A VERY LIGHT GRAVITINO IN A NO-SCALE MODEL

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ABSTRACT

We present a class of no-scale phenomenological $N = 1$ supergravity models in which the gravitino is much lighter than the weak interaction scale. In particular, we present a simple model in which the gravitino mass is fixed dynamically to be $m_{3/2} = m_W^p / m_P^{p-1}$, $1 < p < 2$. In addition, we find that the QCD vacuum parameter θ is determined to be zero, thus solving the strong CP problem. Such models avoid the cosmological problems of models with $m_{3/2} = O(m_W)$, and photino decay into $\gamma + \text{gravitino}$ poses no problem if annihilation suppresses the initial photino density below the baryon density of the Universe.

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In most phenomenological supergravity models the gravitino mass $m_{3/2}$ is of the same order as the W mass¹⁾. This is because generic choices of Kähler potential for the chiral superfields, including those with minimal kinetic terms, give scalar masses of order $m_{3/2}$ ²⁾. Since these scalars include the Weinberg-Salam Higgs boson H, and since $m_W = O(m_H)$, in general $m_{3/2} = O(m_W)$. It is, however, possible³⁾ to construct models in which the gravitino mass-scale is decoupled from that of the scalars, and hence of m_W . In particular, models⁴⁾ based on a $SU(n,1)/SU(n) \times U(1)$ Kähler manifold, where n is the number of chiral superfields, have global supersymmetry in the observable sector and hence vanishing scalar masses even if $m_{3/2} \neq 0$. Since global supersymmetry must be broken in the real world, it is necessary to introduce into the observable sector of these models some source of global supersymmetry breaking. The only possibility⁴⁾ is a non-zero gaugino mass, which is generic if the kinetic term for the gauge fields is non-minimal⁵⁾. If the gaugino mass is non-zero, then radiative corrections will generate non-zero scalar masses⁴⁾. Thus we expect the gaugino mass to be $O(m_W)$, but the gravitino mass could *a priori* be different⁶⁾.

The above remarks could be quite general, but we were led to formulate such scenarios⁶⁾ with $m_{3/2} \neq O(m_W)$ in the context of our no-scale supergravity models^{7,8,4)}. In the first such models^{7,8)}, the gravitino mass was undetermined at the tree level, but radiative corrections determined it dynamically to be $O(m_W)$. Extensions of the no-scale supersymmetric standard model to include a massive GUT sector led us⁴⁾ naturally to consider an $SU(n,1)/SU(n) \times U(1)$ Kähler potential. In this case it is possible to exploit⁶⁾ the decoupling between the gaugino and gravitino masses so that radiative corrections determine dynamically $m_{\tilde{V}} = O(m_W)$, but not $m_{3/2}$ which could be fixed by other considerations. In our first exercise⁶⁾ of this type we fixed $m_{3/2} = O(m_p)$ by hand.

Models in which $m_{3/2} \neq O(m_W)$ are of cosmological interest, since a gravitino which weighs between about 10^4 GeV and 1 keV is a cosmological embarrassment⁹⁾. In most supersymmetric models the lightest supersymmetric particle is a photino which is absolutely stable, while the gravitino is heavier and decays into $\tilde{\gamma} + \gamma$

with a lifetime which is longer than the age of the Universe during primordial nucleosynthesis. Disaster could strike⁹⁾ if the presence of massive gravitinos alters the expansion rate of the Universe during nucleosynthesis, or if they generate excessive entropy when they subsequently decay, or if the showers from photons among their decay products disrupt the previously synthesized light nuclei. Presumably gravitinos with mass $< m_p$ would have been produced copiously at the Planck epoch, and this primordial abundance should be suppressed by a subsequent inflationary epoch¹⁰⁾. Gravitinos with masses less than the reheating temperature T_R after inflation would then be regenerated, and to suppress this regenerated abundance sufficiently^{11,12)} we must demand $T_R < O(10^{10})$ GeV. In typical GUT inflationary models $T_R = O(10^{11} \text{ to } 10^{16})$ GeV, so an acceptable window for the gravitino mass is

$$m_{3/2} \geq T_R = O(10^{11} \text{ to } 10^{16}) \text{ GeV} \quad (1)$$

This was the window explored in our previous paper⁶⁾.

In this paper we explore the other acceptable window when $m_{3/2} < O(1)\text{keV}$. In this case the gravitino has a lifetime much longer than the age of the Universe, but is so light that it can retain today its primordial abundance with no need for an inflationary epoch, and its present mass density is below the closure density¹³⁾. We demonstrate how such a very light gravitino can be reconciled with a much larger scale of global supersymmetry breaking: $m_{\tilde{V}} = O(m_W)$ using non-minimal kinetic terms for the gauge superfields. While the mechanism may be more general, our examples are in the context of no-scale models⁴⁾ with an $SU(n,1)/SU(n) \times U(1)$ Kähler potential. In the simplest example of this type we find that the dynamical determination of the gaugino mass by radiative corrections at the same time fixes the gravitino mass:

$$m_{3/2} = O(m_{\tilde{V}}^P / m_P^{P-1}) \ll m_{\tilde{V}} \quad (2)$$

though other possibilities exist. As an added bonus we find that the QCD θ parameter can be determined dynamically to be zero by non-perturbative QCD effects.

In this class of models the photino becomes unstable against decay into a photon and a gravitino. Its lifetime is too long to be relevant to particle physics experiments, but one might worry that it suffers the same cosmological embarrassments as the unstable gravitino in conventional scenarios. This is not the case, because the electroweak interactions of the photino provide it with a much larger annihilation rate than that enjoyed by the gravitino, and the primordial abundance of photinos is automatically suppressed^{14,15}). Suppression due to annihilation, by the same amount that would have made the present mass density of a *stable* photino astrophysically acceptable, would guarantee that the mass density and the decay products of *unstable* photinos cause no astrophysical problems. Finally, we note that this new class of phenomenological supergravity models with a very light gravitino (2) may have advantages over models with a very heavy gravitino (1) if one tries to compute radiative corrections.

Phenomenological supergravity models permit⁵⁾ a non-minimal kinetic term for the gauge superfields of the form

$$\int d^2\theta h_{ab}(z^i, y^j) V_a V_b \quad (3)$$

where h_{ab} is an analytic function of the chiral superfields z^i and y^j . We denote by y^j the observable chiral superfields (quarks, leptons, Higgses, ...) and by z^i hidden chiral superfields mainly responsible for determining the local supersymmetry-breaking scale. We assume for simplicity the simple symmetric form

$$h_{ab}(z^i, y^j) = \delta_{ab} h(z^i, y^j) = \delta_{ab} h(z) \quad (4)$$

for the chiral function, in which case the universal gaugino mass given by the non-minimal kinetic term (3) is

$$m_{\tilde{V}} = \left| e^{G/2} \frac{G_{,z} (\ln \text{Re} h)_{,z^*}^*}{G_{,zz^*}} \right| \quad (5)$$

In Eq. (5) we have simplified to the case of a single hidden superfield z , introduced the Kähler potential $G(z, y^j)$ and its derivatives $G_{,z} \equiv \partial G / \partial z$, $G_{,zz^*} = \partial^2 G / \partial z \partial z^*$,

as well as $(\ln \text{Re } h), z \equiv \partial/\partial z (\ln \text{Re } h) = h, z/\text{Re } h$. Equation (5) clearly contains a factor of the gravitino mass

$$m_{3/2} = e^{G/2} \quad (6)$$

(hereafter we use natural units: $M = m_p/\sqrt{8\pi} \equiv 1$), which tells that the scales of local ($m_{3/2}$) and of global ($m_{\tilde{V}}$) supersymmetry breaking are proportional. However, there is a coefficient of proportionality $|(G_z/G_{zz^*})(\ln \text{Re } h)_z|$ in Eq. (5) which remains to be specified, and could *a priori* be very different from $O(1)$.

The observable sector contains other global supersymmetry-breaking parameters besides $m_{\tilde{V}}$ (5), notably the scalar masses m_0 . Generic Kähler potentials have $m_0 = O(e^{G/2}) = O(m_{3/2})$ [Eq. (6)], thereby frustrating our goal of a hierarchy between $m_{3/2}$ and the effective global supersymmetry-breaking scale. If we seek $m_{3/2} \gg O(M_W)$ as we did previously, we are forced to choose a Kähler potential which guarantees $m_0 \ll m_{3/2}$ at the tree level, such as the $SU(n,1)/SU(n) \times U(1)$ forms⁴⁾ mentioned previously. In our case, where we seek $m_{3/2} \ll O(m_W)$, it is not essential to choose this form of Kähler potential. It is, however, a desirable ingredient in our no-scale programme for the dynamical determination of $m_{\tilde{V}} = O(m_W)$, so we concentrate on $SU(n,1)/SU(n) \times U(1)$ models as a context for a very light gravitino.

We therefore take

$$G = -3 \ln(z + z^* + \phi(y_j^*, y^j)) \quad (7)$$

which gives

$$G_{12} = -\frac{3}{(z + z^* + \phi)} \quad ; \quad G_{1zz^*} = \frac{3}{(z + z^* + \phi)^2} \quad (8)$$

so that

$$G_{12}/G_{1zz^*} = -(z + z^* + \phi) = -e^{-G/3} \quad (9)$$

Substituting into the expression (5) we find

$$m_{\tilde{V}} = |e^{G/6} (\ln \text{Re } h)_{z^*}^*| \quad (10)$$

which we can rewrite using Eq. (6) as

$$m_{\tilde{V}} = m_{\frac{3}{2}}^{\frac{1}{3}} |(\ln \operatorname{Re} h)_{,z}^*| \quad (11)$$

A priori, the function $(\ln \operatorname{Re} h)_{,z}$ could be arbitrarily complicated, and there need⁶⁾ be no relation between $m_{\frac{3}{2}}$ which is related to $\operatorname{Re} z$, and $m_{\tilde{V}}$ which could depend on both $\operatorname{Re} z$ and $\operatorname{Im} z$. Let us, however, examine the case

$$h(z) = e^{iA z^q} + ic \Rightarrow (\ln \operatorname{Re} h)_{,z} = iqA z^{q-1} [1 + i \tan A \operatorname{Re} z^q] \quad (12)$$

where c and A are real constants. In this case one finds that

$$m_{\tilde{V}} \geq Aq | \operatorname{Re} z^{q-1} - \operatorname{Im} z^{q-1} \tan A \operatorname{Re} z^q | m_{\frac{3}{2}}^{\frac{1}{3}} \quad (13)$$

For $\operatorname{Re} z \gg \operatorname{Im} z$, this results to

$$m_{\frac{3}{2}} \simeq \mathcal{O}(m_{\tilde{V}}^p), \quad p = 1/(1 - \frac{2}{3}q) \quad (14)$$

So far we have not specified $m_{\tilde{V}}$. This can be determined dynamically⁶⁾ by radiative corrections to be $\mathcal{O}(m_W)$, the hidden gravitino sector playing no role. However, once $m_{\tilde{V}}$ is determined to be $\mathcal{O}(m_W)$, we then see that for $p > 1$ $m_{\frac{3}{2}}$ (13) is simultaneously determined dynamically to be of the very small magnitude (2). In order not to have too singular couplings to ordinary matter of the type discussed by Fayet¹⁶⁾, we should take $1 < p < 2$. We do not discuss in detail the phenomenological sparticle spectrum in this class of models as it is similar to those in Refs. 4 and 6.

It is remarkable that the simple model (12) also allows us to solve completely the strong CP problem. The non-minimal kinetic terms (4) will also give rise to a CP-violating coupling of the form⁵⁾

$$\frac{i}{4} \theta F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a \quad (15)$$

However, from expression (12) we find that

$$\theta = \frac{\text{Im } h(z)}{\text{Re } h(z)} = \tan A \text{Re } z^q + \frac{c e^{A \text{Im } z^q}}{\cos A \text{Re } z^q} \quad (16)$$

The dynamical determination of $m_{\tilde{\gamma}}$ determines only some combination of $\text{Re } z$ and $\text{Im } z$. The remaining dynamical degree of freedom is buried in $\text{Im } h(z)$. In fact, we could view $m_{\tilde{\gamma}}$ and $\text{Im } h(z)$ as the true dynamical variables of the theory, rather than $\text{Re } z$ and $\text{Im } z$. One can also check that apart from the supersymmetric partner of the coupling (15), the $N = 1$ supergravity Lagrangian does not contain other renormalizable terms depending on $\text{Im } h$. Therefore $\text{Im } h$, and hence θ , can be determined dynamically independently of the values of $m_{3/2}$ and $m_{\tilde{\gamma}}$. Indeed, the only non-trivial dynamical dependence on θ comes from non-perturbative QCD effects which favour $\theta = 0$. The dynamical freedom accorded to us by the $\text{Im } h$ field enables us to "relax" to $\theta = 0$, thus solving the strong CP problem.

A generic feature of models with a very light gravitino is that now $\tilde{\gamma} \rightarrow \gamma + \text{gravitino}$ decay becomes possible, with a lifetime similar to that of the gravitino $\rightarrow \tilde{\gamma} + \gamma$ in models where $m_{3/2} > m_{\tilde{\gamma}}$. For $m_{\tilde{\gamma}} = 0(100)$ GeV as expected, the most severe constraint on its decays into photons comes from the requirement^{17,12)} that it should not disrupt light elements fabricated during primordial nucleosynthesis:

$$\frac{\beta f m_{\tilde{\gamma}} n_{\tilde{\gamma}}}{E_* n_e} \lesssim 1 \quad (17)$$

where $\beta = 0(0.2)$, the fraction f of the $\tilde{\gamma}$ rest energy carried by the decay photon is $\frac{1}{2}$, $E_* \approx 100$ MeV, and n_e is the density of electrons. The condition (17) can be rewritten as

$$m_{\tilde{\gamma}} n_{\tilde{\gamma}} \lesssim n_e \times 1 \text{ GeV} \approx \rho_B \quad (18)$$

where ρ_B is the mass density in baryons. Therefore, if $\tilde{\gamma}\tilde{\gamma}$ annihilation in the early Universe at temperatures above $0(100)$ MeV is sufficiently efficient to suppress the density of *stable* photinos so far that they would not outweigh baryons in the Universe today, the same suppression is sufficient to avoid the disruption of light elements by *unstable* photinos. Other constraints¹²⁾ on the abundance of unstable photinos are less severe than (17) and are hence obeyed a

fortiori if the mass-density constraint (18) is respected. It is indeed possible^{14,15)} to construct supersymmetric models in which the condition (18) is satisfied.

Finally, we draw attention to one possible advantage of models with a very light gravitino, by comparison with models with $m_{3/2} \gg O(m_W)$. It is well known¹⁸⁾ that there are radiative corrections to the vacuum energy which are proportional to

$$\text{Str } \mathcal{M}^2 = \sum_{\text{spin } J} (-1)^J (2J+1) m_J^2 \quad (19)$$

and that there are radiative corrections to scalar masses which are $O(m_B^2 - m_F^2)$, where B and F stand for the bosons and fermions in a supermultiplet. No-scale models require a flat hidden sector potential, and the corresponding hidden sector spin-zero fields have masses $O(m_W^2/m_P)$. If $m_{3/2} = O(m_P)$, or even if $m_{3/2} = O(m_W)$, radiative corrections in the hidden sector of the form mentioned above (19) look dangerous, and could give large masses to the hidden sector scalars which destroy the flatness needed in no-scale scenarios. Moreover, hidden sector radiative corrections $\delta V = \text{Str } \mathcal{M}^2 \times O(m_P^2)$ are now of order $O(m_W^4)$, which is no larger than the non-gravitational radiative corrections. $N = 1$ supergravity models are to be understood as *effective* theories remaining after all heavy degrees of freedom, gravitational radiative corrections, and hidden sector couplings are evaluated and integrated out. The no-scale hypothesis is that the hidden sector is *effectively* flat: within our philosophy it would be double counting to recalculate loops involving the hidden sector. Nevertheless, if $m_{3/2}$ is as small as Eq. (2), the gravitino and the hidden sector will make a very small contribution to $\text{Str } \mathcal{M}^2$ [Eq. (19)], and loop corrections to scalar masses and to the vacuum energy are correspondingly small. Some readers may find this a reassuring advantage over models with $m_{3/2} \gtrsim O(m_W)$.

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