Hauser Feshbach Calculations in Deformed Nuclei

S. M. Grimes
Ohio University
Athens, Ohio 45701

Abstract

Hauser Feshbach calculations for deformed nuclei are typically done with level densities appropriate for deformed nuclei but with Hauser Feshbach codes which enforce spherical symmetry by not including K as a parameter in the decay sums. A code has been written which does allow the full K dependence to be included. Calculations with the code have been compared with those from a conventional Hauser Feshbach code. The evaporation portion (continuum) is only slightly affected by this change but the cross sections to individual (resolved) levels are changed substantially. It is found that cross sections to neighboring levels with the same J but differing K are not the same. The predicted consequences of K mixing will also be discussed.

1 Introduction

Calculation of spectra and cross sections for reactions proceeding under the condition that the Bohr independence hypothesis is valid were first made by Weisskopf [1, 2]. Hauser and Feshbach [3] and Wolfenstein [4] then modified the formalism to allow for the possibility that the independence hypothesis applies separately to states of given J and π , i.e., the branching may be different for states of large J than those of small J .

In 1972, a revision to the Hauser-Feshbach formalism was proposed [5]. For nuclei with $A > \sim 40$, proton bombardment can produce compound states of two different isospin values (assuming $N > Z$), while alpha bombardment of a target chosen to produce the same compound nucleus only produces one value. Thus, one might expect different proton to alpha branching ratios for compound nuclei formed by alpha bombardment than by proton bombardment. A slight enhancement in the proton to alpha decay ratios for proton bombardment is predicted by the traditional Hauser Feshbach formalism ($\sim 15\%$) because of the different spin distribution in the two entrance channels, but the experimental measurements showed a 40% to 50% enhancement.

If one uses a Hauser Feshbach code which includes isospin [5], the factor increases to 60% to 65%. By invoking isospin mixing, one can infer that letting the upper isospin states mix into lower isospin states before decay $\sim 50\%$ of the time reproduces the experimental result [6]. It was later shown that isospin effects could be seen in alpha- and neutron-induced reactions as well [7].

There is a possibility that the situation is similar for deformed nuclei. Use of a conventional Hauser Feshbach code for calculations in deformed nuclei is forcing the decay of the compound nucleus to be independent of K , the angular momentum projection on the symmetry axis.

2 Development of a Hauser Feshbach Code Including K

It is obvious that the possible effects of K on branching ratios can only be investigated with a new formulation of the Hauser-Feshbach equation. In this case, the cross section will be

$$\sigma_{ab} = \frac{\pi \lambda^2}{(2I_1 + 1)(2I_2 + 1)} \sum_{J_1 K_1 \pi} \frac{\tau_a \tau_b}{\sum_c \tau_c}. \quad (1)$$

Here, λ is the reduced wavelength of the particle in the entrance channel, I_1 is the projectile spin and I_2 is the target spin. Each τ is the appropriate transmission coefficient multiplied by the square of the

appropriate Clebsh-Gordan coefficient to couple to J and K . The sum in the denominator is over all exit channels, outgoing energies and final J' and K' values coupled to the J and K of the compound nucleus. The formalism is entirely parallel to that introduced for isospin in Reference [5]. In that case, the Clebsh-Gordan coefficients are for coupling to a particular $T_z (= (N - Z)/2)$ and $T = T_z$ or $T = T_z + 1$. For the deformed case, the coupling is to J and particular values of $K \leq J$.

It turns out that an important difference between the isospin case and the deformed case is that the deformed case seems to have small effects on the continuum. On the other hand, there are substantial effects on the relative cross sections to resolved final states.

It was originally anticipated that introduction of K in the formalism would lead to differences in the population of final states with the same J and different K values. Some differences of this type are observed but they might be expected to be erased if K mixing is large. This would be analogous to that situation for isospin.

There is a more profound effect which is present even if K mixing is large. A level in a spherical basis is $2J + 1$ degenerate, since each of the states with $-J \leq J_z \leq J$ has the same energy. This results in compound nuclear cross sections that are approximately proportional to $2J + 1$ if J (the spin of the level) is comparable to or less than the spin cutoff parameter, σ . For a deformed nucleus, this degeneracy is changed. A level of spin J is now split into $J + 1$ levels (even A) or $J + 1/2$ levels (odd A). Each level has a degeneracy of 2 unless the spin projection K is zero, in which case the degeneracy is one [8].

The consequence of this change is that the cross sections calculated in a deformed basis have a tendency to vary less rapidly with J than those for a spherical Hauser Feshbach calculation. The use of a standard Hauser Feshbach code even with the deformed levels included in the level density forces a spherical symmetry on the calculations.

3 Calculations

A new Hauser Feshbach code [9] has been written which includes a sum over K as well as J and π and also includes the Clebsh-Gordan coefficients as described in Section 2. The code, because of the inclusion of the sums over K , is approximately eight to ten times slower in execution than a comparable spherical Hauser Feshbach code [10,11]. The code can make calculations in hybrid situations, e.e., one in which some residual nuclei are deformed and others are spherical. Finally, the code can be run with complete K conservation or with fractions of mixing before decay between 0 and 1.

Calculations were done for neutrons incident on ^{168}Er at energies of 1, 2, 4 and 6 MeV. For the three highest energies, the decay was predominately to the continuum. The shape and magnitude of the continuum were only slightly changed between calculations with HF2002 (spherical) and HF2012 (deformed). In each case the level densities used were those appropriate for deformed nuclei, i.e., rotational bands were included.

The results at all the bombarding energies were similar for the cross sections to resolved final states.

1. The removal of the $(2J + 1)$ degeneracy made the cross sections increase for low J and decrease for high J when using the deformed Hauser Feshbach code. This conclusion was not changed when K mixing was introduced. This conclusion was valid in each exit channel (α, p, n).
2. The remaining degeneracy difference is between $K = 0$ and $K \neq 0$ states. This results in an enhancement of $K \neq 0$ compared to $K = 0$ levels of the same J . The degeneracy difference is a factor of two. The observed factor is slightly larger than a factor of two with no mixing. As the K mixing increases, the factor approached two. Thus, the K mixing does *not* remove the dependence of cross sections on K as well as J . For residual nuclei with even A , the dependence on K is weak if $K \neq 0$, but the states of given J with $K \neq 0$ are approximately twice as large as those with the same J and $K = 0$. Residual nuclei with odd A have no levels with $K = 0$; hence, all levels have

Table 1: Low-lying states in ^{168}Er

State	E_x	J	K	π
1	0.0	0	0	+
2	0.08	2	0	+
3	0.264	4	0	+
4	0.549	6	0	+
5	0.862	2	2	+
6	0.895	3	2	+
7	0.93	8	0	+
8	0.99	4	2	+
9	1.12	5	2	+
10	1.13	4	4	-
11	1.19	5	4	-
12	1.22	0	0	+
13	1.26	6	2	+
14	1.26	2	0	+
15	1.31	6	4	-

double degeneracy and the K dependence is very small. In both cases, the difference between no K mixing and complete K mixing is typically no more than 10%.

Table 1 lists the J , K , and π values for the lowest fifteen levels of ^{168}Er . In Table 2, the cross sections for neutron scattering at 2 MeV from ^{168}Er are presented for a Hauser Feshbach code based on spherical symmetry [10, 11] (a conventional Hauser Feshbach code) and the cross sections calculated with a deformed Hauser Feshbach code [9]. In each case, the cross sections are presented as ratios to the calculated elastic cross section. This cross section is about 100% larger for the deformed calculations. All input parameters are identical for the two calculations. Note the strong tendency in the cross sections calculated with the deformed code to enhance the relative cross sections for low J and reduce the cross sections for high J . There is also a tendency for states with the same J but different K to be larger for $K \neq 0$.

Table 3 presents the same results for 6 MeV neutrons. At this energy, the substantial inhibition of the population of states with $J \geq 6$ is largely removed. However, the tendency to reduce cross sections at high J is still observed. Note that for the spherical calculations eight of the cross sections are larger than the elastic; for the deformed calculation, only one cross section is larger than the elastic. Also, note that the spherical calculation predicts very similar cross sections for levels 3, 8 and 10. This is not surprising, since each level has $J = 4$. Since the K values differ, it is also not surprising that the deformed Hauser Feshbach predicts substantially different cross sections for the three states. The results quoted in Table 2 and Table 3 are for no mixing of K values. If one invokes complete K mixing, the trio of states 3, 8 and 10 move towards the asymptotic limit of twice as much cross sections for states 8 and 10 as for 3. Thus, state 8 has a virtually unchanged cross section, while the cross section for state 3 goes up about 12% and that for state 10 goes down 8%.

As has been previously noted, the continuum is virtually unchanged in moving to a deformed calculation. Both the cross section and average energy change by less than 5% for neutrons, protons and alpha particles. There are some higher order changes, however. The average J value of states populated in the continuum goes down about 15% in going to the deformed calculation and the average K value increases. These changes could potentially modify (n,2n) cross sections near threshold.

Table 2: Cross sections for scattering of 2 MeV neutrons from ^{168}Er (normalized to ground state cross section).

Level	Conventional HF Cross Section	Deformed HF Cross Section
1	1.0	1.0
2	1.84	0.39
3	0.28	0.04
4	10^{-9}	7×10^{-9}
5	1.66	0.78
6	0.86	0.31
7	8×10^{-10}	5×10^{-9}
8	0.25	0.08
9	0.08	0.02
10	0.25	0.10
11	4×10^{-6}	9×10^{-6}
12	0.89	0.90
13	2.5×10^{-9}	8×10^{-9}
14	1.56	0.33
15	1.4×10^{-6}	2.8×10^{-6}

Table 3: Cross sections for scattering of 6 MeV neutrons from ^{168}Er (normalized to ground state cross section).

Level	Conventional HF Cross Section	Deformed HF Cross Section
1	1.00	1.00
2	3.04	0.63
3	1.89	0.24
4	0.49	0.05
5	2.93	1.35
6	2.64	0.90
7	0.043	0.004
8	1.85	0.52
9	1.04	0.253
10	1.71	0.64
11	0.95	0.3
12	0.97	0.96
13	0.47	0.104
14	2.66	0.60
15	0.42	0.12

4 Summary

A new Hauser Feshbach code has been developed which includes K as a quantum number in calculating decay of the compound nucleus. Very small effects are found for the continuum. Cross sections for the population of resolved final levels are frequently changed by 40% or more. A systematic tendency is found such that cross sections for levels of large J are reduced and cross sections for low J states are enhanced. Cross sections for $K = 0$ states are reduced relative to those of states of the same J with $K > 0$.

References

- [1] V. Weisskopf, *Phys. Rev.* **52**, 295 (1937).
- [2] V. F. Weisskopf and D. H. Ewing, *Phys. Rev.* **57**, 472 (1940).
- [3] W. Hauser and H. Feshbach, *Phys. Rev.* **87**, 366 (1952).
- [4] L. Wolfenstein, *Phys. Rev.* **82**, 690 (1951).
- [5] S. M. Grimes, J. D. Anderson, A. K. Kerman and C. Wong, *Phys. Rev. C* **5**, 85 (1972).
- [6] C. R. Lux, N. T. Porile and S. M. Grimes, *Phys. Rev. C* **15**, 1308 (1977).
- [7] S. M. Grimes, *Phys. Rev. C* **46**, 1064 (1992).
- [8] S. M. Grimes, *Phys. Rev. C* **78**, 057601 (2008).
- [9] S. M. Grimes, Ohio University Report **INPP 12-01**, “A Hauser Feshbach Code for Deformed Nuclei” (2012)
- [10] S. M. Grimes, J. D. Anderson, J. W. McClure, B. A. Pohl and C. Wong, *Phys. Rev. C* **10**, 2373 (1974).
- [11] S. M. Grimes, Ohio University Report **INPP 04-03** (2004).