seen in Fig. 1 the K^* will only become stable if $\langle m_s \bar{s} s \rangle$ is $0.2 \times 10^{-2} \text{ GeV}^4$ or larger. Then there is some freedom to adjust the mass by adjusting s_0 . We emphasize that we are discussing effects at the 20 MeV level. Changing s_0 by 100 MeV, within the expected range determined by linear trajectory spacings, one can raise $\langle m_s \bar{s} s \rangle$ slightly. This analysis determines the allowed range of $\langle m_s \bar{s} s \rangle$ for this state. There is a very mild dependence on m_s , which is also indicated in Fig. 1.

In the case of ϕ the situation is inverted. Stability is now less constraining. Fig. 2 shows the prediction for the ϕ -meson mass for a few values of the continuum threshold. Especially the physical value of the coupling constant requires low $\langle m_s \bar{s} s \rangle$ and m_s , as can be seen from Fig. 3a. Fig. 3b shows the variation in the ϕ -meson mass.

The analysis for the other states (E,f') is similar but less stringent. The values for the parameters are compatible with those found for the K^* and ϕ mesons. The masses for these states are slightly above the experimental values. In both cases about 40 MeV. Since the sum rules appear to be accurate to this level we interpret this deviation as mixing with other states.

It is remarkable that the compatibility of all the sum rules requires a very narrow window for $\langle m_s \bar{s} s \rangle$. This is indicated in Fig.4. The mass dependence is also strong but mainly determined by ϕ and E.

We therefore conclude that the masses of the mesons and their couplings can be determined to a very high degree of accuracy. As a consequence we have established that

$$\langle m_s \bar{s} s \rangle = (0.20 \pm 0.01) \cdot 10^{-2} \text{ GeV}^4 \quad (14)$$

and

$$m_s(1 \text{ GeV}) = 110 \pm 10 \text{ MeV} \quad (15)$$

These numbers are compatible with current algebra and the accurate ratio for non-strange and strange quark masses. They appear to be low compared to previous determinations [3], though within errors. Determinations from baryon sum rules [7,8] suggest the following value for the condensate, which is compatible with (14)

$$\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle uu \rangle \quad (16)$$

but the extremely low value for the mass suggested in [8] seems to be ruled out.

Finally let us collect all the results for the masses and couplings of the mesons considered in this paper.

$$\begin{aligned} m_\phi &= 1010 \pm 10 \text{ MeV} & (\text{exp: } 1020 \text{ MeV}) \\ g_\phi^2/4\pi &= 13.0 \pm 0.2 & (\text{exp: } 11.7 \pm 0.9) \\ m_{K^*} &= 890 \pm 15 \text{ MeV} & (\text{exp: } 890 \text{ MeV}) \\ g_{K^*}^2/4\pi &= 1.46 \pm 0.1 & (17) \\ m_E &= 1470_{-10}^{+30} & (\text{exp: } 1420 \text{ MeV}) \\ m_{f'} &= 1540_{-10}^{+40} & (\text{exp: } 1520 \text{ MeV}) \end{aligned}$$

We have also included the K^* coupling in (17) although it is not experimentally measurable. It shows the degree of SU(3) breaking predicted by the sum rules. Exact SU(3) symmetry would imply $g_{K^*}^2/4\pi = 1.16$. The experimental values in (17) are from [9], except for the ϕ -meson coupling which is from [10]. The value in [10] is corrected for $p\pi$ branching. The errors in the theoretical predictions are determined from the ranges in $\langle m_s \bar{s}s \rangle$ and m_s as given by (14) and (15) (see also Fig.4).

In conclusion we emphasize that the analysis reported here is much more refined than in our previous paper [6].

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FIGURE CAPTIONS

Figure 1

Plot of the K^* mass against the Borel variable M^2 for different values of the parameters:

- a) $\alpha_s = 0.4$, $m_s = 90$ MeV, $\langle m_s \bar{s}s \rangle = 0.17 \cdot 10^{-2} \text{ GeV}^4$, $s_0 = 1.6 \text{ GeV}^2$;
- b) same as a) but with $\langle m_s \bar{s}s \rangle = 0.20 \cdot 10^{-2} \text{ GeV}^4$;
- c) $\alpha_s = 0.4$, $m_s = 90$ MeV, $\langle m_s \bar{s}s \rangle = 0.17 \cdot 10^{-2} \text{ GeV}^4$, $s_0 = 1.8 \text{ GeV}^2$;
- d) same as c) but with $m_s = 150$ MeV.
- e) (the final solution) $\alpha_s = 0.4$, $m_s = 90$ MeV, $\langle m_s \bar{s}s \rangle = 0.21 \cdot 10^{-2} \text{ GeV}^4$, $s_0 = 1.8 \text{ GeV}^2$, $g^2_{K^*}/4\pi = 1.46$.

Figure 2

Plot of the ϕ mass against the Borel variable M^2 for different values of the parameters. All cases have $s_0 = 2 \text{ GeV}^2$ and $\alpha_s = 0.30$

- a) $m_s = 90$ MeV, $\langle m_s \bar{s}s \rangle = 0.23 \cdot 10^{-2} \text{ GeV}^4$;
- b) $m_s = 100$ MeV, $\langle m_s \bar{s}s \rangle = 0.20 \cdot 10^{-2} \text{ GeV}^4$;
- c) $m_s = 90$ MeV, $\langle m_s \bar{s}s \rangle = 0.17 \cdot 10^{-2} \text{ GeV}^4$.

Figure 3

Variations of the ϕ parameters against m_s . All curves have $s_0 = 2 \text{ GeV}^2$;

- a) $\alpha_s = 0.4$, $\langle m_s \bar{s}s \rangle = 0.17 \cdot 10^{-2} \text{ GeV}^4$;
- b) $\alpha_s = 0.3$, $\langle m_s \bar{s}s \rangle = 0.17 \cdot 10^{-2} \text{ GeV}^4$;
- c) $\alpha_s = 0.4$, $\langle m_s \bar{s}s \rangle = 0.20 \cdot 10^{-2} \text{ GeV}^4$;
- d) $\alpha_s = 0.3$, $\langle m_s \bar{s}s \rangle = 0.20 \cdot 10^{-2} \text{ GeV}^4$.

Figure 4

Allowed values of $\langle m_s \bar{s}s \rangle$ a) and m_s b) from all states. The arrows at the begin- or endpoint of a line indicate that lower, resp. higher values of the parameters are not excluded for those particular states by our analysis.

Figure 1

Figure 2

Figure 3

Figure 4