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DIMENSIONAL REDUCTION AND DYNAMICAL SYMMETRY BREAKING

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ABSTRACT

We present a model in which the electroweak gauge group is broken according to a dynamical scenario based on the chiral symmetry breaking of high colour representations. The dynamical scenario requires also the existence of elementary Higgs fields, which in the present scheme come from the dimensional reduction of a pure gauge theory.

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In the last decade we witnessed the triumph of spontaneously broken gauge theories (SBGT) in the description of electroweak interactions¹⁾. However, it is still an open question what triggers the spontaneous symmetry breaking. It seems rather unlikely that the usual Higgs mechanism²⁾ involving elementary scalar fields is at the heart of the spontaneous breaking of the electroweak sector. A number of unattractive features of the standard electroweak model such as the lack of predictions for fermion masses and mixing angles, the hierarchy problem related to the difference between the mass scales of the fermions and the vector bosons, are due to the existence of Higgs fields. The corresponding hierarchy problem is even worse in grand unified theories (GUTs).

Dynamical scenarios based on the existence of an ultrastrong force and new fermions³⁾ (technicolour, technifermions) or quarks in higher colour representations⁴⁾⁻⁹⁾ suggest that these fermions bind and form among others the Goldstone bosons which are eaten by the W and the Z.

The dynamical scenario based on high colour representations is very attractive since it does not suffer from flavour changing neutral currents in tree order as does the technicolour¹⁰⁾ and moreover, it is supported by Monte Carlo lattice calculations¹¹⁾. However, at some stage it requires also (like the technicolour) elementary Higgs fields.

In another picture the Higgs fields and the symmetry breaking potential have a geometric origin in the dimensional reduction scheme using symmetric gauge fields¹²⁾. This scenario is very attractive because of the few parameters it introduces and moreover it admits naturally chiral fermions which is the big problem in other schemes of dimensional reduction such as Kaluza-Klein theories.

We propose here to combine the dynamical scheme with high colour representations and the scheme of symmetric gauge fields in such a way that the latter makes it unnecessary to introduce Higgs fields by hand in the first scheme. Namely we propose that the electroweak symmetry breaking is due to the dynamical mechanism while the Higgs fields that are needed in the dynamical scheme have "geometrical" origin.

Let us first briefly recall the main ideas of the two approaches. The dynamical scheme we have just mentioned requires the existence of fermions in higher than triplet representations of the $SU(3)_C$ gauge group. Chiral

symmetry breaking in the high representation quark sector is assumed to occur at much larger mass scales than the ordinary one. Monte Carlo calculations of the scales of the chiral symmetry breaking give a strong indication of the validity of the above assumption. Studying an $SU(2)$ -colour model it was found that fermions in different representations indeed do condense at exponentially separated mass scales.

Neglecting current quark masses and electroweak interactions, there exists in the QCD Lagrangian for each of the quark sectors in some r -representations a separate global chiral symmetry

$$SU(N_r)_L \times SU(N_r)_R \times U(1) \quad (1)$$

where N_r is the number of quark flavours in the r -representation ($r = 3, 6, 8, 10, 15$, etc.). At some value of the effective strength of colour forces it is assumed that quark-antiquark condensates are formed giving rise to a vacuum that breaks the axial part of the global $SU(N_r)_L \times SU(N_r)_R$ symmetry and preserves the vector $SU(N_r)$. Failure of the conserved axial charges to annihilate the vacuum implies the existence of $N_r^2 - 1$ Goldstone bosons, which are massless quark-antiquark bound state excitations, and dynamical masses for the quarks. Switching on the electroweak interactions described by the standard model with left-handed doublet and right-handed singlet quarks the local $SU(2)_L \times U(1)$ symmetry is also broken by the vacuum which is induced by the quark condensates to the $U(1)$ of electromagnetism. The would-be Goldstone bosons are pions and other pseudoscalars and their coupling to the weak current is:

$$\langle 0 | J_{5r}^{W_\mu}(0) | \pi_r(p) \rangle = -i f_{\pi_r} p^\mu \quad (2)$$

where f_{π_r} is the r -pion coupling constant and $J_{5r}^{W_\mu}(p)$ is the weak axial current. In the absence of some source for quark masses the "pions" remain massless and some combination of them gives masses to the W and the Z . These masses are of the right magnitude if the "Goldstone pions" are coupled to the vector bosons with an effective $f_\pi \sim 250$ GeV. In fact, this is precisely what we expect to happen in this scheme if we introduce fermions in some appropriate (higher) representations.

This framework involves also a mechanism which provides masses to fermions. The mechanism consists of an enlargement of the colour group to

some gauge group G_s and a subsequent breaking of G_s to $SU(3)_c$. Then the massive $G_s/SU(3)_c$ gauge bosons mediate transitions between colour singlets, colour triplets, and fermions in higher colour representations (echo-fermions) which belong to an appropriate irreducible representation of the group G_s (haplons). Then the light fermions obtain masses through radiative contributions coming from the exchange of the heavy gauge bosons corresponding to the coset space generators. A particularly successful choice of G_s for realistic model building⁷⁾ appears to be $G_s = [SU(3) \times SU(3)]_s$ which breaks to the diagonal $SU(3)_c$. However, the breaking was achieved by introducing elementary Higgs fields. Here we are going to explain the appearance of these Higgs fields with a geometrical mechanism.

In the theory of symmetric gauge fields one starts with a pure Yang-Mills theory with a gauge group G , defined on a spacetime M . M is assumed to be the direct product of the four-dimensional Minkowski spacetime and a compact coset space S/R . Here S is a compact semisimple Lie group and R is a Lie subgroup of S . Therefore we assume that the metric of M is block-diagonal. Dimensional reduction occurs by requiring the gauge fields, A_μ , to be S invariant. This means that the action of an element of S on A_μ is compensated by a gauge transformation. Given a coset space S/R , it is possible to determine explicitly both the compensating gauge transformation and the dependence of A_μ on the coset space co-ordinates. When A_μ is inserted in the Yang-Mills Lagrangian on M , one can integrate over the coset space co-ordinates and obtains a Yang-Mills-Higgs theory in four dimensions.

In order to obtain non-trivial S -invariant gauge potentials, one has to choose a subgroup R_G of G isomorphic to R . Then by fixing the groups G , S , R and the embedding of R to S , the radius parameter of the coset space and the coupling constant of the original Yang-Mills Lagrangian, the theory in four dimensions is completely determined. Let us mention some features of the four-dimensional theory relevant to our discussion.

(i) The gauge group in four dimensions, which we denote by H , is the centralizer of R_G in G . Thus we have the group structure

$$G \supset R_G \times H \quad (3)$$

where H is the maximal subgroup of G commuting with R .

(ii) The Higgs fields can be determined as well as the Higgs potential. Depending on the form of the Higgs potential the gauge group H can break further. However, there exist cases when the breaking of H to some $K \subset H$ is guaranteed, and one can determine the final gauge group after the breaking without working out the Higgs potential¹³⁾. Let us suppose that G has also a subgroup isomorphic to S and we embed S into G . Then denoting by K the centralizer of $S_G \subset G$, we have, recalling Eq. (3), the following structure:

$$G \supset \begin{array}{c} S_G \\ \cup \\ R_G \end{array} \times \begin{array}{c} K \\ \cap \\ H \end{array} \quad (4)$$

In this case the result of dimensional reduction is a four-dimensional Yang-Mills-Higgs theory with gauge group H , and the Higgs multiplets and the Higgs potential are such that H is spontaneously broken to K .

One of the most appealing features of the approach using symmetric gauge fields is that it naturally accommodates chiral fermions to be contrasted with the competing Kaluza-Klein type attempts¹⁴⁾. To incorporate fermions we add to the pure Yang-Mills Lagrangian a Dirac Lagrangian with fermions assigned to some representation of G . The definition of symmetric spinors is analogous to that of the symmetric gauge fields¹⁵⁾, which means that the action of S on the spinor ϕ is compensated by a gauge transformation and a local Lorentz rotation in the tangent space. After dimensional reduction, the entire theory becomes four-dimensional, with spinors being in some multiplets of the H group. If the dimension of the coset space S/R is even, one can start with Weyl spinors on M which contain both left- and right-handed four-dimensional spinors in M_4 . The symmetry constraints can act differently on the left- and right-handed components and therefore parity violation may occur in the final theory¹⁶⁾.

We think that this attractive scheme may be useful to describe the unification of strong, electromagnetic and weak interactions. Unfortunately no realistic models have been constructed so far¹⁷⁾. The model we present in this paper has many nice features, but it cannot be considered as a full theory.

Let us consider a theory in six dimensions defined on a space-time $M = M \times S^2$. The theory in six dimensions contains only pure Yang-Mills fields with gauge group $G = SU(6)_S \times SU(2)_L \times U(1)$.

The fermions (= prehaplons) are assigned to the following representations:

$$\begin{aligned}
 \begin{pmatrix} P^u \\ P^d \end{pmatrix}_L &= (84; 2; 1/3) \\
 P^u_R &= (84; 1; 4/3) \\
 P^d_R &= (84; 1; -2/3) \\
 \begin{pmatrix} P^{\nu_e} \\ P^e \end{pmatrix}_L &= (20; 2; 1) \\
 P^e_R &= (20; 1; -2)
 \end{aligned} \tag{5}$$

The above fermionic representations do not introduce $SU(6)_S$ anomalies. In order to cancel the $SU(2)_L \times U(1)$ anomalies in four dimensions we introduce a prehaplon family having the structure (5) and assigned to the 56 representation of $SU(6)_S$. The hypercharges have to be chosen such that the condition $\Sigma Q = 0$ is satisfied for all the fermions which survive after dimensional reduction. This can be easily done and we are going to see later that the fermions with the unconventional charges that survive after dimensional reduction coming from the 56 representation will be heavy and they will disappear from the low energy fermionic spectrum.

The crucial point for the rest of the discussion is the observation that the $SU(6)$ group has the following maximal subgroup structure:

$$\begin{aligned}
 SU(6) \supset & \begin{matrix} SU(2) & \times & SU(3)_{diag} \\ \cap & & \cap \end{matrix} \\
 & U(1) \times SU(3) \times SU(3)
 \end{aligned} \tag{6}$$

i.e. it has the form of Eq. (4) with $S = SU(2)$, $R = U(1)$, $H = SU(3) \times SU(3)$, $K = SU(3)_{diag}$. Thus by choosing $S/R = SU(2)/U(1)$ the $SU(6)_S$ gauge group becomes after dimensional reduction $[SU(3) \times SU(3)]_S$ while the S and the dimensional reduction act trivially on the $SU(2)_L \times U(1)$ part of the

theory. We can easily find the Higgs content of the dimensionally reduced theory. In general the gauge fields, which become scalars in four dimensions, transform under R as a vector $\underline{v}$ as specified by the embedding

$$\text{ad } S = \text{ad } R + \underline{v} \quad (7)$$

and $\underline{v} = \sum_K S_K$, where each S_K is an irreducible representation of R . Then the Higgs multiplets under H can be found by decomposing the adjoint representation of G under $R_G \times H$:

$$\text{ad } G = \sum_K (r_K, h_K) \quad (8)$$

where r_K is some irreducible representation of R_G and h_K is some irreducible representation of H . For each pair of (r_K, S_j) where r_K and S_j are identical irreducible representations, there is an h_K Higgs multiplet in the four-dimensional theory.

In the present case

$$\text{ad } SU(2) = \text{ad } U(1) + \underline{v} \quad (9)$$

and the vector $\underline{v}$ decomposes under $R = U(1)$ into $(1) + (-1)$. On the other hand

$$3 \otimes 5 = (1, 8) \left(\frac{1}{2}\right) + (8, 1) \left(-\frac{1}{2}\right) + (3, \bar{3})(1) + (\bar{3}, 3)(-1) + (1, 1)(0) \quad (10)$$

so that the Higgs fields transform under $H = SU(3) \times SU(3) \times U(1)$ as

$$(3, \bar{3}) ; (\bar{3}, 3) \quad (11)$$

Because we have the group structure (4) we do not have to work out the Higgs potential as the breaking of $SU(3) \times SU(3) \times U(1)$ to $SU(3)_{\text{diag}}$ is guaranteed by the theorem of ref. 13).

In order to identify the four-dimensional fermion multiplets after dimensional reduction, one has first to embed R into the local Lorentz group, $SO(N)$, of S/R in such a way that the N of $SO(N)$ has the branching rule $N = \underline{v}$ where $\underline{v}$ is given in Eq. (7). This determines the embedding of R into $SO(N)$ uniquely. Next one takes the spinor of $SO(N)$, σ , and decomposes

it into irreps of R; $\sigma = \sum_K \sigma_K$. Then one has to decompose the representation, F, of the gauge group G to which the fermions are assigned, under $R_G \times H$; $F = \sum_K (r_K, h_K)$. Similarly to the case for the Higgs fields, for each pair of (r_K, σ_K) where r_K and σ_K are identical irreps., there is an h_K multiplet of spinor fields in the four-dimensional theory.

If N is even, we can define Weyl spinors on $M_4 \times S/R$ and a Weyl spinor of one chirality is decomposed into four-dimensional spinors of both chiralities. The four-dimensional chirality is linked to the chirality with respect to $SO(N)$, since $\Gamma^{D+1} = \gamma^5 \times \tilde{\gamma}^{D-3}$ where $D = 4+N$. In general the symmetry constraints can act differently on spinors with positive and negative $\tilde{\gamma}^{D-3}$ chirality and this way we can end up with a left-right asymmetry in four dimensions.

In the present case the spinor of $SO(3)$ decomposes under $R = U(1)$ as $(\frac{1}{2}) + (-\frac{1}{2})$. In our example we take the following $SU(6)$ representations for the spinors: 20, 56, 84. Their decomposition under $U(1) \times SU(3) \times SU(3)$ is:

$$SU(6) \supset U(1) \times SU(3) \times SU(3)$$

$$\begin{aligned} 84 = & \begin{aligned} & (1/2) \quad (3,1) \\ & + (-1/2) \quad (1,3) \\ & + (1/2) \quad (\bar{6},1) \\ & + (-1/2) \quad (1,\bar{6}) \\ & + (1/2) \quad (3,8) \\ & + (-1/2) \quad (8,3) \\ & + (3/2) \quad (\bar{3},\bar{3}) \\ & + (-3/2) \quad (\bar{3},3) \end{aligned} \end{aligned}$$

$$\begin{aligned} 56 = & \begin{aligned} & (3/2) \quad (10,1) \\ & + (-3/2) \quad (1,10) \\ & + (1/2) \quad (3,6) \\ & + (-1/2) \quad (6,3) \end{aligned} \end{aligned}$$

$$\begin{aligned} 20 = & \begin{aligned} & (1/2) \quad (\bar{3},3) \\ & (-1/2) \quad (3,\bar{3}) \\ & (3/2) \quad (1,1) \\ & (-3/2) \quad (1,1) \end{aligned} \end{aligned}$$

Applying the previous results it is easy to identify the fermions which survive after dimensional reduction. The only surviving spinors are those

with $(\frac{1}{2})$ and $(-\frac{1}{2})$ under the $U(1)$ group corresponding to, say, left- and right-handed particles (fermionic haplons).

In general, after dimensional reduction the fermions acquire Yukawa couplings, therefore most of them obtain a mass comparable to the vector boson mass after the spontaneous symmetry breaking. In our example the surviving fermions remain massless after the spontaneous symmetry breaking. This phenomenon is rather special and it is due to the fact that coset space is S^2 and that $S \subset G^{18})$.

The $[SU(3) \times SU(3) \times U(1)]_S$ gauge group breaks further to the $SU(3)_{\text{diag}}$ as we have already stated. Then a linear combination $V_\mu^- \equiv (A_\mu - B_\mu)/\sqrt{2}$ of the vector bosons A_μ, B_μ of $SU(3) \times SU(3)$ become massive while the orthogonal combination $V_\mu^+ \equiv (A_\mu + B_\mu)/\sqrt{2}$ emerges as the colour octet of massless gauge bosons of QCD.

Under the diagonal $SU_c(3)$ the fermionic haplons become ordinary fermions in the triplet representation for quarks and in singlets for leptons. The higher representations correspond to what we call echo fermions.

We recall for convenience how some of the haplon representations are decomposed under $SU_c(3)$:

$$[SU(3) \times SU(3) \times U(1)]_S \supset SU_c(3)$$

$$\begin{aligned} (3,8) &= 3 + \bar{6} + 15 \\ (\bar{3},3) &= 1 + 8 \\ (3,6) &= 8 + 10 \end{aligned}$$

The $SU_c(3)$ interactions force the fermions of various representations to condense and the chiral symmetry breaking is expected to take place at different scales. We expect that the highest representation which appears in the left-handed doublet and in the right-handed singlet (i.e., the 15-plet) gives the leading contribution to the breaking of $SU(2)_L \times U(1)$ to $U(1)_{\text{em}}$. Thus its condensation scale should be $O(500)\text{GeV}$. In addition, the broken generators V_μ^- of $SU(3) \times SU(3)/SU_c(3)$ give radiative masses to quarks and leptons. All echo fermions of higher colour representation acquire large dynamical masses and some of them obtain current quark masses

as well. From the 56-plet of $SU(6)_S$ after spontaneous symmetry breaking the only fermions that appear are echo quarks with exotic charges which, however, acquire large dynamical masses. Ordinary quarks acquire the usual dynamical and the following current quark masses in the one- and the two-loop approximations:

$$m^1(R) = \frac{g \alpha_s C_2(R) M_R^3}{4\pi M_{V^-}^2} \ln \frac{M_{V^-}^2}{(3 C_2(R))^{2/3} M_R^2}$$

$$m^2(R) = \frac{g^2 \alpha_s^2 C_2(R) M_R^3}{4\pi^2 M_{V^-}^2} \ln \frac{M_{V^-}^2}{(3 C_2(R))^{2/3} M_R^2}$$

The leptons acquire masses in the one-loop approximation. However, there still remain some massless fermions.

We can estimate the V_μ^- masses, which are related to the Higgs masses and these latter are determined by the length scale of the extra dimensions. We assume that the massless quarks correspond to the u and the d quarks which might obtain a small current mass from another source. Then identifying those quarks which acquire radiative (current) masses with the s and the c we have to fix these masses at $O(1)$ GeV. This way we find that $\mu_6 \sim 20$ GeV, $\mu_{15} \sim 500$ GeV and $M_{V^-} \sim 30$ TeV. Therefore the radius of the S^2 sphere is roughly 10^{-17} cm.

Clearly, despite its attractive features, our approach has also some drawbacks. The most obvious one is that we did not take gravity into account¹⁹⁾. However, by including gravity, one might be able to give a natural explanation to spontaneous compactification. Namely we hope that by considering the coupled Einstein-Yang-Mills equations one can find a classical solution for the metric corresponding to a spacetime $M_4 \times S/R$.

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