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ASPECTS OF CP-VIOLATION IN SUPERSYMMETRIC THEORIES

J.-M. Gérard^{*)}, W. Grimus⁺⁾, A. Masiero,^{*)}
D.V. Nanopoulos and Amitava Raychaudhuri^{*)}

CERN - Geneva

A B S T R A C T

We give a comprehensive study of CP violating effects in low energy softly broken supersymmetric theories. We find that gluino exchange contributions to CP-violation arising from the Kobayashi-Maskawa mixing matrix may provide indirect evidence of supersymmetry at low energy. Indeed, we point out two important departures from the predictions of the standard model: i) in the Kaon system, the experimental value of the ε parameter cannot be reconciled with a large B lifetime ($\tau_B > 10^{-12}$ s) and light top quark ($m_t \lesssim 50$ GeV) in the context of the usual three-generation standard model; this is no longer true when the additional supersymmetry contributions to CP-violation are included; ii) in the Beon system, the presence of supersymmetry enhances $B^0-\bar{B}^0$ mixing as well as off-shell and on-shell CP violating phenomena making more likely an experimental detection of some of the effects. On the other hand, the electric dipole moment of the neutron due to the Kobayashi-Maskawa phase is still very small and ε'/ε continues to be consistent with the experimental bounds.

^{*)} IISN Fellow, on leave of absence from Institut de Physique Théorique, Université de Louvain, Louvain-la-Neuve, Belgium; address after September 1984: Max-Planck Institut für Physik und Astrophysik, Munich, Germany.

⁺⁾ Address from July 1, 1984: Institut für theoretische Physik, Boltzmanngasse 5, Vienna, Austria.

^{*)} On leave (1983-84) from the Department of Pure Physics, Calcutta University, Calcutta, India.

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1. INTRODUCTION

So far supersymmetry [1] (SUSY) gauge theories [2] present appealing properties only from the theoretical side: they soften divergences [3], yielding a possible solution to the gauge hierarchy problem [4], they lead to a unification with gravity [5] and it has been suggested that the most extended version ($N = 8$ supergravity) could represent the eagerly searched for fundamental theory of Nature [6]. This enthusiasm contrasts with the absolute lack of experimental evidence in favour of supersymmetry as yet. There are two ways to put SUSY under phenomenological scrutiny: directly, by seeking the SUSY partners [7] of the ordinary particles which should lie not far from the M_W region if we want SUSY to help in solving the hierarchy problem, and indirectly, by "virtual" effects in the low energy phenomenology which cannot be accounted for by the standard model.

Until now, low-energy SUSY gauge theories [8] have been found to satisfactorily encompass all the experimental constraints [9] which are nicely incorporated in the standard model [10]. Now we have to dare more: we must investigate phenomena where significant departures from the standard model predictions take place. A very sensitive test for any extension of the standard model is its prediction for the rare processes, i.e. flavour-changing neutral interactions whose real and imaginary parts are tested to high accuracy. As for the real parts, careful analyses in the context of SUSY theory have already been performed and SUSY has emerged unscathed [9]. Here, we focus our attention on the imaginary parts, i.e. on CP-violating effects, and we shall try to present the full picture of these phenomena in the framework of SUSY models.

We shall start with a general discussion of the sources of CP-violation in SUSY theories. After discarding the uninteresting ones, we shall concentrate on the main source of CP-violation for the Kaon and Beon systems in these models: the standard CP-violating phase δ of the Kobayashi-Maskawa (KM) [11] mixing matrix K which can now make itself felt, thanks to the strongly coupled but nevertheless flavour-violating fermion-sfermion-gluino gauge vertices [12]. We shall present a study of the relevant CP-violating parameters in the $K^0-\bar{K}^0$ system, ε and ε'/ε ,

and much emphasis will be placed on the connection in SUSY theories between ε , the top mass and the bottom lifetime [13,14]. We also discuss the important issue of the dipole electric moment of the neutron (DEMON)* in the SUSY context. Finally, we shall extend our analysis to the $B^0-\bar{B}^0$ system.

2. SOURCES OF CP-VIOLATION IN THE KAON SYSTEM

To begin with, we consider the box diagrams which are responsible for the $K^0-\bar{K}^0$ transition. In SUSY theories, the different sources of CP-violation can be clearly characterized by the four fundamental classes of diagrams which are depicted in fig. 1a-e. In the non-SUSY case, the internal bosonic lines can be a) the W_L boson [15] or b) some charged Higgs exchange [16]. CP-violation is due to the KM phase which appears in the gauge vertices and possibly on additional phases which are present in multi-Higgs models.

The new SUSY sources of CP-violation for the $K^0-\bar{K}^0$ system are individuated in the three classes c), d) and e) of fig. 1: c) this diagram represents the mere supersymmetrization [17] of the "standard box": the internal quarks are replaced by their scalar partners, the squarks $(\tilde{q}_{L,R})$, whilst the W bosons and the Higgs scalars H are substituted by their fermionic partners, the wino $\tilde{W}$ and the Higgsino $\tilde{H}$. d) Additional complex parameters appearing in the soft-breaking sector [8] of the SUSY Lagrangian, namely gaugino masses and $\tilde{q}_L-\tilde{q}_R$ mixing, lead to a fourth class of diagrams. The dominant contribution comes from gluino ($\tilde{g}$) exchange. e) The main new effect due only to the KM phase [13,14], which appears at the gauge vertices, is induced by gluino exchange between left-handed quark fields.

We now start the CP-selection on the above four classes of candidates. The charged Higgs exchange in b) can give a sufficiently large ε but provides an exceedingly high value [18] for ε'/ε (~ 0.05) ruled out by the recent data [19]:

*) Several of the authors of this paper add a dissenting note to the use of this voodooistic acronym.

$$\epsilon'/\epsilon = -0.0046 \pm 0.0053 \pm 0.0024 \quad (1)$$

Moreover, we should also add that the prediction for the DEMON is dangerously close [20] to the present upper bound [21]

$$(d_n)_{exp.} < 4 \cdot 10^{-25} \text{ e.cm} \quad (2)$$

An analogous quick demise is reserved for the "naïve" supersymmetrization c): the $\tilde{W}$ - $\tilde{H}$ complex mixing alone cannot provide a big enough [17] ϵ and again one is facing catastrophic consequences [22] for ϵ'/ϵ and d_n .

So we are left with the standard CP-violation due to W-exchange in the class a) and to the SUSY contributions in d) and e). One can now wonder whether it is possible to get enough CP-violation in the SUSY case when the KM phase is switched off. In other words, can we get ϵ from diagram d) only? This hope of fully replacing the good old KM phase by the new SUSY phase ϕ (associated with the above quoted soft-breaking terms) is illusory: even taking the generous upper bound of $10^{-24} \text{ e} \cdot \text{cm}$ on d_n , this phase cannot be much greater [23] than 10^{-2} for reasonable values of squark and gluino masses. This implies a too small ϵ parameter [13].

Since our hope for a SUSY CP-violation hinges again on the KM phase [13], it is essential to understand how it sneaks into diagram e). This is discussed in the next section.

3. FLAVOUR MIXING THROUGH THE SQUARK MASS MATRIX AND GLUINO EXCHANGE

In the supersymmetric limit squarks possess the same mass as the corresponding quarks. The spontaneous breaking of supergravity at high energy induces, in principle, soft-breaking terms of the residual global SUSY in the low-energy sector [8]. Two of these soft terms are of interest for our analysis. One is the common, flavour diagonal, mass μ for all the scalar partners of the left-handed and right-handed quarks, $\tilde{q}_L$ and $\tilde{q}_R$, respectively. The second one realizes a "left-right" mixing between $\tilde{q}_L$ and $\tilde{q}_R$.

For the Kaon and Beon systems in which we are interested, only the down squark mass matrix is of relevance. At the tree level it is given by

$$M_{\tilde{d}}^{2(0)} = \begin{pmatrix} \mu^2 \mathbf{1} + M_d M_d^\dagger & A^* \mu M_d \\ A \mu M_d^\dagger & \mu^2 \mathbf{1} + M_d^\dagger M_d \end{pmatrix} \quad (3)$$

where M_d denotes the down quark mass matrix, A is a typical complex soft-breaking parameter which depends on the details of the hidden sector, μ sets the scale for the breaking of global SUSY and should be $O(M_{\tilde{g}})$ (this scale is usually identified with the gravitino mass $m_{3/2}$ or the gaugino mass).

At low energy, one must take into account important quantum corrections; the remarkable consequence for flavour mixing and CP-violation is the fact that a term involving the up quark mass matrix M_u is now introduced into the left-handed sector of the down squark mass matrix [12]:

$$M_{\tilde{d}}^2 \simeq \begin{pmatrix} \mu_L^2 \mathbf{1} + M_d M_d^\dagger + c M_u M_u^\dagger & A^* \mu M_d \\ A \mu M_d^\dagger & \mu_R^2 \mathbf{1} + M_d^\dagger M_d \end{pmatrix} \quad (4)$$

From (4) it is apparent that we can no longer diagonalize M_d and $M_d^\dagger$ by the same transformations, and in this way we can expect the manifestation of the KM CP-violating phase in the $\tilde{d}$ - $\tilde{d}^*$ gauge vertices.

Armed with this qualitative understanding of the situation we can now turn to a more quantitative discussion. First of all we proceed to an estimation of the parameter c for models based on local supersymmetry. In general, this parameter satisfies a rather complicated renormalization group equation (RGE). However, in the minimal SUSY extension of the standard model [24-26] with only two Higgs doublets H_1 and H_2 coupled to the right-handed down and up quarks

respectively, it can be rather easily expressed in terms of mass parameters appearing in the low-energy scalar potential:

$$V_{\text{tree}}(H_1, H_2) = m_1^2 |H_1|^2 + m_2 |H_2|^2 - m_3^2 (H_1 H_2 + \text{h.c.}) + D\text{-terms} \quad (5)$$

The flavour-violating piece appearing in the RGE for the $\tilde{d}_L$ squark masses arises only from the Yukawa sector through charged Higgs exchanges and is dominated by the t quark Yukawa coupling. In this limit, this piece can be easily related to the RGE for the difference $(m_2^2 - m_1^2)$ where the gauge contributions cancel out:

$$\frac{d}{dt}(m_t^2) \simeq \frac{1}{3} \frac{d}{dt}(m_2^2 - m_1^2), \quad t \equiv \ln \frac{M_X^2}{\mu^2} \quad (6)$$

In this equation, the factor 3 is due to the additional colour degrees of freedom in loops and M_X is the high scale defining the boundary conditions $m_1^2(M_X) = m_2^2(M_X)$ and $c(M_X) = 0$. After integration we therefore obtain

$$c(M_w) \simeq \frac{1}{3} \frac{m_2^2 - m_1^2}{m_t^2} \Big|_{M_w} M_w \quad (7)$$

This simple expression allows us to discuss general features of the parameter c .

Since $m_t \gg m_b$ implies a larger, negative, quantum correction to m_2^2 , $m_2^2(M_w) < m_1^2(M_w)$ and we conclude from eq. (7):

$$c(M_w) < 0 \quad (8)$$

This observation provides us with an upper bound for the top quark mass in order that the gauge group $SU(3)_c \times U(1)_{\text{em}}$ be unbroken after quantum corrections. Neglecting for the moment the flavour mixing angles as well as the $\tilde{d}_L$ - $\tilde{d}_R$ mixing, we obtain the following $\tilde{b}_L$ squark mass squared:

$$M_{\tilde{b}_L}^2 \simeq \mu_L^2 - |c| m_t^2 \quad (9)$$

which implies

$$m_t \lesssim \mu_L / \sqrt{|c|} \quad (10)$$

to avoid a non-vanishing vacuum expectation value for the scalar $\tilde{b}_L$.

Secondly, eq. (7) provides us with an estimation of c . Let us briefly review the possible scenario for $SU(2)_L \times U(1)$ breaking through radiative corrections in the minimal model. A straightforward minimization of the potential (5) leads to the following constraints at the M_w scale:

$$\begin{aligned} m_1^2 + m_2^2 &= \frac{\omega^2 + 1}{\omega} m_3^2 \\ m_1^2 - m_2^2 &= \frac{\omega^2 - 1}{\omega^2 + 1} \left(\frac{\omega^2 + 1}{\omega} m_3^2 + M_{\frac{1}{2}}^2 \right) \end{aligned} \quad (11)$$

$$\omega \equiv \langle H_2 \rangle_0 / \langle H_1 \rangle_0 \gg 1$$

where $M_{\frac{1}{2}}$ is the mass of the weak neutral gauge boson.

If the RGE's for mass parameters are controlled by the Yukawa sector [24,25] (in this scenario, $m_t \gtrsim 55$ GeV), m_2^2 gets large, negative, quantum corrections such that an $SU(2) \times U(1)$ -breaking can be induced at the scale where

$$\begin{aligned} m_1^2 + m_2^2 &= 0, \quad m_3^2 = 0 \\ m_1^2 - m_2^2 &\simeq M_{\frac{1}{2}}^2 \quad (\omega \gg 1). \end{aligned} \quad (12)$$

In this case, we obtain

$$c(M_w) \simeq -\frac{1}{3} \frac{M_{\frac{1}{2}}^2}{m_t^2} \quad (13)$$

which supports the assumption of $c \sim 0(1)$.

On the other hand, if RGE's are dominated by the gauge sector [25,26], m_1^2 and m_2^2 have a similar, relatively flat evolution between M_X and M_w :

$$\begin{aligned} m_1^2 + m_2^2 &\simeq 2 m_{3/2}^2 \\ \omega &= 1 + \sigma, \quad \sigma \lesssim 0.1 \end{aligned} \quad (14)$$

where $m_{3/2}$ stands for the gravitino mass in this model, and $SU(2)_L \times U(1)$ can be broken for arbitrary small value of m_t . Since this case corresponds to a light top quark, c can still be of the order of unity, now depending essentially on the gaugino mass:

$$c(M_W) \simeq -\frac{d}{3} \frac{2m_3^2 + M_2^2}{m_t^2} \gtrsim -0.1 \frac{M_2^2}{m_t^2} \quad (15)$$

Our discussion above has been somewhat simplified. A more elaborate estimate in the minimal supergravity model with a light top quark can be obtained from the results of ref. 26

$$c \simeq -\left(\frac{m_{3/2}}{M_W}\right)^2 \left\{ 0.99\xi^2 + 0.30A\xi + 0.046(3+A^2) \right\}, \quad \xi \equiv M/m_{3/2} \quad (16)$$

where A is the soft-breaking term responsible for the $\tilde{d}_L - \tilde{d}_R$ mixing in (3) and M is the gaugino mass, both at M_X . In this framework, the gluino mass ($m_{\tilde{g}}$) and the squark mass parameters μ_L and μ_R are

$$\begin{aligned} m_{\tilde{g}} &\simeq \frac{\alpha_3(M_W)}{\alpha_{GUT}} M \simeq 2.53 M \\ \mu_L^2 &\simeq m_{3/2}^2 + 5.3 M^2 \\ \mu_R^2 &\simeq m_{3/2}^2 + 4.8 M^2 \end{aligned} \quad (17)$$

For illustration, if we take $A = 3$, $m_{3/2} = 15$ GeV and $\xi = 2.2$ (3.1), we obtain

$$c \simeq -0.26 \quad (-0.45)$$

with

$$\begin{aligned} m_t &\simeq 25 \text{ (35) GeV} \\ m_{\tilde{g}} &\simeq 84 \text{ (118) GeV} \\ \tilde{M} &\simeq 75 \text{ (102) GeV} \end{aligned} \quad (18)$$

where $\tilde{M}$ is an average squark mass. From eqs. (16) and (17) one can conclude, using stability arguments [26] ($\xi \gtrsim 2$), that a large value of c always requires rather heavy squarks and gluinos, a result already reflected in eq. (18). However, our subsequent numerical calculations show that such a situation gives rise to very small SUSY effects for CP-violation. Therefore, this minimal supergravity model cannot be tested through CP-violating processes. For the rest of our analysis we consider the SUSY parameters as phenomenological inputs and do not restrict ourselves to this model.

Let us now turn to the implications of such a large flavour non-diagonal piece in the down squark mass matrix. In the physical basis of matter fields, it induces an important flavour-violating strong gauge coupling:

$$\begin{aligned} \lambda_{\tilde{g} d d} &= i\sqrt{2} g_s \tilde{d}_j^\dagger \tilde{g}_a \frac{\lambda^a}{2} \{ \Gamma_L P_L + \Gamma_R P_R \}^{ij} d_j \\ P_L &= \frac{1-\gamma_5}{2}, \quad P_R = \frac{1+\gamma_5}{2} \end{aligned} \quad (19)$$

g_s is the strong coupling constant, λ^a ($a = 1, \dots, 8$) are the Gell-Mann matrices and i, j are flavour indices. The coupling matrices Γ_L, Γ_R are related to the squark mass matrix (4). In the basis in which the down quark mass matrix is diagonal ($\hat{M}_d$), (4) becomes

$$\tilde{M}_d^{-2} \simeq \begin{pmatrix} \mu_L^2 1 + \hat{M}_d^2 + c K^\dagger \hat{M}_u^2 K & |A| \mu \hat{M}_d \\ |A| \mu \hat{M}_d & \mu_R^2 1 + \hat{M}_d^2 \end{pmatrix}. \quad (20)$$

In this expression, a possible phase in A [$A = |A| \exp(-2i\phi_A)$] has been rotated away by a phase transformation. In the interaction, eq. (19), ϕ_A always appears in the combination $\phi = \phi_g - \phi_A$ where ϕ_g is a possible phase in the gluino mass [$m_{\tilde{g}} = |m_{\tilde{g}}| \exp(-2i\phi_g)$]. We can now write down the coupling matrices Γ_L, Γ_R as [13]

$$(\Gamma_L, \Gamma_R) = U^\dagger \begin{pmatrix} e^{i\phi} 1 & 0 \\ 0 & e^{-i\phi} 1 \end{pmatrix}, \quad U^\dagger \tilde{M}_d^{-2} U = \text{diag}(\tilde{M}_j^2). \quad (21)$$

It should be noted that the two 6×3 matrices Γ_L, Γ_R together form a 6×6 unitary matrix.

Using eq. (19), one can now calculate the box diagram (fig. 1e) contribution $M_{ab}(\tilde{g})$ to the $M^0-\bar{M}^0$ mixing where M^0 is a neutral meson of the form $d_a\bar{d}_b$. Here we are interested in the $K^0(\bar{s}d)$, $B_d^0(b\bar{d})$ and $B_s^0(b\bar{s})$. The complete expression for $M_{ab}(\tilde{g})$ can be found in ref. 13. We present below the simplified form that is obtained when $\tilde{d}_L-\tilde{d}_R$ mixing is neglected. (However, in all our numerical calculations we use the complete expression). In this approximation

$$\Gamma_L = \begin{pmatrix} K \\ 0 \end{pmatrix}, \quad \Gamma_R = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (22)$$

and

$$M_{ab}(\tilde{g}) = \langle M^0 | H_{eff} | \bar{M}^0 \rangle = \frac{\alpha_s^2 f_M^2 m_B}{108 m_{\tilde{g}}^2} \sum_{j,m} (\delta I_{jm} + \eta K_{jm}) \lambda_{(ab)}^j \lambda_{(ab)}^m$$

$$\lambda_{(ab)}^j = K j a^* K j b$$

$$I_{jm} = \frac{1}{z_j - z_m} \left[\frac{z_j \ln z_j}{(1-z_j)^2} + \frac{1}{1-z_j} - (z_j \rightarrow z_m) \right] \quad (23)$$

$$K_{jm} = \frac{1}{z_j - z_m} \left[\frac{z_j^2 \ln z_j}{(1-z_j)^2} + \frac{1}{1-z_j} - (z_j \rightarrow z_m) \right], \quad z_j = \frac{M_j^2}{m_{\tilde{g}}^2}$$

In (23), B is the usual correction factor to the vacuum insertion estimate [27] of the hadronic matrix element, $\alpha_s = g_s^2/4\pi$, m_M is the mass of M^0 and f_M is the analogue of the pion decay constant of the M^0 meson. We use the convention $CP|M^0\rangle = -|\bar{M}^0\rangle$.

4. THE KAON SYSTEM, THE TOP QUARK MASS AND THE DEMON

In view of the unexpectedly long B-meson lifetime [28], there has recently been a renewed interest in the CP-violating parameters ε and ε'/ε of the $K^0-\bar{K}^0$ system. These results significantly lower the upper bounds on the θ_2 and θ_3 angles of the KM mixing matrix [29]. In the context of the standard $SU(2)_L \times U(1)$ model, the experimental value of ε therefore requires a rather heavy top quark to compensate for the smallness of these angles [30,31]. Indeed, if one chooses the presently

popular value for the B parameter [32], $B = 1/3$, one finds that a B-meson lifetime greater than 10^{-12} s requires the top quark to be heavier than 50 GeV in the three generation case [31,33], provided one neglects long-distance contributions [34] to the ε parameter which in itself stands not on very firm grounds [35].

We have shown before that attractive candidates to overcome this possible impasse are SUSY theories. Indeed, from fig. 2 which is an update of fig. 2 of ref. 14, it can be appreciated how dramatically things change in this latter class of models; e.g. one can easily accommodate a B lifetime $(1-2) \cdot 10^{-12}$ s with a top quark mass less than 40 GeV. The most recent data [36] on $\bar{R} \equiv \Gamma(b \rightarrow u e^+ \bar{\nu}) / \Gamma(b \rightarrow c e^+ \bar{\nu})$, $\bar{R}^{\text{exp}} < 0.04$ endangers the ordinary model even more than before ($\bar{R} \lesssim 0.05$) and makes the SUSY models more appealing. For the plot of fig. 2 we have taken $\bar{R} = 0.03$, $\alpha_s = 0.1$, $c = -1$, $m_{\tilde{g}} = 40$ GeV, $\mu_L = 50$ GeV, $\mu = 40$ GeV, $\mu_L^2 - \mu^2 = 100 \text{ GeV}^2$, $|A| = 3$ and $\phi = 0$. For our choice of c and μ_L we can only consider a top quark mass less than about 45 GeV for the reason discussed in eq. (10). As stated already, to draw this figure, we also took into account the $\tilde{d}_L-\tilde{d}_R$ mixing in the squark mass matrix whose contributions are, however, smaller than 1% of the pure left-handed one given by eq. (23).

Until now, we have neglected the additional contributions to ε coming from penguin diagrams [15,37]. This approximation, justified in the framework of the standard model, is still valid when we take into account SUSY penguins, as will be clear from our numerical results presented below.

We will now discuss the SUSY penguin contribution to ε'/ε , but let us begin with a short discussion of our procedure. For any choice of the bottom lifetime, τ_B , the KM angle θ_3 is fixed and the angles θ_2 and δ are correlated. For a fixed top quark mass, for every value of τ_B , the experimental value of ε selects two solutions for δ , both with $\sin \delta$ positive, a feature shared by the standard model [38]. As an example, we plot in fig. 3 $\cos \delta$ as a function of τ_B for a top quark mass of 40 GeV. The SUSY input parameters are those which went into fig. 2. The maximum allowed values of τ_B in fig. 3 in both the SUSY and ordinary case can actually be read off from the curves of fig. 2; they simply correspond to the τ_B at which the lower bound on m_t becomes 40 GeV, the value we have chosen.

The details of the superpenguin calculation will be presented elsewhere [39]. Here we restrict ourselves to two plots of ϵ'/ϵ as a function of t_B for $m_t = 30$ and 40 GeV (fig. 4). For comparison, we have also exhibited the non-SUSY curves. We should mention that in this calculation we have assumed that a fraction 1/6 of the total amplitude of $K^0 \rightarrow \pi\pi$ ($I = 0$) is attributed to penguins [38]. From fig. 4 it is evident that the SUSY contribution to ϵ'/ϵ is significant and reduces its magnitude [39]. Finally, since including SUSY makes ϵ'/ϵ even smaller, the neglect of superpenguin contributions to ϵ is justified.

We now turn to the SUSY DEMON [40] d_n . We have already emphasized in the first part of this paper that there are two main new sources of CP-violation in the SUSY case: one is related to the appearance of the KM phase δ in the $\tilde{g}$ - $\tilde{q}$ - $\tilde{q}$ vertices [13], while the other one is due to the presence of the SUSY phase [23] $\phi = \phi_g - \phi_A$. First of all, we consider the contribution to d_n coming from this latter class of "purely" SUSY effects. The gluino contribution to the electric dipole moment of the down quark has been calculated [23,13] to be

$$d_d(\tilde{g}) = -\frac{2e}{q\pi} \frac{\alpha_s}{m_{\tilde{g}}} \sum_j \text{Im} \left(\frac{\Gamma_L^{jd} \Gamma_R^{jd*}}{m_{\tilde{g}}} \right) \frac{1}{z_j} D(z_j) \quad (24)$$

$$D(z) = \frac{1}{2(1-z)^2} \left(1+z + \frac{2z \ln z}{1-z} \right).$$

Then we can use the generous upper bound $|d_d(\tilde{g})| \leq 10^{-24} e \cdot \text{cm}$ to establish an upper bound on ϕ :

$$|\sin(2\phi)| \lesssim 10^{-2} \quad (25)$$

In fact, as we said in the introduction, it is just this bound together with the smallness of the $\tilde{d}_L$ - $\tilde{d}_R$ mixing which prevents ϕ from being sufficiently large to provide the entire source for CP-violation in the K^0 - $\bar{K}^0$ system, when δ is switched off. The next question is obviously whether we can get a large d_n only from δ , in the absence of the phase ϕ . Although the usual $\sin \theta_2 \sin \theta_3 \sin \delta$ factor is close to the upper bound on ϕ (eq. 25) and therefore one could expect a rather large DEMON from a naïve substitution [13] in eq. (24), it can be proved that the one

loop SUSY contribution coming from gluino exchange with $\phi = 0$ vanishes when the squark mass matrix is given by eq. (20). The argument goes as follows: using eq. (21) with $\phi = 0$ and expanding $f(z) \equiv D(z)/z$ in $(z, -z)$, $d_d(\tilde{g})$ is proportional to

$$\begin{aligned} & \text{Im} (U_{j1}^* U_{j4}^T) \left[f(z_1) + (z_j - z_1) f'(z_1) + \frac{1}{2} (z_j - z_1)^2 f''(z_1) + \dots \right] = \\ & = \text{Im} \left[(\tilde{M}_{\tilde{q}}^2 - \tilde{M}_1^2) f'(z_1) / m_{\tilde{g}}^2 + \frac{1}{2} (\tilde{M}_{\tilde{q}}^2 - \tilde{M}_1^2)^2 f''(z_1) / m_{\tilde{g}}^4 + \dots \right]_{41} \end{aligned}$$

To show that this expression is zero, one has to prove that

$$\text{Im} (\tilde{M}_{\tilde{q}}^{2n})_{41} = 0$$

for all $n = 1, 2, 3, \dots$. For $n = 1$, this can immediately be read off from eq. (20) and for all higher n 's it can easily be proved by induction. Thus, in this class of models, a large observed DEMON should imply a complex gaugino mass and/or a complex A. In our subsequent calculations we set $\phi = 0$.

5. BEYOND THE STANDARD MODEL

Even though our emphasis in this paper is on CP-violation, in this section we also discuss the subject of B^0 - $\bar{B}^0$ mixing which turns out to be very interesting. It is known that sizeable B^0 - $\bar{B}^0$ mixing and possibly measurable CP-violation effects are expected in the standard model [15,41-43]. We look for a possible enhancement due to the addition of the main SUSY contribution arising again from the box diagram eq. (23). To search for phenomenologically relevant parameters, we must first distinguish between off-shell transitions (i.e. the analogue of the K^0 - $\bar{K}^0$ mixing) which can be tested through various asymmetries and same-sign di-lepton production in semi-leptonic decays, and on-shell transitions with non-leptonic decays which constitute a possibly privileged place to look for CP-violation effects. We make use here of the careful analysis of ref. 44 and, in particular, we follow their notation.

The mass eigenstates in the Beon system are given by:

$$B_{H,L} = \frac{(1+\epsilon_B) B^0 \pm (1-\epsilon_B) \bar{B}^0}{\sqrt{2(1+|\epsilon_B|^2)}} \quad (24)$$

An important parameter which characterizes CP-violation in $B^0-\bar{B}^0$ mixing is the quantity $\tilde{\eta}$ defined as:

$$\tilde{\eta} = \frac{1-\epsilon_B}{1+\epsilon_B} = \frac{\sqrt{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}}{\sqrt{M_{12} - \frac{1}{2}i\Gamma_{12}}} \quad (25)$$

where M_{12} and Γ_{12} appear in the well-known effective Hamiltonian

$$H \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M_{12} - \frac{1}{2}\Gamma_{12} & M_{12} - \frac{1}{2}\Gamma_{12} \\ M_{12}^* - \frac{1}{2}\Gamma_{12}^* & M_{12}^* - \frac{1}{2}\Gamma_{12}^* \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}. \quad (26)$$

The deviation of $\eta \equiv |\tilde{\eta}|$ from unity is a measure of CP-violation. For further use we define

$$Z = \frac{1-\eta^2}{1+\eta^2}. \quad (27)$$

In SUSY theories, M_{12} in eq. (26) gets an extra contribution from the box diagram fig. 1e of which the dominant piece is given in eq. (23). We assume that there are no important SUSY contributions to Γ_{12} . For the evaluation of Γ_{12} we follow ref. 44, but for the sake of simplicity in the case of the ordinary contribution to M_{12} we employ the popular approximate formula [45].

In the virtual (off-shell) transitions, the relevant parameter for $B^0-\bar{B}^0$ mixing (for odd relative angular momentum L) is [42]

$$R_{\text{odd}} = \frac{N^{++} + N^{--}}{N^{+-} + N^{-+}} = \frac{r + \bar{r}}{2}. \quad (28)$$

N^{++} (N^{+-}) denotes the number of same (opposite)-sign dileptons coming from the production, subsequent mixing and decay of $B^0-\bar{B}^0$ pairs in e^+e^- annihilation, whilst r and $\bar{r}$ are

$$r \equiv \frac{\Gamma(B^0 \rightarrow \ell^+ \nu X^-)}{\Gamma(B^0 \rightarrow \ell^- \bar{\nu} X^+)} = \eta^2 \frac{x^2 + y^2}{2 + x^2 - y^2}$$

$$\bar{r} = \frac{\Gamma(\bar{B}^0 \rightarrow \ell^- \bar{\nu} X^+)}{\Gamma(\bar{B}^0 \rightarrow \ell^+ \nu X^-)} = \eta^{-2} \frac{x^2 + y^2}{2 + x^2 - y^2} \quad (29)$$

$$x = \frac{\Delta M}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma}.$$

The total lepton charge asymmetry of all primary leptons coming from $B^0-\bar{B}^0$, $\ell^{\pm}$, gives directly a measure of CP-violation (L odd):

$$\ell^{\pm} = \frac{N^{+-} - N^{-+}}{N^{+-} + N^{-+}} = \frac{r - \bar{r}}{2 + r + \bar{r}} \quad (30)$$

The alternative possibility to search for CP-violation in the Beon system is to look for final states f into which both B^0 and $\bar{B}^0$ decay [46]; then, the CP-asymmetry which is relevant for $e^+e^- \rightarrow B^0 + \bar{B}^0 + X$ (L even) is

$$a \equiv \frac{\sigma(\ell^+ X f) - \sigma(\ell^- X f)}{\sigma(\ell^+ X f) + \sigma(\ell^- X f)} = \frac{2x y^2 \sin 2\psi}{1 + y^2 + 2y \cos 2\psi} \quad (31)$$

$$u = \frac{1 - y^2}{1 + x^2}, \quad \exp(-2i\psi) = -\frac{1}{\eta} \frac{\langle f | \mathcal{L}(\Delta B=1) | B^0 \rangle}{\langle f | \mathcal{L}(\Delta B=1) | \bar{B}^0 \rangle}.$$

Now we turn to our findings. In the Beon system, the contribution from the pure left-handed graphs, i.e. eq. (23), is still dominant; $\tilde{d}_L - \tilde{d}_R$ mixing contributions turn out to be at most as large as 30% in this case ($|A| = 3$). We use a common B factor for all hadronic matrix elements and for definiteness, we choose the popular value [32] of 0.33. The SUSY input values can be found in section 4 and furthermore, we use $f_B = f_K = 0.16$ GeV.

Before beginning the presentation of our results, we might mention that in addition to the two Beon systems ($B_d^0 = b\bar{d}$ and $B_s^0 = b\bar{s}$) which by themselves contain a lot of variables, we now also have a large number of additional SUSY parameters. Instead of drowning a timid reader in a flood of plots, we choose to present a selective set in which the deviations from the ordinary case are most prominent and which cover a fair range of the spectrum of the SUSY input values.

We present results for $m_t = 40$ GeV only because such a relatively light top quark may be a potential embarrassment for the standard model [31, 33] whilst for the SUSY model (see ref. 14) this is well within the allowed possibilities. We use a somewhat low gluino mass of 40 GeV and squark masses in the same ballpark since for much heavier SUSY particles, the supersymmetric contributions are strongly suppressed.

We plot the different variables that characterize mixing and CP-violation in the Beon system as a function of τ_B for the SUSY case and, for comparison, also the results for the standard model. We choose, as before, $c = -1$, but to illustrate the dramatic dependence on this parameter, for some cases we also exhibit the curves for $c = -1/4$ which turn out to be roughly similar to the ordinary ones. As we already know from section 4, δ is a double-valued function of τ_B (fig. 3) and therefore, all the variables that we now calculate also reflect this double-valued nature.

We start with the mixing parameter R_{odd} . For $c = -1$, we find that for B_d^0 this quantity is remarkably enhanced by almost as much as a factor of 100 and is plotted in fig. 5a. This figure contains curves for both $|A| = 3$ and 0 from which one can observe the relative insensitivity of the result to $\tilde{d}_L - \tilde{d}_R$ mixing. On the other hand, the dependence on c is very strong and for $c = -1/4$, R_{odd} is only slightly enhanced compared to the ordinary case as shown in fig. 5b. In the B_s^0 system, R_{odd} is almost independent of τ_B as in the standard model, but much larger. We summarize below the values for several choices of parameters:

$$\begin{array}{lll} R_{\text{odd}}(B_s^0) \simeq 0.93 & c = -1, |A| = 3 & \\ R_{\text{odd}}(B_s^0) \simeq 0.90 & c = -1, A = 0 & \\ R_{\text{odd}}(B_s^0) \simeq 0.20 & c = -1/4, |A| = 3 & \\ R_{\text{odd}}(B_s^0) \simeq 0.12-0.14 & \text{standard model} & \end{array}$$

$R_{\text{odd}} = 1$ corresponds to maximal mixing and provides an indirect means to look for SUSY effects.

Now we come to quantities which measure CP-violation. In figs. 6 and 7 we have plotted Z for the B_d^0 and B_s^0 systems, respectively. Unfortunately, SUSY effects tend to reduce this CP-violation parameter by a factor of about 1/3 in the B_d^0 case and by 1/10 in the B_s^0 case for $c = -1$.

In fig. 8 we exhibit $\ell^\pm$ for the B_d^0 system. It is clear from the figure that for $c = -1$ one can get a relatively large enhancement for this parameter. Noting that

$$\ell^\pm \simeq -2 \frac{R_{\text{odd}}}{1 + R_{\text{odd}}} Z \quad (32)$$

this enhancement can be explained as due to a compensation of the decrease in Z by a large increase in R_{odd} for the B_d^0 system. We do not plot $\ell^\pm$ for the B_s^0 system since R_{odd} remains almost constant in that case and $\ell^\pm$ mimics the behaviour of Z .

Finally, the variable α which measures on-shell CP-violation [46] is plotted in fig. 9 for the B_d^0 system. For $c = -1$ this again shows nearly an order of magnitude increase for larger τ_B . For B_s^0 , α is very small and therefore uninteresting.

6.

CONCLUSIONS

In this paper we have considered CP-violation in SUSY theories. We have compared different sources of CP-violation and have argued that the only viable mechanism involves the KM phase δ contributing through gluino exchange graphs. Even though gluons conserve flavour, the gluinos do not, since the squark and quark mass matrices are diagonalized by different unitary transformations. The departure from flavour conservation is due to radiative effects and its strength is characterized by a parameter c which is $O(1)$. In the minimal $N = 1$ supergravity model, either $|c|$ is too small or the squarks and gluinos too heavy to provide significant SUSY contribution to CP-violation. Therefore, we follow the more

pragmatic philosophy of choosing the SUSY parameters as phenomenological input values and take $|c| = 1$ and SUSY particles of mass around 40 GeV.

As already widely recognized, in the light of the recent measurements of the bottom lifetime, the standard model might be unable to explain the value of the ϵ parameter if the top quark is light. Of course, this conclusion depends on the size of the B parameter which describes the deviation from the vacuum insertion approximation of the hadronic matrix elements. In particular, the currently favoured $B = 0.33$ constitutes a potential danger for the standard model. This is no longer true if we include SUSY. In addition, we find that for ϵ'/ϵ the new contribution from supersymmetry even diminishes a little bit the ordinary value. Moreover, there is no one loop gluino contribution to the DEMOM coming from the KM phase.

It has been advocated [47] that a large $B^0-\bar{B}^0$ mixing can explain the same-sign di-muon events seen in the $p\bar{p}$ collider. Such a large mixing emerges naturally in SUSY models even when the ϵ parameter constraint from the Kaon system is satisfied, contrary to the non-SUSY case. Furthermore, $B^0-\bar{B}^0$ mixing is also considerably enhanced and may be experimentally detectable at CESR. In general, the directly observable CP-violation parameters for the Beon system are enhanced as well, for large τ_B .

Our results indicate a strong sensitivity to the flavour-violation parameter c ; for $|c| = 1/4$, the SUSY effects already become rather small. The $\tilde{d}_L-\tilde{d}_R$ squark mixing, relatively unimportant for the Beon system, is negligible for the Kaon system.

To summarize our results: SUSY can make significant contribution to CP-violating phenomena in the $K^0-\bar{K}^0$ and $B^0-\bar{B}^0$ systems. The lower bound on the top quark mass from the ϵ parameter is considerably relaxed. CP-violation and mixing parameters in the Beon system are enhanced sometimes even by orders of magnitude. The experimental verification of these results could provide the much needed support for SUSY models.

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Figure captions

Fig. 1 : Box diagrams responsible for CP-violation in the $K^0-\bar{K}^0$ transition.

a) The standard W_L box diagram; b) additional non-SUSY diagrams in multi-Higgs models; c) a typical diagram where CP-violation originates from $\tilde{H}-\tilde{W}$ mixing; d) the diagrams dependent on the SUSY phase ϕ (the crosses on the squark lines denote $\tilde{d}_L-\tilde{d}_R$ mixing while those on the gluino lines are mass insertions); e) the dominant SUSY diagrams which depend on the Kobayashi-Maskawa phase δ .

Fig. 2 : In this and the remaining figures, $B = 0.33$ and all quantities are plotted as a function of β (the bottom lifetime $\tau_B = \beta \times 10^{-12}$ sec). The minimal allowed top quark mass, m_t , in the ordinary and the SUSY case. The SUSY inputs can be found in the text.

Fig. 3 : The two solutions for the KM phase δ as selected by the experimental value of ϵ for $m_t = 40$ GeV. The solid line denotes the ordinary case. The other three curves correspond to SUSY cases: the dotted, dashed and dot-dashed ones represent the choices $c = -\frac{1}{4}$ and $|A| = 3$, $c = -1$ and $|A| = 3$, $c = -1$ and $A = 0$, respectively. (We keep the above notation in all subsequent figures.) The other parameters are the same as in fig. 2.

Fig. 4 : ϵ'/ϵ for $m_t = 30$ GeV and $m_t = 40$ GeV.

Fig. 5 : R_{odd} for the B_d system. In the Beon system we always choose $m_t = 40$ GeV.

Fig. 6 : Z for the B_d^0 system.

Fig. 7 : Z for the B_s^0 system.

Fig. 8 : $1^\pm$ for the B_d^0 system.

Fig. 9 : α for the B_d^0 system.

FIG. 1

FIG. 2

FIG. 3

FIG. 6a

FIG. 5b

FIG. 8

FIG. 7b

FIG. 9