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ON THE SPIN-ZERO PARTNERS OF COMPOSITE W-BOSONS

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A B S T R A C T

The spin-zero partners of the W-bosons can couple to fermion pairs à la Higgs ($\tilde{\pi}$) or in a universal way (U,X). Their parameters are related to the dynamics of some class of QCD and SUSY-like composite models using spectral function sum rules. We show that a 50 GeV mass for these bosons does not need a relatively strong explicit breaking of the preon chiral symmetry. A light $\tilde{\pi}$ decouples in low energy weak interaction physics, in contrast to the pion in hadronic interactions. The best signal for the $\tilde{\pi}$ is its copious pair production in W or Z decays with a width of $O(0.25 \text{ GeV})$. We also show that a strong coupling for the Z-X- γ vertex and a weak one for the Z- e^+e^- vertex are dynamically consistent. However, a vector-meson dominance scenario at the Z-X- γ vertex can imply a $Z \rightarrow e^+e^-\gamma$ rate four times smaller than the present data.

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2. HYPERPIONS

A) Dynamics

We analyze here the dynamical properties of the spin-zero partners of the W-bosons which have a Higgs-type coupling to fermion pairs. These spin-zero partners will hereafter be called "hyperpiions". In terms of the elementary hapiion fields (α, β) , which also form the W-bosons, the hyperpiions $\tilde{\pi}$ are:

$$\tilde{\pi}^- = \alpha\bar{\beta} ; \quad \tilde{\pi}^+ = \bar{\alpha}\beta ; \quad \tilde{\pi}^0 = \frac{1}{\sqrt{2}}(\bar{\alpha}\alpha - \bar{\beta}\beta), \quad (2.1)$$

where the hapiions form a doublet of the weak isospin SU(2) global symmetry^{2,4)}. We have already studied in Ref.7) the case where the hapiions are spin-1/2 fields. Here, we shall study also the case of spin 0 and SUSY-hapiions, using a Finite Energy Sum Rule (FESR) method⁵⁾, which appears to give many more constraints on the boson parameters than the familiar Laplace(or Borel) sum rule⁹⁾, as we have seen for the analysis of the W-channel¹⁾.

1) Spin-1/2 hapiions:

The hyperpiions are associated to the divergence of the charged weak current:

$$\langle 0 | \partial_\mu J_W^\mu | \tilde{\pi}^- \rangle = \sqrt{2} F_\pi M_\pi^2, \quad (2.2a)$$

where F_π and M_π are the hyperpion decay amplitude and mass while:

$$\partial_\mu J_W^\mu = i \frac{1}{2} (m_\alpha + m_\beta) \bar{\alpha} \gamma_5 \beta : \quad (2.2b)$$

The above parameters are related to each other through the PCAC-like relation: $2M_\pi^2 F_\pi^2 = -\frac{1}{2} (m_\alpha + m_\beta) \langle \bar{\alpha} \alpha + \bar{\beta} \beta \rangle$, (2.3) which shows the fact that the global chiral symmetry is realized à la Nambu-Goldstone. $\langle \bar{\alpha} \alpha \rangle$ is the hapiion vacuum condensate which is known to be of the order of $(-5M_\pi)^3$ ¹⁾. The value of the hapiion mass m_α controls the explicit breaking of the global chiral symmetry and so might depend on the less-understood nature of the quarks and leptons. Actually in some models where these latter are composite objects, and are some fermionic partners of some Goldstone bosons, one would prefer a "good" realization of the chiral symmetry. Here, we shall leave the hapiion mass as a free parameter and we shall analyze two different cases. In the first case, we assume that m_α has a value of few tens of MeV, i.e., it is of the order of the light quark mass value and to which will correspond light hyperpiions. In the second case, we shall allow a larger value of m_α of the order of a few GeV, to which will, presumably, correspond much heavier hyperpiions.

1. INTRODUCTION

In a previous paper¹⁾, one of us has analyzed the constraints which can be obtained from a QCD²⁾ and a SUSY³⁾-like composite models of weak interactions in the case where the W-bosons form a triplet of the SU(2) global weak isospin symmetry⁴⁾ like the ρ -mesons of QCD form a triplet of the SU(2) strong isospin symmetry. Using a finite energy sum rule-like approach based on a local duality between the theoretical and the phenomenological sides of the sum rule⁵⁾, it has been shown¹⁾ that the knowledge of the W and Z parameters from the data⁶⁾ fixes the vacuum structure of the underlying hyperstrong theories and the continuum threshold above which one expects to observe the composite structure of the W and Z bosons. In this paper, we shall analyze within the same framework the properties of the spin zero partners of the W-bosons, which can have a Higgs-type (the $\tilde{\pi}$) or a universal (the X) coupling to fermion pairs. The existence of the former is inherent to the idea of having a SU(2)(or its extension) global chiral symmetry realized à la Nambu-Goldstone. So in this way, the $\tilde{\pi}$ can be associated to the divergence of the charged weak current⁷⁾. The existence of the X has a phenomenological motivation because it can explain the anomalous large decay rate of the Z into e^+e^- ⁸⁾. In the first part of the paper we analyze the properties of the $\tilde{\pi}$ using the information on the dynamical parameters obtained in Ref.1) . We also discuss some phenomenological consequences of the $\tilde{\pi}$ at present energies. We show that the $\tilde{\pi}$ decouple from ordinary matter and this fact differs considerably from the case of the pion for hadronic physics. We find that the only relevant effect of the $\tilde{\pi}$ is its copious production from the W and Z decays which can reduce the leptonic branching ratio by 6 to 10%. So, one should consider this section of the paper as an improved and an extended version of the already published paper quoted in Ref.7. In the second part of the paper (section 3), we discuss the property of the X-boson which is assumed to couple universally to fermion pairs. We attempt to analyze the consistency of the assumed property of the X-boson and the Nambu-Goldstone realization of chiral symmetry. We shall show that a mass of the X-boson which is of the order of 50 GeV can be understood if one allows an explicit breaking of the chiral symmetry by a mass term which is 3% of the scale of compositeness which we take, for example, to be of the order of 1 TeV¹⁾ (recall that in QCD the ratio of the up quark current algebra mass over the hadronic scale is of the order of 8%). We shall also discuss the various assumptions of the nature of the X-bosons and we complete our discussion by selecting the most stringent constraints on the X-boson parameters from the present data and by showing the dynamical difference between the Z-X- γ and the Z- e^+e^- vertices.

(i) Exact SU(2) global symmetry :

In this case, m_a is almost zero (we take it to be a few tens of MeV) compared to the compositeness scale . The corresponding Goldstone boson might also be very light^{*)}, and we expect it to be produced from the J/ψ radiative decay . Using a Goldberger-Treiman -like relation for the estimate of the $\tilde{\pi}$ -coupling to fermion pairs⁷⁾ :

$$g_{\tilde{\pi}ff'} \approx \frac{(m_f^2 + m_{f'}^2)^{1/2}}{\sqrt{2}F_{\tilde{\pi}}} U_{ff'} \quad (2.4)$$

where $U_{ff'}$ is a Cabibbo-like mixing angle between the two fermions , we can deduce the branching ratio :

$$\frac{\Gamma(\psi \rightarrow \tilde{\pi}\gamma)}{\Gamma(\psi \rightarrow e^+e^-)} \approx \frac{1}{2\pi\alpha} \frac{g_{\tilde{\pi}ee}^2}{g_{\tilde{\pi}ee}^2} \quad (2.5)$$

which is known experimentally to be less than 10^{-5} ¹⁰⁾. Then we obtain the constraint :

$$F_{\tilde{\pi}} \geq 2.2 \text{ TeV} \quad (2.6)$$

This large value of $F_{\tilde{\pi}}$ implies that a light $\tilde{\pi}$ should decouple from ordinary matters as its coupling with fermions is inversely proportional to $F_{\tilde{\pi}}$ (see Eq.(2.4)) . Using this lower bound of $F_{\tilde{\pi}}$ into Eq.(2.3) , we deduce:

$$M_{\tilde{\pi}} \lesssim 272 \text{ MeV} \quad (2.7)$$

for m_a of the order of 10 MeV . It is easy to see that the presence of this light $\tilde{\pi}$ does not affect the well-established low energy data , and it be interesting to see if its presence affects the cosmology data . In this case of spin-1/2 haptions , the charged hyperpions can also acquire a mass via electromagnetism . We estimate such a contribution using the standard current algebra spectral function sum rule approach¹¹⁾, which should be valid here as we have a good realization of the SU(2) global symmetry . Then , we get:

$$M_{\tilde{\pi}^+}^2 - M_{\tilde{\pi}^0}^2 \approx \left(\frac{q}{g_H}\right) \cdot (3.692) \frac{1}{F_{\tilde{\pi}}^2} M_W^2 F_W^2 \quad (2.8)$$

which is smaller than $(86 \text{ MeV})^2$ for the value of $F_{\tilde{\pi}}$ given in Eq.(2.6) . Eq.(2.8) shows that in the chiral limit where $m_a = 0$, the charged $\tilde{\pi}$ acquires a tiny mass .

*) In some broken SUSY-like models , the $\tilde{\pi}$ might acquire a large mass through the dynamical breaking parameters of supersymmetry .

(ii) Explicitly broken SU(2) global symmetry :

Here , we analyze the case where the $\tilde{\pi}$ can have a mass larger than 15 GeV as required from the PETRA experiment bound on scalar particles¹²⁾ . We can assume that this mass originates from an explicit breaking of the global chiral symmetry . Actually , such a possibility is not a priori ruled out insofar as we do not yet have a clear understanding (theoretically and experimentally) of the exact structure of quarks and leptons as well as of the origin of their masses . So , in this case of a $\tilde{\pi}$ with a large mass , we expect that the $\tilde{\pi}$ -dominance to the spectral function of the two-point function :

$$\Psi_W(q^2 < 0) = i \int d^4x e^{iqx} \langle 0 | T \gamma_\mu J_W^\mu(x) (\partial_\mu J_W^\mu(0))^\dagger | 0 \rangle \quad (2.9)$$

associated to the divergence of the charged current can be a good approximation (at most the higher states contribution to the spectral function is three times the $\tilde{\pi}$ one) . So , we expect a QCD sum rule-like analysis to give a useful information on the $\tilde{\pi}$ -parameters . The Laplace transform sum rule can be obtained from the dispersion relation :

$$\Psi_W(q^2 \equiv -Q^2 < 0) = \int_0^\infty \frac{dt}{t+Q^2} \frac{1}{t} \text{Im} \Psi_W(t) + \text{"subtraction"} \quad (2.10)$$

by applying to both sides the operator⁹⁾ :

$$\int_0^\infty \frac{dt}{t+Q^2} \frac{1}{t} \text{Im} \Psi_W(t) = \int_0^\infty \frac{dt}{t+Q^2} \frac{1}{t} \text{Im} \Psi_W(t) + \text{"subtraction"} \quad (2.11)$$

The sum rule analysis is very similar with the one in Refs.7,9c,13) . It consists on the evaluation of the QHD diagrams given in Fig.1.1 in the deep Euclidean region and on the saturation of the spectral function by the lowest ground state plus its eventual excitation . In this way , one can deduce the following constraint for $SU(N)_H$ and for colour triplet N_c -haptions :

$$\frac{N_c N_H}{16\pi^2} M_H^2(\tau_0) \approx (1 \sim 4) M_{\tilde{\pi}}^2 F_{\tilde{\pi}}^2 \tau_0^2 \quad (2.12)$$

where $\tau_0 \sim 1/2$ is the scale at which one expects the resonance contribution to the sum rule to be optimized . We have shown in Ref.1) that this scale is of the order of the compositeness scale, i.e., it is of the order of 1 TeV. The factor 4 inside parentheses corresponds to the expected maximal effect of the states having higher masses . Solving Eqs. (2.3) and (2.12) , we get :

$$M_{\tilde{\pi}}^2 \approx - \left(\frac{N_c N_H}{16\pi^2} \right) (1 \sim 0.25) \frac{\tau_0^{-2}}{\langle \bar{\alpha} \alpha \rangle} M_H \quad (2.13a)$$

$$F_{\tilde{H}} \approx -\left(\frac{46\pi^2}{M_c M_H}\right)^{1/2} (1 \sim 2) \tau_0 \langle \tilde{A}^* \alpha \rangle . \quad (2.13b)$$

Then, for instance $N_{cH} = 3$, $\tau_0 \approx 1 \text{ TeV}^{-2}$ and $\langle \alpha \rangle \approx (-5M_W)^3$; we deduce :

$$M_{\tilde{H}} \approx (1 \sim 0.5) \cdot 4.7 (m_u M_W)^{1/2}, \quad (2.14a)$$

$$F_{\tilde{H}} \approx (1 \sim 2) 500 \text{ GeV}. \quad (2.14b)$$

If one uses the PETRA bound on the mass of scalar particles, i.e., $M_{\tilde{H}}$ should be larger than 15 GeV [2], we get :

$$m_u \gtrsim (1 \sim 4) (420 \text{ MeV}). \quad (2.15)$$

We conclude from our previous analysis that an α -mass of the order of 60 MeV is enough to give a mass of the order of 15 GeV to the $\tilde{H}$, which has a decay constant given by Eq. (2.14a). It is interesting to notice that even for this case, the α -mass is not unreasonably large for communicating a quite large mass to the $\tilde{H}$. So, at this level we can still consider the SU(2) global chiral symmetry to be "quasi-unbroken". Actually, one can also, for example, see that a $\tilde{H}$ -mass of the order of 50 GeV would require a haplon mass less than 3 GeV, i.e., the ratio of the haplon mass over the compositeness scale is of the order of 3% and it is smaller than the ratio of the up quark mass over the 1 GeV characteristic hadron scale. In addition, we can also deduce from Eq. (2.8) the electromagnetic mass shift between the charged and neutral hyperpions. We obtain :

$$M_{\tilde{H}}^2 - M_{\tilde{H}^0}^2 \approx (1 \sim 0.25) (0.52 \text{ GeV})^2 \quad (2.16)$$

which is still a tiny effect compared to the one which originates from the explicit breaking of the SU(2) chiral symmetry.

2) Spin-0 and SUSY-haplons :

Here, we point out the main difference of the previous case with the one where we have unbroken supersymmetric (SUSY) - like theories. The main features of SUSY-like theories are :

(i) One has a new Dashen formula¹⁴⁾ :

$$M_{\tilde{H}} F_{\tilde{H}}^2 \approx 2 m_u \langle \phi^* \phi \rangle, \quad (2.17)$$

coming from the action of the axial charge to the $\theta=0$ component of the super-

potential $\cdot \langle \phi^* \phi \rangle$ is the vacuum condensate of the scalar fields. One can see from Ref.15) that such a relation combined to other ones gives for small numbers of the haplon flavour (1 or 2) :

$$M_{\tilde{H}} \approx \sqrt{2} m_u, \quad F_{\tilde{H}} \approx \langle \phi^* \phi \rangle^{1/2} \approx \sqrt{2} \langle \phi^* \phi \rangle^{1/2}. \quad (2.18)$$

In this case the scalar condensate like $F_{\tilde{H}}$ is singular in the chiral limit. Then a light $\tilde{H}$ should decouple from ordinary matter as its coupling is given by Eq.(2.4).

(ii) It has been shown¹⁶⁾ that the $\tilde{H}$ does not acquire a mass from electromagnetism in an unbroken SUSY-theory. This result can be viewed as an indirect consequence of the non-renormalization theorem of elementary superfields at the bound state level. However, we should also mention that the $\tilde{H}$ can obtain a mass from the dynamical breaking mechanism.

B) Decay modes of hyperpions

We have seen from Eq.(2.4) that the hyperpions behave like technipions because of their Higgs-type coupling. The properties of hyperpions have been discussed in Ref. 10). The decay of the light hyperpions into ordinary particles should be highly suppressed by the small available phase space and by the large decay amplitude obtained in Eq.(2.6). Such properties make the difference between the hyperpions and the pions of QCD. The two-photon decay of a neutral hyperpion is¹⁷⁾

$$\Gamma(\tilde{H} \rightarrow \gamma\gamma) \approx \left(\frac{g}{f}\right)^2 (Q_u^2 - Q_d^2) \frac{1}{64\pi} \frac{M_{\tilde{H}}^3}{F_{\tilde{H}}^2}, \quad (2.19)$$

which is very small for a light hyperpion. A much heavier hyperpion can have a much greater possibility to decay into quarks and leptons pairs. The decay rate is :

$$\Gamma(\tilde{H} \rightarrow q\bar{q}) \approx \frac{1}{i\pi} M_{\tilde{H}} g^2 \left(1 - 4\frac{m_q^2}{M_{\tilde{H}}^2}\right)^{3/2}, \quad (2.20)$$

where the coupling constant is in Eq.(2.4). So, a decay of a neutral hyperpion into a pair of muons is :

$$\Gamma(\tilde{H} \rightarrow \mu^+\mu^-) \approx (26 \text{ eV})^4 \sim 0.25 \left(\frac{M_{\tilde{H}}}{15 \text{ GeV}}\right)^3, \quad (2.21)$$

from which we deduce the branching ratio :

$$B_{q\ell} \equiv \frac{\Gamma(\tilde{H} \rightarrow q\bar{q})}{\Gamma(\tilde{H} \rightarrow \mu^+\mu^-)} \approx \left(\frac{m_\ell}{m_q}\right)^2 \left(1 - 4\frac{m_\ell^2}{M_{\tilde{H}}^2}\right)^{3/2}, \quad (2.22)$$

which is given in the Table. In the same way, we can also study the branching ratio of the charged hyperpion. The different channels of possible decays of the hyperpion are also given in the Table. One can notice that a hyperpion

*) Q_1, N_c, N_H are respectively the charge and the numbers of colour and hypercolours of the haplon i.

having a mass lighter than the top quark has a total width less than 100-200 keV. If the hyperpion has a mass larger than the top one, one would expect that the most dominant decay proceeds via t and b quarks jets with a $\sin^2\theta$ angular distribution characteristic of a scalar particle decay.

C) Production modes of hyperpions

Let us discuss only the most interesting productions of the hyperpions. Actually, as the hyperpions behave like technipions, we refer to Ref. 10) for a more detailed study of various production modes of the hyperpions. The standard way for producing charged hyperpion pairs is in e^+e^- experiment with the branching ratio:

$$\frac{\Gamma(e^+e^- \rightarrow \tilde{\pi}^+\tilde{\pi}^-)}{\Gamma(e^+e^- \rightarrow \mu^+\mu^-)} \approx \frac{1}{4}\beta^3, \quad (2.23)$$

where β is the hyperpion velocity. The other expected production of the hyperpion is through the radiative decay of the toponium bound state if the hyperpion mass is larger than 15 GeV so that it cannot be produced from the decay of already known quarkonia. The branching ratio is:

$$\frac{\Gamma(T \rightarrow \tilde{\pi}\tilde{\pi})}{\Gamma(T \rightarrow e^+e^-)} \approx \frac{1}{2\pi\alpha} g_{\tilde{\pi}\tilde{\pi}T}^2 \left(1 - \frac{M_{\tilde{\pi}}^2}{M_T^2}\right), \quad (2.24)$$

which is of the order of 0.5 for a top quark mass of 50 GeV and for $F_{\tilde{\pi}}$ of the order of 350 GeV (Eq. (2.14)). Another possibly more exciting production mode is by the W and Z - decay. We estimate the branching ratio as ⁷⁾:

$$\Gamma(W^- \rightarrow \tilde{\pi}^-\tilde{\pi}^+) \approx \frac{1}{48\pi} g_{W\tilde{\pi}\tilde{\pi}}^2 M_W \left(1 - \frac{M_{\tilde{\pi}}^2}{M_W^2}\right)^{3/2}, \quad (2.25)$$

for a hyperpion lighter than 40 GeV. We have estimated the W - $\tilde{\pi}$ - $\tilde{\pi}$ coupling by assuming a universal coupling of the W to fermion pairs and to the hyperpion pairs at zero momentum transfer ⁷⁾. This assumption is very similar to the usual ρ -universality relation which equates the ρ -couplings to e^+e^- , $\pi^+\pi^-$ and NN . In this way, one gets:

$$g_{W\tilde{\pi}\tilde{\pi}} \approx g_{Wee} \approx M_W/F_W \approx 0.6 \quad (2.26)$$

which implies the decay rate given in Fig. 2.1. Then:

$$\Gamma(W \rightarrow \tilde{\pi}\tilde{\pi}) \leq 0.2 \text{ GeV} \quad \text{and} \quad \Gamma(Z \rightarrow \tilde{\pi}\tilde{\pi}) \leq 0.316 \text{ GeV} \quad (2.27)$$

The effect is important and is like the one of one generation. It may reduce the leptonic branching ratio of the W and Z by 6-10% ⁷⁾. But a firm statement is difficult because of many uncertainties (calculation of the Dreil-Yan process which initiates the W and Z productions, other possible exotic decays). However, the production of $\tilde{\pi}\tilde{\pi}$ will induce four quark jets which are harder than the QCD induced one because the latter contain gluon jets.

3. SPIN-ZERO BOSONS COUPLED UNIVERSALLY TO FERMION PAIRS

Such spin-zero bosons (hereafter called $U(X)$) should be contrasted to the hyperpions discussed in the previous section. We have seen that the latter couples to fermion pairs with a Higgs-type coupling $m_f / F_{\tilde{\pi}}$, which is a very small coupling for small m_f and for large $F_{\tilde{\pi}}$. So, this fact renders the $\tilde{\pi}$ -effect inobservable at low energies. On the contrary, one has assumed ¹⁸⁾ that the $U(X)$ couples universally to fermion pairs as do the W and Z bosons. Within this assumption, one can explain the large decay rate of the Z into e^+e^- for a e^+e^- invariant mass which is of the order of 50 GeV ⁸⁾. It is almost obvious that the X cannot be a pion-like but could be, probably, a $U(1)$ particle, as is the η' of QCD ^{18a)} or a parity doublet state ^{18b)}.

A) Phenomenological constraints on the $U(X)$ -boson

Let us summarize the present constraints on the parameters of the X -boson from the data:

(i) The large decay rate Γ_r of the Z into e^+e^- assumed to occur through a virtual X -boson imposes a mass of the X of the order of 50 GeV. Also, one needs a leptonic branching ratio of the X of the order of 6%. So, one roughly expects a total width of the order of:

$$\Gamma_X \rightarrow all \approx 18 \Gamma_X \rightarrow e^+e^- + \Gamma_X \rightarrow \nu\bar{\nu} \quad (3.1)$$

for five quarks and three leptons where the two-photon width should be less than the electronic one ⁹⁾.

(ii) The recent e^+e^- data search for the X -boson ¹⁹⁾, mainly the one from the Bhabha scattering reaction implies the following constraints:

- An X -boson having a mass lower than 42.5 GeV is excluded with a 95% confidence level.

- An X -boson lighter than the Z -boson and for a Γ_r larger than 20 MeV is excluded.

- For Γ_r which is in the range 15 to 20 MeV, an X -boson having a mass between 47 and 68 GeV is allowed provided that its coupling $g^2/4\pi$ to fermion pairs is less than $7.5 \cdot 10^{-4}$.

(iii) The evaluation of the X -contribution to the $(g-2)$ of the electron and muon ²⁰⁾ shows that the main effect is due to the one of Fig. 3.1. Such a contribution can be a hundred times larger than the one allowed by the discrepancy between the theoretical and experimental results of the $(g-2)$ unless the different couplings of the X -boson to fermion pairs and to the Z -photon obey the following constraint:

$$ac = -bd \quad (3.2)$$

+) A more detailed discussion on some phenomenological implications of the X -boson will be done in section 3.E.

where the $X\bar{e}^+e^-$ vertex is proportional to $(\bar{e}\gamma_\mu)_5$ and the $Z\bar{X}\gamma$ vertex is proportional to $(\bar{e}\gamma_\mu)_5$. In particular, the previous constraint shows that a $V\text{-}A$ coupling at the electronic vertex should be accompanied by a $V+A$ one at the three-bosons vertex. One should also notice that the contribution of the diagram in Fig. 3.2 is smaller than the other weak bosons contributions of Fig. 3.3.

B) Can the X-boson be η' -like?

Let us assume that the X-boson is associated to the $U(1)_A$ singlet current as is the η' for the case of QCD. Then, in the limit of the zero hapon mass, the X can be related to the anomaly-like current:

$$\langle 0 | \frac{\partial_\mu}{2\pi} \vec{F}_{\mu\nu} \vec{F}^{\mu\nu} | X \rangle = \sqrt{2} F_X M_X^2, \quad (3.3)$$

for $SU(2) \times SU(3)_H \times U(1)_A$, F_X is the X-decay amplitude and $\vec{F}_{\mu\nu}$ is the dual of the hypergluon field strength tensor. Under the above assumption for the nature of the X-boson, we expect that the QCD sum rule used for a pure Yang-Mills should work here, for a non-supersymmetric theory. Then, one can deduce the sum rule (22):

$$\int_0^\infty dt e^{-t\tau} (1 - t\frac{\tau}{2}) \frac{1}{\pi} \text{Im} \psi(t) \approx \left(\frac{q_H}{\pi} \right)^2 \frac{1}{\pi^2} \tau^{-2} \left\{ \frac{1}{\log \tau \Lambda^2} + 2\pi^2 \langle F^2 \rangle \tau^2 + \dots \right\}. \quad (3.4)$$

Now we introduce the ansatz:

$$\frac{1}{\pi} \text{Im} \psi(t) = 2 M_X^2 F_X^2 \delta(t - M_X^2) + \text{"QHD continuum"} \quad (3.5)$$

for a parametrization of the spectral function. The QHD continuum is assumed to be given by the discontinuity of the lowest order term in the RHS. Using a finite-energy-like form of the sum rule, which is equivalent to leading order by matching the same powers of τ in the two sides of the sum rules, we get the constraint:

$$2 M_X^2 F_X^2 + \left(\frac{q_H}{2\pi} \right)^2 \frac{1}{\pi^2} \frac{t^2}{\log(1/t_0 \Lambda^2)} \approx \left(\frac{q_H}{\pi} \right)^2 \langle F^2 \rangle. \quad (3.6)$$

If, for example, we take $M_X = 50$ GeV and $F_X = 100\text{--}200$ GeV $^+$, $q_H^+ \langle F^2 \rangle = (12 M_W^4)^{1/4}$ and $q_H(\tau_0) = 0.6$, we need for $\Lambda \sim 100$ GeV:

$$\sqrt{t_0} \approx 60 M_W, \quad (3.7)$$

from the constraint in Eq.(3.6). The threshold of the continuum is five times larger than the one for the W -channel¹⁾. This fact indicates, perhaps, some inconsistencies for the assumption of the nature of the X. Now, we can turn the constraint the other way around. One can assume

+) The assumption of the universal coupling to fermions might suggest that F_X has the same order value than F_Z . We give in Section 3.E an alternative way for the estimation of F_X .

that the continuum threshold of the W and the X-bosons channels is approximately the same and is of the order of 1 TeV. Then, for the value of F_X used above, one can deduce from Eq.(3.11) the estimate:

$$M_X \approx (1 \sim 2) \text{ TeV} \quad (3.8)$$

of the mass of the lowest ground state boson. This result is certainly higher than the one used for a phenomenological reason¹⁸⁾. One can conclude from the above discussion that a X-boson having a mass of the order of 50 GeV and a decay amplitude of the order of 100-200 GeV can be a η' -type particle if there is a large gap between the lowest ground state and the continuum threshold.

C) Can the X-boson be η' -like?

Another possible assumption for the nature of the X-boson is the one where it is an isoscalar singlet. This assumption is very similar to the case of the η or of the ϵ of QCD. In this case the X-boson is a $SU(2)$ singlet and there is no extra- $U(1)_A$ piece which contributes to its dynamics. This case corresponds to the one where the $SU(2)$ group is a subgroup of a higher symmetry group. Then, in terms of the hapon fields:

$$|X\rangle \equiv \frac{1}{\sqrt{2}} (\bar{\alpha}\alpha + \bar{\beta}\beta) \quad (3.9)$$

1) Spin-1/2 hapons:

We shall study the two-point function:

$$\psi(q^2) = i \int d^4x e^{iqx} \langle 0 | T \bar{J}_X(x) J_X(0) | 0 \rangle, \quad (3.10)$$

where we normalize the current as:

$$J_X = \frac{m}{\sqrt{2}} (\bar{\alpha}\alpha + \bar{\beta}\beta) \quad \text{and} \quad \langle 0 | J_X | X \rangle = \sqrt{2} F_X M_X^2 \quad (3.11)$$

where F_X is the X-decay amplitude analogous to $f_\pi = 93$ MeV. We use the ansatz in Eq.(3.10) for the parametrization of the spectral function. We apply the Laplace operator in Eq.(2.11) to the two-point function and we use the FESR version of the Laplace sum rule:

$$\int_0^\infty dt e^{-t\tau} \frac{1}{\pi} \text{Im} \psi_X(t), \quad (3.12)$$

which has the fact of being independent of the value of the two-point function at zero momentum transfer. For $SU(3)_H$, one can deduce the theoretical expression of the sum rule from the one in Ref. 23). Then we get the constraint:

$$2 F_X^2 M_X^4 - \frac{3}{16\pi^2} m_\pi^2 t_c^2 \approx m_\pi^2 \frac{F_X}{8} \langle F^2 \rangle \quad (3.13)$$

Using, for instance, that t_c is positive, we deduce:

$$M_X^2 F_X \gtrsim 64 m_\pi M_W^2 \quad (3.14)$$

Then, for $M_X = 50$ GeV, and F_X of the order of 100-200 GeV, we deduce :

$$m_\mu \lesssim (0.6 \sim 1.2) \text{ GeV}. \quad (3.15)$$

If, in addition, one uses t_c of the order of 1 TeV², one can see that the continuum effect on the above constraint is small, so that one may consider the inequality in Eq. (3.15) to be almost saturated.

2) Spin-0 and SUSY-haplon:

In the case of spin-0 haplons, the two-point function defined above has the following asymptotic form¹⁵⁾:

$$\psi_X(q^2) = \frac{3}{16\pi^2} m^2 \left\{ -\log \frac{q^2}{\mu^2} + \frac{22}{3} \pi^2 \langle \phi^* \phi \rangle / q^2 \right\} \quad (3.16)$$

Using familiar techniques, we deduce from a FESR-like sum rule :

$$2 F_X^2 M_X^2 - \frac{3}{16\pi^2} m^2 t_c \simeq 8 m^2 \langle \phi^* \phi \rangle \quad (3.17)$$

For an unbroken SUSY, one knows that²³⁾:

$$2 m^2 \langle \phi^* \phi \rangle \simeq -m_\mu \langle \bar{\alpha} \alpha \rangle. \quad (3.18)$$

Using the positivity of the continuum threshold, one deduces :

$$F_X^2 M_X^2 \gtrsim -2 m_\mu \langle \bar{\alpha} \alpha \rangle \quad (3.19)$$

which is very similar to the PCAC relation in Eq. (2.3). Using the above already-used set of the X-parameters, we deduce :

$$m_\mu \lesssim 181 \text{ MeV} \quad (3.20)$$

for $\langle \bar{\alpha} \alpha \rangle = -(5M_W)^3$. One can notice that an explanation of the X-boson mass does not necessarily need a strong explicit breaking of chiral haplon symmetry contrary to some naive expectation. This fact is due to the relative small value of the X-decay amplitude compared to the decay amplitude of the hyperpions discussed in section 2.

D) Evaluation of the Z-X-γ coupling

We evaluate the Z-X-γ coupling following the QCD sum rules method used in Ref. 25) which consists in evaluating the theoretical expression of the three-meson vertex function in the Euclidian region where all external masses are equal and larger than the scale of the theory. In this way, some eventual mass singularities are automatically absent (see Fig. 3.4a). We approximate the phenomenological side of the three-point function by real poles (see Fig. 3.4b). We recall that within such an approach, the trilinear couplings of ordinary mesons have been obtained with a good accuracy despite the crude approximation used by retaining only the leading contributions to both sides of the sum rule. We would expect that the predictions for the composite models would be quite good. So let us discuss, for definiteness,

the case of spin-1/2 haplons (the extension of the discussion to the other cases can be easily done). The three-point function which controls the Z-X-γ coupling is :

$$T_{\mu\nu}(p, q) = i \int d^4x d^4y e^{i(p \cdot x + q \cdot y)} \langle 0 | T T_\mu^X(x) T_\nu^Y(y) | 0 \rangle \quad (3.21)$$

with :

$$\begin{aligned} T_\mu^Z &= \frac{1}{2} (\bar{\alpha} \gamma^\mu \alpha - \bar{\beta} \gamma^\mu \beta) \\ T_\mu^X &= \frac{m}{\sqrt{2}} (\bar{\alpha} \gamma_\mu^5 \alpha + \bar{\beta} \gamma_\mu^5 \beta) \\ T_\mu^Y &= \frac{1}{2} (\bar{\alpha} \gamma^\mu \alpha + \bar{\beta} \gamma^\mu \beta) \end{aligned} \quad (3.22)$$

where the choice of the current has been dictated by the conservation of the isospin. One can inspect that in the limit of the zero mass haplon, the leading contribution is given by the diagram in Fig. 3.5 which can be easily evaluated. One introduces the boson pole contribution via the decay amplitudes in Eq. (3.11) and :

$$\langle 0 | J_\mu^Z | Z \rangle = F_Z M_Z \epsilon_\mu \quad (3.23)$$

At the symmetrical point ($p+q$)² = q^2 = $-Q^2$, we get the constraint :

$$g_{ZX\gamma} \left(\frac{\sqrt{2} M_X^2 F_X}{q^2 + M_X^2} \right) / \left(\frac{F_Z M_Z}{q^2 + M_Z^2} \right) \simeq \frac{3}{\sqrt{2}} \frac{m \langle \bar{\alpha} \alpha \rangle}{(q^2)^2}. \quad (3.24)$$

We may use the familiar Laplace operator for improving the dominance of the leading terms retained in Eq. (3.24). Then, we obtain within a factor two :

$$g_{ZX\gamma} \simeq \frac{3}{2} \frac{m \langle \bar{\alpha} \alpha \rangle}{M_X^2 F_X M_Z F_Z} (1 \sim 2). \quad (3.25)$$

Using for example the result in Eq. (3.14) with an approximate equality sign, the fact that $\langle \bar{\alpha} \alpha \rangle = -(5M_W)^3$ and $M_Z = 1.1 M_W$, one deduces :

$$g_{ZX\gamma} \simeq 16 / M_Z (1 \sim 2). \quad (3.26)$$

This relation is very similar to the one which we know for the $\omega\pi\pi$ coupling from current algebra and from QCD sum rules²⁵⁾. This above result differs greatly from the one of the Z-e⁺-e⁻ coupling which follows from the normalization of the corresponding form factor to 1 at zero momentum transfer which is a consequence of the universal coupling of the Z to fermion pairs :

$$g_{Ze^+e^-} \approx \frac{M_W}{F_W} \approx 0.6 \quad (3.27)$$

Actually, if we use the above dynamical framework for the evaluation of the $Z-e^+e^-$ coupling, one can see, by assuming for example that the electron is a bound state of a fermion and a scalar fields, that the analogue of the diagram in Fig. 3.5 vanishes in the chiral limit. So the non-vanishing contributions will be higher order in the chiral symmetry breakings and should be smaller than the $Z-X-\gamma$ coupling^{*)}.

E) Some decay and production modes of the X

In this section, we shall not give a complete discussion of all phenomenological consequences of the X-boson. We shall limit ourselves to those implied by the estimate of the coupling constant obtained earlier. The result in Eq. (3.26) combined with the fact that the Z and X have a definite chirality (see the additional diagram in Fig. 3.6) implies for $F_Z \sim F_W \sim 1.6 M_W$ and for $M_X = 50 \text{ GeV}$:

$$\Gamma(Z \rightarrow X\gamma) \approx \frac{\alpha}{8} (g_{ZXR})^2 M_Z^3 \left(1 - \frac{M_X^2}{M_Z^2}\right)^3 \approx (100 \sim 400) \text{ MeV}, \quad (3.28)$$

which can be compatible with the UA_{1,2} data⁸⁾. However a smaller branching ratio appears to be more compatible with a vector meson-dominance of the three-point function. If one compares the above prediction with the one using a non-relativistic approach^{18a)}, one might conclude a haplon "constituent mass" of the order of 28 to 20 GeV. A comparison of the result with the one in Ref. 18b) would imply a value of the compositeness scale of the order of 0.76 to 1.5 TeV.

Now, we can use a vector meson -dominance approach in order to convert the result in Eq. (3.26) into 27, 18b):

$$g_{XX\gamma} \approx \sin\theta_W g_{ZXR}, \quad (3.29)$$

which implies:

$$\frac{\Gamma(X \rightarrow \gamma\gamma)}{\Gamma(Z \rightarrow \gamma\gamma)} \approx \frac{3}{8} \left(\frac{M_X}{M_Z}\right)^3 \left(1 - \frac{M_X^2}{M_Z^2}\right)^{-3} \sin^2\theta_W \approx 0.16, \quad (3.30)$$

i.e.:

$$\Gamma(X \rightarrow \gamma\gamma) \approx (10 \sim 40) \text{ MeV}. \quad (3.31)$$

If one combines the above result with the one derived from a non-relativistic picture:

$$\Gamma(X \rightarrow \gamma\gamma) \approx \frac{18\pi\alpha^2 g_X^4}{M_X} F_X^2, \quad (3.32)$$

*) An attempt to explain the difference of the strength of the $Z-X-\gamma$ and $Z-e^+e^-$ couplings using different context is in Ref. 26).

**) See Ref. 18a) and references therein.

one can deduce the decay amplitude of the X:

$$F_X \approx (10 \sim 110) \text{ GeV}, \quad (3.33)$$

which is of the order of F_W . The other possible decay of the X is the one into gluon pairs if the hapiions are coloured. Such a decay cannot be favoured by a large branching ratio of the X into e^+e^- pairs²⁸⁾ needed by the UA_{1,2} data⁸⁾. Actually for a Z into e^+e^- width of the order of 20 MeV and for a Z into X- γ width of Eq. (3.28), one would need:

$$\frac{\Gamma(X \rightarrow e^+e^-)}{\Gamma(X \rightarrow \text{all})} \approx (5 \sim 20) \% \quad (3.34)$$

The smallest value in Eq. (3.34) might be explained by Eq. (3.1) and the value in Eq. (3.31) provided that the hapiions are uncoloured. The largest value in Eq. (3.34) cannot be explained by the above scheme. So, one should still wait better statistics for the data in order to have a clean status of the scheme.

4. CONCLUSIONS

We have studied, using spectral function sum rules, some properties of the spin-zero partners of the composite W-bosons in a class of QCD and SUSY-like models. We have seen in section 2 that the hyperpions which have a Higgs-type coupling decouple from ordinary matter. Their only relevant effect is their copious production from the W and Z decays so that a careful experimental measurement of the W and Z widths should be useful for clarifying the (non)-existence of these hyperpions. The decay of the W and Z bosons into hyperpions having a mass larger than 15 GeV would induce four quark jet events which are harder than the QCD induced ones, since the latter contain gluon jets. In the case of very light hyperpions, they would appear like scalar muons from the W and Z decays. In section 3, we have analyzed the assumption for having a spin-zero boson which couples universally to fermion pairs. We have seen that the $U(1)_A$ nature of the X-boson may lead to some inconsistencies in the model if one assumes that the continuum threshold for the W and X-channels are approximately the same. The result corresponding to the case where the X is like the η or the ϵ but not like the η' , is more reasonable. In this picture, we have evaluated the $Z-X-\gamma$ coupling and we have shown that it is stronger than the $Z-e^+e^-$ one because the latter is higher order in the chiral symmetry breaking parameters expansion. We have also reexamined some decay and production modes of the X-boson.

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REFERENCES

- 1) S. Narison, "Dynamical properties of composite W-bosons", CERN TH 3871 (1984)
- 2) See for example, H. Fritzsch, Univ. Munich MPI-PAE/Pth 76/83 and references therein H. Harari, Proc of the 1982 SLAC Summer Institute (Stanford).
- 3a) F. Bordi, R. Casalbuoni, D. Dominici and R. Gatto, Phys. Lett. 127B(1983)419.
- 3b) W. Buchmüller, R.D. Peccei and T. Yanagida, Nucl. Phys. 227B(1983)503.
- 3c) R. Barbieri, A. Masiero and G. Veneziano, Phys. Lett. 128B(1983)179.
- 3d) O.W. Greenberg, R.N. Mohapatra and M. Yasue, Phys. Lett. 128B(1983)85.
- 3e) J.C. Pati and A. Salam, Nucl. Phys. 234B(1984)189.
- 4) J.J. Sakurai, Lectures given at the 17th Rencontre de Moriond, March 1982, les Arcs (France) and references therein.
- 5a) R.A. Bertlmann, G. Launer and E. de Rafael, Marseille preprint. CPT84/PI586/1984
- 5b) N.V. Krasnikov, A.A. Pivovarov and N.N. Tavkhelidze, Z. für Phys. C19(1983)301.
- 6a) G. Arnison et al., Phys. Lett. 122B(1983)103.
- 6b) M. Banner et al., Phys. Lett. 122B(1983)476.
- 7a) S. Narison, Annecy LAPP preprint TH68(1982), unpublished.
- 7b) G. Girardi, S. Narison and M. Perrottet, Phys. Lett. 133B(1983)234.
- 8a) G. Arnison et al., Phys. Lett. 126b(1983)398.
- 8b) P. Baganian et al., Phys. Lett. 129B(1983)130.
- 9a) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. 147B(1979)385, 448.
- 9b) J.S. Bell and R.A. Bertlmann, Nucl. Phys. 177B(1981)218.
- 9c) S. Narison and E. de Rafael, Phys. Lett. 103B(1981)57.
- 10) G. Barbiellini et al., DESY report 81-064(1981)
- 11) T. Das, G. Guralnik, V.S. Mathur and S. Okubo, Phys. Rev. Lett. 18(1967)759.
- 12) M. Althoff et al., Phys. Lett. 122B(1983)95.
- 13) C. Becchi, S. Narison, E. de Rafael and F.J. Yndurain, Z. für Phys. C8(1981)335.
- 14) G. Veneziano, Phys. Lett. 128B(1983)199.
- 15) S. Narison, CERN preprint TH 3834(1984) (to appear in Phys. Lett. B).
- 16a) W. Lerche, R.D. Peccei and V. Visnjic, MPI-PAE/Pth 12/84.
- 16b) T.E. Clark and S.T. Love, FNAL-84/39T(1984).
- 17) S. Pakvasa, J. Ellis and D.V. Nanopoulos, CERN TH 3464(1983).
- 18a) U. Baur, H. Fritzsch and H. Faisner, Phys. Lett. 135B(1984)313.
- 18b) R.D. Peccei, Phys. Lett. 136B(1984)121.
- 18c) F.M. Renard, Phys. Lett. 139B(1984)449.
- 18d) F.W. Bopp et al., Preprint SI-83-24(1983).
- 19) Cello collaboration Desy Report 84-020 (1984).
- 20a) F. del Aguila, A. Mendez and R. Pascual, Barcelona report UAB-FT-105(1984)
- 20b) M. Suzuki, LBL-17630 (1984).

21a) For a review on the (g-2) calculation, see e.g.:

J.Calmet, S.Narison, M.Perrottet and E.de Rafael, Rev. Mod. Phys. 49(1977)21.

21b) T.Kinoshita, B.Nizic and Y.Okamoto, Phys. Rev. Lett. 52(1984)717.

21c) T.Kinoshita and W.B.Lindquist, Phys. Rev. Lett. 47(1981)1573.

22) S.Narison, CERN-TH-3796(1984) (to appear in Z.für Phys. C)

23) S.Narison, N.Paver, E.de Rafael and D.Treleani, Nucl. Phys. 212B (1983)365.

24a) E.Witten, Nucl. Phys. 188B(1981)513 and 202B(1982)253.

24b) S.Gecotti and L.Girardello, Phys. Lett. 110B(1982)39.

25) S.Narison and N.Paver, Z. für Phys. C22(1984)69 and Phys. Lett. 135B (1984)159.

26) F.M.Renard, Montpellier preprint MU-PM 84-7 (1984).

27) F.M.Renard, Phys. Lett. 126B(1983)59.

28) D.Düsedau, D.Lüst and D.Zepperfeld, MPI-PAE/Pch 24/84.

TABLE

Hyperpion decay modes normalized to their muonic widths

Neutral Hyperpions				
$B_{\tau\tau}$	$B_{uu}-B_{dd}$ ($m_u=10$ MeV)	B_{ss} ($m_s=250$ MeV)	B_{cc} ($m_c=1.4$ GeV)	B_{bb} ($m_b=4.5$ GeV)
$2.6 \cdot 10^2$	$0.9 \cdot 10^{-2}$	6	$1.7 \cdot 10^2$	$2 \cdot 10^2$
Charged Hyperpions				
$B_{\tau\nu\tau}$	B_{us}	B_{ub}	B_{cd}	B
$2.6 \cdot 10^2$	0.3	< 18	< 10	< 176
	B_{cb}	B_{td}	B_{ts}	B_{tb}
		$(m_t = 40 \text{ GeV})$		
	< 654	$< 2 \cdot 10^3$	$< 10^4$	$< 2 \cdot 10^5$

FIG. 1.1

FIG. 2.1

FIG. 3.1

FIG. 3.2

FIG. 3.3