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STABLE COMPACTIFYING EINSTEIN-YANG-MILLS SYSTEMS

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A B S T R A C T

A stable spontaneously compactifying solution of the six-dimensional $SU(3)$ Einstein-Yang-Mills theory over $M^4 \times S^2$ with massless fermions is exhibited.

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In the last few years we witnessed a revival of interest in using theories based on the old assumption [4] that the physical space-time has more than four dimensions to explain the observed properties of elementary particles. The spontaneously compactified theories [2-3] form a class of these models: they are theories of gravity coupled to Yang-Mills and spinor fields in a 4+N dimensional space-time which are reduced - via a solution of the Einstein-Yang-Mills /EYM/ field equations - to a 4 dimensional low energy theory by compactifying the extra N dimensions into a compact coset space S/R with an appropriately small size R_0 . It is clear that the low energy theory - that consists of the massless fluctuations around the solution - may have physical relevance only if the solution is stable, i. e. if there are no tachyons among the massive excitations.

In constructing the compactifying EYM solution it may be useful to exploit the symmetry of S/R. A general scheme - we call it coset-space dimensional reduction scheme /CSDR/ - was given in [8] to derive S symmetric G gauge fields on $M^4 \times S/R$ - that are a priori no solutions to any equation - if R is embedded into G. The extra components of the symmetric gauge vectors form scalar fields in 4 dimensions, their potential is determined by the extra components of the field strength. Sometimes this potential is of symmetry breaking type: e.g. if S is also embedded into G then this is the case but then the minimum of the potential is at 0 [9], i.e. this minimum corresponds to a pure gauge A_μ on S/R. Recently in [5] a superficially different method was presented to construct S symmetric R gauge fields that solve the EYM equations if S/R is a symmetric space. One of us has quite recently shown [40] that

these gauge fields can be identified with the "unbroken minimum" of the CSDR scheme / i.e. with the state when the 4 dimensional scalar fields vanish / and - using the harmonic expansion [44] for the fluctuations in the pure G gauge theory on $M^4 \times S/R$ - found that the solution is unstable under very general circumstances. In the meantime it was discovered [7] that for $S/R = S^2$ a compactifying $U(1)$ solution when embedded into $SU(2)$ develops instabilities both in the pure 6 dimensional gauge theory and in the coupled EYM theory. It is important to understand if these instabilities mean the complete disintegration of the models or merely signal the transition to a new vacuum state within the framework of EYM theory. The obvious candidate for this new state is the "symmetry breaking" minimum of the scalar potential predicted by the CSDR scheme. / Of course this may happen only when S is not embedded into G as a pure gauge on S/R cannot compactify the extra N dimensions. /

Motivated by these arguments in this paper we consider an $SU(3)$ gauge theory coupled to gravity in 6 dimensions and show that when the extra components of the gauge vectors are proportional to the spherically symmetric "symmetry breaking" minimum of a model constructed by Manton [42] one gets a solution of EYM equations with space-time being $M^4 \times S^2$. Furthermore, by applying the harmonic expansion to the fluctuations around this solution we point out that the solution is stable / i.e. has no tachyons / both in the pure $SU(3)$ gauge theory on $M^4 \times S^2$ and in the EYM theory. In the latter case we find that the number of zero mass vectors is consistent with the gauge group of the massless theory being $SU(2) \times SU(2) \times U(1)$ where one $SU(2)$ comes from the gauging of the invariance group of S^2 while the other $SU(2)$ and the $U(1)$ come from the

stability group in $SU(3)$ of the compactifying background solution. As this background on S^2 corresponds to an $SU(3)/Z_3$ / but not to an $SU(3)$ / monopole we can add fermions only if we put them [43] into triality 0 $SU(3)$ representations / i.e. where the centre of $SU(3)$ is represented trivially /. If we couple one 6 dimensional Weyl fermion belonging to the octet of $SU(3)$ to the model then after compactification in 4 dimensions we get both right and left handed massless fermions that are singlets under the "metric" $SU(2)$ while transforming as 2 and $\bar{2}$ under the other $SU(2)$ and have 3 and $\bar{3}$ $U(1)$ charges respectively. Clearly both the gauge group of the massless theory and the fermion quantum numbers suggest that we can consider this $SU(3)$ EYM theory only as a toy model exhibiting a stable spontaneous compactification with massless fermions on a coset S/R with a gauge group bigger than R .

Let us consider first the pure $SU(3)$ gauge theory on $M^4 \times S^2$ whose action is

$$S = \int d^4x d^2y \sqrt{-g} \left(\frac{1}{2} \text{Tr} F_{\mu\nu}^A F^{\mu\nu A} \right) \quad (1)$$

where x^a $a=0, \dots, 3$ are the Minkowski coordinates and $y^\mu = (\theta, \varphi)$ denote the usual spherical coordinates on S^2 , $F_{\mu\nu}^A$ is the field tensor

$$F_{\mu\nu}^A = \partial_\mu^A \partial_\nu^A - \partial_\nu^A \partial_\mu^A - e [A_\mu^A, A_\nu^A] = F_{\mu\nu}^i \frac{\lambda_i}{2}, \quad \text{Tr} \lambda_i \lambda_j = -2 \delta_{ij}$$

and e is the gauge coupling constant. / We use the $/-, +, +, +/$ signature and follow the notation of [6, 7]/. The field equations stemming from (1) are

$$(-g)^{-\frac{1}{2}} D_\mu \left[(-g)^{\frac{1}{2}} F^{\mu\nu} \right] = 0 \quad (2)$$

The $SU(3)$ gauge vectors that correspond to the symmetric breaking ground state of Manton's model [12] have the form $A_a=0$, $/a=0, \dots, 3/$; $A_\mu = \frac{1}{e} W_\mu$ / $\mu = \theta, \varphi$ / where

$$W_\theta = -\phi_1 = -\frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = -\frac{i}{2} T_2$$

$$\begin{aligned} W_\varphi &= \phi_2 \sin \theta - \phi_3 \cos \theta = \\ &= \frac{\sin \theta}{2i} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} - \frac{i}{2} \cos \theta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = -\frac{i}{2} \sin \theta T_1 - \frac{i}{2} \cos \theta T_3 + \end{aligned} \quad (3)$$

$$+ \frac{i}{6} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cos \theta$$

/ Here the T_i matrices form an $SO(3)$ subalgebra $[T_i, T_j] = i \epsilon_{ijk} T_k$ /. The only nonvanishing component of the field tensor is

$$F_{\theta\varphi} = -\frac{1}{e} H \sin \theta = \frac{i}{6} e^{-1} \text{diag}(-1, 2, -1) \sin \theta \quad (4)$$

and it is straightforward to verify that (3), (4) indeed solve (2). It is interesting to observe that the symmetric ground state of Manton's model corresponds to $W_\theta = 0$, $W_\varphi = -\phi_3 \cos \theta$ which obviously solves (2) as well.

We investigate the stability of the solution (3) by studying the small fluctuations around it, i.e. we write $A_\mu = \bar{A}_\mu + V_\mu$ / where $\bar{A}_\mu$ is the background given by (3) / and treat V_μ as small perturbations. However, before doing so we cast the background,

(3), into a simpler, "pure string" form using the gauge transformation

$$\Omega = \exp\left(\frac{i}{2} \tau_3 \varphi\right) \exp\left(\frac{i}{2} \tau_2 \theta\right) :$$

$$w_\theta = 0, \quad w_\varphi = H \cos \theta$$

The expression for w^φ is singular at both the north / $\theta = 0$ / and south / $\theta = \pi$ / pole of the sphere, however, with /singular/ gauge transformations $\exp(\mp H \varphi)$ we obtain

$$w_\theta^{N,S} = 0, \quad w_\varphi^{N,S} = H(\cos \theta \mp 1) \quad (5)$$

which are free of singularities in the $N / \theta < \frac{3\pi}{2} /$ or $S (\theta > \frac{\pi}{2})$ regions. The transition / patching / function connecting w^S and w^N is $T(\varphi) = \exp(2\varphi H)$ and the quantization condition is that $T(2\pi) = \exp(4\pi H) = k$ belongs to the centre of the gauge group [14]. For our H we find $k = \exp(i \frac{2\pi}{3}) \tau_3$ i.e. the solution (5) is a minimally charged $SU(3) / Z_3$ / but not an $SU(3) /$ monopole on S^2 . Furthermore, it is easy to see that the elements of H are the smallest possible ones consistent with k being the generator of the centre of $SU(3)$. It is in this property where eq.(5) basically differs from the symmetric ground state; for the latter the transition function would be $T = \exp(-2\varphi \phi_3)$ thus k is the same but the elements in ϕ_3 are twice as big as in H . On the basis of the existing monopole-stability analyses [15] we expect that the solution (5) is stable as it describes a "minimal" minimally charged $SU(3) / Z_3$ monopole while this is not so for the symmetric ground state.

Since the solution (5) is $SO(3) \times$ Poincare invariant we expand the fluctuations around it in terms of 4 dimensional fields [6, 7, 11]

$$v_a(x, \theta, \varphi) = \sum_{\ell=|\lambda|}^\infty \sum_m \sqrt{2\ell+1} D_{\lambda m}^\ell(\theta, \varphi) v_{am}^\ell(x); \quad (6)$$

$$v_\pm(x, \theta, \varphi) = \frac{1}{\sqrt{2}} (v_4 \mp i v_5) = \sum_{\ell=|\lambda|}^\infty \sum_m \sqrt{2\ell+1} D_{\lambda m}^\ell(\theta, \varphi) v_{\pm m}^\ell(x)$$

where $D_{\lambda m}^\ell$ are the $2\ell+1$ dimensional unitary irreducible representation matrices of $SU(2)$ and v_4, v_5 denote the tangent space /"flat"/ components of V_μ . The λ helicities are determined from the effect on $v_a, v_\pm$ of the gauge transformation and tangent space rotation that compensate the symmetry transformation (i.e. an $SO(3)$ rotation) on the background

$$v_a \rightarrow e^{H\lambda} v_a e^{-H\lambda}; \quad v_\pm \rightarrow e^{H\lambda} v_\pm e^{-H\lambda} \mp i\lambda$$

where $\lambda = \lambda(\theta, \varphi)$ is such that (5) stays invariant. The various elements of V transform as $e^{-i\lambda\lambda}$ under this transformation and correspondingly are expanded in terms of the various matrix elements $D_{\lambda m}^\ell$. For our H we find

$$\lambda(V) = \begin{pmatrix} 0 & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}; \quad \lambda(v_+) = \begin{pmatrix} 1 & \frac{1}{2} & 1 \\ \frac{3}{2} & 1 & \frac{3}{2} \\ 1 & \frac{1}{2} & 1 \end{pmatrix}; \quad \lambda(v_-) = \begin{pmatrix} -1 & -\frac{3}{2} & -1 \\ -\frac{1}{2} & -1 & -\frac{1}{2} \\ -1 & -\frac{3}{2} & -1 \end{pmatrix}$$

A novel feature of these λ matrices is the appearance of half-integer values; they imply that the expansion of the corresponding V elements contains only half-integer ℓ 's [3].

The linearized YM equations for the $[v_{am}^\ell(x)]_{ij}$ component fields, in an appropriate background gauge, read as

$$\left(\lambda^2 - \frac{\lambda(\ell+1) - \lambda_{ij}}{R_0^2} \right) \left[v_{am}^\ell(x) \right]_{ij} = 0, \quad \ell = |\lambda_{ij}|, |\lambda_{ij}|+1, \dots$$

$$\left(\lambda^2 - \frac{\lambda(\ell+1) - [\lambda_{+ij} \mp 1]^2}{R_0^2} \right) \left[v_{+m}^\ell(x) \right]_{ij} = 0, \quad (7)$$

$$\ell = |\lambda_{+ij}|, |\lambda_{+ij}| + 1, \dots$$

Here R_0 denotes the radius of the sphere. As $\lambda(v_+)$ contains only positive $\lambda(v_-)$ only negative / entries it is easy to see that every fluctuation has a positive or zero mass square i.e. the solution (5) is - at least perturbatively - stable. The only massless excitations are 4 Lorentz vector fields V_a / found for $\ell = \lambda = 0$ /; they form the gauge vectors corresponding to the $SU(2) \times U(1)$ stability group of H in $SU(3)$.

It is instructive to compare these findings with those for the symmetric ground state /i.e. when in (5) we replace H with $-\phi_3$ /. In the latter case all the λ matrices contain only integer entries but $\lambda(v_+)/\lambda(v_-)$ has two zero elements beside the positive / negative / ones and according to the expressions (7) the $\ell = \lambda = 0$ elements / and only those / in v_+ and v_- become tachyons.

Next we consider the coupled EYM theory in 6 dimensions

whose action is

$$S = - \int d^6x \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v_0 \right) \quad (8)$$

where G is the gravitational coupling and v_0 is a cosmological constant. It is straightforward to prove that the coupled EYM equations following from (3) reduce to certain algebraic equations for R_0 and v_0 [4] if we make the following ansatz for $g_{\mu\nu}$: $ds^2 = \gamma_{ab} dx^a dx^b + R_0^2 (\sin^2 \theta d\varphi^2 + d\theta^2)$ while assume that $A_a = 0$, $A_\mu = e^{-1} W_\mu$ with W_μ taken from (5). The solution of these equations is $R_0^2 = 8\pi G / 3e^2$; $v_0 = 3e^2 / (8\pi G)^2$; thus $M^4 S^2$ and (5) really solve the EYM theory.

To test the stability of this solution we must consider the fluctuations coming from the metric as well; i.e. we must put $g_{\mu\nu} = \bar{g}_{\mu\nu} + 4\sqrt{\pi G} h_{\mu\nu}$ and $A_\mu = \bar{A}_\mu + V_\mu$ and linearize the EYM equations in $h_{\mu\nu}, V_\mu$. The elements of $h_{\mu\nu}$ have a harmonic expansion similar to eq. (6); the λ values were obtained in [6] and are the following: $\lambda = 0$ for h_{ab} and $h_{+-} = \frac{1}{2}(h_{44} + h_{55})$; $\lambda = \pm 1$ for $h_{a+} = \frac{1}{\sqrt{2}}(h_{a4} \mp i h_{a5})$; $\lambda = \pm 2$ for $h_{++} = \frac{1}{2}(h_{44} - h_{55} \mp 2i h_{45})$.

The next step is to plug these harmonic expansions for h and (6) into the linearized EYM equations to derive the equations governing the 4 dimensional fields $[v_{am}^\ell(x)]_{ij}, [v_{+m}^\ell(x)]_{ij}, h_{ab}^{(\ell m)}(x), h_{a+}^{(\ell m)}(x)$ and $h_{++}^{(\ell m)}(x)$. In these equations [7] the mixing between the gauge and metric fluctuations depends only on the background gauge field tensor $F_{+-} = -\frac{1}{6eR_0^2} \text{diag}(-1, 2, -1)$ and as this matrix is diagonal we conclude that the inclusion of gravity does not affect the off-diagonal components of the gauge equations, (7). Furthermore, as the harmonic expansion of h_{a+} starts with $\ell=1$, the diagonal $\ell=0$ components of the V_a equation remain unchanged, thus the massless $SU(2) \times U(1)$ gauge vectors seen in the pure $SU(3)$ gauge theory survive the coupling to gravity. For $\ell \geq 1$ the equations

describing the mixing between the transversal vector fields

$\left[\frac{\ell m}{a} (x) \right]_{ii} = v_a^i(\ell m) / i = 1, 2, 3 /$ and $h_{a+}^{\ell m}$ are

$$\begin{aligned} \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) h_{a+}^{\ell m} + i \frac{\sqrt{2}\ell(\ell+1)}{R_0^2} \left(v_a^1(\ell m) + v_a^3(\ell m) \right) &= 0, \\ \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) v_a^{1,3}(\ell m) + i \frac{\sqrt{2}\ell(\ell+1)}{2R_0^2} \left(h_{a-}^{\ell m} - h_{a+}^{\ell m} \right) &= 0. \end{aligned} \quad (9)$$

Here we used that $v_a^2(\ell m) = - \left(v_a^1(\ell m) + v_a^3(\ell m) \right) / \sqrt{2}$. The mass eigenvalues obtained from (9) are $M_1^2 = \ell(\ell+1)/R_0^2$ and $M_2^2 = \left[\ell(\ell+1) \mp \sqrt{2\ell(\ell+1)} \right] / R_0^2$; thus we see that there are no tachyons in the vector spectrum and for $\ell=1$ we get one more triplet of zero mass vectors; they are the $SU(2)$ gauge vectors originating in the gauging of the symmetry group of the background metric.

A look at the general expressions derived in [7] reveals that the equations for the traceless tensor fields $\left(h_{ab} - \frac{1}{2} h_{cc} \delta_{ab} \right)^{\ell m}$ and for the scalars $h_{++}^{\ell m}$ decouple from the rest and give masses to these fields: $\ell(\ell+1)/R_0^2$ with $\ell \geq 0$ and $\ell \geq 2$ respectively.

However, the equations describing the scalars $h_{+-}^{\ell m}(x)$, $\left[\frac{\ell m}{a} (x) \right]_{ii} = v_+^i$; the divergences $\partial_a v^i(\ell m) = v^i$, $\partial_a h_{a+}^{\ell m} = H_+$; and the pure trace $h_{aa}^{\ell m}(x) = h_{aa}$ couple in an intricate way for $\ell \geq 1$ / for $\ell=0$ there is no mixing and they yield some massless fields /

$$\begin{aligned} \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) h_{aa} + 2 \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) h_{+-} + \frac{2i\sqrt{2}\ell(\ell+1)}{R_0^2} \left(v_-^1 + v_-^3 - v_+^1 - v_+^3 \right) &= 0 \\ \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) H_+ + \sqrt{6} R_0^{-1} \partial^2 \left[v_+^1 + v_+^3 \right] + i \frac{\sqrt{2}\ell(\ell+1)}{R_0^2} \partial^2 \left(v_+^1 + v_+^3 \right) &= 0 \end{aligned}$$

$$\begin{aligned} \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) h_{+-} + i \frac{\sqrt{2}\ell(\ell+1)}{R_0^2} \left(v_-^1 + v_-^3 - v_+^1 - v_+^3 \right) &= 0, \\ \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) v_+^{1,3} + \frac{i}{2R_0^2} \left[\frac{\ell(\ell+1)}{3} \left(H_- - H_+ \right) \right] &= 0, \\ \left(\partial^2 - \frac{\ell(\ell+1)}{R_0^2} \right) v_+^{1,3} + \frac{H_+}{R_0 \sqrt{6}} + \frac{i}{2R_0^2} \left[\frac{\ell(\ell+1)}{3} \left(h_{+-} - \frac{1}{2} h_{aa} \right) \right] &= 0. \end{aligned} \quad (10)$$

We say that our solution is stable if the zeros of the determinant of the Fourier transformed eq. (10) / with $\partial^2 \rightarrow -k^2$ / correspond to non-negative mass squares, as these zeros / together with the massless modes found for $\ell=0$ / provide the set from which the actually propagating masses should be selected according to the procedure of ref. [6]. We found that these zeros are at

$$-\frac{\ell(\ell+1)}{R_0^2}, \frac{1}{2R_0^2} \left\{ -\left[2\ell(\ell+1) + 3 \right] \pm \sqrt{12\ell(\ell+1) - \frac{7}{4}} \right\},$$

$$\frac{1}{2R_0^2} \left\{ -\left[\ell(\ell+1) + 1 \right] \pm \sqrt{4\ell(\ell+1) + 1} \right\} \quad \text{with various multiplicities}$$

and this proves the stability of our solution in the framework of EXM theory.

Because of the $SU(3)/Z_3$ nature of the compactifying solution we can add fermions to the model only if they belong to such $SU(3)$ representations that are in fact representations of $SU(3)/Z_3$ as well [13]. The simplest possibility is the octet. / In 6 dimensions we use a realization of Dirac matrices appropriate to the reduction of $SO(1,5)$ spinors into $SO(1,3) \times SO(2)$ ones: $\Gamma_a = \gamma_a \otimes \mathbb{1}_2$, $\Gamma_4 = \gamma_5 \otimes \tau_1$, $\Gamma_5 = \gamma_5 \otimes \tau_2$ with γ_a being the 4 dimensional Dirac matrices and τ_i the Pauli matrices. An irreducible $SO(1,5)$

spinor, ψ , belonging to the +1 eigenvalue of the $SO(1,5)$ invariant matrix $\overline{\Gamma} = \Gamma_0 \Gamma_1 \dots \Gamma_5 = \gamma_5 \otimes \tau_3$ naturally branches into the irreducible pieces under $SO(1,3) \times SO(2)$: ψ_R with $\gamma_5 = 1$ and $\tau_3 = 1$ and ψ_L with $\gamma_5 = -1$ and $\tau_3 = -1$. To obtain the fermion spectrum in 4 dimensions we expand both ψ_R and ψ_L in a way similar to eq. (6) and substitute these expressions into the mass operator $[\delta]/i \overline{\psi}_L \Gamma^\mu D_\mu \psi + \text{h.c.}$ containing the background metric and gauge fields. The λ values for the various components of ψ_L and ψ_R can be read off from the background covariant derivative of ψ ; if ψ is in the octet it has the form

$$D\psi = d\psi - i \left\{ \frac{1-\gamma_5}{2} \left(\frac{1}{2}\psi - i[H, \psi] \right) + \frac{1+\gamma_5}{2} \left(-\frac{1}{2}\psi - i[H, \psi] \right) \right\} (\cos \theta - 1) d\varphi \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

thus

$$(\lambda_L)_{ij} = -\frac{1}{2} - N_{ij}; (\lambda_R)_{ij} = \frac{1}{2} - N_{ij}; N = \begin{pmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Note that again the λ matrices contain both integer and half-integer entries and both λ_L and λ_R have two zeros allowing "symmetric" /i.e. $\ell = 0$ / fermions.

In terms of the harmonic components the mass term finally looks like [6]

$$\frac{i}{R_0} \sum_{lm} \sum_{ij} \sqrt{\left(\ell + \frac{1}{2}\right)^2 - N_{ij}^2} \left(\overline{\psi}_{Li j}^{\ell m} \psi_{Ri j}^{\ell m} - \overline{\psi}_{Ri j}^{\ell m} \psi_{Li j}^{\ell m} \right), \ell \geq |\lambda_{ij}|.$$

This implies that those components of ψ_R / ψ_L that belong to the zero elements of λ_R / λ_L in the $\ell = 0$ mode acquire no mass; thus after compactification we have two left and right handed

massless fermions that are singlets under the "metric" $SU(2)$. An inspection reveals that they transform as 2 and $\overline{2}$ under the other $SU(2)$ while coupling to the $U(1)$ with charges -3 and 3 respectively.

Unlike the massless "Kaluza-Klein" fermions found previously [2-4, 6, 16, 17] these ones are "non-topological" in the sense that - because of the Z_3 nature of the compactifying gauge fields - their appearance is not connected to any known index theorems. /Note also that the number of left handed and right handed zero modes is the same, a fact consistent with the vanishing index /.

All what we found here for $G = SU(3)$ generalizes in a simple way for $SU(N)$, $N \geq 3$. First in eq. (5) we replace H with

$$H = i \text{diag} \left(p, \dots, p, \underbrace{q, \dots, q}_{N-m} \right) \text{ possessing the stability group}$$

$SU(m) \times SU(N-m) \times U(1)$. The minimal values for p and q guaranteeing that the solution (5) is an $SU(N)/Z_N$ monopole on S^2 /i.e. $\exp(4\pi H) = \exp\left(i \frac{2\pi}{N}(N-m) \frac{1}{N}\right)$ are $p = \frac{N-m}{2N}$, $q = -\frac{m}{2N}$.

Computing with these values the λ matrices we find that the elements of $\lambda(V_a)$ are either 0 or $\pm \frac{1}{2}$ while those of $\lambda(V_+)/\lambda(V_-)$ can be $1, \frac{3}{2}, \frac{1}{2} / -\frac{1}{2}, -\frac{3}{2}, -1$ /. As λ_{ij} (7) are still valid for the fluctuations around this background we see that all masses are positive except some zeros in the $\ell = 0$ mode for those components of V_a that belong to the zero elements of $\lambda(V_a)$. /These modes constitute the gauge vectors for $SU(m) \times SU(N-m) \times U(1)$ /. When we couple gravity to the model we expect three more zero mass vectors from the mixing between the metric and $\ell=1$ vector fluctuations while there is no reason to worry that the stability behaviour of this solution would be worse than for $SU(3)$. The simplest

representation in which we can couple fermion is the adjoint of $SU(N)$. If we do this then from a ψ satisfying $\overline{\Gamma}\psi = \psi$ after compactification we get $(\underline{m}, N-m)$ left and right handed massless fermions that are singlets under the metric $SU(2)$ and transform as $(\underline{m}, N-m)$ and $(\overline{m}, N-m)$ under the $SU(m) \times SU(N-m)$ respectively. For $G = SU(4)$ we obtain the desired $SU(2) \times SU(3) \times U(1)$ gauge group for the massless theory, however, the fermion quantum numbers $/SU(2)$ singlets ψ_L in $\mathfrak{3}$ and in $\overline{\mathfrak{3}}$ of $SU(3)$ / are rather unrealistic. Nevertheless, we hope that the possibility discovered here may have some use in solving the notoriously difficult problem of predicting the fermion quantum numbers [16] - possibly in conjunction with more complicated coset spaces S/R .

REFERENCES

- [1] Th. Kaluza, Sitzungsberichte der Preuss. Akad. Wiss./1921/ 966
O. Klein, Nature 118/1926/516; Z. Phys. 37/1926/ 895
J. Rayski, Acta Phys. Pol. 27/1965/ 89
R. Kerner, Ann. Inst. H. Poincaré 9/1968/ 143
W. Thirring, Lecture Notes of 17th Internationale
Universitätswochen für Kernphysik, Schladming 1978.
E. Gremmer and J. Scherk, Nucl. Phys. B103/1976/; B108/1976/
409; B118/1977/ 61
J. F. Luciani, Nucl. Phys. B135/1978/ 111.
Z. Horváth, L. Palla, E. Gremmer and J. Scherk, Nucl. Phys.
B127/1977/ 57.
Z. Horváth and L. Palla, Nucl. Phys. B142/1978/ 327
L. Palla, Proc. of 19th International Conference on High
Energy Physics, Tokyo /1978/p. 629.
S. Randjbar-Daemi and R. Percacci, Phys. Lett. 117B (1982) 41.
S. Randjbar-Daemi, A. Salam and J. Strathdee, Nucl. Phys.
B214 /1983/ 491.
S. Randjbar-Daemi, A. Salam and J. Strathdee, Phys. Lett.
124B /1983/ 345.
P. Forgács and N. Manton, Commun. Math. Phys. 72 /1980/ 15.
D. Olive and P. West, Nucl. Phys. B217 /1983/ 243
J. Harnad, S. Schneider and J. Tafel, Lett. Math. Phys.
4 /1980/ 41.
L. Palla, CERN preprint TH-3614 /1983/ to appear in Z. Phys. C.
A. Salam and J. Strathdee, Ann. Phys. 141 /1982/ 316.

- [12] N. Manton, Nucl. Phys. B158 /1979/ 141.
- [13] Z. F. Ezawa and H. C. Tze, Orsay preprint LPTPE 76/3 /1976/.
- [14] E. Corrigan and D. Olive, Nucl. Phys. B110 /1976/ 237.
- [15] R. A. Brandt and F. Neri, Nucl. Phys. B161 /1979/ 253.
S. Coleman, Proc. of the 19th Int. School of Subnuclear
Physics, Erice 1981, /ed. A. Zichichi, New York, Plenum 1983/.
- [16] E. Witten, Princeton Univ. Preprint 1983 Oct.
- [17] S. Randjbar-Daemi, A. Salam and J. Strathdee, Phys. Lett.
132B /1983/ 56.