ON THE SPIN-ZERO PARTNERS OF COMPOSITE W-BOSONS

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A B S T R A C T

The spin-zero partners of the W-bosons can couple to fermion pairs à la Higgs (π) or in a universal way (U,X). Their parameters are related to the dynamics of some class of QCD and SUSY-like composite models using spectral function sum rules. We show that a 50 GeV mass for these bosons does not need a relatively strong explicit breaking of the preon chiral symmetry. A light π decouples in low energy weak interaction physics, in contrast to the pion in hadronic interactions. The best signal for the π is its copious pair production in W or Z decays with a width of O(0.25 GeV). We also show that a strong coupling for the Z-π vertex and a weak one for the Z-e⁺e⁻ vertex are dynamically consistent. However, a vector-meson dominance scenario at the Z-π vertex can imply a Z → e⁺e⁻γ rate four times smaller than the present data.

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1. INTRODUCTION

In a previous paper\(^1\), one of us has analyzed the constraints which can be obtained from a QCD\(^2\) and a SUGR\(^3\)-like composite model of weak interactions in the case where the W-bosons form a triplet of the SU(2) global weak isospin symmetry\(^4\); like the p-mesons of QCD form a triplet of the SU(2) strong isospin symmetry. Using a finite energy sum rule-like approach based on a local duality between the theoretical and the phenomenological sides of the sum rule\(^5\), it has been shown\(^1\) that the knowledge of the W and Z parameters from the data\(^6\) fixes the vacuum structure of the underlying hyperstrong chiralities and the continuum threshold above which one expects to observe the composite structure of the W and Z bosons. In this paper, we shall analyze within the same framework the properties of the spin zero partners of the W-bosons, which can have a Higgs-type (the $\sigma$) or a universal (the $X$) coupling to fermion pairs. The existence of the former is inherent to the idea of having a SU(2)-or its extension) global chiral symmetry realized à la Nambu-Goldstone. So in this way, the $\sigma$ can be associated to the divergence of the charged weak current\(^7\). The existence of the $X$ has a phenomenological motivation because it can explain the anomalous large decay rate of the Z into $\pi^0\pi^0$\(^8\). In the first part of the paper we analyze the properties of the $\sigma$ using the information on the dynamical parameters obtained in Ref.\(^1\). We also discuss some phenomenological consequences of the $\sigma$ at present energies. We show that the $\sigma$ decouples from ordinary matter and this fact differs considerably from the case of the pion for hadronic physics. We find that the only relevant effect of the $\sigma$ is its copious production from the W and Z decays which can reduce the leptonic branching ratio by 6 to 10%. So, one should consider this section of the paper as an improvement and an extended version of the already published paper quoted in Ref.\(^7\). In the second part of the paper (section 3), we discuss the properties of the $X$-boson which is assumed to couple universally to fermion pairs. We attempt to analyze the consistency of the assumed property of the X-boson and the Nambu-Goldstone realization of chiral symmetry. We shall show that a mass of the $X$-boson which is of the order of 50 GeV can be understood if one allows an explicit breaking of the chiral symmetry by a mass term which is $\Delta M$ of the scale of compositeness which we take, for example, to be of the order of $\Lambda$ TeV\(^1\)(recall that in QCD the ratio of the up quark current algebra mass over the hadronic scale is of the order of $\Delta M$). We shall also discuss the various assumptions of the nature of the $X$-bosons and we complete our discussion by selecting the most stringent constraints on the $X$-boson parameters from the present data and by showing the dynamical difference between the $Z-X-\gamma$ and the $Z-\sigma-\sigma$ vertices.

2. HYPERIONS

A) Dynamics

We analyze here the dynamical properties of the spin-zero partners of the W-bosons which have a Higgs-type coupling to fermion pairs. These spin-zero partners will hereafter be called "hyperions". In terms of the elementary hapyon fields ($\nu, \nu\bar{\nu}$), which also form the W-bosons, the hyperions $\pi$: $\nu - \nu\bar{\nu}$ $\nu - \nu\bar{\nu}$ $\nu - \nu\bar{\nu}$, (2.1)

where the hapyons form a doublet of the weak isospin SU(2) global symmetry\(^2,4\). We have already studied in Ref.\(^7\) the case where the hapyons are spin-1/2 fields. Here we shall study the case of spin 0 and SUSY-hapyons, using a finite energy sum rule (FESR) method\(^5\), which appears to give many more constraints on the boson parameters than the familiar Laplace (or Borel) sum rule\(^9\), as we have seen for the analysis of the W-channel\(^1\).

1) Spin-1/2 hapyons:

The hyperions are associated to the divergence of the charged weak current:

$$\propto \frac{\partial}{\partial \xi W} \int d^4 \eta \propto \frac{\partial}{\partial \xi W} \int d^4 \eta \propto \frac{\partial}{\partial \xi W} \int d^4 \eta$$

(2.2a)

where $W_\eta$ and $H_\eta$ are the hyperion decay amplitude and mass while:

$$\propto \frac{\partial}{\partial W} \int d^4 \eta \propto \frac{\partial}{\partial W} \int d^4 \eta \propto \frac{\partial}{\partial W} \int d^4 \eta$$

(2.2b)

The above parameters are related to each other through the PCAC-like relation:

$$\propto \frac{\partial}{\partial W} \int d^4 \eta \propto \frac{\partial}{\partial W} \int d^4 \eta \propto \frac{\partial}{\partial W} \int d^4 \eta$$

(2.2c)

which shows the fact that the global chiral symmetry is realized à la Nambu-Goldstone. $\langle \Phi \rangle$ is the hapyon vacuum condensate which is known to be of the order of $(-\Phi)^\frac{1}{2}$. The value of the hapyon mass $\eta$ controls the explicit breaking of the global chiral symmetry and so might depend on the less-understood nature of the quarks and leptons. Actually in some models where these latter are composite objects, and are some fermionic partners of some Goldstone bosons, one would prefer a "good" realization of the chiral symmetry. Here we shall leave the hapyon mass as a free parameter and we shall analyze two different cases. In the first case, we assume that $\eta$ has a value of few tens of MeV, i.e., it is of the order of the light quark mass value and to which will correspond light hyperions. In the second case, we shall allow a larger value of $\eta$, of the order of a few GeV, to which will presumably correspond much heavier hyperions.

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(ii) Explicitly broken SU(2) global symmetry:

Here, we analyze the case where the $\tau$ can have a mass larger than 15 GeV as required from the E791 experiment bound on scalar particles [13]. We can assume that this mass originates from an explicit breaking of the global chiral symmetry. Actually, such a possibility is not a priori ruled out insofar as we do not yet have a clear understanding (theoretically and experimentally) of the exact structure of quarks and leptons as well as of the origin of their masses. So, in this case of a $\tau$ with a large mass, we expect that the $\tau$-dominance of the spectral function of the two-point function:

$$\omega_{\tau}/(q^2<0) = \int d\omega <\tau|\hat{J}_\mu(\omega)|\tau>, \quad (q^2<0)$$

which is known experimentally to be less than 10^{-10}. Then we obtain the constraint:

$$\omega_{\tau} \geq 2.2 \text{ TeV}.$$  

This large value of $\omega_{\tau}$ implies that a light $\tau$ should decouple from ordinary matters as its coupling with fermions is inversely proportional to $\omega_{\tau}$ (see Eq.(2.4)). Using this lower bound of $\omega_{\tau}$ into Eq.(2.3), we deduce:

$$\omega_{\text{Higgs}} \lesssim 2.2 \text{ TeV},$$

for $\omega_{\text{Higgs}}$ of the order of 10 MeV. It is easy to see that the presence of this light $\tau$ does not affect the well-established low energy data, and it be interesting to see if its presence affects the cosmology data. In this case of spin-1/2 haptions, the charged hyperons can also acquire a mass via electromagnetism. We estimate such a contribution using the standard current algebra spectral function sum rule approach [11], which should be valid here as we have a good realization of the SU(2) global symmetry. Then, we get:

$$\omega_{\text{Higgs}}^2 - \omega_{\text{Higgs}}^2 \simeq \frac{1}{2} (s + c_2) \frac{M_{\tau}^2}{M_{\text{Higgs}}^2}.$$  

which is smaller than $(85 \text{ MeV})^2$ for the value of $\omega_{\text{Higgs}}$ given in Eq.(2.6). Eq.(2.8) shows that in the chiral limit where $\omega_{\text{Higgs}} = 0$, the charged $\tau$ acquires a tiny mass.

* In some broken SUSY-like models, the $\tau$ might acquire a large mass through the dynamical breaking parameters of supersymmetry.
potential. $<\phi^4>$ is the vacuum condensate of the scalar fields. One can see from Ref.15 that such a relation combined to other ones gives for small numbers of the hyperon flavour (1 or 2):

$$M_H^2 = \sqrt{2\mu} m_N, \quad F_H = <\phi^4>^{1/2} \approx \sqrt{2}\phi^4$$

In this case the scalar condensate like $F_H$ is singular in the chiral limit. Then a light $H$ should decouple from ordinary matter as its coupling is given by Eq.(2.4). (ii) It has been shown that the $H$ does not acquire a mass from electromagnetism in an unbroken SUSY theory. This result can be viewed as an indirect consequence of the non-renormalization theorem of elementary superfields at the bound state level. However, we should also mention that the $H$ can obtain a mass from the dynamical breaking mechanism.

3) Decay modes of hyperons

We have seen from Eq.(2.4) that the hyperons behave like techni-hadrons because of their Higgs-type coupling. The properties of hyperons have been discussed in Ref.10. The decay of the light hyperons into ordinary particles should be highly suppressed by the small available phase space and by the large decay amplitude obtained in Eq.(2.6). Such properties make the difference between the hyperons and the pions of QCD. The two-photon decay of a neutral hyperon is

$$\Gamma(H \rightarrow \gamma\gamma) \approx \left(\frac{\alpha^2}{\pi^2}\right)(1 - A_{\mu}^2)\left(\frac{m_H}{2\pi}\right)^2 \frac{\alpha^2}{\pi} \frac{m_H^3}{F_H^2}$$

which is very small for a light hyperon. A much heavier hyperon can have a much greater possibility to decay into quarks and leptons pairs. The decay rate is

$$\Gamma(H \rightarrow q\bar{q}) \approx \frac{1}{\pi} \frac{\alpha^2}{\pi^2} \frac{m_H^2}{2\pi^2 f_H^2} (1 - A_{\mu}^2)^2 \frac{m_H^3}{F_H^2}$$

where the coupling constant is in Eq.(2.4). So, a decay of a neutral hyperon into a pair of muons is

$$\Gamma(H \rightarrow \mu^-\mu^+) \approx \left(26 eV/\sqrt{2}\right)\left(1/15 GeV\right)^3$$

from which we deduce the branching ratio:

$$\mathcal{B}_H = \left(\frac{\Gamma(H \rightarrow \mu^-\mu^+)}{\Gamma(H \rightarrow \gamma\gamma)} \approx \left(\frac{26 eV}{\sqrt{2}}\right)^2\left(1/15 GeV\right)^3$$

which is given in the Table. In the same way, we can also study the branching ratio of the charged hyperon. The different channels of possible decays of the hyperon are also given in the Table. One can notice that a hyperon

(1) One has a new Dashen formula

$$M_H^2 \approx \frac{1}{2} m_H <\phi^4>$$

coming from the action of the axial charge to the $\theta=0$ component of the super-
having a mass lighter than the top quark has a total width less than 100–
200 keV. If the hyperon has a mass larger than the top quark, one woule
expect that the most dominant decay proceeds via the b quark jet with a sin^2
angular distribution characteristic of a scalar particle decay.

C) Production modes of hyperons

Let us discuss only the most interesting productions of the hyperons. Actual
ly, as the hyperons behave like technihpons, we refer to Ref. 10) for a more
detailed study of various production modes of the hyperons. The standard
way for produced charged hyperon pairs is in an e^+e^- experiment
with the branching ratio:

\[ \frac{\Gamma(W \rightarrow \pi^+\pi^-)}{\Gamma(W \rightarrow \mu^+\mu^-)} \approx \frac{1}{5} \beta \]

(2.23)

where \( \beta \) is the hyperon velocity. The other expected production of
the hyperons is through the radiative decay of the toponium bound state if the
hyperon mass is larger than 15 GeV so that it cannot be produced from the
decay of already known quarkonia. The branching ratio is:

\[ \frac{\Gamma(W \rightarrow \pi^+\pi^-)}{\Gamma(W \rightarrow e^+e^-)} \approx \frac{1}{5} \frac{\alpha^2}{\beta} \frac{\beta^2}{\gamma^2} \left( 1 - \frac{\beta^2 - \gamma^2}{\alpha^2} \right) \]

(2.24)

which is of the order of 0.5 for a top quark mass of 50 GeV and for \( \gamma \)
of the order of 350 GeV (Eq. (2.14)). Another possibility is the excitation
mode by the W and Z - decay. We estimate the branching ratio as:

\[ \frac{\Gamma(W \rightarrow \pi^+\pi^-)}{\Gamma(W \rightarrow e^+e^-)} \approx \frac{1}{5} \frac{\alpha^2}{\beta} \frac{\beta^2}{\gamma^2} \left( 1 - \frac{\beta^2 - \gamma^2}{\alpha^2} \right) \]

(2.25)

for a hyperon lighter than 40 GeV. We have estimated the W-pi coupling
by assuming a universal coupling of the W to fermion pairs and to the hyperon
pairs at zero momentum transfer 7). This assumption is very similar to
the usual quark universality relation which equates the g-couplings to e^+e^-,
\( \pi^+\pi^- \), and \( NN \). In this way, one gets:

\[ \frac{\Gamma(W \rightarrow \pi^+\pi^-)}{\Gamma(W \rightarrow e^+e^-)} \approx \frac{\alpha^2}{\beta} \frac{\beta^2}{\gamma^2} \left( 1 - \frac{\beta^2 - \gamma^2}{\alpha^2} \right) \]

(2.26)

which implies the decay rates given in Fig. 2.1. Then:

\[ \Gamma(W \rightarrow \pi^+\pi^-) < 0.2 \text{ keV} \quad \text{and} \quad \Gamma(Z \rightarrow \pi^+\pi^-) < 0.316 \text{ keV} \]

(2.27)

The effect is important and is like the one of one generation. It may reduce
the leptonic branching ratio of the W and Z by 6-10% 7. But a firm state ment
is difficult because of many uncertainties calculation of the Drell-Yan proce ss
which initiates the W and Z productions, other possible exotic decays). How ever, the production of \( \pi^+\pi^- \) will induce four quark jets which are harder than the
QCD induced one because the latter contain gluon jets.

3. SPIN-ZERO BOSONS COUPLED UNIVERSALLY TO FERMION PAIRS

Such spin-zero bosons (hereafter called U(1)) should be contrasted to
the hyperons discussed in the previous section. We have seen that the latter
couples to fermion pairs with a Higgs-type coupling \( uF \), which is a very
small coupling for small \( u \) and large \( F \). So, this fact renders the \( u \)-
effect inobservable at low energies. On the contrary, one has assumed 18) that
the U(1) couples universally to fermion pairs as do the W and Z bosons.
Within this assumption, one can explain the large decay rate of the Z into
\( e^+e^- \) for a spinless boson which is of the order of 30 GeV. It is as
almost obvious that the X cannot be a pion-like but could be, probably, a U(1)
particle, as is the \( \eta' \) of QCD 18s). If the X is spinless, it is almost
obvious that the X cannot be a pion-like boson.

A) Phenomenological constraints on the U(1)-boson

Let us summarize the present constraints on the parameters of the X-
from the data:

(i) The large decay rate \( \Gamma_Z \) of the Z into \( e^+e^- \) assumed to occur through
a virtual X-boson imposes a mass of the X of the order of 50 GeV. Also, one
needs a leptonic branching ratio of the X of the order of \( 6\% \). So, one roughly
expects a total width of the order of:

\[ \Gamma_X \rightarrow 6\% \approx 4 \Gamma_Z \rightarrow e^+e^- + M_X \rightarrow \gamma \gamma \]

(3.1)

for five quarks and three leptons where the two-photon width should be less
than the electronic one.

(ii) The recent \( e^+e^- \) data search for the X-boson 19), mainly the one
from the Bhabha scattering reaction implies the following constraints:
- The X-boson having a mass lower than 42.5 GeV is excluded with
a 95% confidence level.
- An X-boson lighter than the Z-boson and for a \( \Gamma_X \) larger than
20 MeV is excluded.
- For \( \Gamma_X \) which is in the range 15 to 20 MeV, an X-boson having
a mass between 47 and 68 GeV is allowed provided that its
coupling \( g^2/\alpha \) to fermion pairs is less than 7.5 \( 10^{-4} \).

(iii) The evaluation of the X-contribution to the (g-2) of the electron
and muon 20) shows that the main effect is due to the one of Fig. 3.1. Such a
contribution can be a hundred times larger than the one allowed by the discrepan-
cy between the theoretical and experimental results of the (g-2) unless the
couplings of the X-boson to fermion pairs and to the Z-photon obey
the following constraint:

\[ \alpha_c = \alpha_d \]

(3.2)

A more detailed discussion on some phenomenological implications of the
X-boson will be done in section 3.E.
where the $X-e^+e^-$ vertex is proportional to $(c\bar{b}y_{\gamma})$ and the $Z-Z\gamma$ vertex is proportional to $(c\bar{d}y_{\gamma})$. In particular, the previous constraint shows that a $V-A$ coupling at the electronic vertex should be accompanied by a $V-A$ one at the three-bosons vertex. One should also notice that the contribution of the diagram in Fig. 3.2 is smaller than the other weak bosons contributions of Fig. 3.3.

3) Can the X-boson be $\eta^-$-like?

Let us assume that the X-boson is associated to the $U(1)_L$ singlet current as is the $\eta^-$ for the case of QCD. Then, in the limit of the zero-hadron mass, the $X$ can be related to the anomaly-like current:

$$<0|\frac{\partial}{\partial x}\vec{F}_\mu\vec{F}_\nu|x> = \frac{\sqrt{2}}{\Lambda} \bar{\chi}M_X^2,$$

(3.3)

for $SU(2) \times SU(3)_c \times U(1)_L$. $F_{\mu\nu}$ is the dual of the hypergluon field strength tensor. Under the above assumption for the nature of the X-boson, we expect that the QCD sum rule used for a pure Yang-Mills should work here, i.e., for a non-supersymmetric theory. Then, once one deduces the sum rule (22):

$$\int_0^\infty d\tau e^{-\tau}(1-\tau)^{\frac{b}{2}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}}$$

(3.4)

Now we introduce the ansatz:

$$\int_0^\infty d\tau e^{-\tau}(1-\tau)^{\frac{b}{2}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}}$$

(3.5)

for a parameterization of the spectral function. The QCD continuum is assumed to be given by the discontinuity of the lowest order term in the RMS. Using a finite-energy-like form of the sum rule, which is equivalent to leading order by matching the same poles of $\tau$ in the two sides of the sum rules, we get the constraint:

$$\int_0^\infty d\tau e^{-\tau}(1-\tau)^{\frac{b}{2}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}}$$

(3.6)

If, for example, we take $M_x = 50$ GeV and $m_X = 100-200$ GeV, $
\lambda x_\gamma > = (12 M)^{\frac{1}{2}}$ and $\lambda x_\gamma > = 0.6$, we need for $\Lambda \approx 100$ GeV:

$$\sqrt{\xi} \approx 60 \text{ MeV}.$$

(3.7)

From the constraint in Eq. (3.6), the threshold of the continuum is five times larger than the one for the $W$-channel. This fact indicates, perhaps, some inconsistencies for the assumption of the nature of the $X$. Now, we can turn the constraint the other way around. One can assume

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* The assumption of the universal coupling to fermions might suggest that $F_X$ has the same order value than $F_Z$. We give in Section 3.2 an alternative way for the estimation of $F_X$. 

that the continuum threshold of the $W$ and the $X$-bosons channels is approximately the same and is of the order of 1 TeV. Then, for the value of $F_X$ used above, one can deduce from Eq. (3.11) the estimate:

$$M_X^2 \approx (A \sim 2) \text{ TeV}$$

(3.8)

of the mass of the lowest ground state boson. This result is certainly higher than the one used for a phenomenological reason. One can conclude from the above discussion that a $X$-boson having a mass of the order of 50 GeV and a decay amplitude of the order of 100-200 GeV can be a $\eta^-$-type particle if there is a large gap between the lowest ground state and the continuum threshold.

4) Can the X-boson be $\eta^-$-like?

Another possible assumption for the nature of the X-boson is the one where it is an isoscalar singlet. This assumption is very similar to the case of the $\eta$ or of the $Z$ of QCD. In this case the $X$-boson is a $SU(2)$ singlet and there is no extra-$U(1)$ piece which contributes to its dynamics. This case corresponds to the one where the $SU(2)$ group is a subgroup of a higher symmetry group. Then, in terms of the hadron fields:

$$J_X = \frac{1}{\sqrt{2}} (\bar{\chi}w + \bar{\chi}\rho)$$

(3.9)

1) Splitting hadrons:

We shall study the two-point function:

$$\langle J_x^\dagger \rangle = i \int d^4x e^{i\Phi} \langle 0 | \hat{J}_x | J_x(0) \rangle,$$

(3.10)

where we normalize the current as:

$$\int d^4x e^{i\Phi} \langle 0 | \hat{J}_x | J_x(0) \rangle = \sqrt{F_X} M_X^2,$$

(3.11)

where $F_X$ is the $X$-decay amplitude analogous to $f_{\eta} \approx 93$ MeV. We use the ansatz in Eq. (3.10) for the parameterization of the spectral function. We apply the Laplace operator in Eq. (2.11) to the two-point function and we use the FESR version of the Laplace sum rule:

$$\int_0^\infty d\tau e^{-\tau}(1-\tau)^{\frac{b}{2}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}} \int_0^\infty d\tau' e^{\frac{\tau'}{\tau}}$$

(3.12)

which has the form of being independent of the value of the two-point function at zero momentum transfer. For $SU(3)_c$, one can deduce the theoretical expression of the sum rule from the one in Ref. 23. Then, we get the constraint:

$$2 F_X^2 M_X^4 = \frac{3}{26} M_X^2 \bar{\rho} \bar{\mu} \bar{\nu} \bar{\phi} \frac{1}{\lambda x_\gamma}$$

(3.13)

Using, for instance, that $\lambda x_\gamma$ is positive, we deduce:

$$M_X^2 \approx (AC \sim 2) \text{ TeV}$$

(3.14)
Then, for $M_x > 50$ GeV, and $F_x$ of the order of 100-200 GeV, we deduce:

$$m_x \lesssim 1.0 - 1.2 \text{ GeV}. \quad (3.15)$$

If in addition, one uses $\mu_x$ of the order of 1 GeV, one can see that the continuum effect on the above constraint is small, so that one may consider the inequality in Eq. (3.15) to be almost saturated.

2) Spin-0 and SUSY-hadrons:

in the case of spin-0 hadrons, the two-point function defined above has the following asymptotic form \cite{5}:

$$\Pi_h (q^2) = \frac{3}{16\pi^2} m^2 \left[ \frac{1}{q^2} \Phi_1^2 + \frac{2\pi^2}{\Phi_1^2 \Phi_2^2} \Phi^2 \Phi^2 \right]. \quad (3.16)$$

Using familiar techniques, we deduce from a Fierz-like sum rule:

$$2F_x^2 M_x^2 - \frac{3}{2} \frac{m^2 \Phi^2}{F_x^2 M_x^2} \simeq 1 \frac{m^2 \Phi^2}{\Phi^2 \Phi^2} \quad (3.17)$$

For an unbroken SUSY, one knows that \cite{3}:

$$2m^2 \Phi^2 \Phi^2 \simeq -m_x \langle \bar{\alpha} \alpha \rangle. \quad (3.18)$$

Using the positivity of the continuum threshold, one deduces:

$$F_x^2 M_x^2 \gtrsim 2m_x \langle \bar{\alpha} \alpha \rangle \quad (3.19)$$

which is very similar to the PCAC relation in Eq. (2.3). Using the above already-used set of the $X$-parameters, we deduce:

$$m_x \lesssim 111 \text{ MeV} \quad (3.20)$$

for $\langle \bar{\alpha} \alpha \rangle = - \langle (X_0)^2 \rangle^2$. One can notice that an explanation of the $X$-boson mass does not necessarily need a strong explicit breaking of chiral symmetry contrary to some naive expectation. This fact is due to the relative small value of the $X$-decay amplitude compared to the decay amplitude of the hyperons discussed in section 2.

D) Evaluation of the $Z$-$X$-$\gamma$ coupling

We evaluate the $Z$-$X$-$\gamma$ coupling following the QCD sum rules method used in Ref. 2.2 which consists in evaluating the theoretical expression of the three-measure vertex function in the Euclidean region where all external masses are equal and larger than the scale of the theory. In this way, some eventual mass singularities are automatically absent (see Fig. 3.4a). We approximate the phenomenological side of the three-point function by real poles (see Fig. 3.4b). We recall that within such an approach, the trilinear couplings of ordinary mesons have been obtained with a good accuracy despite the crude approximation used by retaining only the leading contributions to both sides of the sum rule. We would expect that the predictions for the composite models would be quite good. So let us discuss, for definiteness,
Actually, if we use the above dynamical framework for the evaluation of the $Z^{-}\rightarrow e^+ e^-$ coupling, one can see, by assuming for example that the electron is a bound state of a fermion and a scalar fields, that the analogue of the diagram in Fig. 3.5 vanishes in the chiral limit. So the nonvanishing contributions will be higher order in the chiral symmetry breaking and should be smaller than the $Z^{-}\times\gamma$ coupling.

2) Some decay and production modes of the X

In this section, we shall not give a complete discussion of all pheomenological consequences of the $X$-boson. We shall limit ourselves to those already by the estimates of the coupling constant obtained earlier.

The result in Eq. (3.26) combined with the fact that the $Z$ and $X$ have a definite chirality (see the additional diagram in Fig. 3.6) implies for $M_Z \approx M_W = 1.6 M_t$ and for $M_X \approx 50 GeV$:

$$\Gamma(X \rightarrow e^+ e^-) \approx \frac{N_c}{4 \pi} \left( \frac{M_Z^2}{M_X^2} \right)^2 \frac{N_c}{4 \pi} \approx (100 \sim 400) m_{e^+ e^-},$$

which can be compatible with the UA1,2 data. However, a smaller branching ratio appears to be more compatible with a vector meson dominance of the three-point function. If one compares the above prediction with the one using a non-relativistic approach, one might conclude a hadron "constituent mass" of the order of 25 to 20 GeV. A comparison of the result with the one in Ref. 18b) would imply a value of the compositeness scale of the order of 0.76 to 1.7 TeV.

Now, we can use a vector meson dominance approach in order to convert the result in Eq. (3.26) into:

$$\Gamma(X \rightarrow\pi^+ \pi^-) \approx \frac{M_X^2}{M_W^2} \Gamma(Z \rightarrow e^+ e^-),$$

which implies:

$$\Gamma(X \rightarrow\pi^+ \pi^-) \approx \frac{3}{8} \left( \frac{M_X}{M_Z} \right)^2 \frac{N_c}{4 \pi} \approx 0.16 \%,$$

i.e.:

$$\Gamma(X \rightarrow\pi^+ \pi^-) \approx (10 \sim 40) m_{\pi^+ \pi^-}.\tag{3.31}$$

If one combines the above result with the one derived from a non-relativistic picture:

$$\Gamma(X \rightarrow\gamma \gamma) \approx \frac{N_c}{4 \pi} \frac{\alpha^2}{M_X^2} \frac{N_c}{4 \pi} \frac{F_X^2}{M_X^2},$$

one can deduce the decay amplitude of the $X$:

$$F_X \approx (10 \sim 40) GeV,$$

which is of the order of $M_W$. The other possible decay of the $X$ is the one into gluon pairs if the hadrons are coloured. Such a decay cannot be observed by a large branching ratio of the $X$ into $e^+ e^-$ pairs (28) needed by the UA1,2 data. Actually for a $Z$ into $e^+ e^-$ width of the order of 20 MeV and for a $Z$ into $\gamma\gamma$ width of Eq. (3.26), one would need:

$$\frac{\Gamma(X \rightarrow e^+ e^-)}{\Gamma(X \rightarrow all)} \approx (0 \sim 20) \%.$$

The smallest value in Eq. (3.34) might be explained by Eq. (3.1) and the value in Eq. (3.31) provided that the hadrons are uncoloured. The largest value in Eq. (3.34) cannot be explained by the above scheme. So, one should still wait better statistics for the data in order to have a clean status of the scheme.

4. CONCLUSIONS

We have studied, using spectral function sum rules, some properties of the spin-zero partners of the composite $W$-bosons in a class of QCD and SUSY-like models. We have seen in section 2 that the hyperquarks which have a Higgs-type coupling decouple from ordinary matter. Their only relevant effect is their copious production from the $W$ and $Z$ decays so that a careful experimental measurement of the $W$ and $Z$ widths should be useful for clarifying the (non-)existence of these hyperquarks. The decay of the $W$ and $Z$ bosons into hyperquarks having a mass larger than 15 GeV would induce four quark jet events which are harder than the QCD induced ones, since the latter contain gluon jets. In the case of very light hyperquarks, they would appear as scalar mesons from the $B$ and $Z$ decays. In section 3, we have analyzed the assumption for having a spin-zero boson which couples universally to fermion pairs. We have seen that the $U(1)_Y$ nature of the $X$-boson may lead to some inconsistencies in the model if one assumes that the continuum threshold for the $W$ and $X$-channels are approximately the same. The result corresponding to the case where the $X$ is like the $\eta$ or the $c$ but not like the $\chi$, is more reasonable. In this picture, we have evaluated the $Z-X-\gamma$ coupling and we have shown that it is stronger than the $Z-\gamma$ one because the latter is higher order in the chiral symmetry breaking parameter expansion. We have also reexamined some decay and production modes of the $X$-boson.
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REFERENCES

2) See for example, H.Fritzsche, Univ. Munich MPT-FAS/Ph 76/83 and references therein. H.Narison, Proc of the 1982 SLAC Summer Institute (Stanford).
4) J.J. Sakurai, Lectures given at the 17th Rencontre de Moriond, March 1982, Les Arcs (France) and references therein.
7a) S.Narison, Annecy LAPP preprint T86-04 (1982), unpublished.
16a) W.Lerche, R.D.Peccei and V.Vissicci, MPI-PAS/Ph 74/84.
21a) For a review on the $g-2$ calculation, see e.g.:
26) F.M. Renard, Montpellier preprint MU-PM 84-7 (1984).
28) D. Düsedau, D. List, and D. Zeppenfeld, MPT-PAT/Fth 74/84.

### TABLE

Hyperon decay modes normalized to their muonic widths

<table>
<thead>
<tr>
<th>Neutral Hyperons</th>
<th>$B_{VT}$</th>
<th>$B_{w} - B_{sd}$</th>
<th>$B_{os}$</th>
<th>$B_{cc}$</th>
<th>$B_{bb}$</th>
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<tbody>
<tr>
<td></td>
<td>($m_{u} = 10$ MeV)</td>
<td>($m_{u} = 250$ MeV)</td>
<td>($m_{c} = 1.4$ GeV)</td>
<td>($m_{b} = 4.5$ GeV)</td>
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</tr>
<tr>
<td></td>
<td>$2.6 \times 10^3$</td>
<td>$0.3 \times 10^{-2}$</td>
<td>$6$</td>
<td>$1.7 \times 10^2$</td>
<td>$2 \times 10^3$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Charged Hyperons</th>
<th>$B_{VT}$</th>
<th>$B_{us}$</th>
<th>$B_{ub}$</th>
<th>$B_{cd}$</th>
<th>$B_{db}$</th>
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<tbody>
<tr>
<td></td>
<td>($m_{u} = 40$ GeV)</td>
<td>($m_{u} = 60$ GeV)</td>
<td>($m_{c} = 1.4$ GeV)</td>
<td>($m_{b} = 4.5$ GeV)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 554$</td>
<td>$&lt; 2 \times 10^3$</td>
<td>$&lt; 10^9$</td>
<td>$&lt; 2 \times 10^3$</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS:

Fig. 1.1: QED - perturbative and non-perturbative diagrams.
Fig. 2.1: W and Z decays into hyperon pairs versus the hyperon masses.
Figs. 3.1 and 3.2: X-boson contribution to the g-2 of leptons.
Fig. 3.3: W and Z contribution to the g-2 of leptons.
Fig. 3.4: a) Symmetrical configuration for the evaluation of the three-point function.
        b) Vector meson dominance to the three-point function.
Fig. 3.5: Leading QED contribution to the three-point function.
Fig. 3.6: Additional contribution to the Z-X-γ vertex due to the chiral nature of the X-boson.
FIG. 3.1

FIG. 3.2

FIG. 3.3