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VARIANT SOURCE METHODS

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A B S T R A C T

Hot and cold wall source methods for calculating the mass spectrum of lattice gauge theories are investigated. Sources for SU(2) E^+ and A_1^- state are introduced. Preliminary Monte Carlo results at $\beta = 2.25$ are compared with previous Monte Carlo variational results.

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Source methods for investigating the mass spectrum of 4d lattice gauge theories were suggested in Refs. 1)-3). To be definite, let us consider the SU(2) gauge group. The cold wall method³⁾ simply consists in fixing on a spacelike plane all SU(2) matrices to the identity. This is mainly useful for investigating the A1+ representation. Compared to other representations of the cubic group, Monte Carlo (MC) investigations of the A1+ state are in good shape⁴⁾. Sources with strong projections on other representations, corresponding to excited states⁴⁾, are therefore desirable. Sources specialized to the E+ and A1- states are introduced in the following. First numerical results are also presented.

Let me first discuss the fixed wall method (hot or cold) in general. It consists in fixing a wave function $|\psi\rangle$ on a spacelike plane:

$$|\psi\rangle = \prod_b \delta(u(b) - A(b)) |0\rangle \quad (1)$$

with the $A(b)$ given and the product goes over all bonds of the spacelike plane. We may expand $|\psi\rangle$ in terms of eigenstates of the transfer matrix T :

$$|\psi\rangle = \sum_{n=0}^{\infty} a_n |n\rangle, \quad T |n\rangle = e^{-E_n} |n\rangle, \quad n=0,1,\dots \quad (2)$$

where we have chosen the normalization $T|0\rangle = |0\rangle$ of the vacuum.

Let $r = (R, P, C)$ specify a representation of the cubic group and W^r be a Wilson loop in this representation [for details, see Ref. 4)]. Let us define

$$\sigma^r = W^r - \langle 0 | W^r | 0 \rangle \quad (3)$$

The matrix element $\langle \psi | 0^r | \psi \rangle$ is fixed by the source and by means of a MC calculation we may obtain the matrix elements

$$\langle \psi | T^z \sigma^r | 0 \rangle, \quad z=1,2,\dots \quad (4)$$

(If $\langle 0 | W^r | 0 \rangle \neq 0$ this requires also a MC calculation without source.) We define $|\phi^r\rangle = 0^r |0\rangle$ and expand in eigenstates of T :

$$|\phi^r\rangle = \sum_{n=1}^{\infty} b_n |n\rangle. \quad (5)$$

Equations (2) and (5) imply

$$\langle \psi | T^z \sigma^r | 0 \rangle = \sum_{n=1}^{\infty} c_n e^{-z E_n} \quad \text{with} \quad c_n = a_n b_n. \quad (6)$$

The coefficients c_n can be positive or negative real numbers. In contrast to the MC variational method (MCV)⁴⁾ the upper bound property of mass spectrum results is lost. An appropriately chosen source may, however, significantly increase the signal and eventually allow us to go to larger distances.

For increasing the signal of the E+ and the A1- representation, I will introduce two different sources (hot walls). The first source consists in putting on a spacelike plane all link variables $U_\lambda = 1$, except on lines in $\hat{e}_2$ direction indicated in Fig. 1. On these lines, we set the $U_\lambda = -1$. Periodic boundary conditions are taken and the extension of the lattice in the spatial directions has to be even. The contribution of this source to the E+ (and other) representations of 28 Wilson loops was considered. The Wilson loops are numerated in a corresponding figure of Ref. 4) and Ref. 5). Using this notation, some of the operators contributing to the E+ representations are listed in Table 1.

On an N^3N_t lattice, we would like to carry out a MC calculation of momentum zero correlation functions

$$S(t) = \langle \psi | O(t) | 0 \rangle \quad (9)$$

Here O is one of the Wilson loop operators in the E+ representation to which the source contributes. Without source, we have for each operator $\langle O \rangle = 0$. With source, the distance $t = 0$ correlation $\langle \psi O(0) \rangle$ is a measure for the strength of the source and proportional to the spatial volume. For definite comparison with MCV results⁵⁾, we consider an 8^3N_t lattice. In Table 1, the numbers in brackets give the ratio $\langle \psi O(0) \rangle / \langle O(0)^2 \rangle$, in other words the enhancement of the signal as compared with the MCV method on the 8^3N_t lattice.

Using the source, I have obtained preliminary MC results at $\beta = 2.25$, which are based on rather moderate statistics of about 6000 triple sweeps with measurements. Before taking the measurements, 1350 sweeps were done for reaching equilibrium. Let us use the finite distance mass definitions

$$m(t_2, t_1) = \frac{-1}{t_2 - t_1} \ln \left(\frac{S(t_2)}{S(t_1)} \right) \quad (10)$$

In Table 2, the results are compared with high statistics MCV results of Ref. 5). Within our present limited statistics, consistency is found. We are now able to obtain E+ results up to distance $t = 3$, whereas previously one was not able to go beyond distance $t = 2$, even with much higher statistics. This is an improvement

by a factor 100 in CPU time. On the other hand, however, the asymptotic large t behaviour seems to be approached slightly more slowly.

In the strong coupling limit⁶⁾ and in the finite (spatial) volume weak coupling limit⁷⁾, one finds a mass ratio $m(E+)/m(A1+) \approx 1.0$. In contrast, MCV calculations apply to the region between these limits and give much larger values^{4),5)}. Using the introduced source, a considerable extension of our present numerical work is in progress⁸⁾, and we may confirm a large $m(E+)/m(A1+)$ ratio up to distance $t = 3$.

My second source is constructed for giving a large $A1-$ signal. The $A1-$ state is of phenomenological interest, because a low-lying experimental candidate exists. Previous MCV calculations gave, however, a high mass value. As this seems to be to some extent controversial [for a review, see Ref. 4)], further confirmation is desirable. The $A1-$ source is defined as follows: let $n = (n_1, n_2, n_3)$ and take all n_i ($i = 1, 2, 3$) odd. At each such point n , the matrices U_λ on links of the shape of Fig. 2 are set equal to $U_\lambda = -1$. All other U_λ in this spatial plane are set equal to $+1$. We now have contributions to the $A1-$ representations of operators 8, 9 and 21 [in the notation of Refs. 4) and 5)]. For an $8^3 N_t$ lattice, the strength of the source and the enhancement of the signal as compared to MCV calculations are listed in Table 3. We recognize the $A1-$ source to be less efficient than the introduced $E+$ source. Further optimization is desirable.

Based on 2500 triple sweeps at $\beta = 2.25$ on an $8^3 \times 8$ lattice, I have obtained preliminary MC results. The $A1-$ mass comes out large again. Beyond distance $t = 1$ the signal is noise, at least with the present statistics. This happens similarly in the high statistics MCV calculations^{4),5)}.

In conclusion, I have demonstrated that variant sources are useful for glueball spectroscopy in non-Abelian lattice gauge theories. Finally, a combination of sources with multi-hit methods^{9),1)} seems to be natural.

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OP	1. E+ representation	2. E+ representation
1	2048 (≈ 1409)	682.5 (≈ 470)
7	8192 (≈ 695)	2730.6 (≈ 232)
22	4096 (≈ 626)	1365.3 (≈ 209)
26	2048 (≈ 407)	628.6 (≈ 136)

Table 1 - E+ source: distance $T = 0$ correlation on an $N_t^{8^3}$ lattice. In brackets: enhancement as compared with the MCV method on this lattice.

OP	REP	m(0,1)	m(0,2)	m(1,2)
1	1	3.72 ± 0.01	3.58 ± 0.04	3.45 ± 0.09
1	2	3.71 ± 0.01	3.67 ± 0.15	3.62 ± 0.32
7	1	3.90 ± 0.01	3.49 ± 0.06	3.07 ± 0.14
7	2	3.92 ± 0.01	3.43 ± 0.16	2.95 ± 0.31 - 0.43
22	1	3.91 ± 0.01	3.51 ± 0.07	3.12 ± 0.15
22	2	3.92 ± 0.01	3.44 ± 0.12	2.96 ± 0.31 - 0.24
26	1	5.49 ± 0.03	3.78 ± 0.23 - 0.16	2.08 ± 0.50 - 0.35
26	1	5.72 ± 0.09	-	-
MCV A1+ 5)		1.69 ± 0.01	1.53 ± 0.03	1.40 ± 0.08
MCV E+ 5)		3.04 ± 0.04	2.72 ± 0.20	2.40 ± 0.51 - 0.35
OP	REP	m(0,3)	m(1,3)	m(2,3)
1	1	3.12 ± 1.66 - 0.24	2.81 ± 2.50 - 0.35	2.18 ± 5.07 - 0.78
1	2	-	-	-
7	1	2.87 ± 0.28 - 0.16	2.35 ± 0.43 - 0.24	1.63 ± 0.98 - 0.60
7	2	-	-	-
22	1	2.92 ± 0.41 - 0.19	2.42 ± 0.62 - 0.28	1.72 ± 1.38 - 0.70
22	2	-	-	-
26	1	-	-	-
26	1	-	-	-
MCV A+1 5)		1.41 ± 0.07	1.27 ± 0.10	1.15 ± 0.24
MCV E+ 5)		-	-	-

Table 2

OP	Al- representation
8	0.75 ($\approx$ 0.22)
9	216 ($\approx$ 96)
21	126.75 ($\approx$ 37)

Table 3 - Al- source: distance $t = 0$ correlations on an $N_t 8^3$ lattice. In brackets: enhancement as compared with the MCV method on this lattice.

REFERENCES

- 1) K.H. Mütter and K. Schilling - Phys.Lett. 117B (1982) 75.
- 2) M. Falcioni, E. Marinari, M.L. Paciello, G. Parisi, B. Taglienti and Zhang Yi-Cheng - Nucl.Phys. B215 [FS7] (1983) 265;
C. Michael and I. Teasdale - Nucl.Phys. B215 [FS7] (1983) 433.
- 3) G. Parisi - Private communication;
Ph. de Forcrand - To appear.
- 4) For a review, see:
B. Berg - Cargèse Lectures 1983, DESY Preprint 84-012 (1984).
- 5) B. Berg, A. Billoire, S. Meyer and C. Panagiotakopoulos - Saclay Preprint PHT 84-24 (1984), to appear in Comm.Math.Phys. 179.
- 6) G. Münster - Nucl.Phys. B190 [FS3] (1981) 439; Erratum B205 [FS5] (1982) 648.
- 7) M. Lüscher and G. Münster - Nucl.Phys. B232 (1984) 445.
- 8) B. Berg and H. Kamenzki - Work in preparation.
- 9) H. Meyer-Ortmanns and I. Montvay - DESY Preprint 84-34 (1984).

Fig. 1

Fig. 2