THE ANOMALY TERM IN THE $N = 2$ SUPERSYMMETRIC GAUGE THEORY

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A B S T R A C T

The Schwinger's slowly varying field approximation is used to compute the terms in the effective Lagrangian that reproduce the anomalies of the underlying theory. This method has been applied to extract the anomaly term in the case of the pure $N = 2$ supersymmetric Yang-Mills theory.

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1. INTRODUCTION

The existence of the axial anomaly in a theory with fundamental fermions as QCD is a very important ingredient for the understanding of relevant features of the dynamics of the pseudoscalar mesons as for instance the decay $\pi^0 \rightarrow 2\gamma^{(1)}$ and the resolution of the U(1) problem²⁾. What one has learnt is that, in order to take into account the anomaly effects, one must have additional terms in the effective Lagrangian for the pseudoscalar mesons having the property of reproducing the anomalous Ward identities of the underlying quark theory. In this way, one ensures that the underlying quark theory and the effective theory of hadrons have the same anomalous symmetries.

Therefore, if one is interested in the construction of effective Lagrangians incorporating the effect of the anomalies, it is important to devise a way of computing them, that simultaneously gives also the terms transforming according to what is prescribed by the anomaly. If the anomaly terms are not vanishing in the slowly varying field approximation, a method, which is very suitable for such a calculation, is the one discussed by Schwinger³⁾ for the computation of the fermion and scalar determinants for almost constant external fields. This method has already been applied⁴⁾ to study the supersymmetric CP^{N-1} model in two dimensions obtaining the anomalous term proposed by Veneziano and Yankielowicz⁵⁾ in the four-dimensional supersymmetric $N = 1$ pure Yang-Mills theory.

In this paper we apply this method to the computation of the four-dimensional fermion and scalar determinants and we show that the well-known anomalous terms reproducing the axial and the dilatational anomalies are recovered. In particular, we use this procedure to study the anomalous transformations of the $N = 2$ supersymmetric pure Yang-Mills theory^{6,7)}. This theory turns out to have a very rich structure, that makes it worth being studied. It has instanton solutions⁸⁾, whose quantum corrections get a contribution only from the zero modes⁹⁾. The conformal invariance of the classical theory is broken only by one-loop diagrams and therefore, the β -function can be computed¹⁰⁾ exactly. It contains a complex scalar field with a classical potential that allows, in general, for a non-vanishing

vacuum expectation value. As a consequence, the model contains monopole and dyon solutions¹¹⁾ in the limit of Prasad, Sommerfield¹²⁾ and Bogomolnyi¹³⁾, whose classical mass is not affected by quantum corrections¹¹⁾. This is connected to the fact that the supersymmetry algebra is modified to include the topological quantum numbers as central charges¹⁴⁾. In addition also the mass of the $N = 2$ matter has been shown not to be renormalized: $m_{\text{bare}} = m_{\text{ren}}^{(15)}$.

In this paper, we start with the $N = 2$ supersymmetric gauge theory coupled to supermatter first discussed by Fayet⁶⁾. We then perform the functional integration over the matter fields and we are left with an effective Lagrangian containing composites of the fields belonging to the gauge hypermultiplet. We can finally identify the terms of the effective Lagrangian that reproduce the anomalies of the underlying theory. In terms of the superfield strength $W(x;\theta)^{(16)}$ usually used in the $N = 2$ supersymmetric gauge theories, the anomalous contribution is given by:

$$\mathcal{L}_{\text{eff}} = \frac{\beta(g)}{2g} \frac{1}{4} \left\{ W^2 \log W^2 \right\} \Big|_{u(\theta)} + \overline{W}^2 \log \overline{W}^2 \Big|_{u(\theta)} \Big\}$$

that was already guessed¹⁷⁾ using arguments based on the symmetries of the theory. $\beta(g)$ is the β -function of the theory. If we perform a super-anomalous transformation discussed in Section 4, we find that the variation of the action gives precisely the anomaly multiplet discussed by Fischer¹⁸⁾.

The paper is organized as follows. In Section 2, we remind the construction of the $N = 2$ supersymmetric Yang Mills theory coupled with supermatter using both $N = 1$ and $N = 2$ superfield formalism.

Section 3 is devoted to the calculation of the relevant determinants for simplicity in the abelian case and to the extraction of the anomalous terms.

In Section 4, we rewrite the anomalous terms in a manifestly supersymmetric formalism using both $N = 1$ and $N = 2$ superfields.

Finally, Section 5 is devoted to some conclusions and physical considerations. In Appendix A, we discuss in some detail the $N = 2$ superfield formalism.

Appendix B is devoted to the calculation of the fermion and boson determinants in

the approximation of slowly varying fields. Finally, the computation of the quadratic and quartic terms in η is performed in Appendices C and D, respectively.

2. SUPERSYMMETRIC $N = 2$ GAUGE THEORY COUPLED WITH MATTER

The multiplet of the pure $N = 2$ gauge theory contains a gluon A_μ , a complex gluino χ and a complex scalar field ϕ . The Lagrangian which describes the interaction among those particles, has been obtained by using different techniques^{6,7)} and is given by:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - (\mathcal{D}_\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - i \bar{\chi} \not{\partial} \chi + \frac{1}{2} g^2 (\phi^\dagger \phi)^2 + \\ & + \frac{i}{f^2} g f^{abc} \{ \bar{\chi}^a (1 - \gamma_5) \chi^b + \bar{\chi}^a (1 + \gamma_5) \chi^c \} \phi^b \phi^c \end{aligned} \quad (2.1)$$

where

$$(\mathcal{D}_\mu \phi)^a = \partial_\mu \phi^a + g f^{abc} A_\mu^b \phi^c \quad (2.2)$$

and the metric is $g_{\mu\nu} \sim (-1, 1, 1, 1)$. In (2.1) we use the notations of Ref. 11 where the transformations of the fields under supersymmetry are given.

It is convenient to rewrite (2.1) in a manifestly $N = 2$ invariant way by introducing, as in Ref. 16, a superspace described by x_μ, θ_α^i and $\bar{\theta}_{\dot{\alpha}i}$ where α and $\dot{\alpha}$ are the usual Weyl indices and i is a $SU(2)$ spinor index.

In order to rewrite (2.1) in a $N = 2$ superspace, one must introduce the following $N = 2$ chiral superfield strength¹⁶⁾

$$W^a[y, \bar{\theta}] = C^{*a} + \bar{\lambda}_{\dot{\alpha}i}^a \bar{\theta}^{\dot{\alpha}i} - b_A^a C_A^a + a_{\mu\nu} F_{\mu\nu}^a + \bar{\lambda}_{\dot{\alpha}i}^a (\bar{\varphi}^{\dot{\alpha}i})^a + u D^{*a} \quad (2.3)$$

and one has

$$\begin{aligned} D^{*a} = & -4(\mathcal{D}_\mu \mathcal{D}_\mu C)^a - 2f^{abc} (\lambda_i^a)^b (\lambda_i^c)^c + 4g^2 f^{abc} c^{de} C^b C^d C^{*e} \\ (\bar{\varphi}^{\dot{\alpha}i})^a = & -2i(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} (\mathcal{D}_\mu \lambda_\alpha^i)^a + 2g f^{abc} (\bar{\lambda}^{\dot{\alpha}i})^b C^c \end{aligned} \quad (2.4)$$

$$C_A^* = -C_A$$

We use the same notations as in Ref. 16. They are summarized in Appendix A, where $b_A^a, a_{\mu\nu}, \bar{\chi}_{\dot{\alpha}i}$ and u are explicitly given. As explained in Ref. 16, W is not the

most general chiral superfield, but it satisfies additional constraints. Using the $N = 2$ superfield strength (2.3) and its complex conjugate $\bar{W}$, it is possible to compute the following gauge invariant Lagrangian:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{8} W^2|_{\bar{u}} - \frac{1}{8} \bar{W}^2|_u = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - 2(\mathcal{D}_\mu C^*)^a (\mathcal{D}_\mu C)^a \\ & -i \bar{\lambda}^a \cdot \bar{\sigma}^\mu (\mathcal{D}_\mu \lambda)^a + 2g^2 (\phi^\dagger c^{*c})^2 - \frac{1}{2} C_A^a C_A^a \\ & -g f^{abc} (\bar{\lambda}^a \bar{\lambda}^c C^b + \lambda^a \lambda^c C^{*b}) \end{aligned} \quad (2.5)$$

that is equal to (2.1) if $\sqrt{2}C = \phi$ and if the Dirac spinor χ appearing in (2.1) is related to the Weyl and $SU(2)$ spinors λ_α^i and $\bar{\lambda}_{\dot{\alpha}}^i$ by the relation

$$\chi = \begin{pmatrix} \lambda_\alpha^1 \\ i \bar{\lambda}_{\dot{\alpha}}^2 \end{pmatrix} \quad (2.6)$$

It is convenient to rewrite the Lagrangians (2.1) and (2.5) using a $N = 1$ superfield formalism. In terms of $N = 1$ superfields the vector hypermultiplet is represented by a vector multiplet $V(x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$

$$V(x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) = -(\theta \sigma_\mu \bar{\theta}) \mathcal{V}_\mu + i(\theta \theta)(\bar{\theta} \bar{\lambda}) - i(\bar{\theta} \bar{\theta})(\theta \lambda) + \frac{1}{2}(\theta \theta)(\bar{\theta} \bar{\theta}) D \quad (2.7)$$

and by a chiral superfield ϕ :

$$\Phi(y_+, \theta) = \sqrt{2} C + \sqrt{2} \theta^\alpha P_\alpha + i \theta \theta G \quad (2.8)$$

In terms of those $N = 1$ chiral superfields, the Lagrangian (2.1) takes the following form:

$$\mathcal{L} = \frac{1}{4} W^\alpha W_\alpha|_F + \frac{1}{4} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}|_{\bar{F}} + \Phi^{*a} (e^a_V)_{ab} \Phi^b|_D$$

$$\begin{aligned}
&= -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + 2g^2 (f^{abc} c^{*b} c^c)^2 - i \bar{\lambda}^a \bar{\sigma}^{\mu} (\mathcal{D}_{\mu} \lambda)^a - i \bar{p}^a \bar{\sigma}^{\mu} (\mathcal{D}_{\mu} p)^a \\
&- 2(\mathcal{D}_{\mu} c)^{*a} (\mathcal{D}_{\mu} c)^a + 2g f^{abc} (\bar{p}^a \bar{\lambda}^c c^b - c^{*a} \lambda^c p^b)
\end{aligned} \quad (2.9)$$

where W^{α} is the $N = 1$ superfield strength and in the last expression we have eliminated the auxiliary fields by means of their algebraic equations of motion. λ and p represent two Weyl gluinos that are related to the $SU(2)$ spinor λ_{α}^i appearing in (2.5) by the following relations:

$$\lambda_{\alpha}^i = \begin{pmatrix} -p_{\alpha} \\ \lambda_{\alpha} \end{pmatrix}; \quad \bar{\lambda}^{\dot{\alpha}}_i = \begin{pmatrix} -\bar{p}^{\dot{\alpha}} \\ \bar{\lambda}^{\dot{\alpha}} \end{pmatrix}; \quad \bar{\lambda}^{\dot{\alpha}i} = \begin{pmatrix} \bar{\lambda}^{\dot{\alpha}} \\ \bar{p}^{\dot{\alpha}} \end{pmatrix}; \quad \lambda_{\alpha i} = \begin{pmatrix} -\lambda_{\alpha} \\ -p_{\alpha} \end{pmatrix} \quad (2.10)$$

In the last part of this section we will couple the $N = 2$ super Yang-Mills theory with $N = 2$ supermatter following the procedure of Ref. 6. This can easily be done using $N = 1$ superfields introducing two chiral supermultiplets:

$$R_i = z_i + \sqrt{2} \theta \psi_i + \theta \theta F_i; \quad i = 1, 2 \quad (2.11)$$

The Lagrangian that describes the coupling between the gauge fields and the supermatter is given by:

$$\begin{aligned}
\mathcal{L} = & \left[R_1^{*a} (e^g V)_{ab}^b R_1^b + R_2^a (e^g V)_{ab}^b R_2^{*b} \right]_D + \\
& + \frac{g}{\sqrt{2}} \left[R_2^a \Phi_{ab} R_1^b + R_1^{*a} \Phi_{ab}^* R_2^{*b} \right]_F
\end{aligned} \quad (2.12)$$

where Φ and V are given by (2.7) and (2.8), respectively. Equation (2.12) can be written in terms of the component fields and one gets:

$$\begin{aligned}
\mathcal{L} = & F_1^{*a} F_1^a + F_2^{*a} F_2^a - (\mathcal{D}_{\mu} z_1)^{*a} (\mathcal{D}_{\mu} z_1)^a - (\mathcal{D}_{\mu} z_2)^{*a} (\mathcal{D}_{\mu} z_2)^a \\
& - i \bar{\psi}^a (\not{D} \psi)^a + g \left[\frac{1}{2} z_1^{*a} D z_1^a - \frac{1}{2} z_2^{*a} D z_2^a + z_2^a G z_1^{*b} + z_1^{*a} G z_2^b \right] \\
& - \frac{g}{\sqrt{2}} \left[\bar{\psi}^a \frac{1-\gamma_5}{2} \lambda^b z_1^b + z_1^{*b} \bar{\lambda}^a \frac{1+\gamma_5}{2} \psi^b - z_2^{*a} \bar{\lambda}^{ab} \frac{1-\gamma_5}{2} \psi^b - \bar{\psi}^a \frac{1+\gamma_5}{2} \lambda^b z_2^b \right] \\
& - \frac{ig}{\sqrt{2}} \left[\bar{\psi}^a \frac{1+\gamma_5}{2} p^{ab} z_1^b + z_2^{*a} \bar{p}^{ab} \frac{1+\gamma_5}{2} \psi^b - \bar{\psi}^a \frac{1-\gamma_5}{2} p^{ab} z_2^b - z_1^{*a} \bar{p}^{ab} \frac{1-\gamma_5}{2} \psi^b \right] \\
& + g \left[F_2^a C^{ab} z_1^b + z_2^a C^{ab} F_1^b + F_1^{*a} C^{*ab} z_2^{*b} + z_1^{*a} C^{*ab} F_2^{*b} \right] \\
& - ig \bar{\psi}^a \frac{1+\gamma_5}{2} C^{ab} \psi^b + ig \bar{\psi}^a \frac{1-\gamma_5}{2} C^{*ab} \psi^b
\end{aligned} \quad (2.13)$$

where the fields in the adjoint representation have been indicated in matrix notation and:

$$(\mathcal{D}_{\mu} z_{i,2})^a = \partial_{\mu} z_{i,2}^a + \frac{i}{2} g \mathcal{V}_{\mu}^{ab} z_{i,2}^b; \quad (\mathcal{D}_{\mu} \psi)^a = \partial_{\mu} \psi^a + \frac{i}{2} g \mathcal{V}_{\mu}^{ab} \psi^b \quad (2.14)$$

the matter Dirac field ψ and the two Majorana gluinos are given by:

$$\psi = \begin{pmatrix} \psi_{1\alpha} \\ i \bar{\psi}_2^{\dot{\alpha}} \end{pmatrix}; \quad p = \begin{pmatrix} p_{\alpha} \\ i \bar{p}^{\dot{\alpha}} \end{pmatrix}; \quad \lambda = \begin{pmatrix} \lambda_{\alpha} \\ i \bar{\lambda}^{\dot{\alpha}} \end{pmatrix} \quad (2.15)$$

By eliminating the matter auxiliary fields by means of their algebraic equations of motion and by introducing the following $SU(2)$ spinors:

$$z = \begin{pmatrix} i z_1^* \\ z_1 \end{pmatrix} \quad z^{\dagger} = (-i z_2 \quad z_2^*) \quad (2.16)$$

$$\bar{\psi}^i = \begin{pmatrix} \lambda_{\alpha}^i \\ \bar{\lambda}^{\dot{\alpha}i} \end{pmatrix} \quad \bar{\eta}_i = (-\lambda_{\alpha}^i \quad \bar{\lambda}^{\dot{\alpha}i}) \quad i = 1, 2 \quad (2.17)$$

it is possible to rewrite (2.13) in the following form after a partial integration:

$$\begin{aligned} \mathcal{L} = & -z^a \Delta_B^{ab} z^b - \bar{\psi}^a \Delta_F^{ab} \psi^b - \\ & - \frac{g}{\sqrt{2}} \left[i \bar{\psi}^a \gamma_5 (\eta^{ab})^i \epsilon_{ij} (z^i)^b + i (z^i)^a \epsilon^{ij} \bar{\eta}^{ab} \gamma_5 \psi^b \right] \end{aligned} \quad (2.18)$$

where

$$\Delta_B^{ab} = -(\mathcal{D}_\mu \mathcal{D}_\mu)^{ab} - g \vec{\tau} \cdot \vec{A} + \frac{g^2}{2} (C C^* + C^* C)^{ab} \quad (2.19)$$

$$\Delta_F^{ab} = i \not{D}^{ab} - i g \frac{1-\gamma_5}{2} C^{*ab} + i g \frac{1+\gamma_5}{2} C^{ab} \quad (2.20)$$

$$\mathcal{D}_\mu^{ab} = \partial_\mu \delta^{ab} + \frac{i}{2} g \not{v}_\mu^{ab} \quad (2.21)$$

and the fields $\vec{A}^{ab}$ are given by

$$\begin{aligned} A_3^{ab} &= -\frac{1}{2} \not{D}^{ab} - \frac{1}{2} g [C, C^*]^{ab} \\ A_-^{ab} &= i G^{ab} \quad A_+^{ab} = -i G^{*ab} \end{aligned} \quad (2.22)$$

in terms of the fields appearing in (2.13).

Finally, the right- and left-handed spinors are defined by

$$\lambda_L = \frac{1+\gamma_5}{2} \lambda \quad (2.23)$$

The form (2.18) is manifestly invariant under the SU(2) internal transformations discussed by Fayet in Ref. 6. The τ matrices appearing in (2.19) are in fact the usual Pauli matrices relative to this internal SU(2).

3. CONSTANT FIELD APPROXIMATION AND THE ANOMALY TERM

In this section, starting from (2.18), we will integrate over the supermatter fields obtaining an effective action containing only composites of the fields belonging to the gauge hypermultiplet and we will identify those terms which transform according to the anomalous Ward identities.

The integration over the supermatter fields z and ψ can be trivially performed because (2.18) is at most quadratic in those fields and one gets the following effective action

$$S_{eff} = \text{Tr} \log \Delta_F - \text{Tr} \log [\Delta_B^i + \frac{g^2}{2} \bar{\eta}^i \gamma_5 \Delta_F^{-1} \gamma_5 \eta_i] \quad (3.1)$$

It is, in general, not possible to compute explicitly the determinants in (3.1). For the sake of simplicity, we will specialize in the case of an abelian theory and we will compute (3.1) in the approximation of slowly varying fields using a technique discussed by Schwinger in Ref. 3 and used already in Ref. 4 to compute anomalous terms. This approximation in fact can be successfully used because the anomalous terms are not vanishing for slowly varying fields. Since η is a Grassmann variable in the constant field approximation, one can expand the second determinant in (3.1) and get only up to quartic terms with η :

$$\begin{aligned} S_{eff} = & \text{Tr} \log \Delta_F - \text{Tr} \log \Delta_B - \frac{g^2}{2} \text{Tr} [\Delta_B^{-1} \bar{\eta} \gamma_5 \Delta_F^{-1} \gamma_5 \eta] + \\ & + \frac{g^4}{8} \text{Tr} [\Delta_B^{-1} \bar{\eta} \gamma_5 \Delta_F^{-1} \gamma_5 \eta \Delta_B^{-1} \bar{\eta} \gamma_5 \Delta_F^{-1} \gamma_5 \eta] \end{aligned} \quad (3.2)$$

Remember that both $(\Delta_B^{-1})^i_j$ and $\bar{\eta}^i \Delta_F^{-1} \eta_j$ are also matrices with indices of the SU(2) internal symmetry.

In this section we will only give the results of the computation of the four pieces appearing in (3.2), while the detailed calculations are performed in Appendices B, C and D.

For the first two terms of (3.2) we get the following expressions:

$$\text{Tr} \log \Delta_B = -\frac{V}{8\pi^2} \left\{ \int_0^{\infty} \frac{ds}{s^3} e^{-\frac{1}{2}s} \left[\frac{1}{2} \left(A^2 - \frac{F_{\mu\nu} F^{\mu\nu}}{24} \right) \right] \int_0^{\infty} \frac{ds}{s} e^{-\frac{1}{2}s} \left[\frac{1}{2} \left(A^2 - \frac{F_{\mu\nu} F^{\mu\nu}}{24} \right) \right] \right\} -$$

$$-\frac{V}{8\pi^2} \int_0^{\infty} \frac{ds}{s^3} e^{-\frac{1}{2}s} \left\{ \frac{g^2}{8} s^2 \frac{F_{\mu\nu} \tilde{F}^{\mu\nu} \cosh(g A \cdot s)}{\cosh(g s \frac{X}{2}) - \cosh(g s \frac{X^*}{2})} - 1 + \right.$$

$$\left. + \frac{s^2 g^2}{2} \left(\frac{1}{24} F_{\mu\nu} F^{\mu\nu} - A^2 \right) \right\} \quad (3.3)$$

and

$$\text{Tr} \log \Delta_F = -\frac{V}{8\pi^2} \left\{ \frac{g^2}{16} F_{\mu\nu} \tilde{F}^{\mu\nu} \log \frac{C^*}{C} + \int_0^{\infty} \frac{ds}{s^3} e^{-\frac{1}{2}s} \left[\frac{1}{24} \left(1 + s^2 \frac{g^2}{24} F_{\mu\nu} F^{\mu\nu} \right) \right] \right\} -$$

$$-\frac{V}{8\pi^2} \int_0^{\infty} \frac{ds}{s^3} e^{-\frac{1}{2}s} \left\{ \frac{g^2}{16} F_{\mu\nu} \tilde{F}^{\mu\nu} \frac{\cosh(g s \frac{X}{2}) + \cosh(g s \frac{X^*}{2})}{\cosh(g s \frac{X}{2}) - \cosh(g s \frac{X^*}{2})} \right. \\ \left. - 1 - \frac{s^2 g^2}{24} F_{\mu\nu} F^{\mu\nu} \right\} \quad (3.4)$$

where

$$\tilde{X} = \vec{H} + i\vec{E} \quad ; \quad \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} (H^2 - E^2) \quad ; \quad \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = i \vec{E} \cdot \vec{H} \quad (3.5)$$

$$\tilde{F}_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \quad , \quad \text{with} \quad \epsilon_{0123} = 1$$

and V is the volume of the Euclidean space-time.

Because of ultraviolet divergences we have regularized the integral over the proper time by introducing a cut-off $1/\Lambda^2$ in the lower limit of integration. In the last two terms of (3.3) and (3.4), the cut-off has been removed because no

divergence appears. In both determinants we have a quadratic and a logarithmic divergence. As a consequence of supersymmetry the quadratic divergences in (3.3) and (3.4) cancel when we compute the expression (3.2). The logarithmic divergence will be eliminated later on by adding suitable counterterms and renormalizing the theory.

Notice also that (3.4) contains a finite term proportional to $\tilde{F} \log (C^*/C)$ that is known to reproduce the effect of the axial U(1) anomaly in the effective action and that is one of the important ingredients for the solution of the U(1) problem²⁾.

The third term of (3.2) is computed in Appendix C and it is given by^{*)}

$$-\frac{g^2}{2} \text{Tr} \left[\Delta_B^{-1} \gamma_5 \Delta_F^{-1} \gamma_5 \right] = -\frac{g^2 V}{32\pi^2} \frac{g^2}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \int_0^{\infty} \frac{ds}{\cosh(g s \frac{X}{2}) - \cosh(g s \frac{X^*}{2})} e^{-\frac{1}{2}s} C^* C s \\ \cdot \frac{g^2}{2} \left\{ C^* \frac{\cosh(g A) - \cosh(g s \frac{X}{2})}{g^2 \tilde{A}^2 - g^2 \frac{X^2}{4}} \left(-i \vec{A}(\lambda \vec{\tau} \lambda) + \frac{1}{2} F_{\mu\nu} (\lambda \sigma^{\mu\nu} \lambda) \right) + \right. \\ \left. + C \frac{\cosh(g A) - \cosh(g s \frac{X^*}{2})}{g^2 \tilde{A}^2 - g^2 \frac{X^{*2}}{4}} \left(-i \vec{A}(\bar{\lambda} \vec{\tau} \bar{\lambda}) + \frac{1}{2} F_{\mu\nu} (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda}) \right) \right\} \quad (3.6)$$

*) The $o^{\mu} 2 \times 2$ matrices are defined as $o^{\mu} = [1, \vec{\sigma}]$, $\bar{o}^{\mu} = [1, -\vec{\sigma}]$ and $\sigma^{\mu\nu} = 1/4 [\sigma^{\mu}\sigma^{\nu} - \sigma^{\nu}\sigma^{\mu}]$, $\bar{\sigma}^{\mu\nu} = 1/4 [\bar{\sigma}^{\mu}\bar{\sigma}^{\nu} - \bar{\sigma}^{\nu}\bar{\sigma}^{\mu}]$ where the $\vec{\sigma}$ are the Pauli matrices. We used for the γ 's the representation:

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix}$$

No cut-off is needed at the lower limit of integration because no divergence appears. We can expand the various cosh terms in (3.6) and obtain the following expression

$$\begin{aligned}
 -\frac{g^2}{2} \text{Tr} [\bar{\Delta}_B^{-1} \gamma_5 \Delta_F^{-1} \gamma_5 \eta] &= -\frac{g^2 V}{32\pi^2} \left\{ \frac{\vec{C}_A}{4C} (\lambda \vec{\tau} \lambda) + \right. \\
 &+ \frac{\vec{C}_A}{4C^*} (\bar{\lambda} \vec{\sigma} \bar{\lambda}) + \frac{1}{4C} F_{\mu\nu} (\lambda \sigma^{\mu\nu} \lambda) + \\
 &\left. + \frac{1}{4C^*} F_{\mu\nu} (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda}) + \dots \right\}
 \end{aligned} \quad (3.7)$$

where we have used the relation

$$\vec{C}_A = -2i\vec{A} \quad (3.8)$$

In the previous expression we have neglected scale invariant terms, that contain higher powers of C and C^* in the denominator. They come from higher order terms in the expansion in the power of s in the integrand of (3.6). Those terms are of the same order of the ones appearing in the last two terms of (3.3) and (3.4).

Finally, following the procedure of Appendix D we can compute the last term of (3.2) and we get

$$\begin{aligned}
 \frac{g^4}{8} \text{Tr} [\Delta_B^{-1} \gamma_5 \Delta_F^{-1} \gamma_5 \eta]^2 &= -\frac{g^2 V}{32\pi^2} \cdot \\
 &\cdot \left\{ \frac{1}{96C^2} [(\lambda \vec{\tau} \lambda)^2 + (\lambda \sigma^{\mu\nu} \lambda)^2] + \frac{1}{96C^{*2}} [(\bar{\lambda} \vec{\tau} \bar{\lambda})^2 + (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda})^2] \right\}
 \end{aligned} \quad (3.9)$$

where for the sake of simplicity we have evaluated only the lowest order terms in $1/C$ and $1/C^*$. We can now add the previously computed four contributions to get the following expression for the effective Lagrangian coming from (3.1):

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} &= \frac{e_o^2}{32\pi^2} \left\{ F_{\mu\nu} \vec{F}^{\mu\nu} \log \frac{C}{C^*} + (2\vec{C}_A^2 + F_{\mu\nu} F^{\mu\nu}) \log \frac{4e_o^2 C}{\Lambda^2} \right. \\
 &- \frac{1}{C} \vec{C}_A (\lambda \vec{\tau} \lambda) - \frac{1}{C^*} \vec{C}_A (\bar{\lambda} \vec{\tau} \bar{\lambda}) - \frac{1}{C} F_{\mu\nu} (\lambda \sigma^{\mu\nu} \lambda) \\
 &- \frac{1}{C^*} F_{\mu\nu} (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda}) - \frac{1}{24C^2} [(\lambda \vec{\tau} \lambda)^2 + (\lambda \sigma^{\mu\nu} \lambda)^2] \\
 &\left. - \frac{1}{24C^{*2}} [(\bar{\lambda} \vec{\tau} \bar{\lambda})^2 + (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda})^2] \right\} + \text{Higher order terms}
 \end{aligned} \quad (3.10)$$

where $\mathcal{L}_{\text{eff}} = S_{\text{eff}}/V$, $e = g/2$, we have used the following identification

$$\int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-g^2 C^* C s} \sim -\log \left(\frac{g^2 C^* C}{\Lambda^2} \right) \quad (3.11)$$

and we have written e_o to indicate the bare charge. Equation (3.10) contains a logarithmically divergent term proportional to the $N = 2$ free Maxwell action given by

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - 2 \partial_\mu C \partial_\mu C^* - i \bar{\lambda} \vec{\sigma}^\mu \partial_\mu \lambda - \frac{1}{2} \vec{C}_A^2 \quad (3.12)$$

Of course, in the slowly varying field approximation, we are able to see only the first and the last term of (3.12). Equation (3.10) can be renormalized by adding to it the free Maxwell action (3.12) with a coefficient $(1 + e_o^2/8\pi^2 \log \Lambda^2/\mu^2)$ necessary to eliminate the cut-off Λ from (3.10). μ is the subtraction point and the relation between the bare and the renormalized coupling constant is given by:

$$1 + \frac{e_o^2}{8\pi^2} \log \frac{\Lambda^2}{\mu^2} = \frac{e^2}{e^2} \quad (3.13)$$

which implies the following β -function for the $N = 2$ supersymmetric QED

$$\beta(e) = \frac{e^3}{8\pi^2} \quad (3.14)$$

Using (3.14) we can rewrite (3.10) after renormalization as follows

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{i\mathcal{P}(e)}{2ze} \left\{ F_{\mu\nu} \tilde{F}^{\mu\nu} \log \frac{C}{C^*} + \left(2\vec{C}_A^2 + F_{\mu\nu} F^{\mu\nu} \right) \log \frac{4e^2 C^* C}{\mu^2} \right. \\ & - \frac{1}{C} \vec{C}_A (\lambda \bar{\epsilon} \lambda) - \frac{1}{C^*} \vec{C}_A (\bar{\lambda} \epsilon \bar{\lambda}) - \frac{1}{C} F_{\mu\nu} (\lambda \sigma^{\mu\nu} \lambda) \\ & \left. - \frac{1}{C^*} F_{\mu\nu} (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda}) - \frac{1}{24C^2} [(\lambda \bar{\epsilon} \lambda)^2 + (\lambda \sigma^{\mu\nu} \lambda)^2] \right. \\ & \left. - \frac{1}{24C^{*2}} [(\bar{\lambda} \epsilon \bar{\lambda})^2 + (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda})^2] \right\} \quad (3.15) \end{aligned}$$

that is the final form of our effective Lagrangian in the approximation of slowly varying fields. As we have already said, the terms in (3.15) are the ones with the minimal amount of scalar fields C and C^* in the denominator. They contain terms that violate the $U(1)$ axial symmetry and the scale invariance of the classical theory and therefore, we expect that they reproduce the multiplet of anomalies of the $N = 2$ supersymmetric QED, while the higher order terms are scale and $U(1)$ invariant.

Although we started from a supersymmetric expression (3.1), it is easy to convince ourselves that we have lost this property in (3.15) because the constant field approximation does not preserve supersymmetry. This fact was already noticed in the second paper of Ref. 4. In this paper it was also remarked that the supersymmetry of the original theory can be recovered by adding terms that contain derivatives of the fields. In the next section we will write a manifestly $N = 2$ invariant expression that reproduces the anomaly term (3.15) apart from terms that are vanishing in the constant field approximation.

4. THE ANOMALY TERM IN THE SUPERFIELD FORMALISM

In this section we rewrite the anomaly term in a manifest $N = 2$ and $N = 1$ supersymmetric formalism obtaining the terms appearing in (3.15) plus additional terms containing derivatives of the fields, which were neglected in the constant field approximation.

Using the formulas of Appendix A, it is possible to compute the following expression:

$$\begin{aligned} & \frac{1}{4} \left\{ W^2 (\log W^2 - 3) \right\}_{|u(\bar{\theta})} + \bar{W}^2 (\log \bar{W}^2 - 3) \Big|_{|u(\theta)} \left\{ \frac{\mathcal{P}(e)}{2e} \right\} = \\ & = \frac{1}{2} \left\{ \left[4\partial_\mu C^* \partial^\mu C + \vec{C}_A^2 + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + 2i \bar{\lambda} \bar{\sigma}^{\mu\nu} \partial_\mu \lambda \right] \log(C^*) + \right. \\ & \quad + \left[4\partial_\mu C \partial^\mu C^* + \vec{C}_A^2 + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2i (\partial_\mu \bar{\lambda}) \bar{\sigma}^{\mu\nu} \lambda \right] \log(C) + \\ & \quad + \frac{1}{2} F_{\mu\nu} \tilde{F}^{\mu\nu} \log \frac{C}{C^*} - \vec{C}_A^2 \left[\frac{\bar{\lambda} \bar{\epsilon} \bar{\lambda}}{C^*} + \frac{\lambda \epsilon \lambda}{C} \right] - F_{\mu\nu} \left[\frac{\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda}}{C^*} + \frac{\lambda \sigma^{\mu\nu} \lambda}{C} \right] \\ & \quad \left. - \frac{1}{24C^{*2}} [(\bar{\lambda} \epsilon \bar{\lambda})^2 + (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda})^2] - \frac{1}{24C^2} [(\lambda \bar{\epsilon} \lambda)^2 + (\lambda \sigma^{\mu\nu} \lambda)^2] \right\} \frac{\mathcal{P}(e)}{2e} \quad (4.1) \end{aligned}$$

that is equal to (3.15) in the constant field approximation. If we define the composite $N = 2$ chiral superfield

$$\bar{R} = \frac{\mathcal{P}(e)}{2e} W^2 \quad (4.2)$$

it is easy to rewrite the anomaly term as follows

$$\mathcal{L}_{\text{eff}}^{(\text{anomaly})} = \frac{1}{4} \left\{ R \left(\log \frac{R}{M^2} - 3 \right) \right\}_{|u(\theta)} + \bar{R} \left(\log \frac{\bar{R}}{M^2} - 3 \right) \Big|_{|u(\bar{\theta})} \quad (4.3)$$

apart from irrelevant terms arising from the difference between μ^2 and M^2 .

In conclusion, we have shown that starting from the action of the $N = 2$ supersymmetric QED we get the anomaly term (4.3) in terms of the condensates (4.2) of the free Maxwell theory, of course, with the β -function of the $N = 2$ supersymmetric QED. We expect that the anomaly term (4.3) will also be obtained in the non-abelian theory and the β -function appearing in (4.2) will be the one relative to the fields of the pure Yang-Mills $N = 2$ supersymmetric theory, that is given by:

$$\beta(g) = -\frac{N}{8\pi^2} g^3 \quad (4.4)$$

that has been shown to be exact to all orders of perturbation theory¹⁰⁾. N is the index of the $SU(N)$ gauge group. We expect also a factor 1 instead of the factor 3 of Eq. (4.3). In fact, in the non-abelian case we get only one time the pure gauge Lagrangian and not three times as in the abelian case.

Let us check now that the effective Lagrangian (4.3) is not invariant under chiral, scale and superconformal transformations reproducing the correspondent anomalies. Under a chiral transformation the field $R(y_{-1}, 0)$ transforms as follows:

$$R(y_{-1}, 0) \rightarrow e^{2i\beta} R(y_{-1}, 0 e^{-i\beta/2}) \quad (4.5)$$

It is easy to check that the action (2.5) is invariant under (4.5) while the effective Lagrangian (4.3) is transformed in the following way:

$$\mathcal{L}_{\text{eff}}^{(\text{anomaly})} \rightarrow \mathcal{L}_{\text{eff}}^{(\text{anomaly})} + i\beta \frac{\beta(g)}{2g} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad (4.6)$$

Under a scale transformation $R(y_{-1}, 0)$ transforms as follows:

$$R(y_{-1}, 0) \rightarrow e^{2\gamma} (y_{-1}^\mu e^\nu, 0 e^{\gamma/2}) \quad (4.7)$$

This implies that (2.5) is invariant, while from (4.3) we get:

$$\mathcal{L}_{\text{eff}}^{(\text{anomaly})} \rightarrow \mathcal{L}_{\text{eff}}^{(\text{anomaly})} + \gamma \frac{\beta(g)}{2g} [F_{\mu\nu} F^{\mu\nu} + \dots] \quad (4.8)$$

where the dots denote the other terms in pure gauge Lagrangian that are zero if one uses their equations of motion. Finally, the superconformal transformations will be discussed after the formulation with $N = 1$ superfields. By means of an explicit one-loop calculation, we have constructed the anomalous term (4.3), involving the condensate (4.2), that satisfies the anomalous Ward identities of the underlying theory. The condensate (4.2) is the natural generalization of $\bar{W}_\alpha \bar{W}^{\dot{\alpha}}$ introduced in Ref. 5 for the pure $N = 1$ gauge theory. We must, however, remember that the choice of the relevant composite fields in a theory where the fundamental particles are confined is a question that can be answered only when the full dynamics of the theory is under control.

In order to show that there is a certain ambiguity in the choice of the fields that are necessary to satisfy the anomalous Ward identities in the composite theory, let us rewrite Eq. (4.1) in terms of $N = 1$ superfields. This formalism will also allow a comparison with the results already obtained for a $N = 1$ gauge theory with matter¹⁰⁾, since in the formalism $N = 1$ the superfield ϕ plays the role of a matter field transforming according to the adjoint representation.

It can be easily seen that (4.1) and (4.3) can be written in the following way:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \frac{\beta(e)}{2e} \left\{ -W^\alpha W_\alpha \log \Phi^2 \right\}_F - \bar{W}_\alpha \bar{W}^{\dot{\alpha}} \log \bar{\Phi}^2 \Big|_F - 2 \Phi \bar{\Phi} (\log \Phi \bar{\Phi}^2 - 4) \Big|_D^{(4.9)}$$

where the superfields W_α , $\bar{W}_{\dot{\alpha}}$, Φ and $\bar{\Phi}$ are the same that appear in Lagrangian (2.9).

At first sight it seems from (4.9) that together with the chiral condensates $W^\alpha W_\alpha$ and $\bar{\Phi}^2$, we are also forced to introduce the additional non-chiral condensate $\Phi \bar{\Phi}$, that is present in (4.9). Actually, this is not necessary because (4.9) can also be written in the following way:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \frac{\beta(e)}{2e} \left[W^\alpha W_\alpha - \frac{1}{2} \bar{D}_\alpha \bar{D}^{\dot{\alpha}} \bar{\Phi} \Phi \right] \log \frac{\Phi^2}{M^2} \Big|_{\theta\theta} + \bar{D}_\alpha \bar{D}^{\dot{\alpha}} (\bar{\Phi} \Phi) \Big|_{\theta\theta}$$

$$+ \left[\bar{W}_\alpha \bar{W}^\alpha - \frac{1}{2} D_\alpha \bar{D}_\alpha \bar{\Phi} \Phi \right] \log \frac{\bar{\Phi}^2}{M^2} + D^\alpha D_\alpha (\bar{\Phi} \Phi) \Big|_{\bar{\theta} \bar{\theta}} \quad (4.10)$$

that involves only the two chiral superfields ϕ^2 and $\bar{W}_\alpha \bar{W}^\alpha - \frac{1}{2} \bar{D}_\alpha \bar{D}^\alpha \bar{\Phi} \Phi$. The expression (4.10) is related to the anomaly term constructed in Ref. 19 for the case $M = N = 1$ if we make the identification: $T = \phi^2 \beta(e)/2e$ and $S = \beta(e)/2e [W^\alpha W_\alpha - \frac{1}{2} \bar{D}_\alpha \bar{D}^\alpha \bar{\Phi} \Phi]$. We notice that there are, in addition, also the terms $\bar{D}_\alpha \bar{D}^\alpha (\bar{\Phi} \Phi)$ and $D^\alpha D_\alpha (\bar{\Phi} \Phi)$, that, however, are invariant under the anomalous transformations.

In conclusion, if we use a manifestly supersymmetric $N = 2$ formalism, we need to introduce the condensate $R = \beta(e)/2e \bar{W}^2$, while in a $N = 1$ formalism only the condensates S and T previously defined are required. Do the two sets of condensates needed to construct the anomaly term contain the same component fields? One can easily check that $R = \beta(e)/2e \bar{W}^2$ contains additional component fields in comparison with those of S and T . In order to recover all the components of R , one must add to S and T also the composite $W_\alpha \bar{\Phi}$, that, however, in the $N = 1$ formalism is not needed to reproduce the anomalous Ward identities. It is only required from the fact that the anomaly multiplet is a multiplet of an extended

$N = 2$ supersymmetry. We see, therefore, that the anomaly term does not define uniquely the condensates. Some more information on the structure of the theory is needed to fix them. In the formalism $N = 1$ it is easier to study how the effective Lagrangian transforms under superconformal transformations. One has in fact:

$$\delta \left[\int d^4x d\bar{\theta} S \log T + h.c. \right] = -i \frac{\beta(e)}{2e} \int d^4x \varphi_i^\alpha (\sigma_\mu)_{\alpha\dot{\alpha}} \gamma^\mu \bar{\Sigma}^{\dot{\alpha}1} + h.c. \quad (4.11)$$

$\bar{\Sigma}$ being the transformation parameter.

Of course, using the $N = 1$ superfields it is easy to perform only one of the two superconformal transformations. However, the general result can be guessed from (4.11) by requiring the $SU(2)$ invariance. In general, one gets

$$\delta \left[\int d^4x d\bar{\theta} S \log T + h.c. \right] = -i \frac{\beta(e)}{2e} \int d^4x \varphi_i^\alpha (\sigma_\mu)_{\alpha\dot{\alpha}} \gamma^\mu \bar{\Sigma}^{\dot{\alpha}1} + h.c. \quad (4.12)$$

We conclude this section by noticing that an expression analogous to (4.10) is also valid in the non-abelian case. We get the following anomaly term:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{1}{2} \frac{\beta(g)}{g} \left[W^\alpha W_\alpha - \frac{1}{2} \bar{D}_\alpha \bar{D}^\alpha (\bar{\Phi} e^{\beta V} \Phi) \right] \log \frac{\bar{\Phi}^2}{M^2} + \bar{D}_\alpha \bar{D}^\alpha (\bar{\Phi} e^{\beta V} \Phi) \Big|_{\bar{\theta} \bar{\theta}} + \\ & + \left[\bar{W}_\alpha \bar{W}^\alpha - \frac{1}{2} D^\alpha D_\alpha (\bar{\Phi} e^{\beta V} \Phi) \right] \log \frac{\bar{\Phi}^2}{M^2} + D^\alpha D_\alpha (\bar{\Phi} e^{\beta V} \Phi) \Big|_{\bar{\theta} \bar{\theta}} \end{aligned} \quad (4.13)$$

where the coupling constant g used here is equal to $g/2$ of expression (2.13).

5. SUMMARY AND CONCLUSIONS ON THE SCALAR POTENTIAL

In this paper by means of a dynamical calculation in a slowly varying field approximation we have explicitly constructed a term which in the effective Lagrangian reproduces the effect of the anomalous Ward identities in the case of the $N = 2$ supersymmetric pure gauge theory. This term is given by the expression (4.3) in the abelian theory. In order to compare with the anomaly term in the case of a supersymmetric $N = 1$ theory with matter we have rewritten (1.1) using a $N = 1$ formalism and we have obtained in the non-abelian case, the expression (4.13), that has a form similar to the anomaly term proposed by Taylor, Veneziano and Yankielowicz¹⁹⁾ for the $N = 1$ theory with matter in the fundamental representations.

The $N = 2$ Yang-Mills theory has the nice feature that the gauge hypermultiplet contains also a complex scalar field, that can be used as a Higgs field to break

the gauge symmetry as shown in Ref. 11. The classical potential in the case of a $SU(2)$ gauge group is given by:

$$V_{cl}(\phi, \phi^*) = \frac{1}{2} g^2 \left[(\phi \phi^*)^2 - \phi^2 \phi^{*2} \right] \quad (5.1)$$

where $\phi = \sqrt{2}C$ and we have made the substitution $g/2 \rightarrow g$ as we have already done in (4.13) in comparison with (2.13). The stationary points of (5.1) are given by:

$$\phi \phi^* = \phi^2 = \phi^{*2} = v^2/2 \quad (5.2)$$

where the value of the constant v is not determined by the potential. In fact the potential (5.1) is flat as a function of v because of the conformal invariance of the classical theory. In conclusion the vacuum expectation value of ϕ brings a dimensional constant in the theory that is, however, arbitrary. Another dimensional constant M is obtained from the renormalization prescription, that at one-loop level breaks also the conformal invariance of the theory.

If we add to the classical potential the one obtained from the anomaly term

(4.13) we get:

$$V(\phi, \phi^*) = \frac{1}{2} g^2 \left[(\phi \phi^*)^2 - \phi^2 \phi^{*2} \right] \left[1 - \frac{\beta(g)}{2g} \frac{1}{2} \log \frac{\phi^2 \phi^{*2}}{4M^4} \right] \quad (5.3)$$

where the β -function is given by (4.4). It is easy to see that although the potential (5.3) contains the dimensional parameter M , the minimum of the potential is given by (5.2), where v is still arbitrary*). Therefore, at the perturbative level it seems as if the quantum theory will contain two distinct dimensional constants. Since the coefficient of the log term in the anomaly contains also the kinetic term of the $N = 2$ theory, we must require that $\phi^2 = v^2/2 > M^2$ in order to have a kinetic term with the right sign**).

*) We notice that we have been taking into account only the quantum corrections coming from the anomaly term. It would be interesting to consider the full effective potential and see if also in this case v is left arbitrary.

**) This feature has been understood in a discussion with G. Veneziano.

A way to get a relation between v and M is by a non-perturbative calculation with instantons. It has been noticed by Novikov, Shifman, Vainshtein and Zacharov²⁰⁾ that the usual infrared divergence appearing from the integration over the scale of the instanton is not present in a supersymmetric theory. Instanton calculations have also been used by Rossi and Veneziano²¹⁾ in order to confirm the expression for the various condensates in the effective Lagrangian approach.

We have performed instanton calculations in the $N = 2$ supersymmetric theory and computed, for instance, the Green's function $\langle \phi^2(x) \phi^2(y) \rangle$ obtaining a constant result proportional to M^4 . By using the cluster decomposition, this implies that $\langle \phi^2 \rangle \propto M^2$ and one therefore gets a connection between v^2 and M^2 . More details on the instanton calculations in the $N = 2$ super Yang-Mills theory will be presented in a separate publication²²⁾.

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Appendix A

In this appendix we give some details of the $N = 2$ superfield formalism following the notations of Ref. 16. The $N = 2$ superfield strength is a chiral superfield that can be written as follows:

$$W[x_\mu - i\theta\sigma_\mu\bar{\theta} = y_+; \bar{\theta}] = C^*(y_+) + \bar{\lambda}_{\dot{\alpha}i}(y_+)\bar{\theta}^{\dot{\alpha}i} - b_A(\bar{\theta})C_A(y_+) + a^{\mu\nu}(\bar{\theta})F_{\mu\nu}(y_+) + \bar{\lambda}_{\dot{\alpha}i}(\bar{\theta})\bar{\varphi}^{\dot{\alpha}i}(y_+) + u(\bar{\theta})D^*(y_+) \quad (A.1)$$

where

$$b_A(\bar{\theta}) = -\frac{1}{2}\bar{\theta}_{\dot{\alpha}j}(\tau_A)^j_i\bar{\theta}^{\dot{\alpha}i} \quad (A.2)$$

$$a_{\mu\nu}(\bar{\theta}) = \frac{1}{2}\bar{\theta}_{\dot{\alpha}i}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\theta}^{\dot{\beta}i}$$

$$\bar{\lambda}_{\dot{\alpha}i} = \frac{\partial u}{\partial \bar{\theta}^{\dot{\alpha}i}}; \quad u(\bar{\theta}) = \bar{\theta}^{\dot{\alpha}i}\bar{\theta}^{\dot{\beta}j}\bar{\theta}^{\dot{\gamma}k}\bar{\theta}^{\dot{\delta}l} \quad (A.3)$$

The fields $\bar{\varphi}^{\dot{\alpha}i}$ and D^* are not independent fields; they are given by (2.4) in terms of the other fields. The Weyl and $SU(2)$ indices are raised and lowered by the antisymmetric tensors $\epsilon_{\alpha\beta}$ and ϵ_{ij} , such that $\epsilon^{12} = -\epsilon_{12} = 1$.

It is also convenient to write down the superfield strength $\tilde{W}$:

$$\tilde{W}(x_\mu + i\theta\sigma_\mu\bar{\theta} = y_+; \theta) = C(y_+) + \lambda_i(y_+)\theta^i - b_A(\theta)C_A(y_+) + a_{\mu\nu}(\theta)F^{\mu\nu}(y_+) + \chi_i^\alpha(\theta)\varphi_\alpha^i(y_+) + u(\theta)D(y_+) \quad (A.4)$$

where

$$D = -4(D_\mu D^\mu C^*) - 2f^{abc}(\bar{\lambda}_{\dot{\alpha}i})^b(\bar{\lambda}^{\dot{\alpha}i})^c + 4g^2 f^{abc} f^{cde} C^* C^b C^d C^e \quad (A.5)$$

$$(\varphi_\alpha^i) = -2i(\sigma^\mu)_{\alpha\dot{\alpha}}(D_\mu \bar{\lambda}^{\dot{\alpha}i}) - 2g f^{abc}(\lambda_\alpha^i)^b C^{*c} \quad (A.6)$$

and

$$b_A(\theta) = -\frac{1}{2}\theta_i^\alpha(\tau_A)^j_\alpha\theta^j; \quad a_{\mu\nu}(\theta) = \frac{1}{4}\theta_i^\alpha(\sigma_{\mu\nu})_\alpha^\beta\theta_\beta^i \quad (A.7)$$

$$\chi_\alpha^i(\theta) = \frac{\partial u}{\partial \theta^\alpha} \mu(\theta); \quad \mu(\theta) = \theta^1\theta^2\theta^3\theta^4 \quad (A.8)$$

Products of superfields (A.1) and (A.3) can be easily computed by using the following formulae:

$$\theta_i^\alpha\theta_j^\beta = \frac{1}{4}\delta_\beta^\alpha(\tau_A)^j_i(\theta\tau_A\theta) - \frac{1}{4}(\sigma_{\mu\nu})_\beta^\alpha(\theta\sigma^{\mu\nu}\theta)\delta_j^i; \quad (A.9)$$

$$\bar{\theta}_{\dot{\alpha}i}\bar{\theta}_{\dot{\beta}j} = \frac{1}{4}\delta_{\dot{\beta}}^{\dot{\alpha}}(\tau_A)^j_i(\bar{\theta}\tau_A\bar{\theta}) - \frac{1}{4}(\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}}(\bar{\theta}\bar{\sigma}^{\mu\nu}\bar{\theta})\delta_j^i; \quad (A.10)$$

$$(\theta_i^\alpha\lambda_\alpha^j)a_{\mu\nu}(\theta) = \frac{\partial u}{\partial \theta^\alpha}(\sigma_{\mu\nu})_\alpha^\beta\lambda_\beta^j; \quad (A.11)$$

$$(\bar{\theta}_{\dot{\alpha}i}\bar{\lambda}^{\dot{\alpha}j})a_{\mu\nu}(\bar{\theta}) = -\frac{\partial u}{\partial \bar{\theta}^{\dot{\alpha}i}}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\lambda}^{\dot{\beta}j}; \quad (A.12)$$

$$(\theta_i^\alpha\lambda_\alpha^j)b_A(\theta) = \frac{\partial u}{\partial \theta^\alpha}(\tau_A)^j_i\lambda_\beta^i; \quad (A.13)$$

$$(\bar{\theta}_{\dot{\alpha}i}\bar{\lambda}^{\dot{\alpha}j})b_A(\bar{\theta}) = -\frac{\partial u}{\partial \bar{\theta}^{\dot{\alpha}i}}(\tau_A)^j_i\bar{\lambda}^{\dot{\alpha}i}; \quad (A.14)$$

$$a_{\mu\nu}(\theta)a_{\rho\sigma}(\theta)F^{\mu\nu}F^{\rho\sigma} = u(\theta)[F_{\mu\nu}F^{\mu\nu} + F_{\mu\nu}\tilde{F}^{\mu\nu}] \quad (A.15)$$

$$a_{\mu\nu}(\bar{\theta})a_{\rho\sigma}(\bar{\theta})F^{\mu\nu}F^{\rho\sigma} = u(\bar{\theta})[F_{\mu\nu}F^{\mu\nu} - \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}] \quad (A.16)$$

$$b_A(\theta)b_B(\theta) = 2u(\theta)\delta_{AB} \quad (A.17)$$

$$b_A(\bar{\theta})b_B(\bar{\theta}) = 2u(\bar{\theta})\delta_{AB} \quad (A.18)$$

$$(\theta_i^\alpha\lambda_\alpha^j)(\frac{\partial u}{\partial \theta^\alpha}\varphi_\beta^j) = -u(\theta)\lambda_i^\alpha\varphi_\alpha^j; \quad (A.19)$$

$$(\bar{\theta}_{\dot{\alpha}i}\bar{\lambda}^{\dot{\alpha}j})(\frac{\partial u}{\partial \bar{\theta}^{\dot{\alpha}i}}\bar{\varphi}^{\dot{\beta}j}) = u(\bar{\theta})\bar{\lambda}_{\dot{\alpha}i}\bar{\varphi}^{\dot{\alpha}i} \quad (A.20)$$

$$(\lambda^\alpha_i \bar{\theta}^\alpha_i) b_A(\theta) = \chi^\beta_j (\tau_\alpha)^j_i \lambda^\alpha_i \quad (A.19)$$

$$(\bar{\lambda}^\alpha_i \bar{\theta}^\alpha_i) b_A(\bar{\theta}) = -\bar{\chi}^\alpha_i (\tau_\alpha)^j_i \bar{\lambda}^\alpha_j \quad (A.20)$$

$$(\lambda^\alpha_i \bar{\theta}^\alpha_i) a_{\mu\nu}(\theta) = -\chi^\beta_i (\sigma_{\mu\nu})^\alpha_\beta \lambda^\alpha_i \quad (A.21)$$

$$(\bar{\lambda}^\alpha_i \bar{\theta}^\alpha_i) a_{\mu\nu}(\bar{\theta}) = \bar{\chi}^\beta_i (\bar{\sigma}_{\mu\nu})^\alpha_\beta \bar{\lambda}^\alpha_i \quad (A.22)$$

Using the previous formulae, it is easy to compute

$$W^2|_{u(\bar{\theta})} = 2C^*D + 2C_A C_A + F_{\mu\nu} F^{\mu\nu} - F_{\mu\nu} \tilde{F}^{\mu\nu} - 2\bar{\lambda}^\alpha_i \tilde{\varphi}^\alpha_i \quad (A.23)$$

and

$$\bar{W}^2|_{u(\bar{\theta})} = 2CD + 2C_A C_A + F_{\mu\nu} F^{\mu\nu} - F_{\mu\nu} \tilde{F}^{\mu\nu} + 2\lambda^\alpha_i \varphi^\alpha_i \quad (A.24)$$

Using the expressions (2.4), (A.4) and (A.5), we get the following relation between $W^2|_{u(\theta)}$ and $\bar{W}^2|_{u(\bar{\theta})}$:

$$\bar{W}^2|_{u(\theta)} - W^2|_{u(\bar{\theta})} = 2F_{\mu\nu} \tilde{F}^{\mu\nu} \quad (A.25)$$

Appendix B

We give some details of the evaluation of the fermionic and the bosonic determinants^{*}. First we look at

$$\text{Tr} \log \Delta_B \quad (B.1)$$

where Δ_B , in the abelian case, is defined as

$$\Delta_B = (-D_\mu D^\mu + g^2 C^* C) \mathbb{1} - g \vec{\tau} \cdot \vec{A} \quad (B.2)$$

with

$$D_\mu = \partial_\mu + ie \gamma_\mu$$

($e \equiv g/2$) and $\mathbb{1}$ is the unit matrix in the internal space.

In the constant field approximation, we notice that the piece $\hat{\Delta}_B \equiv g \vec{\tau} \cdot \vec{A}$ is an infinite constant matrix, i.e. diagonal in the x space, and commutes with the piece $\tilde{\Delta}_B = (-D_\mu D^\mu + g^2 C C^*) \cdot \mathbb{1}$ which is diagonal in the $\vec{\tau}$ space. The two matrices can be simultaneously diagonalized; the eigenvalues of $\hat{\Delta}_B$ are found at once to be:

$$\Omega_\pm = \pm g |\vec{A}| \quad (B.3)$$

We focus our attention on $\tilde{\Delta}_B$, and in order to carry out the calculations in a simple way, we take a particular choice for the constant field strength $F_{\mu\nu}$, i.e. we keep constant and different from zero only the third components of the electric and the magnetic fields

$$-E_z = F^{03} \neq 0 \quad H_z = F^{12} \neq 0$$

At the end we will restore the explicit covariance of the results introducing the full $\vec{E}$ and $\vec{H}$. To realize such a choice, the potential will be

$$\begin{aligned} V^1 &= -F^{12} x_2 & V^3 &= -F^{30} x_0 \\ V_z &= V_0 = 0 \end{aligned} \quad (B.4)$$

^{*}) We follow closely the procedure of Ref. 4b.

In order to diagonalize $\tilde{\Delta}_B$, we go to the Euclidean space where

$$X^0 = X_4 = i X^0 \quad \partial_0 \equiv \frac{\partial}{\partial X^0} = i \frac{\partial}{\partial X_4}$$

and

$$f^{34} = f_{34} = i F^{30} \quad f^{12} = f_{12} = F^{12} \quad (B.5)$$

Then we get

$$\tilde{\Delta}_B = \left\{ -\partial_4^2 + e^2 f_{34}^2 \left(x_4 + \frac{i}{e f_{34}} \partial_3 \right)^2 - \partial_2^2 + e^2 f_{12}^2 \left(x_2 + \frac{i}{e f_{12}} \partial_1 \right)^2 + g^2 C^* C \right\} \cdot 1 \quad (B.6)$$

In order to have normalizable eigenstates, we consider the system in a box of a finite size L in the x_3 and x_1 directions, with periodic boundary conditions, and we look for eigenstates of $\tilde{\Delta}_B$ of the form

$$\phi(x) = \frac{1}{L} e^{i P_1 x_1} e^{i P_3 x_3} \chi(x_2, x_4) \quad (B.7)$$

where

$$P_1 = \frac{2\pi l}{L} \quad ; \quad P_3 = \frac{2\pi k}{L} \quad (B.8)$$

$$l, k = \dots, -2, -1, 0, 1, 2, \dots$$

and the $1/L$ factor ensures the proper normalization.

On such functions, $\tilde{\Delta}_B$ becomes

$$\tilde{\Delta}_B \phi(x) = \left[-\partial_4^2 + e^2 f_{34}^2 \left(x_4 - \frac{1}{e f_{34}} P_3 \right)^2 - \partial_2^2 + e^2 f_{12}^2 \left(x_2 - \frac{1}{e f_{12}} P_1 \right)^2 + g^2 C^* C \right] \phi(x) \quad (B.9)$$

The (B.9) may be easily solved in terms of two decoupled harmonic oscillators with frequencies $\nu_4 = e f_{34}/2\pi$, $\nu_2 = e f_{12}/2\pi$ and with the oscillations centres displaced with respect to the origin.

The eigenvalues of $\tilde{\Delta}_B$ read

$$\Omega(n, m) = (2n+1)e f_{12} + (2m+1)e f_{34} + g^2 C^* C \quad (B.10)$$

The eigenfunctions χ may be written as

$$\chi_{(n, m)}(x_2, x_4) = (e^2 f_{34} f_{12})^{1/4} H_n(\xi_2) H_m(\xi_4) \quad (B.11)$$

where the $H_n(\xi)$ are the properly normalized eigenfunctions of the harmonic oscillator

$$\left(-\frac{d^2}{d\xi^2} + \xi^2 \right) H_n(\xi) = (2n+1) H_n(\xi)$$

$$\int d\xi H_n^2(\xi) = 1$$

and

$$\xi_2 = \sqrt{e} f_{12} \left(x_2 - \frac{P_1}{e f_{12}} \right), \quad \xi_4 = \sqrt{e} f_{34} \left(x_4 - \frac{P_3}{e f_{34}} \right) \quad (B.12)$$

The factor $(e^2 f_{34} f_{12})^{1/4}$ in the (B.11) again is needed in order to ensure proper normalization.

In conclusion for Δ_B we get the eigenvalues

$$\Omega_{e, n, m}^\pm = (2n+1)e f_{12} + (2m+1)e f_{34} + g^2 C^* C \pm g |\vec{A}| \quad (B.13)$$

and the eigenstates

$$\begin{aligned} \phi_{e, n, m}^\pm(x) &= \frac{(e^2 f_{12} f_{34})^{1/4}}{L} e^{i P_1 x_1} e^{i P_3 x_3} \\ &\cdot H_n \left(\frac{e f_{34} x_4 - P_3}{\sqrt{e} f_{34}} \right) H_m \left(\frac{e f_{12} x_2 - P_1}{\sqrt{e} f_{12}} \right) \cdot \phi^\pm \end{aligned} \quad (B.14)$$

where $\phi^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\phi^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

We are now able to evaluate $\text{Tr} \lg \Delta_B$:

$$\begin{aligned} \text{Tr} \log \Delta_B &= \log \det \Delta_B = \log \prod \Omega_{e, n, m}^\pm \\ &= \sum_{\pm} \sum_{n, m=0}^{+\infty} \log \Omega_{e, n, m}^\pm \end{aligned} \quad (B.15)$$

The sum over ℓ, k gives rise to a divergence which is merely a volume factor. Indeed the two harmonic oscillators are centered for each value of ℓ and k , around $\bar{x}_2 \equiv 2\pi\ell/e f_{12}$ and $\bar{x}_4 \equiv 2\pi k/e f_{34}$. L , then summation over ℓ and k corresponds to integration over x_2 and x_4 . Since the fields are constant, we get a factor which in the limit of $L \rightarrow \infty$ contains the infinite volume of the space time that we call V in the following:

$$\sum_{\ell, k} = \frac{L^2}{4\pi^2} e^2 f_{12} f_{34} \int dx_2 dx_4 = \frac{V}{4\pi^2} e^2 f_{12} f_{34} \quad (\text{B.16})$$

Then we are left with

$$\text{Tr log } \Delta_B = \frac{V e^2 f_{12} f_{34}}{4\pi^2} \sum_{n, m} \left\{ \log [\Omega(n, m) + g|\vec{A}|] + \log [\Omega(n, m) - g|\vec{A}|] \right\} \quad (\text{B.17})$$

In order to regularize this infinite sum, we use a proper time regularization procedure by means of the formula

$$\log x = - \int_{1/x^2}^{\infty} \frac{ds}{s} e^{-xs} \quad (\text{B.18})$$

Λ^2 is a cut-off needed to regularize the determinant (B.17). We can, therefore, rewrite (B.17) as follows:

$$\begin{aligned} \text{Tr log } \Delta_B &= - \frac{V e^2 f_{12} f_{34}}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} \sum_{n, m} \left\{ e^{-[\Omega(n, m) + g|\vec{A}|]s} + e^{-[\Omega(n, m) - g|\vec{A}|]s} \right\} = \\ &= - \frac{V e^2 f_{12} f_{34}}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} 2 \cosh g|\vec{A}|s \sum_{n, m} e^{-\Omega(n, m) \cdot s} \end{aligned}$$

$$\begin{aligned} &= - \frac{V e^2 f_{12} f_{34}}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-g^2 C^* C s} \frac{\cosh(gAs)}{2 \sinh(e f_{12} s) \sinh(e f_{34} s)} \\ &= - \frac{V e^2 f_{12} f_{34}}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-g^2 C^* C s} \frac{\cosh(gAs)}{\cosh e(f_{12} + f_{34})s - \cosh e(f_{12} - f_{34})s} \end{aligned} \quad (\text{B.19})$$

where $A = |\vec{A}|$. Going back to Mikowski space and rewriting the expression in terms of electric and magnetic field, we get:

$$\text{Tr log } \Delta_B = - \frac{V e^2 i \vec{E} \cdot \vec{H}}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} \frac{e^{-g^2 C^* C s} \cosh(gAs)}{\cosh e(H_2 + i E_2)s - \cosh e(H_2 - i E_2)s} \quad (\text{B.20})$$

Since $\text{Tr lg } \Delta_B$ can be only a function of the invariants that we can build up with the electric and magnetic field, we can rewrite a manifestly covariant expression for (B.20) in terms of the complex vector $\vec{X} = \vec{H} + i\vec{E}$

$$\text{Tr log } \Delta_B = - \frac{V e^2 i \vec{E} \cdot \vec{H}}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} \frac{e^{-g^2 C^* C s} \cosh(gAs)}{\cosh(eXs) - \cosh(eX^*s)} \quad (\text{B.21})$$

Because of the properties of the cosh, (B.21) is a function of the only two independent invariants that we can build up with $\vec{H}$ and $\vec{E}$:

$$i\vec{E} \cdot \vec{H} = \frac{1}{4}(\vec{X}^2 - \vec{X}^{*2}) = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \text{and} \quad H^2 - E^2 = \frac{\vec{X}^2 + \vec{X}^{*2}}{2} = \frac{1}{2}F_{\mu\nu}F^{\mu\nu} \quad (\text{B.22})$$

In order to regularize the (B.21), we look at small s behaviour of the integrand:

$$\frac{\cosh(gAs)}{\cosh(eXs) - \cosh(eX^*s)} \sim \frac{2}{s \rightarrow 0} \frac{e^{s^2 A^2 s^2} - \frac{e^{s^2 X^2} - \frac{e^{s^2 X^{*2}}}{2}}{2}}{e^{s^2 X^2} - \frac{e^{s^2 X^{*2}}}{2}} \left\{ 1 + g^2 \frac{A^2 s^2}{2} - \frac{e^{s^2 X^2} - \frac{e^{s^2 X^{*2}}}{2}}{12} + O(s^4) \right\} \quad (\text{B.23})$$

i.e. the integral (B.21) has a quadratic and a logarithmic divergence and we re-write it as

$$\text{Tr} \log \Delta_B = -\frac{V}{4\pi^2} \left\{ \int_0^\infty \frac{ds}{s^3} e^{-\frac{g^2}{4}Cs} \left[\frac{e^2}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \frac{s^2 \cosh(gAs)}{\cosh(eXs) - \cosh(eX^*s)} \right] - \right. \\ \left. - \frac{1}{2} + s^2 \left(\frac{e^2}{24} F_{\mu\nu} F^{\mu\nu} - g^2 \frac{A^2}{4} \right) \right\} \quad (\text{B.24})$$

$$-\frac{V}{8\pi^2} \left\{ \int_0^\infty \frac{ds}{s^3} e^{-\frac{g^2}{4}Cs} + \left(g^2 \frac{A^2}{2} - \frac{e^2}{12} F_{\mu\nu} F^{\mu\nu} \right) \int_0^\infty \frac{ds}{s} e^{-\frac{g^2}{4}Cs} \right\}$$

where in the first term we have now safely taken the limit $\Lambda^2 \rightarrow \infty$. Equation (B.24) coincides with the expression written in (3.3).

In a very similar way, we evaluate now

$$\text{Tr} \log \Delta_F \quad (\text{B.25})$$

where

$$\Delta_F = i\cancel{D} - ig \frac{1-\delta_5}{2} C^* + ig \frac{1+\delta_5}{2} C \quad (\text{B.26})$$

In the constant field approximation using the gauge potential choice (B.4), we get in the Euclidean space

$$\Delta_F = \begin{pmatrix} igC\mathbb{I} & \Delta^- \\ \Delta^+ & -igC^*\mathbb{I} \end{pmatrix} \quad (\text{B.27})$$

$$\Delta^\pm = \mathbb{I} \cdot \partial_4 \pm i[\sigma^1(\partial_1 - ie f_{12} x_2) + \sigma^2 \partial_2 + \sigma^3(\partial_3 - ie f_{34} x_4)] \quad (\text{B.28})$$

As before, the system is considered in a finite box in x_3 and x_1 directions and we look for solutions of the eigenvalue equation

$$\Delta_F \Psi = \omega \Psi \quad (\text{B.29})$$

with

$$\Psi = \frac{1}{L} e^{i p_1 x_1} e^{i p_3 x_3} \begin{pmatrix} \psi_1(x_2, x_4) \\ \psi_2(x_2, x_4) \end{pmatrix} \quad (\text{B.30})$$

$$p_1 = 2\pi\ell/L \quad ; \quad p_3 = 2\pi\kappa/L \quad \kappa, \ell = \dots, -1, 0, 1, \dots$$

where $\psi_{1,2}$ are 2 component spinors. The eigenvalue equation becomes

$$\begin{aligned} igC\psi_1 + \Delta^- \psi_2 &= \omega \psi_1 \\ \Delta^+ \psi_1 - igC^* \psi_2 &= \omega \psi_2 \end{aligned} \quad (\text{B.31})$$

Since, on the functions (B.30), we have:

$$\Delta^+ \Delta^- = 1 \cdot \left\{ \delta_4^2 - e^2 f_{34}^2 \left(x_4 - \frac{p_3}{e f_{34}} \right)^2 + \delta_2^2 - e^2 f_{12}^2 \left(x_2 - \frac{p_1}{e f_{12}} \right)^2 \right\} - e^2 \sigma^3 (f_{12} + f_{34}) \quad (B.32)$$

it is easy to solve the Eqs. (B.31) using the bosonic results. For the two components of $\psi_{1,2}$ we get indeed two equivalent pairs of eigenvalues:

$$\omega^\pm(\eta, m) = i g \frac{C-C^*}{2} \pm i \sqrt{g^2 \frac{(C+C^*)^2}{4} + 2 n e f_{12} + 2 m e f_{34}} \quad (B.33)$$

$$\omega^\pm(\eta, m) = i g \frac{C-C^*}{2} \pm i \sqrt{g^2 \frac{(C+C^*)^2}{4} + 2(m+1) e f_{12} + 2(m+1) e f_{34}} \quad (B.34)$$

After straightforward, though somewhat lengthy, calculations a complete set of normalized eigenvectors is found to be (we have conveniently rearranged the eigenvalue assignment):

$$\psi_{1(\eta, \ell, m, n)}^+ (x) = N_1^+ \begin{bmatrix} -\frac{\sqrt{2} f_{34}(m+1)e}{\omega^+ - i g C} H_n(\xi_2) H_{m+1}(\xi_4) \\ -\frac{\sqrt{2}(m+1)e f_{12}}{\omega^+ - i g C} H_{m+1}(\xi_2) H_m(\xi_4) \\ H_n(\xi_2) H_m(\xi_4) \\ 0 \end{bmatrix} \quad \begin{matrix} \text{Eigenvalue} \\ \omega^+(n+1, m+1) \end{matrix}$$

$$\psi_{2(\eta, \ell, m, n)}^+ (x) = N_2^+ \begin{bmatrix} -\frac{\sqrt{2} n e f_{12}}{\omega^+ - i g C} H_{n-1}(\xi_2) H_m(\xi_4) \\ -\frac{\sqrt{2} m e f_{34}}{\omega^+ - i g C} H_n(\xi_2) H_{m-1}(\xi_4) \\ 0 \\ H_n(\xi_2) H_m(\xi_4) \end{bmatrix} \quad \begin{matrix} \text{Eigenvalue} \\ \omega^+(n, m) \end{matrix}$$

$$\psi_{1(\eta, \ell, m, n)}^- (x) = N_1^- \begin{bmatrix} 0 \\ H_n(\xi_2) H_m(\xi_4) \\ \frac{\sqrt{2} n e f_{34}}{\omega^- + i g C^*} H_n(\xi_2) H_{m+1}(\xi_4) \\ \frac{\sqrt{2}(m+1)e f_{12}}{\omega^- + i g C^*} H_{m+1}(\xi_2) H_m(\xi_4) \end{bmatrix} \quad \begin{matrix} \text{Eigenvalue} \\ \omega^-(n+1, m) \end{matrix}$$

$$\psi_{2(\eta, \ell, m, n)}^- (x) = N_2^- \begin{bmatrix} 0 \\ H_n(\xi_2) H_m(\xi_4) \\ \frac{\sqrt{2} n e f_{12}}{\omega^- + i g C^*} H_{n-1}(\xi_2) H_m(\xi_4) \\ -\frac{\sqrt{2}(m+1)e f_{34}}{\omega^- + i g C^*} H_n(\xi_2) H_{m+1}(\xi_4) \end{bmatrix} \quad \begin{matrix} \text{Eigenvalue} \\ \omega^-(n, m+1) \end{matrix} \quad (B.35)$$

where the N factors include the x_1, x_3 dependence and all the normalization coefficients, i.e.:

$$N = \frac{(e^2 f_{12} f_{34})^{1/4}}{\sqrt{2} L} e^{i p_4 x_4} e^{i p_{34} x_3} \left(1 - i g \frac{C + C^*}{\omega^+(n, m) - \omega^-(n, m)} \right) \quad (B.36)$$

The $H_n(\xi)$ as the ξ variables are the ones previously defined (B.12).

For each value of n and m there are four independent eigenvectors with their corresponding eigenvalues; we see, however, that from the set of eigenvalues the value $\omega^-(0, 0)$ which does not correspond to any acceptable eigenvector, has to be excluded.

We may now evaluate $\text{Tr} \lg \Delta_p$ with the techniques we used in the bosonic case:

$$\text{Tr} \log \Delta_F = \frac{V}{4\pi^2} \int_{12}^{14} \left\{ \log \omega^+(0,0) + \sum_{m=1}^{\infty} \log [\omega^+(0,m) \omega^-(0,m)] + \right. \\ \left. + \sum_{n=1}^{\infty} \log [\omega^+(n,0) \omega^-(n,0)] + 2 \sum_{n,m=1}^{\infty} \log [\omega^+(n,m) \omega^-(n,m)] \right\} =$$

$$= \frac{V}{4\pi^2} \int_{12}^{14} \left\{ \log (igC) + \sum_{n=1}^{\infty} \log (g^2 C^* C + 2ne f_{34}) + \right. \\ \left. + \sum_{n=1}^{\infty} \log (g^2 C^* C + 2ne f_{12}) + \right. \\ \left. + 2 \sum_{n,m=1}^{\infty} \log (g^2 C^* C + 2ne f_{12} + 2ne f_{34}) \right\} \quad (\text{B.37})$$

Using the (B.18) and the procedure previously used, we get

$$\text{Tr} \log \Delta_F = \frac{Ve^2}{8\pi^2} \int_{12}^{14} \left\{ \log \frac{C}{C^*} - \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} e^{-s} \frac{g^2 C^* C s}{\cosh(e f_{12} s) \cosh(e f_{34} s)} \right\} \quad (\text{B.38})$$

This result can be rewritten in an invariant form in the Minkowski space

$$\text{Tr} \log \Delta_F = -\frac{Ve^2}{8\pi^2} i \vec{E} \cdot \vec{H} \left\{ \log \frac{C}{C^*} + \right. \\ \left. + \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} e^{-s} \frac{g^2 C^* C s}{\cosh(e X s) + \cosh(e X^* s)} \right\} \quad (\text{B.39})$$

Extracting the divergent part, we finally get:

$$\text{Tr} \log \Delta_F = -\frac{V}{8\pi^2} \left\{ \frac{e^2}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \log \frac{C}{C^*} + \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s^3} \left(1 + \frac{s^2 e^2}{6} F_{\mu\nu} F^{\mu\nu} \right) e^{-g^2 C^* C s} \right\}$$

$$-\frac{V}{8\pi^2} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s^3} e^{-g^2 C^* C s} \left[\frac{e^2 s^2}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \frac{\cosh(e X s) + \cosh(e X^* s)}{\cosh(e X s) - \cosh(e X^* s)} \right]$$

$$-1 - \frac{s^2 e^2}{6} F_{\mu\nu} F^{\mu\nu} \quad (\text{B.40})$$

that is the expression written in (3.4).

Appendix C

In this appendix we consider the term

$$-\frac{1}{2}g^2 \text{Tr} [\Delta_B^{-1} \gamma_5 \Delta_F^{-1} \gamma_5 \gamma] \equiv Q \quad (\text{C.1})$$

where we remember that the trace is both in the x-space and in the internal τ -space. In order to evaluate the inverse of the fermionic and the bosonic operators we use the general property that, given the eigenvalues and a complete set of eigenvectors of an operator Δ , allows to construct

$$\Delta^{-1}(x, y) = \sum_{\{n\}} \varphi_n(x) \frac{1}{\Omega_n} \varphi_n^\dagger(y) \quad (\text{C.2})$$

Using the result of Appendix B, it is straightforward to compute the elements $[\Delta_F^{-1}(x, y)]_{ab}$ of the inverse fermion operator (here the a, b span the 4×4 γ -space). It is easy to check that because of the trace, terms with a $\neq b$ do not contribute to Q. We are then left with the diagonal elements:

$$\begin{aligned} [\Delta_F^{-1}(x, y)]_{aa} &= \frac{(e^{i p_\mu t_{34}})^{1/2}}{L^2} \sum_{\ell, k=-\infty}^{+\infty} e^{i p_\ell(x_1 - y_1)} e^{i p_k(x_3 - y_3)} \cdot \sum_{n_1, m=0}^{\infty} H_{n_1}(x_2) H_m(x_4) H_{n_1}^*(y_2) H_m^*(y_4) \cdot D_{aa} \\ D_{41} &= \frac{i g C}{\omega(n_1, m) \omega^-(m, 1, m)} = \frac{-i g C^*}{\Omega + X} \\ D_{22} &= \frac{i g C}{\omega^+(n_1, m) \omega^-(n_1, m+1)} = \frac{-i g C^*}{\Omega - X} \\ D_{33} &= \frac{i g C^*}{\omega^+(n_1, m+1) \omega^-(n_1, m)} = \frac{i g C}{\Omega + Y} \\ D_{44} &= \frac{i g C^*}{\omega^+(n_1, m) \omega^-(n_1, m)} = \frac{i g C}{\Omega - Y} \end{aligned} \quad (\text{C.3})$$

where $\Omega \equiv \Omega(n, m)$ is the same as defined by the Eq. (B.10) and $X = e f_{12} - e f_{34}$, $Y = e f_{12} + e f_{34}$. Writing now for Δ_F^{-1} its general expression in terms of diagonal Dirac matrices

$$\Delta_F^{-1} = \Delta_0 \not{1} + \Delta_5 \gamma_5 + \Delta_{\mu\nu} \sigma^{\mu\nu} \quad (\text{C.4})$$

it is easy to find the expressions for Δ_0 , Δ_5 and $\Delta_{\mu\nu}$ in terms of $[\Delta_F^{-1}]_{aa}$.

We then use (C.4) and the results of Appendix B to rewrite

$$\begin{aligned} \bar{\eta}_i \gamma_5 \Delta_F^{-1} \gamma_5 \eta^j &= -\frac{1}{2} (\Delta_0 + \Delta_5) (\bar{\eta})_i^j (\lambda \bar{\tau} \lambda) + \\ &+ \frac{\Delta_0 - \Delta_5}{2} (\bar{\tau})_i^j (\lambda \bar{\tau} \lambda) + i \Delta_{\mu\nu} \delta_i^j [\bar{\lambda} \bar{\sigma}^{\mu\nu} \lambda - \lambda \sigma^{\mu\nu} \lambda] \end{aligned} \quad (\text{C.5})$$

For the bosonic inverse operator using the properties of the τ matrices, we have:

$$\Delta_B^{-1} = \frac{\tilde{\Delta}_B + g^2 \tilde{A}}{\tilde{\Delta}_B^2 - g^2 A^2} \quad (\text{C.6})$$

where $\tilde{\Delta}_B$ is the operator defined in Appendix B. Performing the trace on internal space, we get for Q:

$$\begin{aligned} Q &= -\frac{g^2}{2} \text{Tr}_x \left\{ \frac{2i \tilde{\Delta}_B \Delta_{\mu\nu}}{\tilde{\Delta}_B^2 - g^2 A^2} (\bar{\lambda} \bar{\sigma}^{\mu\nu} \lambda - \lambda \sigma^{\mu\nu} \lambda) \right. \\ &\quad \left. - \frac{\Delta_0 + \Delta_5}{\tilde{\Delta}_B^2 - g^2 A^2} g \tilde{A} (\lambda \bar{\tau} \lambda) + \frac{\Delta_0 - \Delta_5}{\tilde{\Delta}_B^2 - g^2 A^2} g \tilde{A} (\bar{\lambda} \bar{\tau} \lambda) \right\} \end{aligned} \quad (\text{C.7})$$

Since Δ_0 , Δ_5 , $\Delta_{\mu\nu}$ are linear combinations of $[\Delta_F^{-1}]_{aa}$ we have to evaluate two kinds of traces:

$$\text{Tr}_x \frac{[\Delta_F^{-1}]_{aa}}{\tilde{\Delta}_B^2 - g^2 A^2} \quad \text{and} \quad \text{Tr}_x \frac{\tilde{\Delta}_B [\Delta_F^{-1}]_{aa}}{\tilde{\Delta}_B^2 - g^2 A^2}$$

Using the techniques of Appendix B, we obtain:

$$\text{Tr}_X \frac{[\Delta_F^{-1}]_{11}}{\tilde{\Delta}_B^2 - g^2 A^2} = \int d^4x dy \left[\frac{1}{\tilde{\Delta}_B^2 - g^2 A^2} \right] (x, y) \cdot [\Delta_F^{-1}]_{11} (y, x) =$$

$$= \frac{V}{4\pi^2} e^2 \int_{12134} \sum_{n,m=0}^{\infty} \frac{1}{\Omega^2 - g^2 A^2} \cdot \frac{-igC^*}{\Omega + X}$$

$$= \frac{V}{4\pi^2} e^2 \int_{12134} \sum_{n,m} \left[\frac{1}{\Omega - gA} - \frac{1}{\Omega + gA} \right] \frac{1}{2gA} \frac{-igC^*}{\Omega + X} =$$

$$= \frac{V}{4\pi^2} e^2 \int_{12134} \sum_{n,m} \frac{-iC^*}{2A} \left\{ \left[\frac{1}{\Omega - gA} - \frac{1}{\Omega + X} \right] \frac{1}{X + gA} - \right.$$

$$\left. - \left[\frac{1}{\Omega + gA} - \frac{1}{\Omega + X} \right] \frac{1}{X - gA} \right\} =$$

$$= \frac{V}{4\pi^2} e^2 \int_{12134} \sum_{n,m} \frac{-iC^*}{2A} \left\{ \frac{1}{(\Omega - gA)(X + gA)} - \frac{1}{(\Omega + gA)(X - gA)} + \frac{2gA}{(X^2 - g^2 A^2)(\Omega + X)} \right\} = \quad (C.8)$$

$$= \frac{V}{4\pi^2} e^2 \int_{12134} \frac{-iC^*}{2A} \int_0^{\infty} ds \sum_{n,m} e^{-\Omega s} \left\{ \frac{e^{gAs}}{X + gA} - \frac{e^{-gAs}}{X - gA} + \frac{2gA e^{-Xs}}{X^2 - g^2 A^2} \right\} =$$

$$= \frac{V}{4\pi^2} e^2 \int_{12134} \frac{-iC^*}{2A} \int_0^{\infty} ds \left\{ \frac{e^{-g^2 C^* s}}{4 \sinh(e f_{12} s) \sinh(e f_{34} s)} \right.$$

$$\left. \cdot \frac{1}{X^2 - g^2 A^2} [2X \sinh(gAs) - 2gA \cosh(gAs) + 2gA e^{-Xs}] \right\}$$

Because of the symmetry of (C.3), the results for the other elements $[\Delta_F^{-1}]_{aa}$ may

be easily read out from (C.8). Using then

$$\Delta_o + \Delta_5 = \frac{1}{2} \left\{ [\Delta_F^{-1}]_{11} + [\Delta_F^{-1}]_{22} \right\} \quad (C.9)$$

and

$$\Delta_o - \Delta_5 = -\frac{1}{2} \left\{ [\Delta_F^{-1}]_{33} + [\Delta_F^{-1}]_{44} \right\} \quad (C.10)$$

we get

$$\text{Tr}_X \frac{\Delta_o + \Delta_5}{\tilde{\Delta}_B^2 - g^2 A^2} = -\frac{V}{8\pi^2} e^2 \int_{12134} igC^* \int_0^{\infty} ds \left\{ \frac{e^{-g^2 C^* s}}{2 \sinh(e f_{12} s) \sinh(e f_{34} s)} \right. \\ \left. \cdot \frac{\cosh(Xs) - \cosh(gAs)}{X^2 - g^2 A^2} \right\} \quad (C.11)$$

and

$$\text{Tr}_X \frac{\Delta_o - \Delta_5}{\tilde{\Delta}_B^2 - g^2 A^2} = \frac{V}{8\pi^2} e^2 \int_{12134} igC^* \int_0^{\infty} ds \left\{ \frac{e^{-g^2 C^* s}}{2 \sinh(e f_{12} s) \sinh(e f_{34} s)} \right. \\ \left. \cdot \frac{\cosh(Ys) - \cosh(gAs)}{Y^2 - g^2 A^2} \right\} \quad (C.12)$$

In a similar way, we can evaluate the trace:

$$\text{Tr}_X \frac{\Delta_B [\Delta_F^{-1}]_{11}}{\tilde{\Delta}_B^2 - g^2 A^2} = \frac{V}{4\pi^2} e^2 \int_{12134} \sum_{n,m=0}^{\infty} \frac{\Omega}{\Omega^2 - g^2 A^2} \frac{-igC^*}{\Omega + X} =$$

$$= -igC^* \frac{V}{4\pi^2} e^2 \int_{12134} \sum_{n,m} \frac{1}{2} \left\{ \frac{1}{(\Omega + gA)(X - gA)} + \frac{1}{(\Omega - gA)(X + gA)} \right. \\ \left. - \frac{2X}{(\Omega + X)(X^2 - g^2 A^2)} \right\} =$$

$$= -igC^* \frac{V}{8\pi^2} e^{\frac{1}{2}f} t_{12} t_{34} \int_0^\infty ds \left\{ \frac{e^{-g^2 C^* C s}}{2 \sinh(e f_{12} s) \sinh(e f_{34} s)} \right\}.$$

$$\frac{X \cosh(gAs) - gA \sinh(gAs) - X e^{-Xs}}{X^2 - g^2 A^2} \quad (C.13)$$

and using (C.3), the corresponding results for the other elements $[\Delta_F^{-1}]_{aa}$ are, again, easily found.

In order to evaluate (C.7), we need Δ_{12} and Δ_{03} ; they are given by:

$$\Delta_{12} = \frac{1}{g} \{ [\Delta_F^{-1}]_{11} - [\Delta_F^{-1}]_{22} + [\Delta_F^{-1}]_{33} - [\Delta_F^{-1}]_{44} \}$$

$$\Delta_{03} = \frac{1}{g} \{ [\Delta_F^{-1}]_{11} - [\Delta_F^{-1}]_{22} - [\Delta_F^{-1}]_{33} + [\Delta_F^{-1}]_{44} \}$$

and substituting the explicit expressions we get:

$$\begin{aligned} \text{Tr } i\tilde{\Delta}_B \frac{\Delta_{12}}{\tilde{\Delta}_B^2 - g^2 A^2} &= -\frac{V}{32\pi^2} e^{\frac{1}{2}f} t_{12} t_{34} \int_0^\infty ds \left\{ \frac{e^{-g^2 C^* C s}}{2 \sinh(e f_{12} s) \sinh(e f_{34} s)} \right\} \\ &\quad \left[-gC^* X \frac{\cosh(gAs) - \cosh(Xs)}{X^2 - g^2 A^2} + gC Y \frac{\cosh(gAs) - \cosh(Ys)}{Y^2 - g^2 A^2} \right] \end{aligned} \quad (C.14)$$

and

$$\begin{aligned} \text{Tr } i\tilde{\Delta}_B \frac{\Delta_{03}}{\tilde{\Delta}_B^2 - g^2 A^2} &= -\frac{iV}{32\pi^2} e^{\frac{1}{2}f} t_{12} t_{34} \int_0^\infty ds \left\{ \frac{e^{-g^2 C^* C s}}{2 \sinh(e f_{12} s) \sinh(e f_{34} s)} \right\} \\ &\quad \left[-gC^* X \frac{\cosh(gAs) - \cosh(Xs)}{X^2 - g^2 A^2} - \right. \\ &\quad \left. - gC Y \frac{\cosh(gAs) - \cosh(Ys)}{Y^2 - g^2 A^2} \right] \end{aligned} \quad (C.15)$$

We collect the results (C.11), (C.12), (C.14) and (C.15) in the expression (C.7) for Q. Then we go back to Minkowski space and we restore the full covariance of the expression as we did in Appendix B. The final result is:

$$\begin{aligned} Q &= -\frac{V}{32\pi^2} g^2 (e^f F_{\mu\nu} \tilde{F}^{\mu\nu}) \int_0^\infty ds \left\{ \frac{e^{-g^2 C^* C s}}{\cosh(e|\vec{X}|s) - \cosh(e|\vec{X}|s^*)} \right\} \\ &\quad \left[\frac{\cosh(e|\vec{X}|s) - \cosh(gAs)}{e^2 |\vec{X}|^2 - g^2 A^2} \left(-\frac{iC}{2} g^2 \vec{A}(\lambda \vec{\tau} \lambda) + \frac{gC}{2} F_{\mu\nu}(\lambda \sigma^{\mu\nu} \lambda) \right) \right. \\ &\quad \left. + \frac{\cosh(e|\vec{X}|s) - \cosh(gAs)}{e^2 |\vec{X}|^2 - g^2 A^2} \left(-\frac{iC}{2} g^2 \vec{A}(\bar{\lambda} \vec{\tau} \bar{\lambda}) + \frac{gC}{2} F_{\mu\nu}(\bar{\lambda} \sigma^{\mu\nu} \bar{\lambda}) \right) \right] \end{aligned} \quad (C.16)$$

where now $\vec{X} = \vec{R} + i\vec{E}$, and it is what we listed in eq. (3.6) provided $e = g/2$.

Appendix D

In this appendix we will evaluate the quantity

$$R = \frac{1}{8} g^4 \text{Tr} [\Delta_B^{-1} \tilde{\gamma}_5 \Delta_F^{-1}]^2 \quad (\text{D.1})$$

limiting ourselves for the sake of simplicity to the lowest order in $1/C$, $1/C^*$.

Then on dimensional ground one can work within the approximation $A_\mu = 0, \tilde{A} = 0$, so that the inverse bosonic and fermionic operators become

$$\Delta_B^{-1} = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 + g^2 C^* C} \quad (\text{D.2})$$

$$\Delta_F^{-1} = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq(x-y)} - \not{q} + i g \frac{1-\gamma_5}{2} C - i g \frac{1+\gamma_5}{2} C^*}{q^2 - g^2 C^* C} \quad (\text{D.3})$$

and the quantity R reduces to

$$R = -\frac{iV}{8} g^4 \left\{ \text{Tr}_2 [(\tilde{\gamma} \gamma^* \gamma) (\tilde{\gamma} \gamma^* \gamma)] \int \frac{d^4 p}{(2\pi)^4} \frac{p^2}{(p^2 - g^2 C^* C)^4} + \right. \\ \left. + \text{Tr}_2 [(\tilde{\gamma} B_5 \gamma)^2] g^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - g^2 C^* C)^4} \right\} \quad (\text{D.4})$$

where

$$B_5 = i \frac{1-\gamma_5}{2} C - i \frac{1+\gamma_5}{2} C^*$$

The momentum integration can be performed leading to

$$R = \frac{V}{32\pi^2} g^2 \left\{ -\frac{a}{12C^*C} + \frac{b}{24(C^*C)^2} \right\} \quad (\text{D.5})$$

Here V is the Euclidean volume as in previous calculations and the quantities a and b are given by

$$a = \text{Tr}_2 (\tilde{\gamma} \gamma_\mu \gamma)^2 = \text{Tr}_2 [\bar{\lambda}_{\alpha i} (\bar{\sigma}_\mu)^{\dot{\alpha}\beta} \lambda_\beta^i - \lambda_\alpha^i (\sigma_\mu)_{\dot{\alpha}\beta} \bar{\lambda}^{\dot{\alpha}j}]^2 \quad (\text{D.6})$$

and

$$b = \text{Tr}_2 (\tilde{\gamma} B_5 \gamma)^2 = -\text{Tr}_2 [C^* \lambda_\alpha^i \lambda_\alpha^j + C \bar{\lambda}_{\dot{\alpha}i} \bar{\lambda}^{\dot{\alpha}j}]^2 \quad (\text{D.7})$$

By using the standard relations

$$(\sigma^\mu)_{\alpha\dot{\alpha}} (\bar{\sigma})^{\dot{\beta}\beta} = -2 \delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}}$$

and

$$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\beta} \epsilon^{\alpha\beta} (\sigma^\mu)_{\beta\dot{\beta}}$$

we easily see that in R, the terms proportional to $1/C C^*$ disappear, as it should, because the contribution coming from Eq. (D.6) exactly cancels out the crossed term coming from b [Eq. (D.7)].

Then we are left with

$$R = -\frac{V}{32\pi^2} \frac{g^2}{24} \left\{ \text{Tr}_2 (\lambda_\alpha^i \lambda_\alpha^j)^2 \frac{1}{C^2} + \text{Tr}_2 (\bar{\lambda}_{\dot{\alpha}i} \bar{\lambda}^{\dot{\alpha}j})^2 \frac{1}{C^{*2}} \right\} \quad (\text{D.8})$$

In order to recast Eq. (D.8) in a form more suitable to our purpose, we make use of Eqs. (A.7), (A.8) and Eqs. (A.13) ÷ (A.16) reaching the final expression

$$\frac{1}{8} g^2 \text{Tr} [\Delta_B^{-1} \tilde{\gamma}_5 \Delta_F^{-1}]^2 = \\ = -\frac{V}{32\pi^2} \frac{g^2}{24} \left\{ \frac{1}{C^2} [(\lambda \tilde{\tau} \lambda)^2 + (\lambda \sigma^{\mu\nu} \lambda)^2] + \right. \\ \left. + \frac{1}{C^{*2}} [(\bar{\lambda} \tilde{\tau} \bar{\lambda})^2 + (\bar{\lambda} \bar{\sigma}^{\mu\nu} \bar{\lambda})^2] \right\} \quad (\text{D.9})$$

REFERENCES

- 1) For a discussion of the decay $\pi^0 \rightarrow 2\gamma$, see, for instance, V. De Alfaro, S. Fubini, G. Furlan and Rossetti, Currents in hadron physics, p. 175. Suppl XXII (1980) 341 and references therein.
- 2) See, for instance, P. Di Vecchia, Proc. Schlading Institute, Acta Phys. Austr. 32 (1980) 341 and references therein.
- 3) J. Schwinger, Phys. Rev. 82 (1951) 664.
J. Schwinger, Particles, sources and fields, Vol. 2, Ch. 4.8.
- 4) F. Riva, Nuovo Cimento 61A (1981) 69.
A. D'Adda, A. Davis, P. Di Vecchia and P. Salomonson, Nucl. Phys. B222 (1983) 45.
- 5) G. Veneziano and S. Yankielowicz, Phys. Lett. 113B (1982) 231.
- 6) P. Fayet, Nucl. Phys. B113 (1976) 135.
- 7) L. Brink, J. Scherk and J. Schwarz, Nucl. Phys. B121 (1977) 77.
F. Gliozzi, D. Olive and J. Scherk, Nucl. Phys. B122 (1977) 253.
- 8) S. Chada, A. D'Adda, P. Di Vecchia and F. Nicodemi, Phys. Lett. 67B (1977) 103.
B. Zumino, Phys. Lett. 69B (1977) 369.
- 9) A. D'Adda and P. Di Vecchia, Phys. Lett. 73B (1978) 162.
- 10) D.R.T. Jones, Nucl. Phys. B87 (1975) 127; P. Howe, K. Stelle and P. West, Phys. Lett. 124B (1983) 55.
V.A. Novikov, M.A. Shifman, V.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B229 (1983) 381.
A. Parkes and P. West, King's College preprint (1984).
- 11) A. D'Adda, P. Di Vecchia and R. Horsley, Phys. Lett. 76B (1978) 298.
- 12) M.K. Prasad and C.M. Sommerfield, Phys. Rev. Lett. 35 (1975) 760.
- 13) E.B. Bogomol'nyi, Sov. J. Nucl. Phys. 24 (1976) 449.

- 14) D. Olive and E. Witten, Phys. Lett. 78B (1978) 97.
- 15) R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo and C.A. Savoy, Phys. Lett. 115B (1982) 212.
- 16) J. Wess, Acta Phys. Austr. 41 (1975) 409.
R. Grimm, M. Sohnius and J. Wess, Nucl. Phys. B133 (1978) 275.
- 17) E. Rabinovici and G. Veneziano, private communication.
- 18) A.W. Fischer, N = 2 supersymmetric Yang-Mills theory and its supercurrent, Durham preprint (1982).
- 19) T.R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B218 (1983) 493.
- 20) V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B229 (1983) 394, 407.
- 21) G.C. Rossi and G. Veneziano, Phys. Lett. 138B (1984) 195.
- 22) P. Di Vecchia, R. Musto, F. Nicodemi and R. Pettorino, to be published.