SPIN 1 BOUND STATES AND DYNAMICAL BREAKING OF SUPERSYMMETRYJ.-M. Gérard^{*)} and J. Weyers⁺⁾

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A B S T R A C T

We argue that whenever massless fermionic components of composite vector superfields are needed to satisfy 't Hooft's anomaly constraints, it should imply a dynamical breakdown of supersymmetry if complementarity does apply.

^{*)}IISN Fellow, on leave of absence from Institut de Physique Théorique, Univ. Catholique de Louvain, Louvain-la-Neuve (Belgium); address after September 1984, Max-Planck Institut für Physik und Astrophysik, Munich (Germany).

⁺⁾On leave of absence from the University of Louvain (Belgium).

It has been proved¹⁾ that QCD-like gauge theories cannot give rise to flavour non-singlet massless bound states with spin higher than $\frac{1}{2}$. This no-go theorem strongly supports 't Hooft's approach²⁾ towards composite models where anomaly matchings between the fundamental and composite levels for all unbroken flavour symmetries lead to a complete description of the massless spin $\frac{1}{2}$ fermion bound state spectrum. This philosophy can easily be extended³⁾ to $N = 1$ globally supersymmetric hypercolour gauge interactions. However, with massless fermions belonging either to a chiral or to a vector superfield, it is then required that their supersymmetric partners, which are of spin 0 or spin 1 respectively, also remain massless bound states as long as supersymmetry is not broken below the hypercolour confining scale.

This naturally leads to a sufficient condition for a dynamical breakdown of supersymmetry to occur: as we will argue in this letter, anomaly matching as ensured by complementarity⁴⁾ may require massless flavour non-singlet fermionic components of vector superfields. The breaking of supersymmetry is then necessary to comply with the Weinberg-Witten theorem already mentioned. Moreover, let us point out that this result does not contradict Witten's index theorem⁵⁾ either, since we are considering the chiral limit of a hypercolour gauge theory.

For definiteness, let us consider an $SU(N)_{HC}$ hypercolour group with a vector superfield $V = (B_\mu, \lambda)$ in the adjoint representation and M left-handed chiral superfields $\Phi_i = (A_i, \psi_i)$ and $\bar{\Phi}^i = (\bar{A}^i, \bar{\psi}^i)$ in the $\bar{N}+N$ representation. Taking the strong anomaly into account, we are left with the flavour group $G_F = SU(M)_L \times SU(M)_R \times U(1)_V \times U(1)_{\tilde{Q}}$ under which the fields transform as follows:

$$\begin{aligned} B_\mu & (1, 1, 0, 0), \lambda (1, 1, 0, -M) \\ \psi^\alpha & (\square, 1, 1, N), A^\alpha (\square, 1, 1, N-M) \\ \bar{\psi}_\alpha & (1, \square, -1, N), \bar{A}_\alpha (1, \square, -1, N-M) \end{aligned} \quad (1)$$

The existence of an additional unbroken axial $U(1)_{\tilde{Q}}$, arising from the original R-symmetry, implies a new non-trivial anomaly matching constraint when gravitation is switched on⁶⁾:

$$T_{n \text{ fund.}} U(1)_{\tilde{Q}} = T_{n \text{ bound states}} U(1)_{\tilde{Q}} \quad (2)$$

This strongly suggests the use^{*)} of complementarity for solving the anomaly equations since it automatically implies the matching in Eq. (2). For the model we are considering, the solution which corresponds to the largest unbroken G_F' flavour group was already proposed in Ref. 7), in connection with the effective Lagrangian approach⁸⁾.

In the broken gauge symmetry phase,

$$\begin{aligned} \langle A_i^a \rangle &= \nu \delta_i^a, \quad (a = 1, \dots, N) \\ \langle A_i^{\hat{a}} \rangle &= 0, \quad (\hat{a} = N+1, \dots, M) \\ \langle \bar{A}_\alpha^i \rangle &= 0 \end{aligned} \quad (3)$$

corresponds to a supersymmetric minimum for $M > N$ ^{**)}, and the unbroken global symmetry reduces to

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An interesting counterexample is however provided by the case $M = N+1$ with G_F unbroken. All anomaly constraints are then indeed satisfied by the composite chiral superfields $\varepsilon_{i_1 \dots i_N}^{\alpha_1 \dots \alpha_N} \Phi_{i_1}^{\alpha_1} \dots \Phi_{i_N}^{\alpha_N}$, $\varepsilon_{i_1 \dots i_N}^{\bar{\alpha}_1 \dots \bar{\alpha}_N} \bar{\Phi}_{i_1}^{\bar{\alpha}_1} \dots \bar{\Phi}_{i_N}^{\bar{\alpha}_N}$ and $\Phi_i^{\alpha} \bar{\Phi}_\beta^i$. Another counterexample is discussed later on in this letter.

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For $N > M$, (3) is still an extremal solution of the potential if one adds a soft breaking (negative) scalar mass term μ^2 and sets

$$\nu^2 = \left(\frac{N}{M-N} \right) \frac{\mu^2}{g_{HC}^2}$$

The maximal attractive channel principle⁹⁾ shows that all the chiral symmetries are broken.

$$G'_F = SU(N)_{\text{diag.}} \times SU(M-N)_L \times SU(M)_R \times U(1)_A \times U(1)_B^{(4)}$$

where A and B are conserved charges

$$A = Q_L + \tilde{Q}$$

(5)

$$B = Q_L + (N-M)V$$

with

$$Q_L = \left(\begin{array}{c|c} \begin{matrix} M-N & \\ & M-N \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 0 & \\ & -N \end{matrix} & \begin{matrix} 0 \\ -N \end{matrix} \end{array} \right) \left\{ \begin{array}{l} N \\ M-N \end{array} \right.$$

(6)

Under G'_F , the remaining massless fermions transform as follows:

$$\psi_a^{\hat{a}} (\square, \square, 1, 0, -M)$$

$$\psi_a^a (1, 1, 1, M, 0)$$

(7)

$$\bar{\psi}_a^a (\square, 1, \square, N, M-N)$$

while the gauginos pair with matter fields to form an invariant mass term

$$\lambda_b^a \left[\psi_a^b - \frac{1}{N} \delta_a^b \psi_c^c \right] \quad (8)$$

From complementarity⁴⁾, we know that the massless spin $\frac{1}{2}$ bound states in the confining symmetric phase carry the same quantum numbers under $SU(N)_L \times SU(M-N)_L \times SU(M)_R \times U(1)_A \times U(1)_B$:

$$\begin{aligned} \psi_i^a A_a^{*i} & \left(\square, \square, 1, 0, -M \right) \\ \psi_i^a A_a^{*i} & \left(1, 1, 1, M, 0 \right) \\ \bar{\psi}_a^i A_i^a & \left(\square, 1, \square, N, M-N \right) \end{aligned} \quad (9)$$

Let us stress that these fermionic massless bound states, which satisfy the fourteen 't Hooft equations, all contain the scalar fields A_i^a with non-vanishing vacuum expectation values in the broken gauge symmetry phase (correspondence principle). The first two sets of massless fermions behave like the θ -components of the vector superfields $U_a^\alpha \equiv \Phi_a^* (e^V)_\alpha^\alpha \Phi_a$. Let us note, however, that chiral superfields alone can also satisfy the anomaly equations since the scalars A_a^{*i} and $\epsilon^{i i_1 \dots i_{N-1}} \epsilon_{a a_1 \dots a_{N-1}} A_{i_1}^{a_1} \dots A_{i_{N-1}}^{a_{N-1}}$ transform identically under the unbroken flavour group. Here we will take the somewhat naive point of view that the bound states which appear at low energy are as simple as possible, namely two body; this turns out to be the case for the "left-right symmetric" solution ($\langle A \rangle = \langle \bar{A} \rangle$).

Anomaly matching conditions only constrain the quantum numbers of states and therefore cannot distinguish between the θ and the $\theta\theta\theta$ -components of U_a^α ; more precisely, they do not determine which one of them is to remain massless. Complementarity does remove this ambiguity since the correspondence principle fixes $U_{a|\theta}^\alpha = \phi^\alpha A_a^*$ to be the massless composite fermions. Therefore, these fields cannot be gauged away through a Wess-Zumino local transformation. Our interpretation is then that a general clash between complementarity and local gauge symmetries (i.e., massless

vector bosons) arising at the bound state level is resolved by a dynamical breaking of supersymmetry with a metastable vacuum. The spin 1 bound states of the theory which are not protected by chiral symmetries must get a mass in agreement with Ref. 1).

From the broken gauge symmetry picture, Affleck et al.¹⁰⁾ recently argued that in supersymmetric QCD theories non-perturbative effects induce an invariant superpotential

$$F \sim \left(\varepsilon^{\alpha_1, \dots, \alpha_M} \bar{\Phi}_{\alpha_1}^{i_1} \dots \bar{\Phi}_{\alpha_M}^{i_M} \Phi_{i_1}^{\beta_1} \dots \Phi_{i_M}^{\beta_M} \varepsilon_{\beta_1, \dots, \beta_M} \right)^{-1/(N-M)} \quad (10)$$

such that non-renormalization theorems are violated. For $M < N$, this indicates the possibility of getting dynamical supersymmetry breaking in four dimensions^{*)}, but the theory has left-right symmetric vacuum expectation values whose degeneracies imply a shift of the ground state to infinity. For $M > N$ such an F-term does not exist (for $M > N$, it is excluded by Bose statistics) which would mean that the flat direction of the potential corresponding to the vacuum degeneracy in Eq. (3) is not lifted at all.

The effective approach (confining, symmetric picture) also indicates minima running to infinity in the chiral limit for $M < N$. The superpotential is indeed given by⁸⁾

$$f = (N-M) S \left[\log S (\det_M T)^{1/(N-M)} - 1 \right] \quad (11)$$

with

$$T_{\beta}^{\alpha} \equiv \Phi_{i_1}^{\alpha} \bar{\Phi}_{i_1}^{\beta} \quad (12)$$

*)

One chiral model with such a breaking has been advocated in Ref. 11).

such that the behaviour of a supersymmetric ground state in the chiral limit is characterized by

$$\begin{aligned} T &\sim m^{(M-N)/N} \\ S &\sim m^{M/N} \end{aligned} \quad (13)$$

For the case $M > N$ ¹²⁾ the symmetry breaking pattern induced is obviously inconsistent with the approach based on complementarity. This can be understood from the fact that some of the light composite fields are absent in the effective superpotential (11). One possible cure⁷⁾ is provided by the introduction of an additional F-term including the (N-body) composite chiral superfields $R^{\alpha_1 \dots \alpha_N} \equiv \epsilon^{i_1 \dots i_N} \Phi_{i_1}^{\alpha_1} \dots \Phi_{i_N}^{\alpha_N}$ and $\bar{R}_{\alpha_1 \dots \alpha_N} \equiv \epsilon_{i_1 \dots i_N} \bar{\Phi}_{\alpha_1}^{i_1} \dots \bar{\Phi}_{\alpha_N}^{i_N}$ such that supersymmetry is unbroken. In our case, we would have to introduce the vector superfields U_a^α to analyze the ground state at low energy. The interaction of the composite superfields is unfortunately somewhat unconstrained although Ward identities do provide some interesting information¹³⁾. We hope to come back to this problem elsewhere. Anyway these features indicate that the approach of Ref. 10) might fail in the particular framework considered here, where additional light fields appear at low energy.

So far, our arguments were based on complementarity which seems to exclude the possibility of local gauge symmetry at the bound state level. Let us now briefly consider a model where complementarity does not apply anymore. An $SU(2)_{HC}$ hypercolour gauge interaction with n doublets^{*)} of chiral superfields has an $SU(n) \times U(1)_Q$ global symmetry under which the fundamental fields transform as follows:

$$\begin{aligned} B_\mu (1, 0) ; \lambda (1, -n) \\ \psi^a (\square, 4) ; A^a (\square, 4-n) \end{aligned} \quad (14)$$

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The theory is only well defined for n even¹⁴⁾.

Assuming $SU(n) \times U(1)_{\tilde{Q}}$ unbroken, we get interesting solutions to 't Hooft's anomaly constraints together with Eq. (3) for the following fermionic massless bound states:

$$\varepsilon_{\alpha_1 \dots \alpha_6} \psi^{\alpha_1} A^{\alpha_2} (\square, 2) \quad (15)$$

and

$$\begin{aligned} & \varepsilon_{\alpha_1 \alpha_2} \psi^{\alpha_1} A^{\alpha_2} (1, 6) \\ & [\psi^{\alpha} A_{\beta}^* - \frac{1}{2} \delta_{\beta}^{\alpha} \psi^{\gamma} A_{\gamma}^*] (\square, 2) \\ & \psi_{\gamma}^* A^{\gamma} (1, -2) \end{aligned} \quad (16)$$

for $n = 6$ and $n = 2$ respectively. In the latter case, we see that θ or the $\theta\theta\bar{\theta}$ -components of the vector superfields $U_{\alpha}^{\beta} \equiv \Phi_{\alpha}^* e^V \Phi^{\beta}$ are again needed. However, here we cannot exclude the possibility of obtaining a supersymmetric $SU(2) \times U(1)$ gauge symmetry (i.e., massless vector bosons) at the bound state level, such that $U_{\alpha|\theta\theta\bar{\theta}}^{\beta}$ should define the corresponding massless gauginos while $U_{\alpha|\theta}^{\beta}$ would be gauged away, both composite fields transforming like the adjoint representation of the original global symmetry.

In conclusion let us stress once again the main points of this letter:

- complementarity seems to imply a dynamical breaking of supersymmetry in some QCD-like models (in the chiral limit). It is also interesting to point out that in this class of models, massless fermions in chiral representations arise from a fundamental vector-like theory.
- in chiral models where complementarity does not apply, massless vector bosons at the effective level do not seem to be excluded.

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