COMPARISON OF MONTE CARLO AND ANALYTICAL
CALCULATIONS IN QCD

G. Ingelman and D.E. Soper *)

CERN -- Geneva

A B S T R A C T

We compare two approaches to calculate the angular energy-energy correlation function in QCD. One uses an analytical calculation and the other a Monte Carlo simulation of a parton cascade evolution. Good agreement is found also when extrapolating to the TeV energy region.

*) Permanent address: Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403, USA.

One can sometimes derive an analytical expression that gives the prediction (to some accuracy) of quantum chromodynamics for a quantity of experimental interest. In other cases, however, no analytical expression is available. Then one can use a Monte Carlo computer program that approximates the Feynman diagrams of quark and gluon interactions and simulates the production of hadrons from the resulting final state partons.

In this letter, we ask how faithfully the Monte Carlo programs simulate QCD. We do this by comparing the predictions of two leading Monte Carlo programs [1,2] with the predictions of an analytical QCD formula [4] for a quantity that is particularly difficult to predict because it contains "double logarithms" in perturbation theory. Since both the computer algorithms and the analytical formula contain parameters that have been adjusted to fit experiment at presently available energies, we expect good agreement between the two methods at present energies. In order to set a rigorous test, therefore, we make the comparison at one hundred times the presently available energy.

Of course, some readers may be completely convinced of the accuracy of the approximations in the Monte Carlo jet evolution algorithms. These readers can regard the comparison as a test of the differential equation used to treat double logarithms in the analytical approach [4].

The experimentally measurable quantity that we have chosen for the comparison is the energy-energy correlation function, $d\Sigma/d\cos\theta$, in e^+e^- annihilation [3]. This quantity is defined [4] in terms of the inclusive cross section $d\sigma/dx_A dx_B d\cos\theta$ for the process $e^+e^- \rightarrow A+B+X$ where A and B are two detected hadrons, x_A and x_B are the energy fractions (defined as $x_A=2\cdot E_A/Q$ as usual, with Q being the c.m. energy of the annihilation) carried by these hadrons, and θ is the angle between the momentum vectors $\vec{p}_A$ and the negative of $\vec{p}_B$. The definition is

$$\frac{d\Sigma}{d\cos\theta} = \frac{1}{4} \sum_A \int_0^1 dx_A x_A \sum_B \int_0^1 dx_B x_B \frac{d\sigma(e^+e^- \rightarrow ABX)}{dx_A dx_B d\cos\theta} \quad (1)$$

where the sums are over all produced hadrons. This function has a large peak centered at the back-to-back direction $\theta = 0$. The peak arises from two jet final states in which one hadron is detected from

each jet. The width of the peak is thus a measure of the angular width of a quark jet. The $\theta = 0$ peak is strongly affected by soft and collinear gluon bremsstrahlung, which produces the double logarithms, $\ln^2\theta$, mentioned earlier.

In our comparisons, we use the same QCD scale parameter for the two Monte Carlo programs and for the analytical formula, $\Lambda_{\overline{MS}} = 150$ MeV.

We use two Monte Carlo models that approximate perturbative jet evolution, coupled to hadronization described by the Lund string model [7]. Both programs [1,2] are based on the leading logarithm approximation of QCD to simulate the parton bremsstrahlung processes that are expected to occur from an initial quark-antiquark system produced in a large Q^2 process. In the algorithms the offshellnesses, t , of the emitted partons are assumed to be strongly ordered. This results in an iterative cascade which is stopped when the offshellness of all partons are below a chosen cut off value, t_{cut} . Together with Λ_{QCD} , this cut off regulates the amount of bremsstrahlung emitted. One of the models [1] also takes soft gluon interference effects into account by an angular ordering of the partons in the emission process. This leads to fewer soft partons being emitted and certain next-to-leading corrections are also effectively taken into account.

This perturbative jet evolution is followed by a nonperturbative hadronization mechanism to get the final hadronic state. In order not to introduce further differences we apply string fragmentation in both cases. This means that all the partons, whose colour ordering is known from the perturbative cascade, are connected by a colour triplet string-field which then fragments according to the Lund model [7,8]. This method was introduced in [9] and found to provide a stable model [10].

In Figure 1, we show the predictions of the two Monte Carlo models for $Q = 30$ GeV, which is within the PETRA-PEP energy range. We see that the predictions agree very well here. (They also agree with data at nearby energies.)

We now compare the two Monte Carlo predictions at $Q = 100 \times 30$ GeV = 3 TeV, which is far beyond the range of any present experiments (but which may be accessible in parton-parton collisions at future proton-proton colliders now being discussed). The predictions

of the two Monte Carlo programs are shown in Figure 2. We see that there is good agreement over a wide range of large and intermediate angles, even though the correlation function changes by a factor of more than 1000 over this range. At very small angles, however, the predictions differ by a factor of about five. The angle at which one begins to see a difference is roughly $1 - \cos\theta = 10^{-5}$, $\theta = 4 \times 10^{-3}$ radians, which corresponds to a transverse momentum $k_{\perp} = Q \cdot \sin(\theta/2)$ of about $k_{\perp} = 7$ GeV.

The difference between the two Monte Carlo models is not related to different cut offs, since there is not much variation of the result due to the change in t_{cut} as shown by the full and dashed curves in Figure 2. We also note here that at these large energies there is very little difference between the hadron level results shown and the corresponding curves for the underlying parton level, i.e. the smearing due to hadronization is very small. Thus, the observed difference is related to some of the other differences between the two parton shower models. One possibility may be the use of $\alpha_s(m^2)$ in [2] instead of $\alpha_s(p_{\perp}^2)$, which has been argued [1] to be a more correct choice since it leads to the correct form of the Sudakov form factor. Qualitatively, this would have the noted effect since a smaller argument, $p_{\perp}^2$, in α_s , and hence a larger coupling constant, would mean more radiation resulting in a broader jet. (It should be noted however, that the angular ordering works in the opposite direction.)

This difference between the Monte Carlo results is somewhat troubling, but it may be well to provide some perspective by comparing the difference to the range of possibilities that might have seemed plausible without knowing about QCD. At one extreme, one might have imagined that the dimensionless quantity $\sigma_T^{-1} \cdot d\Sigma/d\cos\theta$ scales, so that it has the same value at $Q = 3$ TeV as at $Q = 30$ GeV, namely

$$[\sigma_T^{-1} \cdot d\Sigma/d\cos\theta]_{\theta=0} \approx 10 \quad (\text{scaling}).$$

At the other extreme, one might have imagined that the naive parton model holds at small angles. Then the original quarks produced from the e^+e^- annihilation decay into hadrons with a Q -independent transverse momentum distribution (relative to the jet axis). As a result $\sigma_T^{-1} \cdot d\Sigma/dk_{\perp}^2$ scales, where $k_{\perp} = Q \cdot \sin(\theta/2)$, so $\sigma_T^{-1} \cdot d\Sigma/d\cos\theta$ at $\theta = 0$ is proportional to Q^2 . Thus

$$[\sigma_T^{-1} \cdot d\Sigma/d\cos\theta]_{\theta=0} \approx 10^5 \quad (\text{parton})$$

at $Q = 100 \times 30$ GeV. We may conclude that the factor five difference between the two Monte Carlo results, while troubling, actually represents a significant accomplishment compared to the factor 10^4 uncertainty that would have existed without QCD.

We now turn to the analytical formula. The formula that we will use is derived in [5] and applied to recent data in [4]. A discussion in the analogous Drell-Yan processes is given in [6].

In the analytical formula there appears a Fourier transform integration over a transverse coordinate b , which is a Fourier conjugate to $k_1 = Q \cdot \sin(\theta/2)$. At very small values of θ rather large values of b can be important in the integration. The formula contains certain functions, $f_1(b)$ and $f_2(b)$, that describe physics at distances b that are not small enough for low order perturbation theory to be reliably applied. In ref. [4], these functions are fit to data from the PLUTO and CELLO detectors at PETRA at c.m. energies Q in the range $17 \text{ GeV} \leq Q \leq 34 \text{ GeV}$. We will make use of two separate fits, labelled A and B here and in ref. [4], so as to give some estimate of the uncertainties in the analytical formula.

The analytical formula is based on a differential equation derived in [5] that gives the evolution with Q^2 of the measured k_1 decay functions of the quark. What is important to recognize here is that the derivation is not limited to the leading logarithm or strong ordering approximations that are used in the Monte Carlo algorithms. On the other hand, the derivation lacks sensitivity to any effects, such as colour string formation, that cannot be seen in perturbation theory (except insofar as these effects are contained in the hadron Bethe-Salpeter wavefunctions). It is assumed that any such nonperturbative effects will have the same structure that is found at arbitrary orders of perturbation theory, so that these nonperturbative effects can be absorbed into the two large- b functions $f_1(b)$ and $f_2(b)$. In summary then it is interesting to compare the Monte Carlo and analytical approaches numerically because they are based on quite different approximations and assumptions.

As mentioned earlier, one expects the Monte Carlo algorithms and the (fitted) analytical formula to agree in the presently accessible energy range. In fig. 3 we compare the two analytical curves A and B to one of the Monte Carlo results at $Q = 30$ GeV. (Recall that the two Monte Carlos agree with each other at this energy.) We see that the analytical and Monte Carlo curves do indeed agree well.

We now look at the two versions, A and B, of the analytical formula at $Q = 3$ TeV, as shown in fig. 4. (Ignore the Monte Carlo histogram for the moment.) We see that the two versions of the analytical formula agree with each other except at small angles where the difference in the fitted non-perturbative functions results in a difference between the two analytical curves of about a factor 2. Thus one may estimate that the analytical result is accurate at the factor 2 level at very small angles and somewhat more accurate at larger angles.

We can now compare the analytical and Monte Carlo results at $Q = 3$ TeV. In fig. 4 we see the analytical curves A and B just discussed together with the results of the Webber-Marchesini version of the Monte Carlo algorithm [1]. We see that at all angles, even very small angles, this Monte Carlo result agrees with the analytical result within the expected uncertainty of the analytical result. The Gottschalk version of the Monte Carlo algorithm (shown in fig. 2) agrees with the analytical formula very well for large and intermediate angles, but gives results that are larger than the analytical curves when θ is smaller than about $4 \cdot 10^{-3}$ radians ($1 - \cos\theta < 10^{-5}$).

The energy-energy correlation is in effect a measure of the width of a jet. We have here investigated quark jets only since they are on a firm basis in the analytical approach. A preliminary comparison for gluon jets, which are considerably wider due to more gluon radiation, shows a rough agreement between the Monte Carlo algorithms and an approximate version of the proper analytical formula for gluon jets.

We conclude that the agreement between the Monte Carlo models tested and the analytical formula is better than one might have expected, with very good agreement at large and intermediate angles and some difference at very small angles. The Monte Carlo and analytical methods agree at zero angle, within the factor five uncertainty arising from the difference between the two Monte Carlo algorithms. If we take the analytical result to be accurate to within a factor two, then one of the two Monte Carlo models is favoured as being the more accurate for quantities sensitive to double logarithms. We suggest that a similar test be applied to other jet evolution--hadronisation Monte Carlo programs used to aid in the analysis of experimental data or to predict the structure of final states at projected higher energy machines.

Acknowledgements

We are grateful to J.C. Collins, W.J. Stirling and B.R. Webber for helpful conversations.

REFERENCES

- [1] G. Marchesini and B.R. Webber, Nucl. Phys. B238 (1984) 1
B.R. Webber, Nucl. Phys. B238 (1984) 492
- [2] T.D. Gottschalk, Nucl. Phys. B214 (1983) 201 and Caltech
preprint CALT-68-1083
- [3] C. Basham, L. Brown, S. Ellis and S. Love, Phys. Rev.
Lett. 41 (1978) 1585, Phys. Rev. D19 (1979) 2018, D24 (1981)
2382
Yu.L. Dokshitzer, D.I. D'Yakonov and S.I. Troyan, Phys.
Lett. 78B (1978) 290
- [4] J.C. Collins, S.-C. Chao, D.E. Soper, in preparation
- [5] J.C. Collins and D.E. Soper, Nucl. Phys. B193 (1981) 381
with erratum Nucl. Phys. B213 (1983) 545, Nucl. Phys. B197
(1982) 446
- [6] J.C. Collins, D.E. Soper, G. Sterman, Ref.TH.3923-CERN
- [7] B. Andersson, G. Gustafson, G. Ingelman, T. Sjostrand,
Phys. Rep. 97 (1983) 31
- [8] T. Sjostrand, DESY preprint DESY T-84-01
- [9] G. Ingelman, Ref.TH.3926-CERN to be published in Z. Phys.
- [10] G. Ingelman, Ref.TH.3969-CERN

FIGURE CAPTIONS

- Figure 1: The energy-energy correlation of eq. (1) for a quark-antiquark system at $Q = 30$ GeV obtained from Monte Carlo simulations using the parton evolution model [1] with $t_{\text{cut}} = 0.5$ GeV² (full line) and $t_{\text{cut}} = 5$ GeV² (dashed line); together with that of [2] with $t_{\text{cut}} = 5$ GeV² (dotted line). In all cases $\Lambda = 150$ MeV and string fragmentation is applied.
- Figure 2: Same as fig. 1 but with $Q = 3$ TeV.
- Figure 3: Analytical calculations of the energy-energy correlation for a quark-antiquark system at $Q = 30$ GeV using two separate fits; A (full line) and B (dashed line); compared to the Monte Carlo calculation (full histogram as in fig. 1).
- Figure 4: Same as in fig. 3 but with $Q = 3$ TeV.

FIG. 1

FIG. 2

FIG. 3