ON THE ORIGIN OF SCALING VIOLATIONS
IN FRAGMENTATION FUNCTIONS

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A B S T R A C T

We show that the scaling violations observed in the fragmentation functions of neutrino scattering can be mostly of a kinematical origin and not necessarily from a QCD Q^2 -evolution. Due to confinement effects, the application of such evolution equations is nontrivial at non-asymptotic energies, but these QCD effects can be taken into account by a dynamical simulation of the parton branching processes followed by a string fragmentation method.

1 Introduction

In deep inelastic scattering, Q^2 -dependence has been observed in fragmentation functions both in the case of neutrino [1] and muon [2] scattering and fits to QCD evolution equations have given reasonable results, although in some cases a rather large Λ -value was needed. The observed variations of x (factorisation breaking) and Q^2 (scaling violation) in the EMC data [2], although being similar to that expected from a next-to-leading order QCD calculation [3], can equally well be represented by a dependence on W^2 alone. It is clear that at low energies various kinematical effects should also play a role. This was studied in [4] for e^+e^- annihilation, but for deep inelastic scattering it has not been taken fully into account until recently [5]. In section 2 we discuss the origin and result of these kinematical effects and also the dependence on different hadronization models.

The Altarelli-Parisi evolution equations [6] for the parton structure functions are well established on both theoretical and experimental grounds. Concerning the similar equations [7,3] for the fragmentation functions the situation is less clear however. The partonic branching processes are the same but the problems of confinement are more serious in this case, and after all this is what the fragmentation functions are supposed to effectively describe. In fact, by using the evolution equations directly one is assuming that partons originating from a branching process fragment into hadrons independently of each other, an assumption which may be unjustified at non-asymptotic energies. Of course, at larger energies (or Q^2 rather) the QCD evolution is expected to become important and we suggest, in section 3, how it can be properly taken into account by a dynamical simulation of the partonic branching processes combined with the string fragmentation model for the final hadronization step.

2 Kinematical effects

To study scaling violations in the fragmentation functions; defined as usually as $D(z)=1/N_{ev} \times dN/dz$ where $z=E_h/v$ is the fraction of the total energy transfer carried by a final hadron; one can study the Q^2 - and W^2 -dependencies defined by

$$D_Q(Q^2) = \frac{1}{N_{ev}(Q^2)} \cdot \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} \frac{dN(z, Q^2)}{dz} dz \quad (1)$$

$$D_W(W^2) = \frac{1}{N_{ev}(W^2)} \cdot \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} \frac{dN(z, W^2)}{dz} dz \quad (2)$$

Fig. 1 and 2 shows these dependencies for the particles with leading charge in neutrino and antineutrino interactions in deuterium as observed in BEBC by the WA25 collaboration at CERN [5]. Of course, the Q^2 - and W^2 -dependencies are not independent but related through the kinematical relation $W^2 = 2mv - Q^2 + m^2$. As can be seen the W^2 -dependencies are in general larger and also show definite, charge dependent structure. The curves are the predictions from the Lund Monte Carlo for simulating deep inelastic lepton-nucleon scattering [8,9] taking the neutrino energy spectrum into account and using standard parton model weak interaction cross sections. The agreement is remarkable in view of the fact that no QCD effects are included in this simulation (except Q^2 -evolved structure functions), so that the result is only due to the simple parton model and a fragmentation procedure that also takes kinematical effects properly into account. If first order QCD processes (hard gluon emission and boson-gluon fusion as is also available in the Monte Carlo program [8]) are included, the curves become slightly steeper but the experimental accuracy is not sufficient to distinguish the two possibilities.

The main effect is thus a W -dependence of a kinematical origin related to the production of a baryon in the target remnant fragmentation and the "cross-talk" between the forward and backward jet, i.e. they do not fragment independently of each other. Protons produced in the target fragmentation is also the cause of the bump structure observed in the W -dependence for the positive particles in fig. 2, but absent for the negative ones. At large energies these protons will populate the lowest z -bin, but with decreasing energy (v or W) they will partly migrate into a higher z -bin simply because of their mass being non-negligible and an energy fraction variable ($z = E_h/v$) is being used. As can be seen in fig. 3 this bump structure disappears if the target remnant diquark is replaced by an antiquark so that no baryon is produced.

In the Lund fragmentation model [10] a very basic assumption is that coloured partons never hadronize independently, but are part of a colour singlet jet system including some colour charged partons and their intermediate colour force fields (strings) and that it is these fields that break up to form the hadrons. Thus, in the hadronic CM frame of deep inelastic scattering, there will be a forward jet (the struck quark) and a backward jet (the target remnant diquark) with a colour flux tube stretched in between. The hadronization process is then most conveniently described using the light-cone variables $W_+ = E + p_z$ and $W_- = E - p_z$ (the z -axis being along the jet system) and a hadron formed from the backward end will take a fraction, z , of W_- leaving a rest system having (see [10] for details)

$$W'_- = (1-z) \cdot W_- \quad (3)$$

$$W'_+ = W_+ - m_\perp^2 / (z \cdot W_-) \quad (4)$$

where $m_\perp$ is the transverse mass of the produced hadron. From this it is clearly seen that the production of a hadron in the backward jet will also influence the forward jet through the $m_\perp^2$ -term in eq. (4). This influence will be particularly strong if a heavier particle is produced in a system with small energy, like the baryon from the target remnant diquark in neutrino scattering. However, even if the remnant diquark is replaced by an antiquark so that no baryon is produced there is still some, but much smaller, scaling violations at these low energies as can be seen from the dashed curves in fig. 3.

In a model where the forward and backward partons fragment independently, i.e. the $m_\perp^2$ -term in eq. (4) is absent, one expects no scaling violations, at least in a symmetric situation with equally heavy partons. Having a quark and a diquark, however, gives an asymmetric case with an unequal energy sharing in the center of momentum frame. The result shown by the dotted curves in fig. 3 is a very small W -dependence which do not reproduce the characteristic structure at small z discussed above.

Going up in energy, e.g. to the EMC muon scattering experiment at the CERN SPS, these kinematical effects are expected to die out. However, they do not do that as quickly as is perhaps often expected. This is demonstrated by the dotted lines in fig. 4 which are from a simulation of the quark-diquark system, neglecting QCD effects, so

that only kinematical effects are taken into account, as above using the Lund model [8,9]. In particular at large z , these effects are sizable even up to $W^2 \approx 200 \text{ GeV}^2$. Even if the kinematical effects alone does not account for the whole observed effect at these energies, they must still be considered. Including also the first order QCD processes of hard gluon emission and photon-gluon fusion as described in [11], gives more scale-breaking effects as shown by the dashed lines in fig. 4. We have not made any attempts to tune the Monte Carlo simulation or take possible experimental problems into account, since we are here mostly interested in qualitative features rather than making a detailed quantitative comparison. The general trends of the observed x - and Q^2 -dependencies (not shown here) are also represented fairly well when both kinematics and first order QCD is included in the model, in particular for the high- z bin which is most sensitive to scale breaking effects. There is, however, some room for an additional source of scaling violations, to which we now turn our attention.

3 QCD evolution of fragmentation functions

A natural consequence of QCD is the occurrence of parton branching processes which are expected to become important at higher energies. The problem is how to take them properly into account for the fragmentation process. In the Altarelli-Parisi evolution [6] of the parton structure functions, confinement is no real problem since, after a possible branching process the parton is probed by a large momentum transfer and the further development of the system is not considered.

For the fragmentation functions, which are supposed to effectively describe the confinement induced hadronization process, the problems of confinement have to be faced as well. By assuming that confinement does not upset the picture one can, however, write similar evolution equations [7,3], which for the quark fragmentation function reads

$$\frac{d}{dt} D_q^h(z, t) = \frac{\alpha_s(t)}{2\pi} \int_{\frac{z}{2}}^1 \frac{dy}{y} \left[P_{qq}(y) D_q^h\left(\frac{z}{y}, t\right) + P_{gq}(y) D_g^h\left(\frac{z}{y}, t\right) \right] \quad (5)$$

where as usual $t = \ln(Q^2/Q_0^2)$ with Q_0^2 being the renormalization point. Such equations are certainly asymptotically correct, the problem is rather to know at what energies they can be safely applied. In principle, all variations below the cut off (Q_0^2) could be absorbed in the definitions of the D-functions, which thus could have some Q^2 - or kinematical W^2 -dependence in them from the start, but in practice this would be difficult.

When using these equations one is in fact assuming that the partons originating from a branching process, fragment into hadrons independently of each other. This is so because the above integral equation for D_q^h contains two terms expressing:

- 1) the probability P_{qq} to find a quark (with energy fraction y) after a branching, times the quark fragmentation function D_q^h giving the probability for that quark to give a hadron
- 2) the probability P_{gq} to find a gluon after the branching, times the gluon fragmentation function D_g^h .

If the two partons from the branching, fragments as a system rather than independently, one can not make this separation into the two terms above. In a string model, like [10], where the hadrons are produced from the field connecting the partons this is clearly the case. In particular, a sufficiently collinear quark and gluon may enter the same hadron if the energy in the field between them is not enough for particle creation, as depicted in fig. 5. The produced hadron will then have a larger energy than either of the quark and the gluon separately. This can, of course, never happen in a model with independently fragmenting partons. Consequently, for the evolution of the fragmentation functions one is faced with the problems of confinement and further assumptions has to be made. In a model where partons do not fragment independently this kind of evolution equations are thus difficult to use.

A more satisfactory way of taking the QCD evolution into account is provided by the possibility to simulate the parton branching processes dynamically using a leading log QCD shower formalism as implemented in terms of a Monte Carlo generation procedure by e.g. [12,13]. The partons thus generated can then be connected by colour fields and the final, necessary hadronization step be performed by the

string model as discussed in [14], thus taking the above mentioned confinement effects into account as well as the kinematical effects as emphasized in the previous section.

We have here made the first attempts to introduce these effects in deep inelastic scattering, by letting the bremsstrahlung from the struck quark (before and after being struck by the virtual photon) be simulated by the program of [13]. At first, one may naively take the Q^2 of the virtual photon as the scale for this parton shower, but due to higher order corrections it turns out that W^2 is the more proper choice [15]. The full lines in fig. 4 shows the resulting W^2 -dependence from a simulation where the possible multiple gluon emission from the struck quark is taken into account using the shower Monte Carlo. First order photon-gluon fusion is also included as before using the exact matrix elements, since this process actually is of importance for the studied dependencies. Again, we have not tried to do fine tuning since we are mostly interested in showing trends and effects from different scale breaking sources. The exact position of the curves will, as before, depend on e.g. the cut off used for the exact treatment of the first order QCD processes and Λ_{QCD} which is here taken to be 300 MeV.

The effects from the leading log parton shower can here be seen. At these energies, however, similar results can also be obtained without the parton shower but having weaker cuts on the matrix elements for the exact simulation of the first order processes, so that one gluon (often then soft or collinear) is emitted in most events. At higher energies these multiple gluon emission effects become more important. To give an example we show in fig. 6 the fragmentation function for a quark jet from a quark-antiquark system of invariant mass 30 GeV. This could then represent the forward jet in a deep inelastic scattering event at $W = 30$ GeV or a 15 GeV quark jet at zero rapidity in the ISR. As can be seen, the high- z tail of the distribution changes somewhat when the parton shower is included. Another consequence of this gluon emission is that the jet becomes wider, i.e. the typical $p_{\perp}$ within the jet increases from about 0.4 GeV to about 0.6 GeV in this case.

For a more detailed discussion of these gluon radiation effects for high energy jets (at present and future collider energies) we refer to [14].

4 Conclusions

We have discussed various effects causing scaling violations in fragmentation functions. At low energies, kinematical effects are very important and was shown to give the dominant contribution at neutrino scattering energies, but are still present at higher energies. In particular, in a string model where the forward and backward jet hadronize as a system, the production of a baryon from target remnant diquark causes disturbances also in the forward jet. If the forward and backward jets fragment independently this disturbance is much weaker, but such a model is hardly realistic at low invariant masses of the system.

The direct application of Altarelli-Parisi type evolution equations is nontrivial at non-asymptotic energies due to confinement problems, but the expected scale breaking effects from QCD can be taken into account by a dynamical simulation of the parton branching processes followed by the string hadronization model taking confinement effects and kinematical constraints into account. The gluon radiation process will then give sizable effects at higher energies.

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See also contributions in the report from the Lausanne workshop on the feasibility of a hadron collider in the LEP tunnel.
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FIGURE CAPTIONS

- Figure 1: Q^2 -dependence of fragmentation functions, as defined in eq. (1), for different z -intervals for positive particles in νd and negative particles in $\bar{\nu} d$ interactions. Data, with $W^2 > 2 \text{ GeV}^2$, are from [5] and the full lines are the Lund model predictions without QCD effects. The z -intervals are 1: $0 < z < 0.1$, 2: $0.1 < z < 0.2$, 3: $0.2 < z < 0.3$, 4: $0.3 < z < 0.4$, 5: $0.4 < z < 0.5$, 6: $0.5 < z < 0.7$, 7: $0.7 < z < 1$.
- Figure 2: W^2 -dependence of fragmentation functions as defined in eq. (2), in data [5] and Lund model without QCD effects. $Q^2 > 2 \text{ GeV}^2$ and z -bins are as in fig. 1.
- Figure 3: W^2 -dependence of fragmentation functions with z -bins as in fig. 2 for different model calculations. Full lines: normal Lund model, dashed curves: Lund model where target remnant diquark is replaced by an antiquark, dotted curves: independent fragmentation of the forward quark and backward diquark.
- Figure 4: W^2 dependence of the fragmentation functions in μp scattering from EMC [2] and model calculations. Dotted curves: Lund model without QCD, dashed curves: Lund model including first order hard QCD processes and full lines is when a parton shower is also included to simulate the leading log gluon emission from the struck quark.
- Figure 5: The parton branching process $q \rightarrow qg$ with the resulting string configuration and possible breakup to form a leading hadron including both the quark and gluon from the branching.
- Figure 6: Fragmentation function for a quark jet from a 30 GeV quark-antiquark system without (dashed curve) and with (full curve) leading log gluon radiation simulated by [13].

Fig.1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6