DIMENSIONAL REDUCTION AND DYNAMICAL SYMMETRY BREAKING

P. Forgacs and G. Zoupanos

CERN - Geneva

ABSTRACT

We present a model in which the electroweak gauge group is broken according to a dynamical scenario based on the chiral symmetry breaking of high colour representations. The dynamical scenario requires also the existence of elementary Higgs fields, which in the present scheme come from the dimensional reduction of a pure gauge theory.

CERN-TH.3883
April 1984
In the last decade we witnessed the triumph of spontaneously broken
gauge theories (SBGT) in the description of electroweak interactions\(^1\).
However, it is still an open question what triggers the spontaneous symmetry
breaking. It seems rather unlikely that the usual Higgs mechanism\(^2\)
involving elementary scalar fields is at the heart of the spontaneous
breaking of the electroweak sector. A number of unattractive features of
the standard electroweak model such as the lack of predictions for fermion
masses and mixing angles, the hierarchy problem related to the difference
between the mass scales of the fermions and the vector bosons, are due to
the existence of Higgs fields. The corresponding hierarchy problem is even
worse in grand unified theories (GUTs).

Dynamical scenarios based on the existence of an ultrastrong force and
new fermions\(^3\) (technicolour, technifermions) or quarks in higher colour
representations\(^4\)-\(^9\) suggest that these fermions bind and form among others
the Goldstone bosons which are eaten by the W and the Z.

The dynamical scenario based on high colour representations is very
attractive since it does not suffer from flavour changing neutral currents
in tree order as does the technicolour\(^10\) and moreover, it is supported by
Monte Carlo lattice calculations\(^11\). However, at some stage it requires
also (like the technicolour) elementary Higgs fields.

In another picture the Higgs fields and the symmetry breaking potential
have a geometric origin in the dimensional reduction scheme using symmetric
gauge fields\(^12\). This scenario is very attractive because of the few
parameters it introduces and moreover it admits naturally chiral fermions
which is the big problem in other schemes of dimensional reduction such as
Kaluza-Klein theories.

We propose here to combine the dynamical scheme with high colour
representations and the scheme of symmetric gauge fields in such a way that
the latter makes it unnecessary to introduce Higgs fields by hand in the
first scheme. Namely we propose that the electroweak symmetry breaking is
due to the dynamical mechanism while the Higgs fields that are needed in the
dynamical scheme have "geometrical" origin.

Let us first briefly recall the main ideas of the two approaches. The
dynamical scheme we have just mentioned requires the existence of fermions
in higher than triplet representations of the SU(3)\(_c\) gauge group. Chiral
symmetry breaking in the high representation quark sector is assumed to occur at much larger mass scales than the ordinary one. Monte Carlo calculations of the scales of the chiral symmetry breaking give a strong indication of the validity of the above assumption. Studying an SU(2)-colour model it was found that fermions in different representations indeed do condense at exponentially separated mass scales.

Neglecting current quark masses and electroweak interactions, there exists in the QCD Lagrangian for each of the quark sectors in some $r$-representations a separate global chiral symmetry

$$SU(N_r)_L \times SU(N_r)_R \times U(1)$$

where $N_r$ is the number of quark flavours in the $r$-representation ($r = 3, 6, 8, 10, 15$, etc.). At some value of the effective strength of colour forces it is assumed that quark-antiquark condensates are formed giving rise to a vacuum that breaks the axial part of the global $SU(N_r)_L \times SU(N_r)_R$ symmetry and preserves the vector $SU(N_r)$. Failure of the conserved axial charges to annihilate the vacuum implies the existence of $N_r^2 - 1$ Goldstone bosons, which are massless quark-antiquark bound state excitations, and dynamical masses for the quarks. Switching on the electroweak interactions described by the standard model with left-handed doublet and right-handed singlet quarks the local $SU(2)_L \times U(1)$ symmetry is also broken by the vacuum which is induced by the quark condensates to the $U(1)$ of electromagnetism. The would-be Goldstone bosons are pions and other pseudoscalars and their coupling to the weak current is:

$$\langle 0 | J^{\mu}_{5r}(0) | \pi_r(p) \rangle = -i f_{\pi_r} p^\mu$$

where $f_{\pi_r}$ is the $r$-pion coupling constant and $J^{\mu}_{5r}(p)$ is the weak axial current. In the absence of some source for quark masses the ''pions'' remain massless and some combination of them gives masses to the $W$ and the $Z$. These masses are of the right magnitude if the ''Goldstone pions'' are coupled to the vector bosons with an effective $f_{\pi} \sim 250$ GeV. In fact, this is precisely what we expect to happen in this scheme if we introduce fermions in some appropriate (higher) representations.

This framework involves also a mechanism which provides masses to fermions. The mechanism consists of an enlargement of the colour group to
some gauge group $G_s$ and a subsequent breaking of $G_s$ to $SU(3)_c$. Then the
massive $G_s/SU(3)_c$ gauge bosons mediate transitions between colour singlets,
colour triplets, and fermions in higher colour representations (echo-
fermions) which belong to an appropriate irreducible representation of the
group $G_s$ (hapions). Then the light fermions obtain masses through radiative
contributions coming from the exchange of the heavy gauge bosons corres-
ponding to the coset space generators. A particularly successful choice of
$G_s$ for realistic model building\(^7\) appears to be $G_s = [SU(3) \times SU(3)]_s$ which
breaks to the diagonal $SU(3)_c$. However, the breaking was achieved by
introducing elementary Higgs fields. Here we are going to explain the
appearance of these Higgs fields with a geometrical mechanism.

In the theory of symmetric gauge fields one starts with a pure Yang-
Mills theory with a gauge group $G$, defined on a spacetime $M$. $M$ is assumed
to be the direct product of the four-dimensional Minkowski spacetime and a
compact coset space $S/R$. Here $S$ is a compact semisimple Lie group and $R$
is a Lie subgroup of $S$. Therefore we assume that the metric of $M$ is block-
diagonal. Dimensional reduction occurs by requiring the gauge fields, $A^ \mu$
to be $S$ invariant. This means that the action of an element of $S$ on $A^ \mu$
is compensated by a gauge transformation. Given a coset space $S/R$, it is
possible to determine explicitly both the compensating gauge transformation
and the dependence of $A^ \mu$ on the coset space co-ordinates. When $A^ \mu$
is inserted in the Yang-Mills Lagrangian on $M$, one can integrate over the coset
space co-ordinates and obtains a Yang-Mills-Higgs theory in four
dimensions.

In order to obtain non-trivial $S$-invariant gauge potentials, one has to
choose a subgroup $R_G$ of $G$ isomorphic to $R$. Then by fixing the groups $G$, $S$,
$R$ and the embedding of $R$ to $S$, the radius parameter of the coset space and
the coupling constant of the original Yang-Mills Lagrangian, the theory in
four dimensions is completely determined. Let us mention some features of
the four-dimensional theory relevant to our discussion.

(i) The gauge group in four dimensions, which we denote by $N$, is the
centralizer of $R_G$ in $G$. Thus we have the group structure

$$G = R_G \times H$$

where $H$ is the maximal subgroup of $G$ commuting with $R$. 
(ii) The Higgs fields can be determined as well as the Higgs potential. Depending on the form of the Higgs potential the gauge group \( H \) can break further. However, there exist cases when the breaking of \( H \) to some \( K \subset H \) is guaranteed, and one can determine the final gauge group after the breaking without working out the Higgs potential\(^{13}\). Let us suppose that \( G \) has also a subgroup isomorphic to \( S \) and we embed \( S \) into \( G \). Then denoting by \( K \) the centralizer of \( S \subset G \), we have, recalling Eq. (3), the following structure:

\[
G \supset \begin{array}{c}
S_G \\
\cap \\
R_G \\
\times K
\end{array}
\]

\( (4) \)

In this case the result of dimensional reduction is a four-dimensional Yang-Mills-Higgs theory with gauge group \( H \), and the Higgs multiplets and the Higgs potential are such that \( H \) is spontaneously broken to \( K \).

One of the most appealing features of the approach using symmetric gauge fields is that it naturally accommodates chiral fermions to be contrasted with the competing Kaluza-Klein type attempts\(^{14}\). To incorporate fermions we add to the pure Yang-Mills Lagrangian a Dirac Lagrangian with fermions assigned to some representation of \( G \). The definition of symmetric spinors is analogous to that of the symmetric gauge fields\(^{15}\), which means that the action of \( S \) on the spinor \( \psi \) is compensated by a gauge transformation and a local Lorentz rotation in the tangent space. After dimensional reduction, the entire theory becomes four-dimensional, with spinors being in some multiplets of the \( H \) group. If the dimension of the coset space \( S/R \) is even, one can start with Weyl spinors on \( M \) which contain both left- and right-handed four-dimensional spinors in \( M_4 \). The symmetry constraints can act differently on the left- and right-handed components and therefore parity violation may occur in the final theory\(^{16}\).

We think that this attractive scheme may be useful to describe the unification of strong, electromagnetic and weak interactions. Unfortunately no realistic models have been constructed so far\(^{17}\). The model we present in this paper has many nice features, but it cannot be considered as a full theory.
Let us consider a theory in six dimensions defined on a space-time \( M = M \times S^2 \). The theory in six dimensions contains only pure Yang-Mills fields with gauge group \( G = \text{SU}(6)_S \times \text{SU}(2)_L \times \text{U}(1) \).

The fermions (= prehaplons) are assigned to the following representations:

\[
\begin{align*}
(p^u_L) &= (84; 2; 1/3) \\
p^u_R &= (84; 1; 4/3) \\
p^d_R &= (84; 1; -2/3) \\
(p^\nu_e)_L &= (20; 2; 1) \\
p^e_R &= (20; 1; -2)
\end{align*}
\]

(5)

The above fermionic representations do not introduce \( \text{SU}(6)_S \) anomalies. In order to cancel the \( \text{SU}(2)_L \times \text{U}(1) \) anomalies in four dimensions we introduce a prehaplon family having the structure (5) and assigned to the 56 representation of \( \text{SU}(6)_S \). The hypercharges have to be chosen such that the condition \( \Sigma Q = 0 \) is satisfied for all the fermions which survive after dimensional reduction. This can be easily done and we are going to see later that the fermions with the unconventional charges that survive after dimensional reduction coming from the 56 representation will be heavy and they will disappear from the low energy fermionic spectrum.

The crucial point for the rest of the discussion is the observation that the \( \text{SU}(6) \) group has the following maximal subgroup structure:

\[
\text{SU}(6) \supset \text{SU}(2) \times \text{SU}(3)_{\text{diag}}
\]

\[\begin{array}{cc}
\text{SU}(2) \\
\text{U}(1) \\
\text{SU}(3) \times \text{SU}(3)_{\text{diag}}
\end{array}\]

(6)

i.e. it has the form of Eq. (4) with \( S = \text{SU}(2) \), \( R = \text{U}(1) \), \( H = \text{SU}(3) \times \text{SU}(3)_{\text{diag}} \). Thus by choosing \( S/R = \text{SU}(2)/\text{U}(1) \) the \( \text{SU}(6)_S \) gauge group becomes after dimensional reduction \( [\text{SU}(3) \times \text{SU}(3)]_S \), while the \( S \) and the dimensional reduction act trivially on the \( \text{SU}(2)_L \times \text{U}(1) \) part of the
theory. We can easily find the Higgs content of the dimensionally reduced theory. In general the gauge fields, which become scalars in four dimensions, transform under \( R \) as a vector \( \nu \) as specified by the embedding

\[
ad S = \text{ad } R + \nu \tag{7}
\]

and \( \nu = \sum K S_K \), where each \( S_K \) is an irreducible representation of \( R \). Then the Higgs multiplets under \( H \) can be found by decomposing the adjoint representation of \( G \) under \( R \times H \):

\[
ad G = \sum K (r_K, h_K) \tag{8}
\]

where \( r_K \) is some irreducible representation of \( R \) and \( h_K \) is some irreducible representation of \( H \). For each pair of \( (r_K, S_j) \) where \( r_K \) and \( S_j \) are identical irreducible representations, there is an \( h_K \) Higgs multiplet in the four-dimensional theory.

In the present case

\[
ad SU(2) = \text{ad } U(1) + \nu \tag{9}
\]

and the vector \( \nu \) decomposes under \( R = U(1) \) into \( (1) + (-1) \). On the other hand

\[
3 \cdot S = \left(1^1, \bar{8}ight) \left(4^2, 1\right) + \left(8, 1\right) \left(-\frac{1}{2}, \frac{1}{2}\right) + \left(3, \bar{3}\right) (1) + \left(\bar{3}, 3\right) (-1) + (1, 1) (0) \tag{10}
\]

so that the Higgs fields transform under \( H = SU(3) \times SU(3) \times U(1) \) as

\[
(3, \bar{3}) ; \ (\bar{3}, 3) \tag{11}
\]

Because we have the group structure (4) we do not have to work out the Higgs potential as the breaking of \( SU(3) \times SU(3) \times U(1) \) to \( SU(3)_{\text{diag}} \) is guaranteed by the theorem of ref. 13).

In order to identify the four-dimensional fermion multiplets after dimensional reduction, one has first to embed \( R \) into the local Lorentz group, \( SO(N) \), of \( S/R \) in such a way that the \( N \) of \( SO(N) \) has the branching rule \( N = \nu \) where \( \nu \) is given in Eq. (7). This determines the embedding of \( R \) into \( SO(N) \) uniquely. Next one takes the spinor of \( SO(N) \), \( \sigma \), and decomposes
it into irreps of $\mathbb{R}$; $\sigma = \sum_K \sigma_K$. Then one has to decompose the representation, $F$, of the gauge group $G$ to which the fermions are assigned, under $R_G \times H; F = \oplus (r_K, \sigma_K)$. Similarly to the case for the Higgs fields, for each pair of $(r_K, \sigma_K)$, where $r_K$ and $\sigma_K$ are identical irreps., there is an $h_K$ multiplet of spinor fields in the four-dimensional theory.

If $N$ is even, we can define Weyl spinors on $\mathbb{M}_4 \times S/R$ and a Weyl spinor of one chirality is decomposed into four-dimensional spinors of both chiralities. The four-dimensional chirality is linked to the chirality with respect to $SO(N)$, since $\Gamma^{D+1} = \gamma^5 \times \tilde{\gamma}^{D-3}$ where $D = 4+N$. In general the symmetry constraints can act differently on spinors with positive and negative $\tilde{\gamma}^{D-3}$ chirality and this way we can end up with a left-right asymmetry in four dimensions.

In the present case the spinor of $SO(3)$ decomposes under $R = U(1)$ as $(\frac{1}{2}) + (-\frac{1}{2})$. In our example we take the following $SU(6)$ representations for the spinors: 20, 56, 84. Their decomposition under $U(1) \times SU(3) \times SU(3)$ is:

$$SU(6) \supset U(1) \times SU(3) \times SU(3)$$

84 = \begin{align*}
(1/2) & (3,1) \\
+ (-1/2) & (1,3) \\
+ (1/2) & (\bar{3},1) \\
+ (-1/2) & (1,\bar{3}) \\
+ (1/2) & (3,8) \\
+ (-1/2) & (8,\bar{3}) \\
+ (3/2) & (\bar{3},3) \\
+ (-3/2) & (\bar{3},\bar{3})
\end{align*}

56 = \begin{align*}
(3/2) & (10,1) \\
+ (-3/2) & (1,10) \\
+ (1/2) & (3,6) \\
+ (-1/2) & (6,3)
\end{align*}

20 = \begin{align*}
(1/2) & (3,3) \\
-(-1/2) & (3,\bar{3}) \\
(3/2) & (1,1) \\
(-3/2) & (1,1)
\end{align*}

Applying the previous results it is easy to identify the fermions which survive after dimensional reduction. The only surviving spinors are those
with \((\frac{1}{2})\) and \((-\frac{1}{2})\) under the \(U(1)\) group corresponding to, say, left- and right-handed particles (fermionic hadrons).

In general, after dimensional reduction the fermions acquire Yukawa couplings, therefore most of them obtain a mass comparable to the vector boson mass after the spontaneous symmetry breaking. In our example the surviving fermions remain massless after the spontaneous symmetry breaking. This phenomenon is rather special and it is due to the fact that coset space is \(S^2\) and that \(S \subset G^\text{18}\).

The \([\text{SU}(3) \times \text{SU}(3) \times U(1)]_s\) gauge group breaks further to the \(\text{SU}(3)\text{\textunderscore diag}\) as we have already stated. Then a linear combination \(V^\mu_{\mu} \equiv (A^\mu - B^\mu)/2\) of the vector bosons \(A, B\) of \(\text{SU}(3) \times \text{SU}(3)\) become massive while the orthogonal combination \(V^\mu_{\mu} \equiv (A^\mu + B^\mu)/2\) emerges as the colour octet of massless gauge bosons of QCD.

Under the diagonal \(\text{SU}_c(3)\) the fermionic hadrons become ordinary fermions in the triplet representation for quarks and in singlets for leptons. The higher representations correspond to what we call echo fermions.

We recall for convenience how some of the hadron representations are decomposed under \(\text{SU}_c(3)\):

\[
[\text{SU}(3) \times \text{SU}(3) \times U(1)]_s \rightarrow \text{SU}_c(3)
\]

\[
(3,8) = 3 + \bar{6} + 15
\]

\[
(\bar{3}, 3) = 1 + 8
\]

\[
(3,6) = 8 + 10
\]

The \(\text{SU}_c(3)\) interactions force the fermions of various representations to condense and the chiral symmetry breaking is expected to take place at different scales. We expect that the highest representation which appears in the left-handed doublet and in the right-handed singlet (i.e., the 15-plet) gives the leading contribution to the breaking of \(\text{SU}(2)_L \times U(1)\) to \(U(1)_{\text{em}}\). Thus its condensation scale should be \(O(500)\text{GeV}\). In addition, the broken generators \(V^\mu_{\mu}\) of \(\text{SU}(3) \times \text{SU}(3)/\text{SU}_c(3)\) give radiative masses to quarks and leptons. All echo fermions of higher colour representation acquire large dynamical masses and some of them obtain current quark masses.
as well. From the 56-plet of $SU(6)$, after spontaneous symmetry breaking the
only fermions that appear are echo quarks with exotic charges which,
however, acquire large dynamical masses. Ordinary quarks acquire the usual
dynamical and the following current quark masses in the one- and the two-
loop approximations:

\[ m^4(R) = \frac{g^2 C_2(R) M_R^3}{4\pi M_V^-} \ln \frac{M_V^-}{(3 C_2(R))^{2/3} M_R^2} \]

\[ m^2(R) = \frac{g^2 C_2(R) M_R^3}{4\pi M_V^-} \ln \frac{M_V^-}{(3 C_2(R))^{2/3} M_R^2} \]

The leptons acquire masses in the one-loop approximation. However, there
still remain some massless fermions.

We can estimate the $\nu^\mu$ masses, which are related to the Higgs masses
and these latter are determined by the length scale of the extra dimensions.
We assume that the massless quarks correspond to the $u$ and the $d$ quarks
which might obtain a small current mass from another source. Then
identifying those quarks which acquire radiative (current) masses with the $s$
and the $c$ we have to fix these masses at $O(1)$ GeV. This way we find that
$\mu_6 \sim 20$ GeV, $\mu_{15} \sim 500$ GeV and $M_V^- \sim 30$ TeV. Therefore the radius of the $S^2$
sphere is roughly $10^{-17}$ cm.

Clearly, despite its attractive features, our approach has also some
drawbacks. The most obvious one is that we did not take gravity into
account. However, by including gravity, one might be able to give a
natural explanation to spontaneous compactification. Namely we hope that by
considering the coupled Einstein-Yang-Mills equations one can find a
classical solution for the metric corresponding to a spacetime $M_4 \times S/R$.

ACKNOWLEDGMENTS

We would like to thank K. Barnes, R. Decker, D. Lüst, Y. Meurice,
O. Napoly and especially J.P. Derendinger for many discussions concerning
the group theoretical problems. We would like also to thank S. Ferrara,
E. Floratos and H. Leutwyler for discussions and encouragement.
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