QCD, BAGS, AND HADRON MASSES

A.T.M. Aerts

CERN -- Geneva

and

Johann Rafelski

Institute of Theoretical Physics and Astrophysics,
University of Cape Town, South Africa

and

CERN -- Geneva

A B S T R A C T

Hadronic mass spectra are systematically fitted to a QCD and bag motivated expression in which the quark interaction matrix elements are allowed to assume best values. A statistically significant fit is found with $B^{1/4} \approx 170$ MeV. The inclusion of surface energy σ has been studied and we have found a second, equally significant fit with $B^{1/4} \approx 210$ MeV and $\sigma \approx 40$ MeV/fm². However, the corresponding hadrons are unacceptably small i.e. $R \approx 0.5$ fm. Selected strange multiquark states are recomputed and found to have generally a higher mass than currently believed.

Hadronic spectra contain valuable information about the underlying structure of the Quantum Chromodynamical interactions^[1], yet little effort has been invested in determining quantities, such as the bag constant or the strange quark mass by a 'confidence level' fit to the hadron spectrum.

In this note we intend to extract these and other quantities in an as much model independent way as possible with assumptions based on well established phenomenological and QCD inspired observations^[1,2,3] such as :

1) The quark content of hadrons has been established and the momentum of quarks in a hadron is assumed to be proportional to $1/R$, R being the size of the hadronic state. Light quarks ($q = u, d$) are taken to be approximately massless, but strange quarks (s) can have a mass, m_s .

2) The QCD-structure of the quark-quark interactions : the hadronic spectrum is split by the hyperfine interactions $\sim (\sigma\lambda/2)_i \cdot (\sigma\lambda/2)_j h_{ij}$ where the associated matrix elements h_{qq} , h_{qs} , and h_{ss} determine the strength of the respective interactions.

3) The 'specific heat' of the perturbative vacuum inside the hadrons is given by a positive energy density B . We later also allow for a surface energy as a test of this hypothesis.

4) The recoil correction to remove the resulting momentum of three or two independently moving quarks is taken to be $\sim 1/R$ but independent of the hadronic structure. In our parametrization this correction is identical to the MIT Z_0/R zeropoint energy term.

The main difference from the conventional bag models which results from these assumptions is (as we will see) that the qq , qs , and ss matrix elements h_{ij} of the QCD current-current interactions (hyperfine interactions) do not follow the ratios dictated by the use of unperturbed quark bag wave functions.

One should not have expected such a perturbative behavior in the first place, since an interaction capable of modifying the hadron energies by up to 20% will also strongly affect the wavefunctions and this is even further amplified in the hyperfine matrix element. The simplest and most pragmatic approach (as taken here) is to ignore the bag-model imposed relations between these hyperfine interaction matrix elements. Then, however, one may attempt a full description of the s -quark carrying hadrons, since the two new parameters h_{qs} and h_{ss} allow for a substantial new freedom in this sector of the hadronic spectrum. Since h_{ss} is, as could be expected, the smallest of all terms and hence the one most likely to be well represented by the MIT model, we can extract α_s from it.

As a direct consequence of our approach we present the most likely values of the important phenomenological quantities such as the bag constant. Furthermore, the knowledge of the matrix elements $h_{qq'}$, $h_{qs'}$, and h_{ss} also immediately permits a more reliable calculation of exotic bound multiquark states in so far as no virtual quark-antiquark annihilation contribution to the interaction appears as is the case in the particular state $(qqqq\bar{s})$ but not in the state $(qqqs\bar{q})$.

Our initial hypothesis (dictated by assumptions 1-4) is that the mass of a hadron is described by the relation :

$$M(R) = BV + \sigma A + \sum_{i=1}^n (x_i^2/R^2 + m_i^2)^{1/2} - E_0/R - \sum_{i < j}^n h_{ij}/R (\sigma\lambda/2)_i (\sigma\lambda/2)_j \quad (1)$$

BV is the bag volume term, σ the surface energy term with $A = 4\pi R^2$, and $m_i \neq 0$ only for strange quarks.

M , as a function of R , is always taken at its minimum value. In our approach R is a variational and hence an insignificant parameter not to be directly related to the radius of a hadron-bag, quite in the spirit of Stevenson's principle of minimum sensitivity^[4]. But $M|_{R_{min}}$ is expected to be near the physical value of interest. We are forced into this point of view, since we do not invoke any dynamics model to justify Eq.1. Consequently, we do not have any means of relating R directly to the underlying structures and must view it as a size parameter and not as a physical quantity. Nevertheless, we get a reasonable range of R of about 20% less than in the MIT bag and will take the point of view that $R \approx 0.5$ fm is not acceptable, while $R = 0.8$ fm is palatable.

Given a certain set of weights (errors) for the spectrum we search for the best fit. Here, however, judicious choice of procedure is essential since the confidence level achieved by the fit will be strongly affected by the assumed scale of the weights. However, the relative significance among different fits is not affected by scale changes, but could be affected by choosing a differently biased set of weights. When choosing our set we accounted for many effects visible in the hadronic spectrum, such as mass differences within isospin multiplets which compounds the

difference in light quark masses with mass corrections of electromagnetic origin, but which are not incorporated in Eq.(1). We have associated a weight of 7 MeV with this uncertainty, which also is accounting for various corrections which do not scale exactly as $1/R$.

The eight parameters : B , x_q , x_s , E_0 , h_{qq} , h_{qs} , h_{ss} and m_s are varied in order to fit the masses of $N(939)$, $\Delta(1232)$, $\omega(783)$, $\rho(769)$, $K(496)$, $K^*(892)$, $\phi(1020)$, $\Sigma(1193)$, $\Lambda(1116)$, $\Sigma^*(1385)$, $\Xi(1318)$, $\Xi^*(1533)$, and $\Omega(1672)$, i.e., 13 states. Out of these thirteen states, five have a large natural width Γ . Considering the fact that our calculation ignores the coupling to the decay channels^[5], we have chosen not to fit the mass m , but $m + \Gamma$. This then qualitatively accounts for the fact that, would the coupling to the decay channels be turned on, the masses of these states would be shifted from their computed 'primitive' position^[6] to their physical position. These wide states (Δ , K^* , ρ , Σ^* , Ξ^*) are also weighted with their intrinsic full width, i.e. Δ with 115 MeV, ρ with 154 MeV, K^* with 51 MeV, Σ^* with 40 MeV, and Ξ^* with 17 MeV, where the last value also includes the 7 MeV referred to above. Finally, we decided to include in the fitting the vague knowledge we have of the values of the strange quark mass ($m_s = 240 \pm 60$ MeV). During the fitting procedure a minimum was found subject to this weak constraint. However, we omit in Table 1 the related contribution to the χ^2 and quote only the value for the thirteen hadronic states considered. We have fitted 13 hadronic states with 8, or including the surface energy 9, parameters. This gives us 5, or 4, degrees of freedom. We have consciously excluded the pion from our considerations. A nearly as significant fit (we find $\chi^2=3.7$ for 6 degrees of freedom, that is a 70% CL) results, when $m_\pi = 139$ MeV is fitted. However, we felt it not justified to consider the pion as a bag and calculate its mass as a difference of GeV sized contributions.

In the individual columns of Table 1 the values of our parameters are given as found by MINUIT (CERN Fortran Library). The first column was obtained with $M(R)$ (Eq.1) without the surface energy term. The second and third columns allow for a surface term. Fit I is in the neighbourhood of $\sigma=0$. Fit II is a second, independent minimum. We also quote the χ^2 of the spectrum shown in Table 2, and the estimated confidence level of the fits, together with an average hadronic radius.

For comparison, we have used the MIT bag model to fit the hadronic spectrum. The $\chi^2=71.5$ found is an order of magnitude larger. However, there are four less parameters in the MIT bag model since our three matrix elements h_{qq} , h_{qs} and h_{ss} are parametrized by α_s (and a given function of the strange mass) only and we have allowed x_q and x_s to

assume best values though they turn out to be quite close to the MIT values. Thus the MIT bag fit has 9 degrees of freedom and a $\chi^2/\text{d.o.f.} = 7.9$. While our confidence level is $>75\%$ the MIT bag's confidence level is negligible. In our approach the spectrum is fitted within one standard deviation and the scheme as well as its parameters appear to have a definite physical meaning.

Inclusion of σ reduces both the χ^2 and the number of degrees of freedom, but as it turns out the statistical significance remains unchanged. Our attitude is here that there is no evidence for surface energy in the hadron spectrum, discounting Fit II for its small hadronic radii. However, we find that the spectrum is compatible with any surface energy between 5 and -5 MeV/fm². The σ -B correlation analysis yields $\sigma = -0.74(B^{1/4} - 171)$ MeV/fm². The bag constant remains extremely stable, i.e. independent of the scheme, and is $B^{1/4} \approx 170$ MeV or $4B = 0.45$ GeV/fm³. Having done many different fits which invariably yielded closely this value, we strongly believe in the significance of this result, barring the possible significance of Fit II.

In the absence of surface energy, the eigenvalues x_i are quite close to the quark bag values, e.g., the MIT bag value 2.04 is to be compared to $x_q = 1.97$. This change of 4% is compatible with the expected influence of the interactions, indicating that the wave functions change by about 20%. The hypothesis of strongly deformed bag wave functions is consistent with the modified h_{qq} , h_{qs} and h_{ss} matrix elements, which follow the ratios 5 : 3.5 : 1. We note that for $m_s = 266$ MeV and $m_s R = 1.12$ the MIT bag model would lead to the ratios 1.4 : 1.2 : 1.0.

Extracting α_s from the h_{ss} matrix element, we find $\alpha_s = 0.6 \pm 0.3$ (to be compared with $\alpha_s^{\text{MIT}} = 2.2$). The largest source of error is the fitting error of h_{ss} . In addition, we have not allowed an $m_s R$ dependence of the matrix element and so we must compare at an average value of R . Notice also that the matrix elements h_{ij} allow for a more general dependence on α_s than the simple proportionality assumed here.

We should emphasize that since the strange quark mass enters the fit only in combination with x_s/R (see Eq.1) we can achieve fits to the spectrum of similar statistical significance by changing x_s and m_s simultaneously with h_{ss} . Therefore our approach cannot be considered as an independent determination of the strange quark mass. To underline this point we show in Fig.1 the χ^2 contours on the a) x_s - m_s and b) x_s - h_{ss} surface which indicates a value for $m_s = 266 \pm 20$ MeV if x_s and h_{ss} are adjusted accordingly, i.e. as indicated by contours '1' in Fig.1. It is

worth recording that a fit with a fixed value of m_s , chosen at e.g., 200 MeV, has comparable significance (70%), since an increase in χ^2 is compensated by a larger number of degrees of freedom : 6.

Concerning the parameter E_0 we first note that it is near the average value of the momenta carried by the quarks i.e., $E_0 \approx (x_q + x_s)/2$, suggesting that it might be dominated by recoil corrections. However, an attempt to set $E_0 = (n_q x_q + n_s x_s)/(n_q + n_s)$ reduces the statistical significance (we find $\chi^2=14.$), even if another "zeropoint" term is added, which then is found nearly zero ($E_0=0.05$). The same remark holds if a fit of $M^2 = E^2 - \langle P^2 \rangle$ is attempted^[2], where now E is given by Eq.1 rather than M (we now find $\chi^2=63.$). Since such theoretical considerations have substantially decreased the statistical significance of the fits to the spectrum, we have not further pursued this line of thought.

From these considerations one may draw the conclusion that the success of the fits described here is significant. This is an important remark as one may take the point of view that the 8 parameters at our disposal are used to fit 8 narrow states, while the large width of the remaining five states (degrees of freedom) allows the small χ^2 we find. Then the fact that other, similar formulas do not work is reassuring us that our approach is physically meaningful.

In view of these remarks we have also fitted the eight narrow states with the eight parameters and a fit with $\chi^2=0.5$ is possible. The resulting parameters are within the errors for the fit given in Table 1. From this point of view the masses of the wide states given below can be regarded as predictions rather than as fitted values.

Turning now to the discussion of the fits to the spectrum, we notice that all fits a) give a near massless pion, b) reproduce $\Sigma^* - \Xi^* - \Omega$ splittings in the decuplet and the $\Sigma - \Xi$ and $\Lambda - \Sigma$ splittings in the octets. In Table 2, we show the 13 states considered, their assumed mass and width for the fit, the masses fitted with volume energy only as well as Fits I and II with surface energies included. We do not show a fit with only surface energy ($B=0$), since a) its statistical significance is noticeably smaller ($\chi^2/d.o.f.=0.95$) than that of Fit II which yields almost the same value of σ , and b) a 'surface only' fit has an average hadron radius of 0.65 fm. Thus a 'surface only' bag appears less attractive than a mixed bag, while a 'volume only' bag is on combined statistical and physical grounds most significant.

Given the improved parameters of the bag model, we can compute the masses of exotic states carrying strange quarks better and these turn out to be generally higher than predicted in the original bag model. Since our approach has no knowledge of qq virtual annihilation interactions^[7], which become possible in the color octet channel, we only consider here a) strange dibaryons, b) mesobaryons of type $q^4\bar{s}$ and c) baryonia of type $q^2\bar{s}^2$ in which such interactions would play only a negligible role. Our initial findings are summarized in Table 3. The bag values are taken from Ref.[8,9]. However, here also one should keep in mind that the physical masses of these states may be considerably lower when the coupling to decay channels is taken into account.

For completeness we should add that not just the masses of the strange exotica turn out to be higher. Also the q^6 dibaryons are heavier. For instance the lightest q^6 dibaryon (with deuteron quantum numbers $I=0, J=1$) is now nearly degenerate with the lightest q^5s dibaryon (see Table 3), which means it is calculated to be about 200 MeV heavier than before^[9]. Note that nearby at a mass of 2360 MeV a dibaryon signal has been reported by H. Ikeda et al.^[10] with the right quantum numbers.

The new parameters obtained here are well compatible with those used in computing properties of quark gluon plasmas^[11]. In particular $B^{1/4} \approx 170$ MeV leads to a critical temperature of circa 155 MeV for baryonless plasma, a value much desired given the known critical Hagedorn temperature of this magnitude, which is connected via the bootstrap hypothesis with the slope of the exponential growth of the hadronic mass spectrum^[12].

Our analysis supports the prevailing attitude that QCD and the picture of two vacua is the underlying dynamical structure of the strong interactions and provides us with a first consistent set of parameters viz. : $B^{1/4} = 170$ MeV, $\alpha_s = 0.6 \pm 0.3$, $m_s = 260 \pm 20$ MeV, and the matrix elements of the current-current interactions. Furthermore, we find that the masses of the exotic multiquark hadrons are generally higher than previously anticipated.

We thank G. Karl and T.H. Hansson for their interest in this work and R. Hagedorn for constructive remarks and a careful reading of the manuscript.

Parameters	Volume Energy Bag	Volume and Surface Energy	
		Minimum I	Minimum II
$B^{1/4}$ [MeV]	171 ± 2	172 ± 2	211 ± 4
$4B$ [MeV/fm ³]	450 ± 20	460 ± 20	1040 ± 90
σ [MeV/fm ²]	0.0	-1.0 ± 0.7	39.9 ± 4.4
m_s [MeV]	266 ± 13	263 ± 14	286 ± 12
h_{qq}	0.40 ± 0.01	0.40 ± 0.01	0.25 ± 0.01
h_{qs}	0.27 ± 0.01	0.27 ± 0.01	0.17 ± 0.01
h_{ss}	0.075 ± 0.027	0.074 ± 0.029	0.044 ± 0.025
x_q	1.97 ± 0.02	1.97 ± 0.02	1.23 ± 0.02
x_s	2.44 ± 0.02	2.45 ± 0.02	1.50 ± 0.02
E_0	2.25 ± 0.04	2.24 ± 0.03	1.45 ± 0.03
χ^2	2.56	2.13	2.13
$\chi^2/\text{d.o.f.}$	0.53	0.53	0.53
Confid. Level	80%	80%	80%
R_{had}	≈ 0.83 fm	≈ 0.83 fm	≈ 0.46 fm

Table 1. Parameters to fits of hadron spectra: a bag-like fit and two fits allowing for surface energy: I is compatible with $\sigma=0$, II is a second minimum with a small R_{had} .

Particle and Mass	Fitted masses $M+\Gamma \pm \Gamma$	Volume Energy Bag	Volume and Surface Energy	
			Minimum I	Minimum II
N : 939	939 $\pm$ 7	940.9	940.4	940.2
Δ : 1232	1347 $\pm$ 115	1322.8	1315.7	1303.6
Λ : 1116	1115.6 $\pm$ 7	1112.7	1113.4	1113.8
Σ : 1193	1193.1 $\pm$ 7	1197.4	1195.7	1195.7
Σ^* : 1385	1425 $\pm$ 40	1446.3	1444.5	1435.5
Ξ : 1318	1318.1 $\pm$ 7	1315.1	1314.7	1316.9
Ξ^* : 1533	1550.4 $\pm$ 17	1561.7	1563.1	1557.7
Ω : 1672	1672.4 $\pm$ 7	1669.3	1671.4	1670.0
ω : 783	782.6 $\pm$ 7	779.2	780.5	781.1
ρ : 769	923 $\pm$ 154	779.2	780.5	781.1
K : 496	495.7 $\pm$ 7	495.4	496.5	495.3
K^* : 892	940 $\pm$ 51	911.2	913.2	913.5
ϕ : 1020	1019.6 $\pm$ 7	1024.1	1021.5	1022.6
π : 140	---	31.9	67.0	15.3

Table 2. Hadronic states of our fit with assumed masses and weights (in MeV), and the resulting fits.

EXOTICA	MIT-Bag	Volume Energy Bag	Volume and Surface Energy	
			Fit I	Fit II
Dibaryons				
q^4s^2 $l=0$ $J=0$	2.20	2.286	2.287	2.263
q^5s $l=1/2$ $J=1$	2.21	2.384	2.386	2.343
q^5s $l=1/2$ $J=2$	2.29	2.483	2.455	2.436
Meso-Baryons				
$q^4\bar{s}$ $l=0$ $J=1/2$	1.700	1.840	1.840	1.820
$l=1$ $J=3/2$	1.800	2.000	2.000	1.975
Baryonium				
$q^2\bar{s}^2$ $l=1$ $J=0$	1.520	1.640	1.640	1.600
$l=2$ $J=0,1,2$	1.610	1.630	1.630	1.630

Table 3. Masses of selected strange exotic states (in GeV).

REFERENCES

- [1] T.A. DeGrand et al, Phys.Rev.D12, 2060(1975)
- [2] K. Johnson, Acta Phys. Polonica B6, 865(1975)
J.F. Donoghue and K. Johnson, Phys.Rev.D21, 1975(1980)
- [3] N. Isgur and G. Karl, Phys.Lett. 72B, 109(1977)
- [4] P Stevenson, Phys.Rev.D.23, 2916(1981)
- [5] A. Badalyan et al., Phys.Rep. 82, 31(1982)
- [6] R.L. Jaffe and F.E. Low, Phys.Rev.D19, 2105(1979)
- [7] P. Zimak and R. D. Viollier, private communication
and to be published
- [8] D. Strottman, Phys.Rev.D20, 748(1979)
- [9] A.T.M. Aerts, P.J.G. Mulders and J.J. de Swart,
Phys.Rev.D17, 260(1978) and D21, 1370(1980)
A.T.M. Aerts, Nijmegen thesis (unpublished)
- [10] H. Ikeda et al., Phys.Rev.Lett.42, 1321(1979)
- [11] See e.g. J. Rafelski and M. Danos, 'Perspectives in High
Energy Nuclear Collisions' NBSIR-2725, Washington D.C. (1983)
- [12] R. Hagedorn, I. Montvay and J. Rafelski, in Proceedings of
the Workshop on 'Hadronic Matter at Extreme Energy Density'
Erice 1978, eds. N. Cabibbo and L. Sertorio, Plenum Press
(1980)

FIGURE CAPTION

Figure 1: χ^2 -contours as a function of a) m_s and x_s and b) m_s and h_{ss} .
The curves are labelled by the number of standard deviations one is away
from the minimum.