VACUUM CONDENSATES IN THE BAG MODEL

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A B S T R A C T

The consequences of having quark and gluon condensates inside a MIT bag are investigated. We show that one naturally expects a state dependent bag constant and a colour-magnetic interaction term $\sim R^2$. This gives the possibility of having a small strong coupling constant α_s inside the bag.

Although QCD is by now a rather well established theory of strong interaction, we must still rely upon various model assumptions in order to do most phenomenology. Two rather successful approaches have been the bag model [1], and the QCD sum rule scheme [2]. The basic assumption of the bag model is that hadrons are bubbles of perturbative vacuum immersed in the full physical vacuum. Inside these bubbles, valence quarks and gluons are interacting only weakly. In the QCD sum rule approach one does not have any explicit space-time model for hadrons, but extracts masses and widths from properties of various current-current correlation functions. A basic ingredient in this scheme is the assumption that the QCD vacuum is characterized by nonvanishing expectation values of various gauge invariant operators like $G^2 = \alpha_s/\pi G_{\mu\nu}^a G_a^{\mu\nu}$, and $\bar{q}q$. These condensates give important contributions to the correlation functions via diagrams like those in Fig.1. One also implicitly assumes that the vacuum remains unchanged in the presence of sources.

The aim of this paper is to consider the possibility that the inside of a bag is not simply perturbative vacuum but contains quark and gluons condensates of the type just mentioned. We first give a short recapitulation of the MIT bag model, and then proceed to consider the effects of condensates inside the bag. In the end we give an explicit example of a model based on these ideas.

Even though the MIT bag model can be given a Lagrangian formulation, there is no simple way leading from the Lagrangian to the hadron spectrum. The equations of motion are very complicated, and can only be solved by making some drastic simplifications. The most commonly used is the static cavity approximation, where one assumes that the bag is a static sphere [3]. The only parameter describing the surface is the radius R , which is fixed by balancing the universal bag pressure B_{MIT} , with the inside pressure exerted by valence particles. It can be shown that the pressure balance condition is equivalent to minimizing the total bag energy with respect to the radius. Essentially all successful phenomenology that has been obtained from the MIT bag model has been based on this approximation and on ignoring loop diagrams. All these calculations (for light quarks) have a disconcerting feature in common, namely a large effective strong coupling constant $\alpha_s \approx 1 - 2$ inside the bag. For a long time there was a hope that the relevant perturbation expansion parameter should anyway turn out to be small (e.g. α_s/π^2), but the recent calculation of the quark self-energy

unfortunately shows that for $\alpha_s = 1$, the first order corrections to the quark energy is almost 50% [4]. Although this might partly be attributed to the sharp bag surface, it still casts serious doubts on the convergence of the perturbation expansion. By assuming a residual interaction between the valence particles and the vacuum condensates, as mentioned above, one can obtain large enough level splittings with a much smaller α_s .

As in the QCD sum rules we shall assume that the vacuum is characterized by nonvanishing matrix elements of gauge invariant operators such as G^2 , $\bar{q}q$ etc.. Inside hadrons, i.e. in the presence of valence particles, the vacuum is modified, so the values of $\langle G^2 \rangle$ etc. are changed but do not necessarily vanish. To calculate the interaction energy between the valence quarks and the vacuum condensates we must make several assumptions to be discussed in some detail below. The modified model we propose can not be derived from a Lagrangian, but is defined only in the static cavity approximation. We shall assume that the energy of a hadronic bag can be written as

$$E_{\text{had}}(R) = E_{\text{val}}(R) + E_{\text{int}}(R) + E_{\text{bag}}(R) \quad , \quad (1)$$

with E_{val} the usual MIT bag valence energy as computed using the standard rules of cavity perturbation theory [5]. The division of the nonperturbative contributions to E_{had} into a bag piece, E_{bag} , coming from the modification of the vacuum, and a condensate - valence interaction contribution, E_{int} , is by no means unambiguous. In fact, E_{int} as calculated below, will include pieces identical in form to the conventional $E_{\text{bag}} = BR^2 + CR^3$ used in bag model phenomenology [6]. There is, however, a class of contributions to E_{int} which are spin dependent and thus cannot be included in E_{bag} . In this paper we will concentrate on these spin dependent parts and assume that the spin independent ones can be incorporated in an effective E_{bag} as discussed below⁺⁾ .

⁺⁾ One might hope, that in a more elaborate treatment, it would make sense to split E_{had} into $E_{\text{bag}} + E_{\text{int}}$ even for the spin independent parts by assuming E_{bag} to be purely geometrical, while E_{int} could depend on, for example, the number and type of valence particles.

To compute the valence-condensate interaction energy E_{int} in Fig.2, first recollect how the usual perturbative diagrams in Fig.3 are treated in the context of the bag model. Specifically, using standard methods (and supressing colour indices) we get for the interaction diagram in Fig.3b

$$\Delta E^{\text{pert}} = - \int_0^\infty d\tau \int d^3x d^3y j_\mu(\vec{x}, \tau) \langle 0 | A^\mu(\vec{x}, \tau) A^\nu(\vec{y}, 0) | 0 \rangle j_\nu(\vec{y}, 0) , \quad (2)$$

where j_μ is the expectation value of the current $\bar{q}\gamma_\mu q$ in a cavity quark state, and where q and A^μ fulfill the usual bag boundary conditions. The vacuum expectation value in Eq.(2) is the usual cavity Feynman propagator continued to imaginary time, $\tau = it$, by a Wick rotation. For quarks in the same cavity mode, the currents are time independent, and performing the τ -integration we obtain the usual result

$$\Delta E^{\text{pert}} = \int_{\text{Bag}} d^3x d^3y j_\mu(\vec{x}) D^{\mu\nu}(\vec{x}, \vec{y}, \omega=0) j_\nu(\vec{y}) . \quad (3)$$

The effects of a background gluon condensate is incorporated by modifying the gluon propagator $D^{\mu\nu}$, as illustrated in Fig.2b. This is done by splitting the vector potential into a background and a quantum part, $A^\mu = A_b^\mu + A_q^\mu$, and substituting in Eq.(2). Using the equations of motion and integrating by parts using the gauge invariant bag boundary condition $n_\mu F^{\mu\nu} = 0$ (n_μ is the outward normal of the bag) one gets

$$\Delta E^{\text{cond}} = -1/4 \int_0^\infty d\tau \int_{\text{Bag}} d^3x d^3y F_{\mu\nu}(\vec{x}, \tau) \langle 0 | G^{\mu\nu}(\vec{x}, \tau) G^{\sigma\lambda}(\vec{y}, 0) | 0 \rangle F_{\sigma\lambda}(\vec{y}, 0) \quad (4)$$

where $G^{\mu\nu}$ is the background field strength and $F^{\mu\nu}$ the field generated by the external quark currents i.e. $\partial_\mu F^{\mu\nu} = j^\nu$. In deriving Eq.(4) we also used

$$\langle E_i^a(x) \rangle = \langle B_i^a(x) \rangle = 0 \quad (5)$$

which follows from colour symmetry, and where i and a are spin and colour indices respectively.

We must now make some assumptions about the correlation function in Eq.(4). First recollect that in the physical vacuum, colour and Lorentz invariance implies

$$\begin{aligned}\alpha_s/\pi\langle B_i^a(x)B_j^b(y)\rangle &= -\alpha_s/\pi\langle E_i^a(x)E_j^b(y)\rangle = \\ &= 1/96\delta^{ab}\delta_{ij}\langle G^2\rangle f((\vec{x}-\vec{y})^2+\tau^2)\end{aligned}\quad (6)$$

where $\tau = i(x_0 - y_0)$, f is a function satisfying $f(0) = 1$, and $\langle G^2\rangle \approx (340 \text{ MeV})^4$, from the QCD sum rules^[2]. Since the bag defines a specific Lorentz frame, and we furthermore expect the condensates to be modified because of the presence of valence particles, we cannot simply use Eq.(6) in our calculation. In spite of this, we shall for simplicity assume that the condensates are homogeneous even inside the bag, but allow for different values for the electric and magnetic contributions, i.e.

$$\begin{aligned}-\alpha_s/\pi\langle E_i^a(x)E_j^b(y)\rangle &= 1/24\delta^{ab}\delta_{ij}\langle E^2\rangle f_E((\vec{x}-\vec{y})^2,\tau) \\ \alpha_s/\pi\langle B_i^a(x)B_j^b(y)\rangle &= 1/24\delta^{ab}\delta_{ij}\langle B^2\rangle f_B((\vec{x}-\vec{y})^2,\tau)\end{aligned}\quad (7)$$

For the simple model presented in this paper only the magnetic correlation function will be of importance. Further assumptions are needed concerning the function f_B in Eq.(7). For simplicity, we shall neglect the space-dependence, but allow for an exponential decay of the correlation function in imaginary time^[7] i.e.

$$f((\vec{x}-\vec{y})^2,\tau) = e^{-\omega\tau}\quad (8)$$

where ω as well as $\langle E^2\rangle$ and $\langle B^2\rangle$ in Eq.(7) are taken as free parameters. Although we have made some very drastic assumptions about the correlation functions, one might hope that the effects to be discussed below would not be qualitatively different for some other form

of the correlations. The sign of $\langle B^2 \rangle$, which we take according to Eq.(6), is admittedly very important but a different space-time dependence in the function f should have no effect.

By substituting Eqs.(7) and (8) into Eq.(4) and performing the integrals, we get for a total colour singlet state of quarks in the same cavity mode,

$$\Delta E^{\text{cond}} = -\pi^2/6 \mu^2 R^2 \frac{1}{\omega} \langle B^2 \rangle \sum_{n < m} \langle \vec{s}_n \cdot \vec{s}_m \vec{T}_n \cdot \vec{T}_m \rangle \quad (9)$$

Where $\vec{s}_n$ and $\vec{T}_n$ are the generators of spin and colour respectively for the n :th quark. In deriving Eq.(9), one finds that the two space integrals factorize, and using the bag boundary conditions, the integral over F^{ij} can be expressed in the usual MIT magnetic moment μ defined by $\int d^3r \frac{1}{2} \vec{r} \times \vec{j}(r) = \mu \vec{s}$. Note that the colour-spin structure of Eq.(9) is exactly that of the usual colour magnetic interaction in the bag model, but the R dependence is quadratic rather than $\sim 1/R$. This is possible since the interaction is characterized by an additional dimensional quantity $1/\omega \langle B^2 \rangle$.

We can proceed in a similar way to calculate the self-energy diagrams in Fig.2. The result is

$$\Delta E^{\text{self}} = -b \langle B^2 \rangle R^2 + c \langle E^2 \rangle R^3 \quad , \quad (10)$$

where b and c are computable positive constants. Making similar assumptions about the quark condensate $\langle \bar{q}q \rangle$ as were made about $\langle B^2 \rangle$ and $\langle E^2 \rangle$, one can also derive

$$\Delta E^{\bar{q}q} = \tilde{b} \langle \bar{q}q \rangle R^2 \quad . \quad (11)$$

Although these calculations have been performed [8], the values of the coefficients b , c , and $\tilde{b}$ will be of no particular interest here, since they will only contribute to the phenomenological term E_{bag} , as already

pointed out. Note, however, that the total contribution to a state from Eqs.(10) and (11), is proportional to the number of quarks.

In the MIT model, where the inside of the bag is assumed to be perturbative, one takes $E_{\text{bag}} = B_{\text{MIT}}V$ where B_{MIT} is the bag constant (i.e. the energy difference between the perturbative and physical vacuum), and V is the volume of the bag. In the present model, where the nonperturbative vacuum is assumed to be only partially destroyed in the inside of the bag, it is natural to assume that E_{bag} depends on the number and kind of valence particles. We have no way of knowing the form of E_{bag} , but for states containing only valence quarks we shall take

$$E_{\text{bag}} = n(BR^2 + CR^3) \quad (12)$$

where n is the number of quarks. This n dependence gives the correct large N_{colour} behaviour for mesons and baryons if the constants B and C are independent of N_c ^{*)}, but the extrapolation to $n=2$ and 3 is of course quite bold. The form $BR^2 + CR^3$ is a guess consistent with earlier work on the bag model, and is also of the same form as the self-energy contributions in Eqs.(10) and (11), so these can be incorporated in E_{bag} without introducing any new free parameters. Thus, in this crude version, E_{int} in Eq.(1) is simply given by Eq.(9). Also note, that even if for some reason E_{bag} would be rather state independent, Eqs.(10) and (11) would anyway introduce an effective state dependence.

The intention of this paper is not to make detailed and comprehensive fits to data, but it is obviously important to show that it is possible to construct a realistic model based on the above ideas. Our aim will be very modest; we shall only try to fit ρ , N and Δ . Since, as pointed out in the previous section, we have several free parameters, this is in general no great achievement. We will, however, put two rather restrictive constraints on our fit namely:

- a) α_s small, specifically we shall fix $\alpha_s = 0.5$.

^{*)} One can easily convince oneself that the usual bag model with a state independent bag constant $B_{\text{MIT}} \sim (\text{energy of the nonperturbative vacuum}) \sim N_c^2$, does not have the correct $N_c \rightarrow \infty$ limit.

b) No geometrical (or "Casimir") term of the form Z/R .

The first requirement is important, since there are strong indications that the perturbation expansion does not converge for $\alpha_s \geq 1$. The second requirement is based on the simple fact that a term like Z/R is purely ad hoc. It is well known, that the conventional bag model cannot satisfy neither a) nor b). The Δ - N splitting implies $R \approx 1 \text{ GeV}^{-1}$ for $\alpha_s = .5$, which implies a kinetic energy of $\sim 6 \text{ GeV}$ for p and Δ . Even allowing for a much bigger α_s , a term Z/R is still needed to fit both p and the baryons. This short recollection of old results was to point out that even with free parameters it is not so easy to satisfy the constraints a) and b) above.

We now describe our model. The masses are calculated by minimizing

$$m(R)^2 = E_{\text{had}}(R)^2 - n(x/R)^2 \quad (13)$$

with respect to R , where $x = 2.04$, n the number of valence quarks, and $E_{\text{had}}(R)$ given by Eq.(1). $E_{\text{val}} = A/R$ is parametrized as in Ref.(9), except that we fix $\alpha_s = 0.5$ and use the value for the quark self-energy (Fig.3a) from Ref.(4). We did not make any systematic search for a best fit, but the parameters

$$1/\omega\langle B^2 \rangle = (145 \text{ MeV})^3$$

$$B = (159 \text{ MeV})^4$$

$$C = (110 \text{ MeV})^4 ,$$

give the quite acceptable results

$$m_{\Delta} = 1224 \text{ MeV} \quad R_{\Delta} = 7.30 \text{ GeV}^{-1}$$

$$m_p = 960 \text{ MeV} \quad R_p = 8.00 \text{ GeV}^{-1}$$

$$m_p = 782 \text{ MeV} \quad R_p = 6.98 \text{ GeV}^{-1}$$

We have thus succeeded in constructing a simple model satisfying the conditions a) and b) above. The uncomfortably large hadron radii are easily understood as a consequence of the large perturbative energy term A/R . For example, $A_{\text{proton}} \approx 6.6$ immediately implies $R > 7 \text{ GeV}^{-1}$.

(the large A is almost entirely due to the kinetic energy $= 3 \times 2.04/R$). In other models one gets a smaller A by subtracting a Z/R term^[3] or by assuming negative quark self energies^[9]. The latter is definitely wrong, and the former entirely ad hoc. A third possible way to lower R (without destroying the mass spectrum) is to add an external elementary pion field as in the chiral bag models^[10]. This possibility will not be pursued here.

Although, as already pointed out, we cannot directly compare our inside condensates with the ones extracted from the QCD sum rules, it is nevertheless interesting to make a rough order of magnitude comparison. Using Eq.(6), the sum rule value for $\langle G^2 \rangle$, and the lattice estimate $\omega = 1.6 \text{ GeV}$ ^[7] we get $1/\omega \langle B^2 \rangle \approx (128 \text{ MeV})^3$, to be compared with the above value $(145 \text{ MeV})^3$.

In summary, we have shown how the assumption of having residual vacuum condensates inside the bag naturally leads to a state dependent bag constant as well as hyperfine interactions $\sim R^2$. We have also presented a simple example of a model for light nonstrange hadrons with a small α_s .

Clearly one should make a more detailed analysis of the light hadron spectrum, and at the same time consider also other quantities like magnetic moments, charge radii, and axial coupling constants. There are also obvious extensions to heavy - light quark systems, glueballs, and multi-quark states. Since the effective bag constant depends on colour, spin, and number of valence particles, one might expect both glueballs and multiquark states to be heavier than in a conventional bag model. In fact, one of the main lessons to be drawn from this paper is that simple minded applications of the bag model to such states might be quite misleading since the bag parameters might have a strong state dependence.

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FIGURE CAPTIONS

- Figure 1: Nonperturbative contributions to current-current correlation functions in the QCD sum rule scheme. The crosses denote interactions with the vacuum condensates.
- Figure 2: Diagrams contributing to the valence-condensate interaction energy.
- Figure 3: Lowest order cavity perturbation diagrams contributing to the valence energy.

FIG. 1

FIG. 2

FIG. 3