CERN-TH.3907/84

INSTANTON EFFECTS IN SUPERSYMMETRIC GAUGE THEORIES

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A B S T R A C T

We investigate how in supersymmetric gauge theories non-perturbative effects are able to generate non-trivial vacuum properties otherwise forbidden by perturbative non-renormalization theorems. This conclusion can be reliably drawn since the constancy of certain Green's functions - due to supersymmetry (SUSY) - allows one to connect vacuum-dominated large distances with short-distance behaviour which is reliably computed by instanton methods. In all the cases we discuss (without matter, with massive or massless matter in real representations and, finally, with matter in complex representations) instanton calculations imply the occurrence of a variety of condensates. For the pure SUSY gauge theory, a gluino condensate induces the spontaneous breaking of Z_{2N} . For massive super quantum chromodynamics (SQCD) we find a peculiar mass dependence of matter condensates whose origin is traced to mass singularities of non-zero mode instanton contributions. These contributions force the massless limit of SQCD to differ from the strictly massless case, in which the spontaneous breaking of chiral symmetries is induced. Inconsistency with an anomaly equation forces either infinite matter condensates or spontaneous SUSY breaking in the massless cases. For non-constant Green's functions, instantons are shown to provide new calculable short-distance singularities of an obvious non-perturbative nature.

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CERN-TH.3907/84

May 1984

1. INTRODUCTION

During the last few years, a considerable effort has been made with the aim of extending our perturbative knowledge of supersymmetric gauge theories to non-perturbative phenomena.

If we take ordinary QCD as a historical precedent, some understanding of phenomena, such as confinement, chiral-symmetry breaking, the $U_A(1)$ problem, is clearly necessary before deciding if and where SUSY can play a role in elementary-particle physics.

The most pressing issues to tackle appear to be the following:

- i) Are SUSY gauge theories confining?
- ii) Is SUSY spontaneously broken by non-perturbative effects?

Or, more generally

- ii)' Are some perturbative vacua lifted or shifted by non-perturbative effects?
- iii) What are the flavour symmetries of the vacuum?

And the related question:

- iv) What is the low-lying spectrum of SUSY gauge theories?

After a few years of investigations, cautious optimism seems to prevail on the possibility of answering some of these questions. Supersymmetry indeed seems to be of great help in constraining the possible answers and in rendering feasible calculations which would appear hopeless in non-SUSY gauge theories. The most effective tools to study the above questions can be classified as follows:

- a) Witten index considerations [1]
- b) Arguments based on explicit effective Lagrangians [2-4]
- c) Supersymmetric and chiral Ward identities [5-7]
- d) Explicit calculations involving topologically non-trivial field configurations [8-12].

A common assumption of all these approaches, which we shall share in this paper, is that SUSY is not explicitly broken by quantum effects even at non-perturbative levels.

Methods of type (a) have been of great value in assessing the absence of spontaneous SUSY breaking in several cases. Methods (b) and (c) have shown explicitly how the value of Witten's index could be realized, when it was known. Furthermore, they have suggested, as an unexpected peculiarity of SUSY gauge theories, the runaway of certain order parameters in the limit of zero mass. More recently, method (d) has allowed the actual proof (modulo some reasonable assumptions) of many of the results suggested by the previous techniques and the extension of them. In this way, breaking of the non-renormalization theorem [13] by non-perturbative effects and, for chiral theories, even spontaneous breaking of SUSY have been essentially established.

In this paper, we shall deal with several aspects of this last method*), giving some details on the technique as well as a discussion of its reliability and possible pit falls. Our overall conclusion is that the (unconstrained) instanton method emerges as a powerful and self-consistent approach to vacuum properties of SUSY gauge theories.

Let us briefly start here by recalling the general ideas underlying the method. One considers Green's function made of gauge invariant field combinations $A_i(x_i)$

$$G(x_1 \dots x_n) = \langle 0 | T(A_1(x_1) \dots A_n(x_n)) | 0 \rangle \quad (1.1)$$

assuming the vacuum to be supersymmetric. Any contradiction arising from that assumption can be the signal of spontaneous SUSY breakdown. The following steps are then taken:

- i) The function G is chosen so that it is x_i -independent and has the correct chirality to be saturated by the one-instanton sector.
- ii) G is computed at short distances through a single instanton calculation.
- iii) Clustering, or better vacuum dominance, is used at large distances.

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*) However, we shall not discuss the constrained instanton method developed in refs. [11], limiting ourselves to comparing the conclusions where possible.

Let us now discuss these steps one by one. Step (i) requires the use of the lowest components of chiral superfields for the operators A_i . In this case, a supersymmetric Ward identity ensures x -independence. In order to have only the contribution from the one-instanton sector, the axial quantum numbers of the A_i 's will have to satisfy some further constraints.

Step (ii) is the crucial point of the approach and rests on the statement that, for a given topological charge sector, the lowest action configuration should dominate at short distances. It is not easy to justify this assumption completely, but strong arguments can be given. Consider, for instance, the correction due to one additional instanton-antiinstanton pair. Contributions of this type are known to be needed, in general, in order to satisfy cluster properties. Only when the two instantons and the anti-instanton are well separated, i.e. only if

$$(y_i - y_j)^2 \gg \rho_i^2 + \rho_j^2 \quad (1.2)$$

where y and ρ are the instanton (or anti-instanton) positions and sites, can we really talk about a new contribution. If condition (1.2) is not satisfied the effect is not distinguishable from that of a single instanton. On the other hand, under (1.2), the total action is, to a good approximation, three times that of a single instanton or $3 \times 8\pi^2/g^2$. Furthermore, in order for the new contribution not to cancel against instanton-anti-instanton effects in $\langle 0|0 \rangle$, the pair will have to be located near the positions x_i , which, by definition of short distances, satisfy the relation:

$$|x_i - x_j| \ll \Lambda^{-1} \quad (1.3)$$

Λ being the renormalization group invariant scale. In conclusion, the three instantons have to be small ($\rho_i \lesssim |x_i - x_j| \ll \Lambda^{-1}$) and well separated. Their contribution to G will thus be depressed relative to the single-instanton contribution by a factor of the order of

$$\exp\left(-2\frac{8\pi^2}{g^2(x_i, x_j)}\right) \sim (\Lambda|x_i - x_j|)^{2\beta_1}; \quad \beta(g) = -\beta_1 \frac{g^3}{16\pi^2} + O(g^5), \quad \beta_1 > 0 \quad (1.4)$$

and the correction due to pairs will be tiny in the regime (1.3), provided that the instanton gives a non-vanishing result.

As we said, in most of our calculations, we shall consider constant Green's functions. In these cases, contributions of other configurations should not just be small but actually add up to zero since terms like (1.4) are forbidden in this case.

In section 4 we shall also compute certain non-constant correlations and exhibit light-cone singularities to be attributed to new operators appearing with non-perturbative coefficients in Wilson's expansion.

In the last step (iii) the constancy of G allows the identification of its value with the contribution of zero-energy intermediate states, the SUSY vacua of the theory:

$$G = \text{const.} = \sum_{\Omega} \prod_i \langle \Omega | A_i | \Omega \rangle ; \quad |\Omega\rangle = \text{SUSY vacuum} \quad (1.5)$$

where we have assumed, as usual, that the local operators $A_i(x_i)$ cannot induce transitions among different vacua. Since the original theory had global symmetries, a single vacuum saturation of eq. (1.5) implies absence of spontaneous symmetry breaking, the opposite being true for a non-trivial sum.

From the explicit form of G we shall be able to distinguish between these two alternatives and to identify the patterns of spontaneous symmetry breaking.

When we add SUSY masses m_i arguments similar to those enforcing x -independence of G give its m_i -dependence. In some cases G has to be also m_i -independent, but the constant value at $m_i \neq 0$ has to differ from the $m_i = 0$ value owing to the change occurring in the vacuum degeneracy when chirality-breaking masses are turned on. This sudden jump in G propagates to short distances because of x -independence and one is faced with the problem that, for small m , corrections to the $m = 0$ result can be $O(1)$. We believe to have identified the origin of this peculiar feature in the singular behaviour of the non-zero modes in the instanton background. We shall see that the picture stemming from our analysis looks consistent for massive QCD Vacuum expectation values (v.e.v.'s)-contrasting with perturbative non-renormalization theorems-are generated and reflect in some cases the singular mass dependence we have just discussed.

The massless case, besides differing from the limit of the massive theory, has a theoretical difficulty in that the outcome of the instanton calculation appears to contradict an anomalous Ward identity except in the improper limit of an infinite v.e.v. This conflict may signal a difficulty with regularizing supersymmetrically theories with massless matter superfields (hence with massless scalars).

Another question, which goes beyond the scope of this paper, is the identification of topological configurations assuring the clustering properties of Green's functions that receive instanton contributions. Indeed, clustering in eq. (1.5) implies v.e.v.'s for the gauge-invariant operators A_i , which carry chiralities unsuitable for instantons to contribute. Other configurations with non-integer Pontryagin number appear to be necessary. It has recently been proposed [9] that these configurations might be "torons", defined as field configurations in a space-time box with twisted boundary conditions [14]. It is hard to visualize how these constant field configurations -- going to zero in the infinite volume limit -- can cluster instanton-dominated amplitudes since the lack of scale in the toron prevents the definition of a superposition of torons reminiscent of the dilute instanton gas invoked for integer winding number configurations.

The outline of the paper is as follows. In section 2 we define a class of Green's functions suitable for instanton calculation and for predicting vacuum properties. In section 3 we give the formal machinery (largely based on the work of ref. [15]) needed to actually perform the computation of those Green's functions. Section 4 deals with the calculations which pertain to the supersymmetric Yang-Mills (SYM), massless SQCD and supersymmetric Georgi-Glashow models, while section 5 is concerned with the peculiar features due to non-zero mass and with the techniques needed to cope with that case. Section 6 contains a discussion of the results and some conclusions.

2. INSTANTON GREEN'S FUNCTIONS

As discussed in refs. [8,10] and [12], non-perturbative instanton-type calculations of certain Green's functions seem to provide interesting information on the properties of the vacua of SUSY theories. In particular, they suggest patterns of flavour symmetry breaking which, while in agreement with effective Lagrangians [2-7] and index theorem considerations [1], are in contrast with perturbative results. This indicates a violation of the non-renormalization theorem [13].

In this section we want briefly to recall which are the relevant Green's functions in this context and how they can be computed to the lowest non-trivial order in the semiclassical expansion around the instanton solutions.

Let us limit ourselves to the case of unextended*) SUSY gauge theories with N colours [$SU(N)_c$ gauge group] and M flavours.

The SQCD Lagrangian in superfield notation reads:

$$\mathcal{L} = \mathcal{L}_{SYM} + \mathcal{L}_{matter} + \mathcal{L}_{mass} \quad (2.1)$$

$$\mathcal{L}_{SYM} = \frac{1}{4g^2} \left[\int d^2\theta W_a^{\alpha b} W_{\alpha b}^a + \int d^2\bar{\theta} \bar{W}_a^{\dot{\alpha} b} \bar{W}_{\dot{\alpha} b}^a \right] \quad (2.2a)$$

$$\mathcal{L}_{matter} = \int d^4\theta \left[\bar{\Phi}^{ai} (e^{2V})_a^b \Phi_{bi} + \tilde{\Phi}^{ai} (e^{-2V})_a^b \tilde{\Phi}_{bi} \right] \quad (2.2b)$$

$$\mathcal{L}_{mass} = \int d^2\theta m_i^j \tilde{\Phi}^{ai} \Phi_{aj} + \int d^2\bar{\theta} m_i^{*j} \bar{\Phi}^{ai} \tilde{\Phi}_{aj} \quad (2.2c)$$

where we have indicated by greek letters $\alpha, \beta, \dots = 1, 2$ the two-valued spin indices, by latin letters $a, b, \dots = 1, \dots, N$ the colour indices and by $i, j, \dots = 1, \dots, M$ the flavour indices.

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*) For the case of $n = 2$ extended pure SYM theories see refs. [8] and [16].

The superfields can be written in components in the Wess-Zumino gauge in the following way:

$$W_\alpha = i\lambda_\alpha + i(F^{\mu\nu}\sigma_{\mu\nu\alpha}{}^\beta + D\delta_\alpha^\beta) + \theta^2\sigma_{\alpha\dot{\alpha}}{}^\mu\mathcal{D}_\mu\bar{\lambda}^{\dot{\alpha}} \quad (2.3a)$$

$$\bar{W}^{\dot{\alpha}} = -i\bar{\lambda}^{\dot{\alpha}} - i(F^{\mu\nu}\bar{\sigma}_{\mu\nu}{}^{\dot{\alpha}}{}_\beta + D\delta_\beta^{\dot{\alpha}}) + \bar{\theta}^2\bar{\sigma}^{\mu\dot{\alpha}\alpha}\mathcal{D}_\mu\lambda_\alpha \quad (2.3b)$$

$$\Phi_{ai} = \varphi_{ai} + \sqrt{2}\theta^\alpha\psi_{\alpha,ai} + \theta^2 F_{ai} \quad (2.3c)$$

$$\bar{\Phi}^{ai} = \varphi^{*ai} + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha},ai} + \bar{\theta}^2\bar{F}^{ai} \quad (2.3d)$$

$$\tilde{\Phi}^{ai} = \tilde{\varphi}^{ai} + \sqrt{2}\theta^\alpha\tilde{\psi}_\alpha^{ai} + \theta^2\tilde{F}^{ai} \quad (2.3e)$$

$$\bar{\tilde{\Phi}}_{ai} = \tilde{\varphi}_{ai}^* + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\tilde{\psi}}^{\dot{\alpha}}_{ai} + \bar{\theta}^2\bar{\tilde{F}}_{ai} \quad (2.3f)$$

where we have used the notation $F_{\mu\nu} = F_{\mu\nu}^A T^A$, $\lambda = \lambda^A T^A$ ($A = 1, \dots, N^2-1$). The covariant derivative is defined as

$$\mathcal{D}_\mu = \partial_\mu + i\tau^A V_\mu^A \quad (2.4)$$

where τ^A are the $SU(N)$ generators in the appropriate representation and are called T^A in the fundamental. Also, as usual

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + i[V_\mu, V_\nu] \quad (2.5)$$

V_μ being the gauge potential.

The superfield V is given by

$$V = -\theta(\bar{\sigma}_\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}V^\mu + i\theta^2\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - i\bar{\theta}^2\theta^\alpha\lambda_\alpha + \dots$$

Φ and $\tilde{\Phi}$ are chiral superfields, while $\bar{\Phi}$ and $\bar{\tilde{\Phi}}$ are antichiral superfields.

Lower and higher latin indices ($a, b, \dots$, and $i, j, \dots$) are meant to indicate that the field belongs, respectively, to the fundamental or its conjugate representation

of the gauge- or of the flavour-symmetry group. Where unnecessary, colour and flavour indices as well as spin indices will be suppressed. Finally, we note that a factor of g has been absorbed in the definition of the vector potential V_μ and of the gluino's fields λ and $\bar{\lambda}$.

Performing the θ and $\bar{\theta}$ integrations and eliminating the auxiliary fields allows one to write in components the Lagrangian (2.1), obtaining in the Euclidean region the expression in Appendix A.

The massless theory possesses a classical global symmetry $[SU(M)_L \times SU(M)_R \times U_V(1) \times U_A(1)] \times U_\lambda(1)$, where the first four factors commute with SUSY, while the last $U_\lambda(1)$ transformation is the R symmetry acting only on ϕ , $\tilde{\phi}$ and λ :

$$U_\lambda(1) : \quad \lambda \rightarrow e^{i\alpha} \lambda \quad ; \quad (\varphi, \tilde{\varphi}) \rightarrow e^{i\alpha} (\varphi, \tilde{\varphi}) \quad (2.6)$$

Since λ is massless and scalar mass terms $(\phi^* \phi, \tilde{\phi}^* \tilde{\phi})$ are invariant, $U_\lambda(1)$ is not broken by $\mathcal{L}_{\text{mass}}$. At the quantum level, however, both $U_A(1)$ and $U_\lambda(1)$ are anomalous. The subgroup of $U_A(1) \times U_\lambda(1)$, which is left unbroken at $m = 0$, turns out to be $\hat{U}_A(1) \times Z_{2N}$, where the $\hat{U}_A(1)$ factor is defined in ref. [3] and Z_{2N} is the discrete subgroup of $U_\lambda(1)$ given by $\alpha = 2\pi k/2N$ in eq. (2.6). At $m \neq 0$ only Z_{2N} remains as true axial symmetry. Another important linear combination of $U_A(1)$ and $U_\lambda(1)$ is the axial $U(1)$ which resides in the supercurrent. Following ref. [3] we shall call it $U_R(1)$.

For convenience we give in the table the charges of the various elementary fields under the axial $U(1)$'s we have mentioned, together with their naïve mass dimensions. Notice that for the lowest components of chiral superfields the mass dimension coincides with the $U_R(1)$ charge.

After these preliminaries we want now to establish what kind of Green's functions can be reliably computed in the massless theory by using instantons*). The massive case will be discussed in section 5.

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*) From now on, we restrict ourselves to self-dual gauge fields. The changes for the anti-self-dual case are obvious.

Necessary conditions are:

- i) gauge invariance,
- ii) fulfilment of the "chirality selection rule" imposed by the anomalous $U_\lambda(1)$ symmetry on instanton-dominated amplitudes,
- iii) invariance under the non-anomalous $\hat{U}_A(1)$ symmetry^{*}).

Condition (i) requires that one has to consider Green's functions built up of gauge-invariant products such as $\text{Tr}(FF)$, $\text{Tr}(\lambda\lambda)$, $\text{Tr}(\lambda F_{\mu\nu}\lambda)$, ..., $\tilde{\phi}\phi$, $\tilde{\phi}F_{\mu\nu}\phi$, ..., $\tilde{\psi}\psi$, $\tilde{\psi}F_{\mu\nu}\psi$, ... Conditions (ii) and (iii) give the relations (see table):

$$\begin{aligned} n_\lambda + n_\psi &= N\nu \\ -M n_\lambda + N n_\psi + (N-M) n_\phi &= 0 \end{aligned} \quad (2.7)$$

if n_λ , n_ψ , and n_ϕ are, respectively, the numbers of $\lambda\lambda$, $\tilde{\phi}\phi$, and $\tilde{\psi}\psi$ pairs appearing, and ν is the winding number. From the relations (2.7) one finds

$$\begin{aligned} n_\lambda &= (N-M)\nu + n_\psi \\ n_\phi &= M\nu - n_\psi \end{aligned} \quad (2.8)$$

At this point, as noticed already in refs. [8] and [10] an interesting possibility arises which explains why we focus on the semiclassical instanton approximation. Let us see what happens if we impose the further constraint that the naïve mass dimension of the Green's function be equal to $\nu\beta_1$, where $\beta_1 = 3N-M$ is the first coefficient of the β function. Clearly in this situation, the full mass dimension of the Green's function is provided by the subtraction point mass scale, μ , appearing in the 't Hooft determinant [17] of the quantum fluctuations around the instanton configuration ($\mu^{\nu\beta_1}$). When this factor is combined with the instanton action ($e^{-8\pi^2\nu/g^2}$), the Green's function appears to be proportional to the renormalization group invariant combination (see Appendix A):

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^{*}) The other $U_A(1)$'s mentioned above, being combinations of $U_\lambda(1)$ and $\hat{U}_A(1)$, do not impose further constraints.

$$\mu^{\nu\beta_1} \exp(-\frac{8\pi^2\nu}{g^2}) = \left[\mu \exp(-\frac{8\pi^2}{g^2}) \right]^{\nu\beta_1} = \Lambda^{\nu\beta_1} \quad (2.9)$$

which is precisely what is expected for a physical quantity of mass dimension $\nu\beta_1$.

To see what is the structure of these Green's functions, we recall that besides eq. (2.8) their mass dimension d must obey the constraint

$$d = 3n_\lambda + 2n_\varphi + 3n_\psi + 2n_F = (3N - M)\nu \quad (2.10)$$

where n_F is the number of $F_{\mu\nu}$ insertions. Solving for n_λ , n_ϕ , and n_ψ , one finds*)

$$\begin{cases} 2n_\psi + n_F = 0 \\ n_\lambda = (N - M)\nu + n_\psi \\ n_\varphi = M\nu - n_\psi \end{cases} \Rightarrow \begin{cases} n_\psi = n_F = 0 \\ n_\lambda = (N - M)\nu \\ n_\varphi = M\nu \end{cases} \quad (2.11)$$

From eq. (2.11) one recognizes that in these particular Green's functions only lowest components of chiral superfields ($\lambda\lambda$ and $\tilde{\phi}\phi$) are allowed. The Green's functions we are considering enjoy the remarkable property of being independent of space-time as a consequence of SUSY.

In fact, if A_i and χ_i are components of chiral superfields ($\Xi_i = A_i + \sqrt{2}\theta\chi_i + \dots$) one has

$$\begin{aligned} (\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} \frac{\partial}{\partial x_i^\mu} G(x_1, \dots, x_m) &\equiv \partial_i^{\dot{\alpha}\alpha} \langle 0 | T(A_i(x_1) \dots A_m(x_m)) | 0 \rangle = \\ &= \langle 0 | T(A_i(x_1), \dots, \{ \bar{Q}^{\dot{\alpha}}, \frac{1}{\sqrt{2}} \chi(x_i) \}, \dots, A_m(x_m)) | 0 \rangle = 0 \end{aligned} \quad (2.12)$$

The last equality follows if $\bar{Q}|0\rangle = 0$ and $[A_i(x_i), A_j(x_j)]_{\text{e.t.}} = 0$, since $[\bar{Q}, A_i] = 0$.

*) From eqs. (2.11), it appears that there is no solution for $M \geq N$ (since $n_\lambda \geq 0$). However, as discussed in section 5, one can also deal with this situation in the massive case [18].

As a consequence of the previous observations we can argue that these constant Green's functions can be reliably computed in the semiclassical instanton approximation (see discussion in section 1). In fact, if at short distances ($|x_i - x_j| \ll \ll \Lambda_{\text{SQCD}}^{-1}$) where instanton calculations are trustworthy, we find for G a non-zero finite space-time independent result, then we must identify it with the expected constant supersymmetric value of G .

In section 4 we will give various examples of Green's functions obeying eqs. (2.11) in the $v = 1$ case and we will show that in the one-instanton approximation they are in fact non-zero, finite, and constant after the instanton collective coordinate integrations. It is very remarkable that in SUSY gauge theories the integration over the dilatation collective coordinate is, unlike the typical QCD case, infrared convergent [8].

Obviously not all Green's functions are expected to be constants. If one does not impose eq. (2.10), on purely dimensional grounds one immediately concludes that instanton contributions to Green's function behave at short distances as $\Lambda^{\beta_1} / |x_i - x_j|^{4m_\psi - 2n_F}$. This result seems to indicate the existence of contributions to the light-cone Wilson expansion different from those guessed from perturbation theory. This point will be discussed in Section 4.

The preceding analysis refers to the strictly massless case where the selection rules (2.7) are exact. For the massive case it is important to recall what can be said in general on the mass dependence of various correlation functions.

Without lack of generality, we can take the mass matrix in eq. (2.2d) to be diagonal by redefining the superfields ϕ_i, ϕ_j i.e.:

$$m_i^j = m_i \delta_{ij}, \quad m_i^{*j} = m_i^* \delta_{ij} \quad (2.13)$$

while keeping in general $m_i \neq m_i^*$.

As far as the x -dependence is concerned, eq. (2.12) continues to be obviously true. In order to get the mass dependence we follow (and restate) the arguments of ref. [5].

Consider again the Green's functions made out of lowest components A_i of chiral superfields. It is easy to show that they are m^* -independent. In fact using (2.2c),

$$\begin{aligned} \frac{\partial}{\partial m_i^*} G(x_1, \dots, x_n) &\equiv \frac{\partial}{\partial m_i^*} \langle 0 | T(A_1(x_1) \dots A_n(x_n)) | 0 \rangle = \\ &= \int d^4y \langle 0 | T(A_1(x_1) \dots A_n(x_n)) d^2\bar{\theta} \bar{\Phi}^{a_i} \tilde{\Phi}_{a_i}(y, \bar{\theta}) | 0 \rangle = \\ &= \int d^4y \langle 0 | T(A_1(x_1) \dots A_n(x_n)) \{ \bar{Q}, \bar{\Phi}^{a_i} \tilde{\Phi}_{a_i}(y, \bar{\theta}) \}_{\bar{\theta} \text{ component}} | 0 \rangle = 0 \end{aligned} \quad (2.14)$$

Using this result the m_i -dependence of G can be obtained from a non-anomalous chiral Ward identity (i not summed)

$$\begin{aligned} m_i \frac{\partial}{\partial m_i} G(x_1, \dots, x_n) &= \left(m_i \frac{\partial}{\partial m_i} - m_i^* \frac{\partial}{\partial m_i^*} \right) G = \\ &= \langle 0 | T(A_1(x_1) \dots A_n(x_n)) \int d^4y (m_i \tilde{\psi}^i \psi_i - m_i^* \bar{\psi}^i \bar{\psi}_i) | 0 \rangle \end{aligned} \quad (2.15)$$

To go on we now need to construct, for each i , a non-anomalous axial transformation which is only broken by the i^{th} mass. The unique choice is

$$\hat{U}_A^{(i)}(1): \lambda \rightarrow e^{-i\frac{\alpha_i}{N}} \lambda; (\psi_j, \tilde{\psi}_j) \rightarrow e^{i\alpha_j} (\psi_j, \tilde{\psi}_j); (\bar{\psi}_j, \bar{\tilde{\psi}}_j) \rightarrow e^{i(\alpha_j - \frac{1}{N})} (\bar{\psi}_j, \bar{\tilde{\psi}}_j) \quad (2.16)$$

The zero-momentum insertion in eq. (2.15) is the integral of the divergence of the current associated with $\hat{U}_A^{(i)}(1)$. Since $\hat{U}_A^{(i)}(1)$ is explicitly broken by $m_i \neq 0$ and there are no anomaly-free $U_A(1)$'s not broken by masses, we can safely assume that there are no massless particles. Thus by standard current algebra techniques we get

$$m_i \frac{\partial}{\partial m_i} G = -\frac{1}{2} G \cdot \sum_{\ell} \hat{\chi}_i(A_{\ell}) \quad (2.17)$$

where $\chi_i(A_{\ell})$ are the $\hat{U}_A^{(i)}(1)$ chiralities of the various operators A_{ℓ} appearing in G with the normalization of the table: $\hat{\chi}_i(\lambda\lambda) = -2/N$. Clearly the combination of eqs. (2.14) and (2.17) gives us the complete mass dependence of G .

3. A PRIMER FOR INSTANTON CALCULATIONS IN SU(N) MASSLESS SQCD

In order to explain the full consequences and the consistency of non-perturbative instanton calculations in SUSY theories, it is first of all necessary to extend to any SU(N) gauge group the formalism already developed for SU(2) in the case of pure SYM theory as well as in the case in which M matter supermultiplets in the fundamental colour representation are present (SQCD).

This is most easily done by employing the powerful and elegant formalism set up in ref. [15]^{*)}, which is indeed applicable to any winding number sector and provides explicit expressions for the fermion zero modes in any colour representation. To cope more easily with this approach, we have found it more useful to work in components rather than in an explicitly SUSY invariant form.

For completeness we give below a brief "compendium" of the formulae necessary for our subsequent calculations, without ever pretending to be complete or rigorous. All the theoretical proof and details can be found in the original literature [15]. We also restrict ourselves to the winding number $\nu = 1$ sector of the theory.

The fundamental object to be considered is the $(N+2) \times N$ matrix in colour space

$$V(x) = f^{1/2} \left(\begin{array}{cc|c} \overbrace{\begin{matrix} -\rho & 0 \\ 0 & -\rho \end{matrix}}^2 & & \overbrace{\begin{matrix} & & & & \end{matrix}}^{N-2} \\ \hline x_L^+ & x_T^+ & 0 \\ -x_T^- & x_L^- & \\ \hline 0 & & f^{-1/2} \mathbb{I}_{(N-2) \times (N-2)} \end{array} \right) \equiv f^{1/2} \left(\begin{array}{ccc} -\rho & 1 & 0 \\ \bar{x} & 1 & \\ 0 & 1 & f^{-1/2} \end{array} \right) \quad (3.1)$$

^{*)} We are particularly indebted to S. Templeton for patiently explaining to us this approach as well as for many useful conversations.

where

$$f(x) = [(x-a)^2 + \rho^2]^{-1} \quad (3.2a)$$

$$x_L^\pm = (x_0 - a_0) \pm i(x_3 - a_3) \quad (3.2b)$$

$$x_T^\pm = (x_2 - a_2) \pm i(x_1 - a_1) \quad (3.2c)$$

a_μ and ρ are the translation and dilatation instanton collective coordinates^{*)}.

In the second equality of eq. (3.1) we have used a simplified block notation with

$$\bar{x} \equiv \bar{\sigma}_\mu (x_\mu - a_\mu) \quad (3.3a)$$

$$(\bar{\sigma}_\mu)^{\alpha\alpha} = (\mathbb{1}_{2 \times 2}, i\vec{\sigma}) \quad (3.3b)$$

where the $\vec{\sigma}$'s are the Pauli matrices. We also define for future use^{**)}

$$x \equiv \sigma_\mu (x_\mu - a_\mu) \quad (3.4a)$$

$$(\sigma_\mu)_{\alpha\alpha} = (\mathbb{1}, -i\vec{\sigma}) \quad (3.4b)$$

Notice that

$$V^+(x) V(x) = \mathbb{1}_{N \times N} \quad (3.5)$$

while

$$V(x) V^+(x) = f \left(\begin{array}{cccc|c} \rho^2 & 0 & -\rho x_L^- & \rho x_T^+ & \\ 0 & \rho^2 & -\rho x_T^- & -\rho x_L^+ & 0 \\ -\rho x_L^- & -\rho x_T^+ & x^2 & 0 & \\ \rho x_T^- & -\rho x_L^+ & 0 & x^2 & \\ \hline 0 & & & & f^{-1} \mathbb{1}_{N \times N} \end{array} \right) = f \left(\begin{array}{ccc|c} \rho^2 & -\rho x_L & & 0 \\ -\rho x_T & x^2 & & \\ \hline 0 & & & f^{-1} \end{array} \right) \quad (3.6)$$

*) In the following, for simplicity of notation the translation collective coordinate a_μ will often be understood when no confusion can arise.

**) We use throughout this paper a Euclidean version of the Wess-Bagger [19] notations.

Using the matrices $V(x)$ and $V^+(x)$ one has for the one-instanton field written as an $N \times N$ matrix in colour space:

$$V_\mu^I(x) = V^+ \partial_\mu V = 2f(x) x_\nu \sigma_{\mu\nu} \quad (3.7a)$$

$$F_{\mu\nu}^I(x) = 4g^2 f(x)^2 \sigma_{\mu\nu} \quad (3.7b)$$

with

$$\sigma_{\mu\nu} = \frac{1}{4} (\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu) \quad (3.8)$$

and for the covariant derivatives in the background instanton field

$$D_\mu^{(N)} = (\partial_\mu + V^+ \partial_\mu V) = V^+ \partial_\mu (V) \equiv D_\mu \quad (3.9)$$

and

$$D_\mu^{(\bar{N})} = (\partial_\mu - (V^+ \partial_\mu V)^T) = (V^*)^+ \partial_\mu (V^*) \quad (3.10)$$

in the colour representation N and $\bar{N}$, respectively.

The normalized $2N$ gluino zero mode in the one-instanton background field are given by the formula

$$\lambda_{0,\alpha}^{r,R} = \frac{1}{\pi} f V^+ (c^{r,R} b_\alpha^+ - b_\alpha (c^{r,R})^+) V \quad (3.11)$$

where the indices $r = 1, 2, \dots, N$ and $R = 1, 2$ specify the $2N$ independent solutions. Remember that the gluinos are Majorana spinors (for which we adopt the Weyl representation, $\alpha = 1, 2$) belonging to the adjoint representation of the colour group. In eq. (3.11) they are written as $N \times N$ matrices in colour space with a factor of g absorbed in their definition (as for the gauge field). The two-vector b^α and the $2N$ -vector $c^{r,R}$ are specified below:

$$b^1 = c^{1,1} = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{array} \right) \left\{ \begin{array}{l} 4 \\ \\ \\ N-2 \end{array} \right. \quad b^2 = c^{1,2} = \left(\begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{array} \right) \left\{ \begin{array}{l} 4 \\ \\ \\ N-2 \end{array} \right. \quad (3.12)$$

$$C^{2,1} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad C^{2,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \quad (3.13a)$$

(3.13b)

$$C^{i,1} = C^{i,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \quad i = 3, \dots, N$$

← at the $i+2$ place

(3.13c)

The corresponding row vectors are indicated by b_α^+ and c_R^{r+} . The indices α and R are lowered and raised by the two-dimensional antisymmetric Levi-Civita tensor with the conventions ($\epsilon^{12} = -\epsilon_{12} = 1$, $\epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta_\gamma^\alpha$)

$$\begin{cases} b_\alpha = \epsilon_{\alpha\beta} b^\beta \\ c_R^r = \epsilon_{RR'} c^{rR'} \end{cases}, \quad \begin{cases} b^{+\alpha} = b_\beta^+ \epsilon^{\beta\alpha} \\ c^{+rR} = c_{R'}^{r+} \epsilon^{R'R} \end{cases} \quad (3.14)$$

The $\lambda_o^{r,R}$ of eq. (3.11) are normalized according to

$$\int d^4x \text{Tr} [\bar{\lambda}_{o\dot{\alpha}}^{r,R} (\sigma_0)^{\dot{\alpha}\alpha} \lambda_{o,\alpha}^{r,R}] = 1 \quad (3.15)$$

where

$$\bar{\lambda}_{o\dot{\alpha}}^{r,R} (\sigma_0)^{\dot{\alpha}\alpha} = \frac{1}{\pi} \int V^\dagger (b^\alpha c_R^{r+} - c_R^r b^{\alpha+}) V = -\lambda_{bR}^{r\alpha} \quad (3.16)$$

and the trace is over colour indices.

For the spinor zero modes in the N and $\bar{N}$ representation one has, respectively:

$$\psi_o^\alpha = \frac{1}{\pi} \int V^\dagger b^\alpha \quad (3.17a)$$

$$\tilde{\psi}_{o\alpha} = \frac{1}{\pi} \int b_\alpha^+ V \quad (3.17b)$$

for each flavour index with the normalization condition [notice that $\bar{\psi}_{0\dot{\alpha}}(\bar{\sigma}_0)^{\dot{\alpha}\alpha} = \bar{\psi}_0^\alpha$]:

$$\int d^4x \bar{\psi}_{0\dot{\alpha}}(\bar{\sigma}_0)^{\dot{\alpha}\alpha} \psi_{0\alpha} = \int d^4x \bar{\psi}_{0\dot{\alpha}}(\bar{\sigma}_0)^{\dot{\alpha}\alpha} \tilde{\psi}_{0\alpha} = 1 \quad (3.18)$$

For reasons already made clear in refs. [8] and [10] (see Appendices A and B) one also needs to solve to lowest non-trivial order in the coupling constant the classical field equations for the scalar fields ϕ and $\tilde{\phi}$ (SUSY partners of ψ and $\tilde{\psi}$) in the one-instanton background field, namely the equations:

$$D^2 \varphi^{r,R} = -i\sqrt{2} \lambda_0^{r,R\alpha} \varphi_{0\alpha} \quad (3.19a)$$

$$(D^2 \tilde{\varphi}^{r,R*})^* = -i\sqrt{2} \tilde{\varphi}_0^\alpha \lambda_{0\alpha}^{r,R} \quad (3.19b)$$

Using the explicit expression [15] for the inverse of D^2 :

$$(D^2)^{-1}_{(x,y)} = - \frac{V(x)^\dagger U(y)}{4\pi^2 (x-y)^2} \quad (3.20)$$

and eqs. (3.11) and (3.17) a straightforward calculation gives (again for each flavour index):

$$\varphi_0^{r,R} = \frac{i\sqrt{2}}{4\pi^2} \int V^\dagger C^{r,R} \quad (3.21a)$$

$$\tilde{\varphi}_0^{r,R} = \frac{i\sqrt{2}}{4\pi^2} \int C^{r,R\dagger} V \quad (3.21b)$$

In this formalism one can also give an explicit closed form for the inverse of the operators $\mathcal{D} = \sigma_\mu D_\mu$ and $\bar{\mathcal{D}} \equiv \bar{\sigma}_\mu D_\mu$. Let us, in fact, consider the two sets of equations:

$$\mathcal{D}_{\alpha\dot{\beta}} \{ \}^\beta_n = k_n \gamma_{n\alpha} \quad n = 1, 2, \dots \quad (3.22a)$$

$$\bar{\mathcal{D}}^{\dot{\alpha}\beta} \gamma_{m\beta} = \bar{k}_m \{ \}^{\dot{\alpha}}_m \quad m = 0, 1, 2, \dots \quad (3.22b)$$

with $\bar{k}_0 = 0$ and $\eta_0 = \psi_0$ as given by eq. (3.17a). We recall that in a self-dual background field there exists no μ_0 , i.e. no zero mode for the operator $\not{D}$, so that only in the completeness relation for the η 's does the $n = 0$ mode appear. Then

the solution of the equations

$$\begin{aligned} (\not{D}_{\alpha\beta} S^{\beta\beta})_{(x,y)} &= \delta_{\alpha}^{\beta} \delta(x-y) - \gamma_{0\alpha}^{(x)} \tilde{\gamma}_0^{\beta}(y) \\ (\not{D}^{\dot{\alpha}\beta} \bar{S}_{\beta\dot{\beta}})_{(x,y)} &= \delta_{\dot{\beta}}^{\dot{\alpha}} \delta(x-y) \end{aligned} \quad (3.23)$$

can be seen to have the following expression:

$$\begin{aligned} S^{\dot{\alpha}\alpha}_{(x,y)} &\equiv \sum_{n=1}^{\infty} \frac{\bar{\gamma}_{n\dot{\alpha}}^{(x)} \gamma_n^{\alpha}(y)}{k_n} = \frac{V_{(x)}^+ V_{(y)} (x-y)_{\mu} (\bar{\sigma}_{\mu})^{\dot{\alpha}\alpha}}{2\pi^2 (x-y)^4} + \\ &+ \frac{V_{(x)}^+ (x-y)_{\mu} (\bar{\sigma}_{\mu})^{\dot{\alpha}\beta} b_{\beta} f(y) b^{\alpha+} V_{(y)}}{2\pi^2 (x-y)^2} \end{aligned} \quad (3.24a)$$

$$\begin{aligned} \bar{S}_{\alpha\dot{\alpha}}(x,y) &\equiv \sum_{n=1}^{\infty} \frac{\gamma_{n\alpha}^{(x)} \bar{\gamma}_{n\dot{\alpha}}^{*(y)}}{k_n} = \frac{V_{(x)}^+ V_{(y)} (x-y)_{\mu} (\sigma_{\mu})_{\alpha\dot{\alpha}}}{2\pi^2 (x-y)^4} + \\ &+ \frac{V_{(x)}^+ b_{\alpha} f(x) (x-y)_{\mu} b^{\beta+} (\sigma_{\mu})_{\beta\dot{\alpha}} V_{(y)}}{2\pi^2 (x-y)^2} \end{aligned} \quad (3.24b)$$

where the $*$ means conjugation in colour space.

As noted by several authors [20], the scalar propagator $(D^2)^{-1}$, and the fermion propagators $S = \not{D}^{-1}$ and $\bar{S} = \bar{\not{D}}^{-1}$ in an instanton background field decrease at large $|x-y|$ separations more slowly than the corresponding free propagators, thus leading to more severe infrared problems.

In unified theories, matter superfields belonging to representations of the gauge group other than the fundamental or the adjoint may appear in the Lagrangian. Expressions like (3.17) and (3.21) can be generalized to various cases, although the formulae tend immediately to become rather cumbersome.

As an example, we give below some equations useful in the particularly interesting case of the Georgi-Glashow SUSY chiral model [21,12]. In this model one

also needs to consider matter in the antisymmetric colour representation. There are $N-2$ spinor zero modes in this representation given up to a normalization constant by ($t = 3, 4, \dots, N$)

$$\left(\psi_0^{t,x} \right)_{ab} = f \left[(V^+ b^x)_a (V^+ c^{t,1})_b - (a \leftrightarrow b) \right] \quad (3.25a)$$

$$\left(\tilde{\psi}_0^t \right)^{ab} = f \left[(b_a^+ V)^a (c_i^{t+} V)^b - (a \leftrightarrow b) \right] \quad (3.25b)$$

Correspondingly, for the scalars one finds, using the analogue of eqs. (3.19)

($R = 1, 2; r = 1, \dots, N; t = 3, 4, \dots, N$):

$$\left(\varphi_0^{r,R,t} \right)_{ab} = f \left[(V^+ c^{r,R})_a (V^+ c^{t,1})_b + (-1)^R \delta_{rt} (V^+ b^x)_a (V^+ b^x)_b - (a \leftrightarrow b) \right] \quad (3.26a)$$

$$\left(\hat{\varphi}_0^{r,R,t} \right)^{ab} = f \left[(c_R^{r+} V)^a (c_i^{t+} V)^b - (-1)^R \delta_{rt} (b^{+x} V)^a (b_a^{+x} V)^b - (a \leftrightarrow b) \right] \quad (3.26b)$$

In all the formulae given above one can notice the special role played by the first four entries of matrices and vectors acting in colour space. This is due to the fact that the instanton solutions essentially reside in an $SU(2)$ subgroup of the gauge group.

4. INSTANTON COMPUTATION OF GREEN'S FUNCTIONS AND VACUUM PROPERTIES IN MASSLESS SUSY THEORIES

In this section we apply the formalism presented in section 3 to compute the Green's functions that, as discussed in sections 1 and 2, are dominated by instanton contributions.

We will first discuss the case of SYM, then the case of massless matter with real (SQCD) and complex fermions (supersymmetric Georgi-Glashow $SU(5)$ model).

4.1 SYM

We consider in this case the correlation function

$$G_N^0 = \langle 0 | \lambda \lambda(x_1) \lambda \lambda(x_2) \dots \lambda \lambda(x_N) | 0 \rangle \quad (4.1)$$

where $\lambda\lambda$ is shorthand for $\epsilon^{\alpha\beta} \lambda_a^\alpha \lambda_b^\beta$ and we remember that a factor of g has been included in our definition of λ . In pure SYM theory with an $SU(N)$ gauge group (4.1) is the generalization of the equation studied in ref. [8] for $SU(2)$. This correlation function is expected to be constant, as explained in section 2, and receives a contribution from the $v = 1$ sector.

In Appendix A we have shown that in the semiclassical one-instanton approximation G_N^0 can be expressed in terms of the $2N$ gluino's zero modes given in formula (3.11), through eq. (A.14). We need then to compute the gauge invariant composite operator $\lambda\lambda$ for each pair of the zero modes given in eq. (3.11). It is easy to see that [eq. (3.2a)]:

$$\begin{aligned} \lambda_0^{1,1} \lambda_0^{1,2}(x) &= 2f(x) \rho^4 \\ \lambda_0^{2,1} \lambda_0^{2,2}(x) &= f(x) \rho^2 x^2 \\ \lambda_0^{1,1} \lambda_0^{2,1}(x) &= -\sqrt{2} f(x) \rho^3 x_T^- \\ \lambda_0^{1,1} \lambda_0^{2,2}(x) &= -\sqrt{2} f(x) \rho^3 x_L^+ \\ \lambda_0^{1,2} \lambda_0^{2,1}(x) &= \sqrt{2} f(x) \rho^3 x_L^- \\ \lambda_0^{1,2} \lambda_0^{2,2}(x) &= -\sqrt{2} f(x) \rho^3 x_T^+ \\ \lambda_0^{r,R} \lambda_0^{s,S}(x) &= 0 \quad r=3,4,\dots,N \quad ; \quad s=1,2 \\ \lambda_0^{r,R} \lambda_0^{s,S}(x) &= \delta_{rs} \epsilon_{RS} f(x) \rho^2 \quad r,s=3,4,\dots,N \end{aligned}$$

(4.2)

It follows that (N-2) of the $\lambda\lambda$ pairs have to be given by the zero modes with $r, s = 3, 4, \dots, N$, while the remaining two are given by the zero modes with $r, s = 1, 2$. Since

$$\sum_{\substack{r, r', s, s' = 1, 2 \\ R, R', S, S'}} \lambda_{0, (X_1)}^{r, R, s, S} \lambda_{0, (X_2)}^{r', R', s', S'} = f^4(X_1) f^4(X_2) \rho^6(X_1 - X_2)^2 \quad (4.3)$$

inserting expressions (4.2) and (4.3) in eq. (A.14) gives, up to an overall multiplicative constant:

$$\begin{aligned} G_N^0 &= \Lambda^{3N} \int \frac{d^4 a d\rho}{\rho^5} (\rho^2)^{2N} \left[(X_1 - X_2)^2 \rho^6 f^4(X_1) f^4(X_2) (\rho^2)^{N-2} \prod_{i=3}^N f^3(X_i) + \text{permutations} \right] \\ &= \Lambda^{3N} \int d^4 a d\rho^2 (\rho^2)^{3N-2} \sum_{i \neq j} \left[(X_i - X_j)^2 f^4(X_i) f^4(X_j) \prod_{k \neq i, j} f^3(X_k) \right] \end{aligned} \quad (4.4)$$

This integral can easily be evaluated by looking at $f(x)$ as a particle propagator and introducing Feynman parameters, thus obtaining:

$$\begin{aligned} G_N^0 &\propto \Lambda^{3N} \int \prod_{i=1}^N d\alpha_i \delta(1 - \sum \alpha_i) \int d^4 a d\rho^2 (\rho^2)^{3N-2} \frac{\prod_i \alpha_i^2 \sum_{j \neq k} (X_j - X_k)^2 \alpha_j \alpha_k}{\left[a^2 + \rho^2 + \sum \alpha_i X_i^2 - (\sum \alpha_i X_i)^2 \right]^{3N+2}} \\ &\propto \Lambda^{3N} \int \prod d\alpha_i \delta(1 - \sum \alpha_i) \frac{\prod \alpha_i^2 \sum_{j \neq k} (X_j - X_k)^2 \alpha_j \alpha_k}{\sum \alpha_i X_i^2 - (\sum \alpha_i X_i)^2} = C_N^0 \Lambda^{3N} \quad (4.5) \end{aligned}$$

In eq. (4.5) C_N^0 is indeed a constant since the expression in square brackets in the second equality is precisely 1 due to the constraint $\sum \alpha_i = 1$.

As explained before, the result of eq. (4.5) implies that the order parameter $\lambda\lambda$ has a non-zero vacuum expectation value: $\langle \lambda\lambda \rangle \neq 0$. As recalled in section 2, the theory possesses a Z_{2N} invariance [the discrete subgroup of the $U_\lambda(1)$ symmetry,

preserved by instantons]. Since $\langle \lambda \lambda \rangle \neq 0$ breaks Z_{2N} spontaneously down to Z_2 , this implies the existence of N degenerate vacua characterized by:

$$\langle \lambda \lambda \rangle_k = (C_N^0)^{1/N} \Lambda^3 \exp \frac{2\pi i k}{N} \quad k=1, 2, \dots, N \quad (4.6)$$

This is in agreement with the value of the Witten's index [1] $\Delta = N$ and with the effective Lagrangian analysis [2]. The instanton calculation (4.5), being Z_{2N} invariant corresponds to averaging over the N vacua. They contribute equally to G_N^0 since from eq. (4.6)

$$\left(\langle \lambda \lambda \rangle_k \right)^N = C_N^0 \Lambda^{3N} \quad \text{independently of } k \quad (4.7)$$

We shall see below cases in which this averaging over degenerate vacua has less trivial effects.

4.2 SQCD

The Green's functions to be considered are now [10] [see eq. (2.11)]

$$\left(G_N^M \right)_{j_1 \dots j_M}^{i_1 \dots i_M} = \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_M) \tilde{\psi}^{i_1} \psi(y_1) \dots \tilde{\psi}^{i_M} \psi(y_M) | 0 \rangle \quad (4.8)$$

where i and j are flavour indices and, as usual, colour and spin indices are understood.

As explained in Appendix A:

$$\left(G_N^M \right)_{j_1 \dots j_M}^{i_1 \dots i_M} = \varepsilon^{i_1 \dots i_M} \varepsilon_{j_1 \dots j_M} G_N^M \quad (4.9)$$

where G_N^M is given by eq. (A.25) with λ_0 , $\tilde{\phi}_0$ and ϕ_0 given by eqs. (3.11) and (3.21).

The calculation of G_N^M parallels that of G_N^0 described above. The expression of $\tilde{\phi}_0^{r,R} \phi_0^{s,S}(x)$ is identical with that of $\lambda_0^{r,R} \lambda_0^{s,S}(x)$, as given by eqs. (4.2), apart from an overall factor of $1/6$. This means that G_N^M has a form analogous to eq. (4.4), containing, besides $(x_i - x_j)^2$, terms proportional to $(x_i - x_j)^2$ and $(y_i - y_j)^2$ with different coefficients. After going over to a Feynman parametrization of the integral, these coefficients turn out to be precisely the right ones to give the

obvious generalization of the square bracket appearing in eq. (4.5), which is again equal to 1. We thus conclude that the single instanton contribution gives:

$$\left(\Gamma_N^M \right)_{j_1 \dots j_M}^{i_1 \dots i_M} = \varepsilon^{i_1 \dots i_M} \varepsilon_{j_1 \dots j_M} \Lambda^{3N-M} C_N^M \quad (4.10)$$

with C_N^M a known, non-zero constant.

The interpretation of eq. (4.10) (remember that we are in the massless case) is that, by clustering at $|x_i - x_j|, |x_i - y_j|, |y_i - y_j| \rightarrow \infty$, $\lambda\lambda$ and $\tilde{\phi}^i \phi_j$ acquire a non-zero v.e.v. Let us discuss properties and problems of this interpretation.

First of all, the lack of factorization of the flavour structure in eq. (4.10) can be attributed to a spontaneous symmetry-breaking mechanism of flavour symmetry. The instanton calculation (as in the SYM case for Z_{2N}) is again averaging over all the degenerate vacua which are connected by the symmetry transformations. In the case at hand, there is an $SU(M) \times SU(M)$ non-anomalous symmetry and the $\varepsilon \cdot \varepsilon$ structure of eq. (4.10) is just the one expected by averaging over $SU(M) \times SU(M)$ -related vacua.

In fact, if in one reference vacuum we write:

$$\langle 0 | \tilde{\phi}^i \phi_j | 0 \rangle = v^i_j; \quad \langle 0 | \lambda \lambda | 0 \rangle = u \quad (4.11)$$

in a transformed one we have

$$\begin{aligned} \langle \Omega | \tilde{\phi}^i \phi_j | \Omega \rangle &= \Omega^i_{i'} v^{i'}_{j'} \tilde{\Omega}^{j'}_j; \\ \langle \Omega | \lambda \lambda | \Omega \rangle &= \langle 0 | \lambda \lambda | 0 \rangle = u \end{aligned} \quad \Omega, \tilde{\Omega} \in SU(M) \quad (4.12)$$

It follows that

$$\begin{aligned} \sum \langle \Omega | \lambda \lambda(x_1) \dots \lambda \lambda(x_{N-M}) \tilde{\phi}^{i_1} \phi_{j_1}(y_1) \dots \tilde{\phi}^{i_M} \phi_{j_M}(y_M) | \Omega \rangle &= \\ = \int d\Omega \Omega^i_{i_1} \dots \Omega^i_{i_M} \int d\tilde{\Omega} \tilde{\Omega}^{j_1}_{j_1'} \dots \tilde{\Omega}^{j_M}_{j_M'} v^{i_1'}_{j_1'} \dots v^{i_M'}_{j_M'} u^{N-M} &= \\ = \varepsilon^{i_1 \dots i_M} \varepsilon_{j_1 \dots j_M} u^{N-M} \det v \end{aligned} \quad (4.13)$$

This has exactly the flavour structure of eq. (4.10) and leads to the identification

$$u^{N-M} \det v = C_N^M \Lambda^{3N-M} \quad (4.14)$$

A serious problem with the above interpretation is related to the fact that the anomalous commutator [6]:

$$\{\phi, \bar{\psi} \phi\} = m \bar{\psi} \psi + \frac{1}{32\pi^2} \lambda\lambda \quad (4.15)$$

implies at $m = 0$:

$$\langle \lambda\lambda \rangle = 0 \quad (4.16)$$

in a SUSY vacuum.

The result (4.16) is incompatible with eq. (4.14), except in the improper limiting situation $v_j^i = \infty$. If one does not accept this as a possible value for $\langle \bar{\phi}^i \phi_j \rangle$, one has to conclude that, in the massless case we are considering, there is a clash between SUSY and instanton calculations. In section 5 we will see that in the massive case there is no such problem, as $\langle \lambda\lambda \rangle$ needs not to vanish as a consequence of eq. (4.15). Yet the values of v_j^i run off to infinity [3,5,10] in the limit $m_j^i \rightarrow 0$.

2) M = N Another special situation in which this problem does not arise is the case with $M = N$, where in the relevant Green's function [see eq. (2.11)]

$$(G_N^N)_{j_1 \dots j_N}^{i_1 \dots i_N} = \langle 0 | \bar{\psi}^{i_1} \psi_{j_1}(x_1) \dots \bar{\psi}^{i_N} \psi_{j_N}(x_N) | 0 \rangle \quad (4.17)$$

no $\lambda\lambda$ operator appears. Following the discussion just given we find:

$$(G_N^N)_{j_1 \dots j_N}^{i_1 \dots i_N} \propto \epsilon^{i_1 \dots i_N} \epsilon_{j_1 \dots j_N} \Lambda^{2N} \quad (4.18)$$

The situation is analogous to the one discussed before for $N > M$, except that eq. (4.14) is replaced by

$$\det v = C_N^N \Lambda^{2N} \quad (4.19)$$

which is not incompatible with eq. (4.16). In this case, we can therefore conclude safely that instanton effects spontaneously break the $SU(N) \otimes SU(N)$ flavour symmetry.

Since the full degeneracy of the perturbative vacuum is lost (we lose, for instance, the vacuum with $\langle \tilde{\phi}^i \rangle = \langle \phi_j \rangle = 0$) the non-renormalization theorem is violated^{*}). Furthermore, we disagree with the conclusions of ref. [11], in which the full degeneracy of perturbative vacua is claimed to survive instanton effects.

The case $N = M$ has also been considered for some peculiar virtues it has in connection with composite models of quarks and leptons [22]. One important question arising there was whether the $U_V(1)$ symmetry is spontaneously broken or not. We can answer that question here by considering the two-point function:

$$G = \langle 0 | \det_{i,a} \tilde{\psi}^{i,a}(x) \cdot \det_{j,b} \psi_{j,b}(y) | 0 \rangle \quad (4.20)$$

where colour and flavour indices are combined to construct an overall singlet.

A straightforward calculation (similar to the one of G_N^N) gives

$$G = \text{const.} \Lambda^{2N} \quad (4.21)$$

We thus conclude that

$$\langle \det \psi \rangle \neq 0, \langle \det \tilde{\psi} \rangle \neq 0 \quad (4.22)$$

which means (at $m = 0$) spontaneous breaking of $U_V(1)$ (as assumed in ref. [22]), since $\det \tilde{\psi}$ and $\det \psi$ have $U_V(1)$ charges $-N$ and N , respectively.

3) $M > N$ In this case, there are no Green's functions that can receive contributions from instantons. In the spirit of our philosophy, which is to deduce vacuum properties by clustering of instanton amplitudes, we should conclude that $\langle \tilde{\phi}^i \phi_j \rangle = \langle \lambda \lambda \rangle = 0$. We will have a better understanding of this conclusion when treating the massive case. There we will find that $\tilde{\phi} \phi$ and $\lambda \lambda$ indeed acquire v.e.v.'s, which, however, vanish in the limit in which all masses go to zero together.

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^{*}) Some degeneracy beyond the one implied by the $SU(N) \otimes SU(N)$ symmetry may persist, since eq. (4.19) only fixes the value of $\det v$.

4.3 Matter with fermions in complex representations

As another example of constant non-vanishing correlation functions we shall consider the case studied in ref. [12] i.e. a generalized supersymmetric Georgi-Glashow SU(5) mode with one or two "families" of matter fields each one consisting of a $\bar{5}$ plus a 10 representation.

In the one-family case the correlation which we need to consider is the three-point function

$$G = \langle 0 | \lambda \lambda(x) \lambda \lambda(y) \chi(z) | 0 \rangle \quad (4.23)$$

where

$$\chi(z) = \varepsilon^{abcde} \lambda_a^{\alpha'} \lambda_{a'}^{b'} (\bar{5})^{c'} (10)_{b'c} (10)_{bc} (10)_{ca} \quad (4.24)$$

and $(\bar{5})^a$, $(10)_{ab}$ represent the scalar components of the corresponding matter superfields.

The fact that this correlation function receives a non-zero instanton contribution follows from the usual rule according to which every fermion zero mode must appear once and only once either explicitly or implicitly through its SUSY scalar partners in the way explained in Appendix A. In the present case besides 10 gluino zero modes, labelled by $r = 1, \dots, 5$; $R = 1, 2$, we have 1 zero mode for the spinor in the $\bar{5}$ representation and 3 for the spinor in the (antisymmetric) 10 representation, labelled by $t = 3, 4, 5$. Since each scalar counts, through eq. (A.24), as a gluino and a matter spinor zero mode, we need to have 3 scalars in the 10 representation, 1 in the $\bar{5}$ and $10-4 = 6$ more λ 's. Gauge invariance fixes finally the way colour indices have to be saturated.

Replacing in eq. (4.23) the gluinos by the formula (3.11), the $\bar{5}$ by (3.21) and the 10 by (3.23), in analogy with the previous cases, the calculation of G is, in principle, straightforward, but certainly tedious. We shall sketch here the calculation of the contribution to G which originates from inserting the λ -zero modes with $r = 1, 2$ in $\lambda\lambda(x)\lambda\lambda(y)$ of eq. (4.23). They produce the usual factor (4.3) which multiplies the contribution to χ coming from the λ -zero modes and the scalar solutions labelled by all the remaining indices. As usual, one has to add

up all permutations of the zero modes with the appropriate $(-1)^P$ sign. Many permutations give a vanishing contribution, but a few contribute and all of them with the same sign. Examples of non-vanishing contributions are:

$$\begin{aligned} \chi &\propto \lambda^{3,1} \lambda^{4,1} \bar{5}^{5,1} 10^{5,2,3} 10^{3,2,4} 10^{4,2,5} \\ \chi &\propto \lambda^{3,1} \lambda^{5,1} \bar{5}^{3,2} 10^{5,2,3} 10^{4,1,4} 10^{4,2,5} \end{aligned} \quad (4.25)$$

Each of them gives the same function of z :

$$\chi \Big|_{\substack{\Gamma=3,4,5 \text{ gluino} \\ \text{modes}}} \propto p^2 f(z) \quad (4.26)$$

which, when combined with formula (4.3), provides a term in G of the form

$$G \propto \Lambda^{13} \int \frac{d^4 x d^4 y}{p^5} p^{20} f(x) f(y) (x-y)^2 p^6 p^2 f(z) \quad (4.27)$$

This term is finite but not constant by itself. As in the previous cases, all other permutations of the zero modes are needed to produce the desired result, which comes out to be

$$G = c \cdot \Lambda^{13} \quad (4.28)$$

We find here again an incompatibility between eq. (4.28) and the result $\langle \lambda \lambda \rangle = 0$, implied by eq. (4.16), except if we take $\langle \chi \rangle = \infty$. This infinite v.e.v. is even less acceptable in this case than is massless SQCD. In fact, in SQCD the perturbative scalar superpotential possesses flat directions, i.e. minima in which the modulus of the scalar fields is arbitrary. In the present case, there are no flat directions and the perturbative vacuum is at zero value of the scalar fields. In ref. [12], excluding the possibility $\langle \chi \rangle = \infty$ on the basis of the previous argument, the clash between eqs. (4.28) and (4.16) has been taken as an indication of spontaneous SUSY breaking induced by instantons. The fact that matter appears in complex representations forbids the introduction of any mass term, which in SQCD

was a way to circumvent this problem. In the two-families case, one is led to consider correlation functions of the type:

$$G = \langle 0 | \lambda \lambda(x) Y(y) Z(z) | 0 \rangle \quad (4.29)$$

$$Y = \varepsilon^{abcde} (\bar{5}_1)^f (10_1)_{fa} (10_2)_{bc} (10_2)_{de} \quad (4.30a)$$

$$Z = \varepsilon^{abcde} (\bar{5}_2)^f (10_2)_{fa} (10_1)_{bc} (10_1)_{de} \quad (4.30b)$$

where the index 1, 2 on the $\bar{5}$'s and the 10's labels the families. By a reasoning similar to that described for the one-family case we arrive at the expression:

$$G \propto \Lambda'' \int \frac{d^4 x d^4 y d^4 z}{g^5} g^{20} f(x)^4 f(y)^4 f(z)^4 \left[g^4 f(y) (x-y)^2 + g^4 f(z) (x-z)^2 \right] + \text{permutations} = \mathcal{O}(\Lambda'') \quad (4.31)$$

Again there is a clash with $\langle \lambda \lambda \rangle = 0$ except for $\langle Y \rangle, \langle Z \rangle = \infty$. Here, as for massless SQCD and contrarily to the one-family case discussed above, this improper value cannot be *a priori* excluded owing to the presence of flat directions in the potential. However, this ceases to be the case if the interaction term

$$\mathcal{L}_{int.} = \sum_{i=1}^2 \eta_i \int d^2 \theta \bar{5}_1^a \bar{5}_2^b 10_{ab} \quad (4.32)$$

is introduced. But for $\eta_i \neq 0$ Konishi-type reasoning gives not only $\langle \lambda \lambda \rangle = 0$, but also $\langle Y \rangle = \langle Z \rangle = 0$ in a SUSY vacuum. One thus concludes that, if G of eq. (4.31) gets only corrections $\mathcal{O}(\eta)$ at (small) finite values of η_i , SUSY is spontaneously broken. Of course, this conclusion necessitates some kind of smoothness assumption in the behaviour of G when the couplings η_i are turned on. We shall see in the next section some examples in SQCD where a non-smooth behaviour is expected as a result of power behaved mass singularities, when masses are turned on. Hopefully, the situation here is safer since η is a dimensionless coupling constant. Thus the conclusions of ref. [12] are presumably correct, in agreement with the

constrained instanton calculations of ref. [11].

4.4 Absence of condensates carrying spin

Up to now, we have investigated properties of the vacuum by clustering amplitudes made up of gauge-invariant composite scalar operators constructed with the lowest components of chiral superfields. By studying the case in which composite operators carry spin, we want now to show that clustering of instanton contributions does not lead to any breaking of Lorentz invariance^{*)}.

We may indeed construct overall scalar correlation functions that might receive contributions from instantons, made up of products of local gauge-invariant operators that carry spin. Examples of this kind are:

$$\langle 0 | \text{Tr}(\lambda_\alpha \lambda_\alpha \lambda_\alpha \lambda_\alpha) \text{Tr}(\lambda_\beta \lambda_\beta \lambda_\beta) | 0 \rangle \varepsilon^{\alpha\beta} \quad \text{for } SU(3) \text{ SYM} \quad (4.33a)$$

$$\langle 0 | \tilde{\psi} \lambda_\alpha \psi(x) \tilde{\psi} \lambda_\beta \psi(y) | 0 \rangle \varepsilon^{\alpha\beta} \quad \text{for } SU(3) \text{ SQCD with } M=2 \quad (4.33b)$$

These amplitudes are expected to be $|x-y|$ -independent and therefore, if non-zero, they would imply the existence of a non-zero v.e.v. for a spin $3/2$ composite operator in the first case or for a spin $1/2$ composite operator in the second, which is physically untenable. Fortunately, the explicit instanton calculation, totally analogous to the ones described previously, gives an identically zero result for both amplitudes (4.33). Although this check was proved to hold in all cases we have tried, we have not been able to find a general proof of it.

4.5 Non-constant Green's functions

As the last point of this section we want to discuss the case of correlation functions which do not only contain lowest components of chiral superfields.

They are not expected to be constant, but still reliably calculable at short distances in the semiclassical one-instanton approximation. This calculation does not bear any information on their long distance behaviour and therefore on vacuum properties, but can provide information on the light-cone behaviour of the theory.

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*) We thank M. Testa for raising this question with us.

The calculation method developed in Appendix A is straightforwardly applicable also in this case. As before, all collective coordinate integrations are finite, thus leading to well-defined numerical results. For SU(2) we find, for instance:

$$\langle 0 | \text{Tr} (\lambda F_{\mu\nu} \sigma_{\mu\nu} \lambda(x)) \text{Tr} (\lambda \lambda(y)) | 0 \rangle = \frac{c \Lambda^6}{(x-y)^2} \quad (4.34a)$$

$$\langle 0 | \text{Tr} (\lambda F_{\mu\nu} \sigma_{\mu\nu} \lambda(x)) \text{Tr} (\lambda F_{\rho\tau} \sigma_{\rho\tau} \lambda(y)) | 0 \rangle = \frac{c' \Lambda^6}{(x-y)^4} \quad (4.34b)$$

Looking at eqs. (4.34), we may ask whether such non-perturbative results can be interpreted in terms of an operator product expansion. It seems to us that this is indeed the case, if one adds to the usual terms established looking at perturbation theory, other operators with non-perturbative coefficients (such as powers of Λ). Their chiralities are determined by the instanton selection rules. For instance, the analysis of various matrix elements of the operator in eq. (4.34a) leads to the following light-cone expansion:

$$\text{Tr} (\lambda F_{\mu\nu} \sigma_{\mu\nu} \lambda(x)) \text{Tr} (\lambda \lambda(y)) \underset{|x-y| \sim 0}{=} \frac{a_1 g^2}{(x-y)^2} \lambda \lambda \lambda \lambda(x) + \frac{a_2 \Lambda^6}{(x-y)^2} \mathbb{1} + \dots \quad (4.35)$$

where the a_i 's are calculable constants. The first term in the r.h.s of eq. (4.35) is the usual one expected on the basis of perturbation theory. The other is dictated by consistency with the value of instanton dominated matrix elements. Notice that in the matrix element of eq. (4.34) the instanton contribution is dominant (by a factor $1/g^2$) over the perturbative one.

Similarly, it is possible to analyse the short-distance behaviour of multi-local operators. We have found instanton contributions which imply new singularities which only occur when more than two points coalesce. For instance, we obtain:

$$\langle 0 | \lambda F(x) \lambda F(y) \lambda \lambda(z) | 0 \rangle \xrightarrow{x=y \rightarrow z} \frac{\Lambda^6}{(x-z)^6} \quad (4.36)$$

while the product of the three operators has a perturbative singularity even when two points coincide.

5. MASS DEPENDENCE AND FLAVOUR SYMMETRY

Treating the massless case we found that constant Green's functions with the appropriate "chiralities" were calculable in the semiclassical instanton approximation. The results obtained explicitly realize the chiral $SU(M) \times SU(M)$ flavour symmetry and are interpreted as the existence of a continuum of degenerate vacua connected by chiral transformations (spontaneous symmetry breaking).

If supersymmetric matter masses are introduced, the chiral $SU(M) \times SU(M)$ symmetry is explicitly broken. If all (flavour) masses are equal, a vector $SU(M)$ survives, while with different masses even this residual symmetry is broken down to M vector $U(1)$'s. Since the vacuum structure is correspondingly modified, correlation functions at large distances do change abruptly as masses are switched on. In turn, x -independence will imply that even at short distances the massless limit is non-smooth. Combining this with the peculiar mass dependence stemming from SUSY (see end of section 2) one is led to the expectation that, in general, the limit $m \rightarrow 0$ does *not* coincide with the massless case, even at short distances. This requires that mass corrections to the semiclassical instanton calculations of the preceding section are sometimes affected by power-like mass singularities whose origin we shall try to identify below.

An illuminating example of what is happening is given by the $N = M = 2$ case. Consider the Green's function

$$G_{j_1 j_2}^{i_1 i_2}(x_1, x_2) = \langle 0 | \tilde{\varphi}^{i_1} \varphi_{j_1}(x_1) \tilde{\varphi}^{i_2} \varphi_{j_2}(x_2) | 0 \rangle \quad (5.1)$$

as discussed before $G_{j_1 j_2}^{i_1 i_2}$ must be independent of $|x_1 - x_2|$, both in the massless and in the massive case. For the massless case in section 4, we have obtained:

$$G_{j_1 j_2}^{i_1 i_2 (m=0)} = c \Lambda^4 \varepsilon^{i_1 i_2} \varepsilon_{j_1 j_2} = c \Lambda^4 \left(-\delta_{j_1}^{i_1} \delta_{j_2}^{i_2} + \delta_{j_2}^{i_1} \delta_{j_1}^{i_2} \right) \quad (5.2)$$

i.e.

$$G_{1,2}^{1,2 (m=0)} = - G_{2,1}^{1,2 (m=0)} = - c \Lambda^4 \quad (5.3)$$

$$G_{1,1}^{1,1 (m=0)} = G_{2,2}^{2,2 (m=0)} = 0 \quad (5.4)$$

Let us now consider the massive case, denoting by m_1 and m_2 the two flavour masses. Let us first recall that, on the basis of the general argument given at the end of section 2, $G_{12}^{12(m_1, m_2)}$ and $G_{21}^{12(m_1, m_2)}$ belong to the class of Green's functions that are mass independent. Since for $m_1 \neq m_2$ the flavour symmetry is reduced to $U_V(1) \times U_V(1)$, we expect^{*}

$$\langle 0 | \tilde{\varphi}^1 \varphi_2 | 0 \rangle = 0 \quad (5.5)$$

implying via clustering

$$G_{12}^{1,2(m_1, m_2)} = c \quad (5.6a)$$

while

$$G_{2,1}^{1,2(m_1, m_2)} = 0 \quad (5.6b)$$

where c is a non-zero constant except if either $\langle \tilde{\phi}^1 \phi_1 \rangle$ or $\langle \tilde{\phi}^2 \phi_2 \rangle$ vanish. Clearly, the massless limit of the (mass-independent) eqs. (5.6) is different from the zero-mass result of eq. (5.3).

The difference between massive and massless results appears also for Green's functions that are not expected to be mass-independent. This is the case of G_{11}^{11} and G_{22}^{22} , which in the massless case are strictly zero, while in the massive case, using the Ward identity (2.17), are expected to behave as:

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^{*}) This is confirmed by the use of Konishi's anomaly arguments [eq. (4.15)].

$$G_{11}^{11(\mu_1, \mu_2)} = \frac{\mu_2}{\mu_1} c'' \quad , \quad G_{22}^{22(\mu_1, \mu_2)} = \frac{\mu_1}{\mu_2} c''' \quad (5.7)$$

when again c'' and c''' are mass-independent constants, different from zero if $G_{12}^{12(m_1, m_2)} \neq 0$.

We will be able to attribute this puzzling difference to non-zero mode contributions to propagators, which are strictly absent in the massless case but are present in the massive one and generate power-behaved infrared mass singularities*) which modify the zero-mode result (5.2) by terms of $O(1)$ in the masses.

A (small) mass expansion is therefore, in general, not valid. This leads to a difficult problem in the calculation of instanton effects in massive theories, since there exist no closed expressions for the massive matter propagators in a background instanton field. (They exist in the massless case and have been given in section 3).

We will, therefore, follow a different procedure: we shall first try to identify a set of Green's functions for which we will be able to show that the above-mentioned mass singularities are absent or under control. This will, nevertheless, allow us to draw a lot of interesting conclusions when combined with the anomaly eq. (4.15). We shall discuss later the Green's functions expected to possess a singular $m \rightarrow 0$ limit.

In order to study the mass behaviour, we start by writing a formal mass-expansion for the massive matter propagators in terms of the massless ones given in section 3. For the scalar fields ϕ and $\tilde{\phi}$, the propagators (diagonal flavour indices are omitted) read

$$\begin{aligned} \langle 0 | \phi(x) \phi(y) | 0 \rangle &= \langle 0 | \tilde{\phi}(x) \tilde{\phi}^*(y) | 0 \rangle = - \frac{1}{D^2 - m^2} = \\ &\simeq - \frac{1}{D^2} - m^2 \frac{1}{D^2} \frac{1}{D^2} + O(m^4) \end{aligned} \quad (5.8)$$

where $1/D^2$ is given in eq. (3.20).

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*) The possibility that instantons may worsen infrared singularities had already been suggested in ref. [20].

For the fermion matter field we will be interested in the "effective" $\tilde{\psi}\psi$ propagator introduced by the mass term (see Appendix B). Separating the zero-mode contributions (3.17) we obtain

$$\langle 0 | \tilde{\psi}(x) \psi(y) | 0 \rangle = m^* \left(\frac{\tilde{\psi}_0(x) \psi_0(y)}{m m^*} + \frac{1}{\not{D}} \frac{1}{\not{D}} + O(m m^*) \right) \quad (5.9)$$

where $1/\not{D} = \bar{S}$ and $1/\not{D} = S$ are given in eqs. (3.24). Let us first notice that both the convolutions $1/\not{D}^2 \cdot 1/\not{D}^2$ and $1/\not{D} \cdot 1/\not{D}$ in eqs. (5.8) and (5.9) have an infrared logarithmically divergent term coming from the integration over the intermediate convolution variable. But, as we shall see later, these two divergent contributions will cancel each other owing to the combination of boson and fermion propagators that will always appear as a consequence of SUSY.

As an example of a "safe" Green's function, let us start by considering the gluino's correlation function

$$G_N^0 = \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_N) | 0 \rangle \quad (5.10)$$

Its value in pure SYM is given by eq. (4.5). In the presence of massless SUSY matter it vanishes owing to the zero modes of the matter fermion determinant. We want to show that nevertheless its mass dependence is calculable by considering, in the spirit of ref. [18], the derivative of eq. (5.10) with respect to masses at mass equal zero. In the one-fermion ($M = 1$) case, one gets from eqs. (5.8), (5.9), (A.14), and (A.19):

$$\begin{aligned} \frac{\partial}{\partial m} G_N^0 &= \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_N) \int d^4 z [-\tilde{\psi} \psi(z) + m^* (\psi^\dagger \psi(z) + \tilde{\psi} \tilde{\psi}^\dagger)] | 0 \rangle = \\ &= m m^* \int e^{-S_{inst.}} dE_{inst.} \sum_{\substack{\lambda\text{-zero} \\ \text{modes}}} (-1)^F \lambda_0 \lambda_0(x_1) \dots \lambda_0 \lambda_0(x_N) \left[\frac{\int d^4 z \tilde{\psi}_0 \psi_0(z)}{m m^*} + \right. \\ &\quad \left. + \text{Tr} \left(\frac{2}{\not{D}^2} - \frac{1}{\not{D} \not{D}} \right) + O(m m^*) \right] \quad (5.11) \end{aligned}$$

The overall factor of m in the second equality in eq. (5.11) comes from the fermion determinant as discussed in appendix B [see eq. (B.1)], while the m^* originates

from eq. (5.9). The symbol Tr refers to the trace over space and colour indices, while tr refers to spin indices only.

The calculation of the $\tilde{\psi}_0\psi_0$ zero mode contribution to eq. (5.11) is straightforward [16]. By using the normalization condition (3.18), one is left with the evaluation of the pure SYM theory gluino but with the β_1 appropriate for a theory with one flavour correlation function given by eq. (4.5). The non-zero mode contribution is easily seen to vanish in this case. Indeed, recalling that in a self-dual background field

$$\bar{D}D = -D^2 \quad (5.12)$$

it is easy to check that (in the space orthogonal to the zero modes)

$$\frac{1}{\bar{D}D} = \frac{1}{\bar{D}} \frac{1}{D} = -D \frac{1}{D^2} \frac{1}{D^2} \bar{D} \quad (5.13)$$

and therefore

$$\text{Tr} \left(\frac{2}{D^2} - \frac{1}{\bar{D}D} \right) = 0 \quad (5.14)$$

Higher derivatives of eq. (5.10) with respect to m vanish at $m = 0$. In fact, the zero-mode contribution to eq. (5.11) is absent because $\psi_0\tilde{\psi}_0$ appear more than once. The contribution of the non-zero modes is seen to cancel by the repeated use of eq. (5.13). Therefore, we conclude that for $M = 1$:

$$G_N^0 = \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_N) | 0 \rangle = c_N \Lambda^{3N} \frac{m}{\Lambda} \quad (5.15)$$

In the same way, one can show that for arbitrary M :

$$\langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_N) | 0 \rangle = c_N' \Lambda^{3N} \prod_{i=1}^M \frac{m_i}{\Lambda} \quad (5.16)$$

By cluster properties, exactly as discussed in section 4.1 for pure SYM, we find that in the presence of M massive matter fields

$$\langle \lambda \lambda \rangle = \Lambda^{3N} \left[c_N' \prod_{i=1}^M \frac{m_i}{\Lambda} \right]^{1/N} \quad (5.17)$$

The N possible values of the root in eq. (5.17) correspond to the existence of N vacuum states signalling the spontaneous breaking of Z_{2N} to Z_2 . This provides, as already discussed in ref. [3], an explication for the value of Witten's index in massive SQCD. We stress that the result (5.17) is valid for arbitrary values of N and M . This is in contrast with the massless case in which no constant Green's functions with the correct chiralities exist for $M > N$.

If we now use the anomaly constraint (4.15) we obtain

$$\langle \tilde{\varphi}^i \varphi_j \rangle = \delta_j^i \frac{\Lambda^3}{m_i} \left[c_N' \prod_{k=1}^N \frac{m_k}{\Lambda} \right]^{1/N} \quad (5.18)$$

which is the effective Lagrangian result [3]. Our derivation has followed the safe path of i) calculating mass derivatives of pure gluino correlations using instantons and ii) extracting $\langle \lambda \lambda \rangle$ and relating it to $\langle \tilde{\phi} \phi \rangle$ by the constraint (4.15).

Equations (5.17) and (5.18) give us the complete set of condensates one needs in order to obtain Green's functions such as (5.1). We get indeed

$$G_{j_1 j_2}^{i_1 i_2}(m_1, m_2) = c_2 \delta_{j_1}^{i_1} \delta_{j_2}^{i_2} m_{i_1}^{-1} m_{i_2}^{-1} m_1 m_2 \Lambda^4 \quad (5.19)$$

in agreement with the expected values (5.6), (5.7) but in contrast with the pure zero-mode contribution yielding eq. (5.2). Thus, in certain Green's functions, such as $G_{j_1 j_2}^{i_1 i_2}$, the non-zero modes should not cancel [as was the case in eq. (5.11)], and should modify the zero-mode result in a mass-independent (or even sometimes in a mass-singular) way. This signals the existence of infrared mass singularities associated with non-zero modes.

In order to expose the mathematical origin of those singularities we have to write an expression for the mass corrections to eq. (5.2). To this end, we need the "effective" $\tilde{\phi} \phi$ propagator whose formal $|m|^2$ expansion is given in appendix B [eq. (B.4)]. We thus find:

$$G_{j_1 j_2}^{i_1 i_2 (u_1, u_2)}(x_1, x_2) = \int e^{-S_{\text{inst.}}} d\epsilon_{\text{inst.}} \left[\delta_{j_1}^{i_1} \delta_{j_2}^{i_2} \text{Tr} P_{i_1}(x_1 x_2) \text{Tr} P_{i_2}(x_2 x_2) + \right. \\ \left. + \delta_{j_2}^{i_1} \delta_{j_1}^{i_2} \text{Tr} (P_{i_1}(x_1 x_2) P_{i_2}(x_2 x_1)) \right] \quad (5.20)$$

where the trace is over colour indices.

We first notice that the zero-mode contribution to P , as given by eq. (B.6), just reproduces the massless result (5.2). Besides it we find corrections proportional to $|m_i|^2$ that, before integration over the instanton's collective coordinates (ρ and a_μ), are finite as a consequence of the cancellation^{*)} between the infrared logarithmic divergences occurring in $1/D^2 \cdot 1/D^2$ and $1/\bar{p} \cdot 1/\bar{p}$ as mentioned before. Unfortunately, although well-defined, the expression (5.20) is extremely complicated and its explicit evaluation has not been attempted. It does not seem to vanish identically, as happened for the non-zero mode contribution in the Green's function of eq. (5.11).

If indeed, the coefficient of these terms does not vanish, the final integration over the dilatation collective coordinates will show a quadratic divergence (at $\rho \rightarrow \infty$) introduced artificially by our $|m|^2$ expansion. The presence of a non-zero mass should regulate this divergence and convert it into a $1/|m|^2$ enhancement factor cancelling the $|m|^2$ coefficient in eq. (5.20). This is how we can see the possible emergence of mass-independent corrections to the zero-mode contribution (5.2).

Looking at eq. (5.20), it is clear that, for consistency with eq. (5.6) and (5.7), the mass singularities discussed above must only be present in the term with the flavour structure $\delta_{j_2}^{i_1} \delta_{j_1}^{i_2}$ and give a contribution that exactly cancels the one coming from the zero modes.

Of course, the expansion in $|m|^2$ that we have performed is formal and higher order terms may equally contribute. The mechanism illustrated here shows

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*) We remark that this cancellation occurs thanks to the boson-fermion mass degeneracy. A SUSY-breaking mass term would spoil this cancellation and give us an x -dependent term with a logarithmically divergent coefficient.

nevertheless how m -independent Green's functions can be different in the massive and massless case.

We notice that a counterpart of the above mechanism would have been found in x -independent Green's functions if analysed in a short distance expansion. Consider, for instance, the Green's function (4.2) for $N = 2$. For any fixed value of the instanton parameter ρ , the Green's function has a zero at $x_1 = x_2$ [cf. eq. (4.3)] and the final non-vanishing constant result would appear as coming from the quadratic ultraviolet divergence in the integration over ρ .

In conclusion, we have seen that, using the anomaly constraint (4.15) and the instanton calculation of the mass derivative of the correlation function G_N^0 , we can determine the value of the condensates $\langle \lambda\lambda \rangle$ and $\langle \tilde{\phi}^i \phi_j \rangle$ and, in particular, their mass dependence (even for $M > N$). We have also found that in correlation functions involving the composite operators $\tilde{\phi}\phi$, mass singularities have to appear in order that a picture consistent with the expected vacuum symmetry is obtained. This will also explain why in some cases $\tilde{\phi}\phi$ condensates are driven off to infinity as $m \rightarrow 0$.

Since mass corrections are typically $O(1)$ in the masses, the massless limit of the massive theory fails to coincide with the strictly zero mass case. Thus, owing to a singular infrared behaviour, massive SQCD cannot really be regarded as a regularization of the massless case.

6. CONCLUSIONS

We shall summarize here our main results:

i) In SYM we have established, for any $SU(N)$ gauge group, the occurrence of a gluino-gluino condensate, implying the spontaneous breaking of the discrete Z_{2N} symmetry down to Z_2 .

Our belief in the consistency of the method has been reinforced by the check that instantons induce no breaking of space-time symmetries, i.e. we do not find condensates carrying spin.

New short distance behaviours were also obtained in a number of cases, requiring a non-perturbative modification of the light-cone expansion.

ii) The method has been found to give consistent results also in the case of massive SQCD. Here the gluino-gluino condensate was computed directly from the method, while squark-antisquark condensate calculation needed the further use of an anomaly constraint.

The consistency of the results with several general constraints was shown to require (*a priori* unexpected) power-behaved mass singularities in the instanton contribution to certain amplitudes. We were able to identify the origin of these singularities, giving rise to peculiar mass dependences of certain condensates and their runaway behaviour in the small mass limit. In that limit ($m_i \ll \Lambda$, $m_i \neq 0$) the theory preserves the vacuum structure of the massive case and differs from the strictly massless situation in which a larger vacuum degeneracy is present.

iii) Indeed for massless SQCD the instanton method is straightforward and requires condensates that bring in spontaneous breaking of flavour symmetries. However, for $M < N$ (M is the number of flavours) this fact is shown to conflict with the anomaly constraint that requires the gluino-gluino condensate to vanish.

One way out, suggested by the limit from the massive case, as well as by the presence of flat directions in the perturbative potential, would be that the (degenerate) vacua do indeed lie at infinity in correspondance with the massless limit taken with all possible structures of the mass matrix.

An alternative is, of course, that SUSY is spontaneously broken at $m = 0$ and that the massless theory has really nothing to do with the limit of the massive one.

iv) In SUSY chiral models of the Georgi-Glashow type this latter possibility (spontaneous SUSY breakdown) is often favoured [12]. This is so in the one-family case because of the absence of flat directions and in the two-family case if a superpotential is added (otherwise there are again flat directions). Other arguments [11] as well as some recent effective Lagrangian studies [23] seem to confirm this first example of non-perturbative breaking of SUSY in four dimensions.

We would like to conclude with a word of warning on our simple-minded way of handling the regularization of SUSY gauge theories. In view of the apparent

conflicts in the massless situations we should at least keep in mind two more options:

- i) There could be a clash between the ultraviolet regularization chosen at $m = 0$ (i.e. the well-known [24] cancellation of non-zero modes between boson and fermion determinants) and that needed to establish the standard form of the anomaly [6].
- ii) Regularization of the massless theory could lead to explicit SUSY breaking. This possibility looks, however, disfavoured by the constancy of G_N that results from the calculation: there is obviously no reason for such constancy if SUSY is broken.

Besides studying these other possibilities, it seems to us that an important task for the future would be to understand *physically* the non-perturbative mechanism through which SUSY vacua are lifted, move to infinity or, sometimes, completely disappear.

Acknowledgements

We are grateful to S. Templeton for introducing us to the formalism of ref. [15] and for many useful conversations. We also wish to thank Y. Frishman and M. Testa for their interest in our work and for various interesting discussions.

Appendix A

In this appendix we want to recall, for completeness and to fix our notation, how to compute Green's functions in the one-instanton ($v = 1$) semiclassical approximation.

The SQCD Lagrangian can be written in components as follows:

$$\mathcal{L}_{\text{SQCD}} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{mass}} \quad (\text{A.1})$$

$$\mathcal{L}_{\text{SYM}} = -\frac{1}{2g^2} \text{Tr}(F_{\mu\nu} F_{\mu\nu}) + \frac{1}{g^2} \bar{\lambda} \not{D} \lambda \quad (\text{A.2a})$$

$$\mathcal{L}_{\text{matter}} = -(\not{D}_\mu \psi)^* (\not{D}_\mu \psi) - (\not{D}_\mu \tilde{\psi}^*)^* (\not{D}_\mu \tilde{\psi}^*) + \bar{\psi} \not{D} \psi + \bar{\tilde{\psi}} \not{D} \tilde{\psi} \quad (\text{A.2b})$$

$$\mathcal{L}_{\text{Yukawa}} = -iV_2 (\bar{\psi} \lambda \psi - \psi^* \lambda \psi + \bar{\tilde{\psi}} \lambda \tilde{\psi} - \tilde{\psi} \lambda \tilde{\psi}^*) \quad (\text{A.2c})$$

$$\mathcal{L}_{\text{scalar}} = -\frac{g^2}{2} (\psi^* T^A \psi - \tilde{\psi} T^A \tilde{\psi}^*) (\psi^* T^A \psi - \tilde{\psi} T^A \tilde{\psi}^*) \quad (\text{A.2d})$$

$$\mathcal{L}_{\text{mass}} = -m\psi^* (\psi + \tilde{\psi} \tilde{\psi}^*) + m\tilde{\psi} \psi + m^* \bar{\psi} \bar{\tilde{\psi}} \quad (\text{A.2e})$$

where T^A ($A = 1, \dots, N^2-1$) are the generators of the gauge group in the fundamental representations, normalized to $\text{Tr}(T^A T^B) = \frac{1}{2} \delta^{AB}$ and we have used the shorthand $F_{\mu\nu} = F_{\mu\nu}^A (T^A)_a^b$, $\lambda = \lambda^A (T^A)_a^b$, $\bar{\lambda} = \bar{\lambda}^A (T^A)_a^b$. Fields belonging to the representation N are represented by vectors with lowered indices ($\phi_a, \tilde{\phi}_a^*, \psi_a, \tilde{\psi}_a$), fields belonging to the presentation $\bar{N}$ by vectors with raised indices ($\phi^{*a}, \tilde{\phi}^a, \bar{\psi}^a, \bar{\tilde{\psi}}^a$). Similarly for the flavour indices which in eqs. (A.2) are understood. For spin indices and other related conventions we refer to ref. [19]^{*} except that we work in the Euclidean region. Finally, a factor of g has been included in the definition of V_μ , λ and $\bar{\lambda}$.

^{*}) For instance, we use $\lambda_\psi = \lambda^\alpha \psi_\alpha = \lambda^{\alpha\beta} \epsilon_{\alpha\beta} = \lambda\psi$; $\bar{\lambda}\bar{\psi} = \bar{\lambda}_\alpha \bar{\psi}^{\dot{\alpha}} = \bar{\lambda}_\alpha \bar{\psi}_\beta \epsilon^{\dot{\alpha}\dot{\beta}} = \bar{\psi}\bar{\lambda}$.

We recall that

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + i [V_\mu, V_\nu] \quad (\text{A.3a})$$

$$\bar{\lambda} \bar{D} \lambda = \bar{\lambda}^A \bar{\sigma}_\mu (\partial_\mu \delta_{AC} - f_{ABC} V_\mu^B) \lambda^C \quad (\text{A.3b})$$

$$(D_\mu \varphi)_a = \left[\partial_\mu \delta_a^b + i V_\mu^A (T^A)_a^b \right] \varphi_b \quad (\text{A.3c})$$

Let us start by considering the case of pure SYM. The Green's function obeying eq. (2.11) for $v = 1$ is given

$$G_N^0(x_1, \dots, x_N) = \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_N) | 0 \rangle \quad (\text{A.4})$$

The classical solution

$$V_\mu = V_\mu^{\text{inst}}, \quad \lambda = \bar{\lambda} = 0 \quad (\text{A.5})$$

is a saddle point of the action. Quadratic fluctuations around this solution of the classical equations of motion can be computed using:

- i) The 't Hooft results on the calculation of determinants in the one-instanton background field [17].
- ii) The observation that in SUSY gauge theories the contributions to these determinants coming from non-zero modes exactly cancel between fermions and bosons [17,24]. (Of course this implies the assumption of the existence of a supersymmetric regularization of chiral fermion determinants.)
- iii) The Berezin functional integration rules for fermions.

The gluon integration leads, remembering (i) and (ii), to an effective integration measure over the instanton translation (a_μ) and dilatation (ρ) collective coordinates, given by:

$$e^{-S_{\text{inst}}} d\varepsilon_{\text{inst}} = c e^{-\frac{8\pi^2}{g^2}} \mu^{2N} (\rho^2)^{2N} \frac{d^4 a}{g^5} d\rho \quad (\text{A.6})$$

where c is a calculable constant (irrelevant for our purposes) and μ is the subtraction point mass scale of the theory.

As for the fermion integration, it is well known that in a self-dual background field the Dirac operator possesses only left-handed zero modes (and only right-handed in an anti-self-dual field), whose number depends on the colour representation to which the fermions belong.

For instance (for $v = 1$), in the fundamental representation there is 1 zero mode, in the antisymmetric $N-2$, in the adjoint $2N$, etc.

For the Majorana spinor λ which belongs to the adjoint representation the Berezin integration rules immediately give:

$$\begin{aligned} Z(\eta, \bar{\eta}) &= \int d\lambda d\bar{\lambda} \exp \left[\int d^4x \left[\bar{\lambda} \not{D} \lambda + \bar{\lambda} \bar{\eta} + \eta \lambda \right] \right] = \\ &= \Delta \prod_{n=1}^{2N} \left(\int d^4x \, \eta \lambda_0^{(k)} \right) \exp \left(- \int d^4x d^4y \, \eta(x)^\alpha \Sigma_{\alpha\dot{\alpha}}(x,y) \bar{\eta}(y)^{\dot{\alpha}} \right) \quad (A.7) \end{aligned}$$

where η and $\bar{\eta}$ are the auxiliary external sources associated with λ and $\bar{\lambda}$, respectively and, defining analogously to eqs. (3.22),

$$\not{D} \bar{\xi}_n = h_n \chi_n, \quad \not{D} \chi_n = \bar{h}_n \bar{\xi}_n, \quad n = 1, 2, \dots \quad (A.8)$$

we have put^{*)}

$$\Delta = \prod_{n=1}^{\infty} \bar{h}_n \quad (A.9)$$

$$\Sigma_{\alpha\dot{\alpha}}(x,y) = \sum_{n=1}^{\infty} \frac{\chi_n(x) \bar{\xi}_n(y)}{\bar{h}_n} \quad (A.10)$$

^{*)} In the formalism of ref. [15] it is possible to give the explicit expression of Σ , but we will not need it here.

The $\lambda_0^{(k)}$'s are the $2N$ zero modes of the operator $\overline{D}$ [given in eq. (3.11)]:

$$\overline{D} \lambda_0^{(k)} = 0 \quad k=1, 2, \dots, 2N \quad (\text{A.11})$$

normalized according to

$$\int d^4x \text{Tr} (\overline{\lambda}_0^{(k)} \overline{\sigma}_0 \lambda_0^{(k')}) = \delta_{kk'} \quad (\text{A.12})$$

Using eqs. (A.6) and (A.7), and exploiting the supersymmetric cancellation of non-zero mode contributions to boson and fermion determinants, we get the formula:

$$G_N^0(x_1 \dots x_N) = c e^{-8\pi^2/g^2} \mu^{3N} \int \frac{d^4q d\rho}{g^5} (g^2)^{2N} \cdot \left. \frac{\int \int \dots \int \frac{\delta}{\delta \eta(x_1)} \frac{\delta}{\delta \eta(x_1)} \dots \frac{\delta}{\delta \eta(x_N)} \frac{\delta}{\delta \eta(x_N)} \left(\prod_{k=1}^{2N} \int d^4x \eta \lambda_0^{(k)} \right) \exp \left(- \int \eta \varepsilon \bar{\eta} d^4x \right) \right|_{\eta = \bar{\eta} = 0} \quad (\text{A.13})$$

From eq. (A.13) it immediately follows that i) there are no possible perturbative contributions to G_N^0 and ii) that G_N^0 has a non-vanishing instanton contribution just because there are exactly $2N$ functional derivatives with respect to η as there are zero modes, so as to eliminate all the η 's in the factor $[\prod_{k=1}^{2N} \int \eta \lambda_0^{(k)}]$. This is indeed the origin of the "chirality selection rule" (2.11).

Remembering that the η 's are Grassman variables one arrives at the final formulae:

$$G_N^0(x_1, \dots, x_N) = \Lambda \int \frac{d^4q d\rho}{g^5} (g^2)^{2N} \sum_{\{k_i\}} (-1)^P \lambda_0^{(k_1)} \lambda_0^{(k_2)} \dots \lambda_0^{(k_{2N-1})} \lambda_0^{(k_{2N})} \quad (\text{A.14})$$

where the sum is extended over the $(2N)!$ permutations of the indices $k_1, \dots, k_{2N}$ and P is their parity. In eq. (A.14) we have indicated by Λ the renormalization group invariant combination

$$\Lambda = \mu e^{-8\pi^2/\beta g^2} \quad (\text{A.15})$$

$\beta_1 = 3N$ being the first coefficient of the SYM β function. As expected from the general discussion of section 2, the factor Λ^{3N} completely accounts for the physical dimensions of G_N^0 , so the integral in the r.h.s. of eq. (A.14) is a dimensionless number.

Let us now go over to the case in which we have M massless matter superfields. The relevant object to consider according to eqs. (2.11) is:

$$G_N^M(x_1, \dots, x_{N-M}, y_1, \dots, y_M) = \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_{N-M}) \tilde{\varphi}^1(y_1) \dots \tilde{\varphi}^M(y_M) | 0 \rangle \quad (A.16)$$

where all different flavours appear once.

To compute eq. (A.16), we now have also to perform the functional integration over the quadratic fluctuations of the matter fields around the one-instanton field configuration which analogously in eq. (A.4) is given by

$$\begin{aligned} V_M &= V_M^{inst.} \\ \lambda &= \bar{\lambda} = 0 \\ \text{all matter fields} &= 0 \end{aligned} \quad (A.17)$$

This is done in the standard way by introducing the auxiliary external sources $K, \tilde{K}, J, \tilde{J}$ and their conjugates, associated respectively with $\psi, \tilde{\psi}, \phi, \tilde{\phi}$ and their conjugate fields.

In the presence of $2M$ Majorana spinors ψ_i and $\tilde{\psi}^i$ (i is the flavour index) in the fundamental representation of the colour group, the instanton integration measure becomes

$$e^{-S_{inst.}} d\Sigma_{inst.} = c' e^{-8\pi^2/g^2} \mu^{3N-M} (g^2)^{2N} \frac{d^4 a d\rho}{g^5} \quad (A.18)$$

The net result for (A.16) is then:

$$G_N^M(x_1, \dots, x_{N-M}, y_1, \dots, y_M) = c' e^{-8\pi^2/g^2} \mu^{3N-M} \int \frac{d^4 q d\varphi}{g^5} (g^2)^{2N} \cdot$$

$$\frac{\delta}{\delta \psi(x_1)} \frac{\delta}{\delta \psi(x_1)} \dots \frac{\delta}{\delta \psi(x_{N-M})} \frac{\delta}{\delta \psi(x_{N-M})} \frac{\delta}{\delta \tilde{J}(y_1)} \frac{\delta}{\delta \tilde{J}(y_1)} \dots \frac{\delta}{\delta \tilde{J}(y_M)} \frac{\delta}{\delta \tilde{J}(y_M)} \cdot \quad (A.19)$$

$$\cdot Z_{SQCD}(\psi, \bar{\psi}; K, \bar{K}, \tilde{K}, \tilde{\bar{K}}; J, J^*, \tilde{J}, \tilde{J}^*) \Big|_{\text{all sources}=0}$$

where Z_{SQCD} is formally given by

$$Z_{SQCD} = \left(\prod_{k=1}^{2N} \int d^4 x_k \psi_k \right) \left(\prod_{i=1}^M \int d^4 x_i K^i \psi_{0,i} \right) \left(\prod_{i=1}^M \int d^4 x_i \tilde{\psi}_0^i \tilde{K}_i \right) \cdot$$

$$\cdot \left(e^{-S_{Yuk.}} \right) \left(e^{-S_{scalars}} \right) \exp - \int d^4 x \left(\psi \tilde{\psi} + \sum_i K^i \tilde{S}_i + \sum_i \tilde{K}^i \tilde{S}_i \right) \quad (A.20)$$

$$\cdot \exp - \int d^4 x \sum_i \left(J^i \frac{1}{D^2} J_i^* + \tilde{J}^{*i} \frac{1}{D^2} \tilde{J}_i \right)$$

In eq. (A.20) S_{Yukawa} and $S_{scalars}$ are obtained from formulae (A.2c) and (A.2d) by appropriately replacing each field with the functional derivative with respect to the corresponding source.

To get a non-zero contribution to eq. (A.19) one has to bring down from $\exp(-S_{Yukawa})$ exactly the 2M vertices

$$e^{-S_{Yuk.}} \rightarrow (i\sqrt{2})^M \prod_{i=1}^M \int d^4 x_i \frac{\delta}{\delta \tilde{J}_i^*(x_i)} \frac{\delta}{\delta \psi(x_i)} \frac{\delta}{\delta K^i(x_i)} \cdot$$

$$\cdot (i\sqrt{2})^M \prod_{j=1}^M \int d^4 x_j \frac{\delta}{\delta \tilde{K}_j(x_j)} \frac{\delta}{\delta \psi(x_j)} \frac{\delta}{\delta \tilde{J}^{*j}(x_j)} \quad (A.21)$$

In this way the number of fermionic $(\lambda, \psi, \tilde{\psi})$ functional derivatives is precisely such as to eliminate completely the corresponding external sources from the first three factors in eq. (A.20), and the Green's function G_N^M becomes:

$$\begin{aligned}
 G_N^M(x_1 \dots x_{N-M}, y_1 \dots y_M) &= c' e^{-8\pi^2/g^2} \mu^{3N-M} \int \frac{d^4 q d p^2}{p^5} (p^2)^{2N} \\
 &\cdot \sum_{\{k_i\}} (-1)^P \lambda_0^{(k_1)} \lambda_0^{(k_2)} \dots \lambda_0^{(k_{2N-2M-1})} \lambda_0^{(k_{2N-2M})} (x_{N-M}) \\
 &\cdot (iV_2)^2 \tilde{\psi}_0^{(k_{2N-2M+1})} \lambda_0^{(k_{2N-2M+2})} \frac{1}{D^2}(y_1) \cdot (y_1) \frac{1}{D^2} \lambda_0^{(k_{2N-2M+2})} \psi_0 \dots (iV_2)^2 \tilde{\psi}_0^{(k_{2N-1})} \lambda_0^{(k_{2N})} \frac{1}{D^2}(y_M) \cdot (y_M) \frac{1}{D^2} \lambda_0^{(k_{2N})} \psi_0
 \end{aligned} \tag{A.22}$$

In eq. (A.19) a scalar propagator $1/D^2$ appears for each pair of functional derivatives such as $\delta/\delta J \cdot \delta/\delta J^*$ or $\delta/\delta \tilde{J}^* \cdot \delta/\delta \tilde{J}$ and we have used the symbolic notation

$$\begin{aligned}
 \tilde{\psi}_0 \lambda_0 \frac{1}{D^2}(y) &\equiv \int d^4 x \tilde{\psi}_0(x) \lambda_0(x) \left(\frac{1}{D^2} \right)_{(x,y)} \\
 (y) \frac{1}{D^2} \lambda_0 \psi_0 &\equiv \int d^4 x \left(\frac{1}{D^2} \right)_{(y,x)} \lambda_0(x) \psi_0(x)
 \end{aligned} \tag{A.23}$$

As shown in section 3 the explicit form of the kernel $1/D^2$ [eq. (3.20)] allows one to solve the equations

$$\begin{aligned}
 D^2 \phi_{0i}^{(k)} + iV_2 \lambda_0^{(k)} \psi_{0i} &= 0 \\
 (D^2 \tilde{\phi}_0^{(k) i*})^* + iV_2 \tilde{\psi}_0^i \lambda_0^{(k)} &= 0
 \end{aligned} \tag{A.24}$$

where, we recall that i is a flavour index and k labels the $2N$ gluino zero modes.

Using $\phi_{0i}^{(k)}$ and the $\tilde{\phi}_0^{(k) i}$, as defined by eqs. (A.24), eq. (A.22) can be rewritten in the form

$$\begin{aligned}
 G_N^M(x_1 \dots x_{N-M}, y_1 \dots y_M) &= \mu^{3N-M} \int \frac{d^4 q d p^2}{p^5} (p^2)^{2N} \sum_{\{k_i\}} (-1)^P \lambda_0^{(k_1)} \lambda_0^{(k_2)} \dots \\
 &\dots \lambda_0^{(k_{2N-2M-1})} \lambda_0^{(k_{2N-2M})} \tilde{\phi}_0^{(k_{2N-2M+1})} \phi_0^{(k_{2N-2M+2})} \frac{1}{D^2}(y_1) \dots \tilde{\phi}_0^{(k_{2N-1})} \phi_0^{(k_{2N})} \frac{1}{D^2}(y_M)
 \end{aligned} \tag{A.25}$$

where we have used the fact that now $\beta_1 = 3N-M$.

Again the full physical mass dimensions are brought by the renormalization group invariant factor Λ^{3N-M} and the remaining integral is a dimensionless number.

We have obtained the result (A.25) for the flavour configuration of eq. (A.16). It is easy to obtain the expression for the general flavour configuration. This is given by

$$\begin{aligned}
 G_N^{M(i_1 \dots i_M)} &= \langle 0 | \lambda \lambda(x_1) \dots \lambda \lambda(x_M) \bar{\psi}^{i_1} \psi(y_1) \dots \bar{\psi}^{i_M} \psi(y_M) | 0 \rangle = \\
 &= \varepsilon^{i_1 \dots i_M} \varepsilon_{j_1 \dots j_M} G_N^M
 \end{aligned}
 \tag{A.26}$$

The $\varepsilon \cdot \varepsilon$ factor in eq. (A.26) may be understood as a consequence of the fact that the flavour index is finally carried by the (anticommuting) fermionic matter zero modes.

Appendix B

In this appendix we want to show how the formulae of appendix A are modified by the presence of the supersymmetric mass term $\mathcal{L}_{\text{mass}}$ of eq. (A.2e).

What we have in mind is, for the reasons explained in the text, a formal m, m^* expansion of our Green's functions G_N^M and a consequent treatment of the contributions of non-zero ψ and $\bar{\psi}$ modes to it. For supersymmetry they will go together with the mass expansion of the massive scalar propagator $1/(D^2 - mm^*)$.

To derive the kind of formulae used in the text, we first notice that the gauge supermultiplet is unmodified so the functional integration over the V_μ , λ , and $\bar{\lambda}$ fields is as in appendix A. This means that in the calculation of G_N^M the instanton integration measure (A.18) is unchanged and that the $2N$ λ 's can be, as before, replaced by the corresponding zero modes.

As for the scalars, the only change in eqs. (A.20) and in the following, is the replacement of the massless propagator $1/D^2$ by the massive one $1/(D^2 - mm^*)$. The point which requires some discussion is the $(\psi, \bar{\psi})$ and $(\tilde{\psi}, \tilde{\bar{\psi}})$ functional integration, in particular how the result (A.20) is recovered in the $mm^* \rightarrow 0$ limit.

The matter fermion functional integration is again performed à la Berezin by expanding the fermion fields in terms of the functions defined by eqs. (3.22). After a little algebra, one formally finds:

$$\begin{aligned} Z(K, \bar{K}, \tilde{K}, \tilde{\bar{K}}) = & \prod_{i=1}^M m_i \Delta_i \exp \int d^4x \left[- \tilde{K}^i \not{D} \frac{1}{\not{D} \not{D} - m_i m_i^*} \tilde{\bar{K}}_i - \right. \\ & \left. - K^i \not{D} \frac{1}{\not{D} \not{D} - m_i m_i^*} \bar{K}_i + m_i^* K^i \frac{1}{\not{D} \not{D} - m_i m_i^*} \tilde{\bar{K}}_i + m_i \tilde{K}^i \frac{1}{\not{D} \not{D} - m_i m_i^*} \bar{K}_i \right] \quad (B.1) \end{aligned}$$

where

$$\Delta_i = \prod_{n=1}^{\infty} \left(\bar{k}_n k_n - m_i m_i^* \right) \quad (B.2)$$

and the eigenvalues k_n , $\bar{k}_n$ were defined in eqs. (3.22).

Remember that

$$\begin{aligned} (\cancel{D}\bar{D})_{\alpha}^{\beta}{}^b &= -(D^2)_{\alpha}^{\beta} \delta_{\alpha}^b - i(\sigma_{\mu\nu})_{\alpha}^{\beta} (F_{\mu\nu})_{\alpha}^b \\ (\bar{D}\cancel{D})_{\dot{\alpha}}^{\dot{\beta}}{}^b &= -(D^2)_{\alpha}^{\beta} \delta_{\dot{\alpha}}^{\dot{\beta}} - i(\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}} (F_{\mu\nu})_{\alpha}^b \end{aligned} \quad (B.3)$$

In a self-dual gauge field $\cancel{D}\bar{D}$ possesses zero modes while $\bar{D}\cancel{D} = -D^2$ does not (and vice versa in an anti-self-dual gauge field).

For the inverse of D^2 , $\bar{D}$, and $\cancel{D}$ a closed expression can be given [17]. They are recorded in section 3 in the case of a one-instanton background field.

In order to see how eq. (A.22) will be modified by possible mass corrections, it is useful to define an "effective propagator", $P_{i\ b}^a(x, x')$, for the quantity $m_i \tilde{\psi}_{(x)}^{ia} \psi_{ib}(x')$ as:

$$\begin{aligned} P_{i\ b}^a(x, x') &= -2 m_i m_i^* \int d^4y d^4y' \left[\left(\frac{1}{D^2 - m_i m_i^*} \right)_{(x, y)} \lambda_0(y) \right. \\ &\quad \cdot \left. \left(\frac{1}{\cancel{D}\bar{D} - m_i m_i^*} \right)_{(y, y')} \lambda_0(y') \left(\frac{1}{D^2 - m_i m_i^*} \right)_{(y', x')} \right]_{\ b}^a \end{aligned} \quad (B.4)$$

The reason for this definition is that in the $m m^* \rightarrow 0$ limit, one formally obtains [see eq. (A.19)]:

$$\begin{aligned} \lim_{m_i, m_i^* \rightarrow 0} P_{i\ b}^a(x, x') &= -2 \int d^4y \left(\frac{1}{D^2} \right)_{(x, y)} \lambda_0(y) \psi_{ib}(y) \\ &\quad \cdot \int d^4y' \tilde{\psi}_{ib}(y') \lambda_0(y') \left(\frac{1}{D^2} \right)_{(y', x')} = P_{i\ b}^{(0)a}(x, x') \end{aligned} \quad (B.5)$$

since $\lim_{m m^* \rightarrow 0} m m^* (\cancel{D}\bar{D} - m m^*)^{-1}$ selects the projector on the subspace spanned by the zero modes of the $\cancel{D}\bar{D}$ operator.

Barring possible infrared divergences (discussed in section 5), a formal $m m^*$ expansion of eq. (B.4) can be written down in the form:

$$\begin{aligned}
 P_i(x, x') \approx & P_i^{(0)}(x, x') - 2\mu_i^2 \left[\left(\frac{1}{D^2} \frac{1}{D^2} \lambda_0 \psi_i \right)_{(x)} \left(\tilde{\psi}_0^i \lambda_0 \frac{1}{D^2} \right)_{(x')} + \right. \\
 & \left. + \left(\frac{1}{D^2} \lambda_0 \psi_i \right)_{(x)} \left(\tilde{\psi}_0^i \lambda_0 \frac{1}{D^2} \frac{1}{D^2} \right)_{(x')} + \left(\frac{1}{D^2} \lambda_0 \frac{1}{(\cancel{D}\bar{D})'} \lambda_0 \frac{1}{D^2} \right)_{(x, x')} \right] \quad (B.6)
 \end{aligned}$$

where now $1/(\cancel{D}\bar{D})'$ is the non-zero mode contribution to the effective $\tilde{\psi}_i^i \psi_i$ propagator.

The first term in eq. (B.6) reproduces the result given in eq. (A.20). The term in square brackets represents the sought mass corrections.

REFERENCES

- [1] E. Witten, Nucl. Phys. B188 (1981) 513 and B202 (1982) 253;
S. Ceccotti and L. Girardello, Phys. Lett. 110B (1982) 39.
- [2] G. Veneziano and S. Yankielowicz, Phys. Lett. 113B (1982) 321.
- [3] T.R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B218 (1983) 493.
- [4] M. Peskin, SLAC-Pub. 3061 (1983).
A.C. Davis, M. Dine and N. Seiberg, Phys. Lett. 125B (1983) 487.
H.P. Nilles, Phys. Lett. 129B (1983) 103.
J.M. Gérard and H.P. Nilles, Phys. Lett. 129B (1983) 243.
- [5] G. Veneziano, Phys. Lett. 124B (1983) 357 and 128B (1983) 199;
G. Shore, Nucl. Phys. B231 (1984) 139.
- [6] K. Konishi, Phys. Lett. 135B (1984) 439.
See also:
T.E. Clark, O. Piguet and K. Sibold, Nucl. Phys. 159B (1979) 1;
S.J. Gates, N. Grisaru, M. Roček and W. Siegel, Superspace (Benjamin-Cummings, Reading, Mass. 1983), Chapter 6.
- [7] T.R. Taylor, Phys. Lett. 125B (1983) 185 and 128B (1983) 403;
J.M. Gérard, J. Govaerts, Y. Meurice and J. Weyers, Phys. Lett. 116B (1982) 29.
- [8] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Nucl. Phys. B229 (1983) 394;
V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B229 (1983) 381 and *ibid.* 407.
- [9] E. Cohen and C. Gomez, Phys. Rev. Lett. 52 (1984) 237.
C. Gomez, University of Salamanca preprint DFTUS 03/84 (1984).
- [10] G.C. Rossi and G. Veneziano, Phys. Lett. 138B (1984) 195.

- [11] I. Affleck, M. Dine and N. Seiberg, Phys. Rev. Lett. 51 (1983) 1026;
"Dynamical SUSY breaking in SQCD" Princeton IAS preprint (Nov. 1983);
Phys. Rev. Lett. 52 (1984) 1677.
- [12] Y. Meurice and G. Veneziano, Phys. Lett. 141B (1984) 69.
- [13] J. Wess and B. Zumino, Phys. Lett. 49B (1974) 52;
J. Illiopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
S. Ferrara, J. Illiopoulos and B. Zumino, Nucl. Phys. B77 (1977) 413;
M.T. Grisaru, M. Roček and W. Siegel, Nucl. Phys. B159 (1979) 429.
- [14] G. 't Hooft, Commun. Math. Phys. 81 (1981) 267.
- [15] E.F. Corrigan, D.B. Fairlie, S. Templeton and P. Goddard, Nucl. Phys. B140
(1978) 31;
E. Corrigan, P. Goddard and S. Templeton, Nucl. Phys. B151 (1979) 93.
- [16] P. di Vecchia, R. Musto, F. Nicodemi and R. Pettorino, preprint CERN TH.3905
(1984).
- [17] G. 't Hooft, Phys. Rev. Lett. 37 (1976) 9 and Phys. Rev. D14 (1976) 3432.
- [18] M. Schmid, preprint CERN TH.3836 (1984).
- [19] J. Wess and J. Bagger, Supersymmetry and supergravity (Princeton University
Press, Princeton NJ, 1983).
- [20] L.S. Brown, R.D. Carlitz, D.B. Creamer and C. Lee, Phys. Lett. 70B (1977)
180.
- [21] See for instance:
S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
N. Sakai, Z. Phys. C11 (1981) 153.
- [22] R. Barbieri, A. Masiero and G. Veneziano, Phys. Lett. 128B (1983) 179.

- [23] K. Konishi, University of Pisa preprint IEUP TH-11/84;
Y. Meurice, private communication.

- [24] A. D'Adda and P. Di Vecchia, Phys. Lett. 73B (1978) 162.

Table

Properties of the various $U_A(1)$ defined in the text.
Chiral charges and dimensions of the relevant elementary fields.
(Chiral charges are additive).

	$U_A(1)$	$U_\lambda(1)$	$\hat{U}_A(1)$	$\hat{U}_A^{(k)}(1)$	$U_R(1)$	
Anomaly	2MQ	2NQ	0	0	(3M-N)Q	
Mass breaking	Yes	No	Yes	Yes	Yes	Mass dimensions
λ	0	1	-M	-1/N	3/2	3/2
$\psi_i, \tilde{\psi}^i$	1	0	N	δ_{ik}	-1/2	3/2
$\phi_i, \tilde{\phi}^i$	1	1	N-M	$-1/N + \delta_{ik}$	1	1