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SPLIT LIGHT COMPOSITE SUPERMULTIPLETS

A. Masiero and G. Veneziano

CERN - Geneva

ABSTRACT

The splitting induced by soft supersymmetry (SUSY) breaking inside the light composite supermultiplets of confining SUSY gauge theories is studied by effective Lagrangian methods. Examples with and without unbroken chiral symmetries are considered. In the former case, for a suitable breaking, the lightest states are spin $\frac{1}{2}$ fermions. A prototype model for one leptonic family is discussed.

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1. - INTRODUCTION

Supersymmetric (SUSY) composite models for quarks and leptons have been advertised in the past few years¹⁾ for possessing several advantages over their non-SUSY competitors. It is becoming clear, however, that a SUSY preonic dynamics may exhibit one further, crucial property: theoretical predictivity. This comes from the recent observation²⁾⁻⁵⁾ that the condensates characterizing the vacuum properties of SUSY gauge theories can be computed by convergent, reliable, small instanton calculations. These have confirmed and extended the results on SUSY gauge theory vacua previously obtained⁶⁾⁻⁸⁾ by effective Lagrangian methods.

Of course, any realistic model of quarks and leptons must contain in one way or another a SUSY breaking mechanism allowing one to push the unseen scalar partners of the ordinary fermions to a sufficiently high mass. Since the road of spontaneous SUSY breaking⁵⁾ appears to be unavailable in the presently considered preon models¹⁾, one is led to use instead some sort of soft SUSY breaking, as originates for instance from the supergravity Higgs effect⁹⁾.

Without committing ourselves to a particular origin of the soft SUSY breaking, we want to study here its interplay with the above-mentioned non-perturbative effects. The purpose of the exercise is that of gaining some general understanding of the low-lying spectrum of these theories as a function of the SUSY breaking parameters. This could eventually enable one to construct realistic and essentially calculable models for composite quarks and leptons.

Unfortunately, the direct calculational method of Refs. 2)-5) cannot be easily extended to include soft SUSY breakings. For this reason, we shall rather employ the effective Lagrangian techniques which have proven so useful in predicting the properties of SUSY vacua.

We shall actually present three quite extreme examples.

- 1) Supersymmetric Yang-Mills (SYM) theory with a gaugino mass term (Section 2). This theory does not possess any continuous chiral symmetry (even in the SUSY limit) and is therefore expected not to contain a light supermultiplet. Nevertheless, this case is interesting insofar as it allows us to see quite easily vacuum angle effects and splittings inside supermultiplets due to SUSY breaking.

- ii) Supersymmetric QCD (SQCD) with two colours and one flavour (Section 3.2). This is the simplest case with a continuous chiral symmetry which is, however, spontaneously broken. A light Goldstone supermultiplet is thus expected, and splittings therein can be studied. Because of the chiral symmetry breaking, we expect the pseudoscalar member of the supermultiplet to be better protected than the scalar and the fermion, making life difficult for phenomenological applications.
- iii) SQCD with three colours and three flavours (Section 3.3). This is a scaled-down version of the model of Ref. 10) in that it has only "leptons" and no "quarks"^{1*}. Yet the model turns out to be very instructive. It has a $U(1)$ axial R-symmetry which is unbroken in the SUSY limit and, consequently, some fermion masses enjoy a double protection [cf. Ref. 10)]. The splitting inside the light supermultiplets is now expected to favour the fermions and, indeed, for a wide range of SUSY breaking parameters, one is able to get rid of the bosonic partners of the leptons. The model allows for further gauging of an $SU(2)_L \times U(1)_Y$ subgroup of the flavour group with technicolour-type breaking down to $U(1)_{em}$ and, finally, a non-vanishing tree-level photino mass is able to induce radiatively a mass for the charged (but not for the neutral) lepton.

2. - SOFTLY BROKEN SUPER YANG-MILLS (SYM) THEORY

We shall start by illustrating the general considerations of Section 1 in the case of softly-broken SYM theory which is defined by the Lagrangian:

$$L = L_{inv} + L_{break} \quad . \quad (2.1)$$

L_{inv} is the usual¹¹⁾ SYM Lagrangian with gauge group $SU(N)$ and L_{break} is given by

$$L_{break} = -m_\lambda \lambda_a^\alpha \lambda_a^\alpha + h.c. \quad ; \quad \begin{array}{l} \alpha = 1, 2 \\ a = 1, \dots, N^2 - 1. \end{array} \quad (2.2)$$

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(1*)

This simplified version of the model of Ref.10) was conceived through discussions with R. Barbieri.

L_{break} is the only possible gauge invariant SUSY breaking term, a gaugino mass. The phase of m_λ is arbitrary. By a $U(1)_R$ transformation

$$\lambda_\alpha \rightarrow \lambda'_\alpha = \exp\left(\frac{i}{2} \arg m_\lambda\right) \lambda_\alpha, \quad (2.3)$$

the phase of m_λ can be rotated away. Since $U(1)_R$ is anomalous, one generates, however, a vacuum angle θ_v :

$$\theta_v = N \arg m_\lambda \quad (2.4)$$

Without loss of generality, we can work with a real $m_\lambda > 0$ and a vacuum angle θ_v defined by

$$L_{\text{inv.}} = -\frac{1}{4} \left(1 + i \frac{g^2}{8\pi^2} \theta_v\right) W^2 \Big|_F + \text{h.c.}, \quad (2.5)$$

where W_α is the gauge superfield and F denotes, as usual, the θ^2 component.

The low-energy physics of the supersymmetric theory ($m_\lambda = 0$) appears to be well described by the effective Lagrangian⁶⁾

$$L_{\text{eff}}^{\text{SUSY}} = \frac{g}{\alpha} (S^* S)^{1/3}_D + \left[\left(S \left(\log \frac{S^N}{\Lambda^{3N}} - N \right) \right)_F + \text{h.c.} \right] \quad (2.6)$$

where D denotes the $\theta^2 \bar{\theta}^2$ component and the renormalization group and gauge invariant composite superfield S is defined by^{1*}

$$S = \frac{g^2}{32\pi^2} W_\alpha^a W^{\alpha,a}. \quad (2.7)$$

Clearly, the effects of a non-zero vacuum angle θ_v and of a non-zero mass m_λ can be accounted for by writing a new L_{eff} :

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Note that our normalization of S follows Ref. 7) rather than Ref. 6).

$$L_{\text{eff}} = \frac{g}{\alpha} (S^* S)^{1/3} + \left[\left(S \left(\log \frac{S^N}{\Lambda^{3N}} - N - i \theta_v \right) \right) + \text{h.c.} \right] + \left[m_\lambda S_{\theta=0} + \text{h.c.} \right], \quad (2.8)$$

where m_λ now includes some g -dependent normalization factors that make it the physical (i.e., renormalization group invariant) SUSY breaking parameter.

The scalar potential resulting from L_{eff} is simply:

$$V = \alpha \left(\pi_\lambda^* \pi_\lambda \right)^{2/3} \left| \log \frac{\pi_\lambda^N}{\Lambda^{3N}} - i \theta_v \right|^2 - m_\lambda \left(\pi_\lambda + \pi_\lambda^* \right), \quad (2.9)$$

where π_λ is the lowest component of S , i.e., $(g^2/32\pi^2)\lambda_\alpha^a \lambda^a_\alpha$. For $m_\lambda = 0$, the N SUSY minima are located at:

$$\langle \pi_\lambda \rangle = \Lambda^3 \exp \left(i \frac{\theta_v}{N} \right) \exp \left(- \frac{2i\pi K}{N} \right); \quad K = 1, \dots, N. \quad (2.10)$$

This degeneracy corresponds to the spontaneous breaking of the non-anomalous Z_{2N} subgroup of $U(1)_R$ down to a $Z_2 (\lambda \rightarrow \pm \lambda)$. A small mass term breaks explicitly the Z_{2N} symmetry and, indeed, it shifts the N local minima, leaving one of them as the true ground state. This is given by the value of K minimizing:

$$E_K \underset{m_\lambda \ll \Lambda}{\simeq} - m_\lambda \left(\pi_\lambda + \pi_\lambda^* \right)_K = - 2 m_\lambda \Lambda^3 \cos \left(- \frac{2\pi K + \theta_v}{N} \right). \quad (2.11)$$

At $\theta_v = 0$, the minimum with $K = 0$ clearly gives the ground state. From Eq. (2.10), we see that it also gives a real $\langle \pi_\lambda \rangle$ and, hence, CP conservation. Indeed, $K = 0$ is the true vacuum in the range $-\pi < \theta_v < \pi$. At $\theta_v = \pi$, the $K = 0$ and $K = +1$ levels cross:

$$E_0 = \text{const.} \cdot \cos \theta_v / N; \\ E_{+1} = \text{const.} \cdot \cos \left(\frac{\theta_v - 2\pi}{N} \right) = E_0; \quad \theta_v = \pi. \quad (2.12)$$

The $K = +1$ vacuum becomes the true ground state up to $\theta_v = 3\pi$, where it crosses (and is replaced by) the $K = +2$ vacuum. This pattern continues up to $\theta_v = (2N-1)\pi$ where the $K = 0$ vacuum comes back again. Thus the whole pattern repeats itself

with θ_v periodicity $2\pi N$, while the physics is periodic with period 2π (it is the same at $\theta_v = 0$ in the $K = 0$ vacuum, at $\theta_v = 2\pi$ in the $K = +1$ vacuum, etc.). The situation is analogous to the one known to hold in QCD¹²⁾ for N degenerate flavours, and is depicted in Fig. 1 for the case $N = 3$. Notice that at $\theta_v = \pi$, one gets a complex $\langle \pi_\lambda \rangle$, with

$$\langle \text{Im } \pi_\lambda \rangle \simeq \langle i \bar{\lambda} \gamma_5 \lambda \rangle \simeq \sin\left(\frac{\pi}{N}\right) \Lambda^3, \quad (2.13)$$

corresponding to a violation of CP. This violation is "spontaneous" for N odd [Dashen's phenomenon¹³⁾], since it corresponds to having just changed the sign of m_λ ($\theta_v = N\pi$ is the same as $\theta_v = \pi$ for N odd), and thus to a CP-conserving Lagrangian. Again, the situation is similar to that of ordinary QCD¹²⁾ for an odd number of degenerate flavours.

Let us now consider the expectation value of the auxiliary fields of S which are related to $F^2 + iF\tilde{F}$. By SUSY, they were zero in the $m_\lambda = 0$ case. It is straightforward to see that they now become $O(m_\lambda)$. More precisely, one finds:

$$\text{Re } \langle S_F \rangle = \left\langle \frac{\alpha}{2} \left(\pi_\lambda \pi_\lambda^* \right)^{2/3} \log \left(\frac{\pi_\lambda^* \pi_\lambda}{\Lambda^{6N}} \right)^N \right\rangle \simeq \frac{m_\lambda}{m_\lambda \ll \Lambda} \frac{m_\lambda}{2N} \langle \pi_\lambda + \pi_\lambda^* \rangle \quad (2.14)$$

and

$$\begin{aligned} \text{Im } \langle S_F \rangle &= \left\langle \frac{\alpha}{2i} \left(\pi_\lambda \pi_\lambda^* \right)^{2/3} \left[\log \left(\frac{\pi_\lambda^*}{\pi_\lambda} \right)^N - 2i\theta_v \right] \right\rangle \simeq \\ &\simeq \frac{m_\lambda}{m_\lambda \ll \Lambda} m_\lambda \langle \pi_\lambda + \pi_\lambda^* \rangle / (2iN). \end{aligned} \quad (2.15)$$

Equation (2.14) shows that $\text{Re } S_F = (g^2/32\pi^2)F^2$ gets a positive expectation value of $O(m_\lambda \Lambda^3)$ [the minimization of (2.11) yields a positive value for $\langle \pi_\lambda + \pi_\lambda^* \rangle$]. This is consistent with the fact that for $m_\lambda \sim \Lambda$, the broken SYM theory should approach the ordinary YM theory (QCD without quarks) for which $\langle F^2 \rangle$ is believed to be positive¹⁴⁾ (condensate of magnetic monopoles).

Turning to Eq. (2.15), one obtains from it at small θ_v :

$$\text{Im } \langle S_F \rangle = g^2 \langle F\tilde{F} \rangle / (32\pi^2) \simeq \frac{m_\lambda \Lambda^3 \theta_v}{N^2} \quad \theta_v \ll 1 \quad (2.16)$$

The result (2.16) again interpolates smoothly between the supersymmetric case $m_\lambda = 0$ and the ordinary YM case ($m_\lambda = \Lambda$), where it is known¹⁵⁾ from the large N solution to the U(1) problem that^{1*}:

$$\frac{2}{\partial\theta_\nu} \left\langle \frac{g^2}{32\pi^2} F\tilde{F} \right\rangle \bigg|_{\substack{\theta_\nu=0 \\ \text{YM}}} \simeq \frac{1}{6} F_\pi^2 m_{\eta'}^2 = O(\Lambda^4) \quad (2.17)$$

Our last point about SYM concerns the mass splitting inside the massive Wess-Zumino supermultiplet which, in the SUSY limit, contains the would-be Goldstone P of the anomalous $U(1)_R$ symmetry, a scalar S and a fermion F. A straightforward calculation yields, to first order in m_λ :

$$M_P^2 = \alpha^2 N^2 \Lambda^2 \quad ; \quad M_P = \alpha N \Lambda \quad (2.18a)$$

$$M_S^2 = \alpha^2 N^2 \Lambda^2 + \frac{8}{3} \alpha m_\lambda \Lambda \quad ; \quad M_S = \alpha N \Lambda + \frac{4}{3} \frac{m_\lambda}{N} \quad (2.18b)$$

$$M_F^2 = \alpha^2 N^2 \Lambda^2 + \frac{4}{3} \alpha m_\lambda \Lambda \quad ; \quad M_F = \alpha N \Lambda + \frac{2}{3} \frac{m_\lambda}{N} \quad (2.18c)$$

We observe that

$$\text{Str } M^2 = \sum_J (2J+1) (-)^J |M_J|^2 = 0 \quad (2.19)$$

This result is a special case of a general supertrace mass formula which is derived in the Appendix.

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Note that, with our normalizations, Λ^3 and m_λ are proportional to N in the large N limit, while $\alpha \sim N^{-4/3}$ [see Ref. 7)].

3. - SOFTLY BROKEN SQCD

3.1 We are now coming to the physically more interesting case of softly broken SQCD. Such a theory is defined by the Lagrangian:

$$L = L_{inv} + L_{break} , \quad (3.1)$$

where L_{inv} is the usual SQCD Lagrangian¹¹⁾ containing, besides the gauge superfield W , the chiral matter superfields Φ_α^i and $\tilde{\Phi}_j^\alpha$ ($\alpha = 1, \dots, N$; $i, j = 1, \dots, M$) in the $N + \bar{N}$ representation of $SU(N)$. L_{inv} contains a SUSY mass term

$$L_{inv}^{mass} = - \sum_{i,j} m_{ij} \Phi_\alpha^i \tilde{\Phi}_j^\alpha \Big|_F + h.c. \quad (3.2)$$

The possible gauge invariant soft breaking terms have been classified in Ref. 16) for a general gauge theory. In the case of SQCD, they can be of one of the following types:

a) a gaugino mass term:

$$L_{break}^{(a)} = - m_\lambda \lambda \lambda + h.c. \quad (3.3a)$$

b) scalar mass terms:

$$L_{break}^{(b)} = \sum_{i,j} \left(\mu_{ij}^2 \phi_i^* \phi_j + \tilde{\mu}_{ij}^2 \tilde{\phi}_i^* \tilde{\phi}_j \right) \quad (3.3b)$$

where ϕ and $\tilde{\phi}$ are the scalar lowest components of Φ and $\tilde{\Phi}$ respectively;

c) bilinear bosonic mass terms:

$$L_{break}^{(c)} = \sum_{i,j} \hat{\mu}_{ij}^2 \phi_i \tilde{\phi}_j + h.c. \quad (3.3c)$$

Unlike the SYM theory, SQCD possesses, in the absence of mass terms, continuous chiral symmetries which, a priori, may or may not be spontaneously broken. This question has been studied, in the SUSY limit, both by the construction of effective Lagrangians^{7),8)} and by direct (instanton) calculations^{3),4)}, with consistent results.

In any event, massless SQCD is expected to exhibit a rich structure of massless bound states, since spontaneous chiral symmetry breaking implies Nambu-Goldstone bosons (and their fermionic SUSY partners), whilst unbroken chiral symmetries imply massless anomaly matching fermions ('t Hooftons) and their bosonic SUSY partners.

Following the discussion of Section 1, we are interested in the splitting inside these supermultiplets in the presence of soft SUSY breakings. In particular with an eye on possible applications to composite models, we would like to understand the circumstances under which the fermionic bound states (quarks and leptons) remain considerably lighter than their bosonic partners (squarks and sleptons). For this purpose, we shall analyze two somewhat extreme situations in SQCD:

- i) $N = 2$, $M = 1$ and
- ii) $N = 3$, $M = 3$.

In the first case, the massless theory has just a chiral $U(1)$ symmetry which is spontaneously broken by a $\tilde{Q}\tilde{Q}$ condensate. As a consequence, we expect bosonic masses to be better protected than the fermionic ones when SUSY is broken. In the second case, on the contrary, some axial symmetries are expected to remain unbroken and this should lead to a more favourable situation for keeping the fermions lighter than their bosonic partners. Indeed, as we shall see, model ii) can be seen as a simplified (i.e., purely leptonic) version of the model of Ref. 10).

3.2 SQCD with $N = 2$ and $M = 1$

An effective Lagrangian for the SUSY limit of this theory has been constructed in Ref. 7). It reads:

$$L_{eff}^{inv.} = \frac{g}{\alpha} (S^* S)_{\mathcal{D}}^{2/3} + \frac{4}{\beta} (T^* T)_{\mathcal{D}}^{1/2} + \left[S \left(\log \left(\frac{S \bar{T}}{\Lambda^5} \right) - 1 \right) - m \bar{T} \right]_F + h.c., \quad (3.4)$$

where besides S of Eq. (2.7), we have introduced the composite superfield⁷⁾:

$$T = \Phi \tilde{\Phi} = \pi + \theta \chi + \dots \quad (3.5)$$

This Lagrangian at $m \neq 0$ possesses two SUSY vacua with order parameters:

$$\begin{aligned} \langle \pi_\lambda \rangle &= \pm \Lambda^3 \left(m/\Lambda \right)^{1/2}, \\ \langle \pi \rangle &= \pm \Lambda^2 \left(\Lambda/m \right)^{1/2} = m^{-1} \langle \pi_\lambda \rangle, \end{aligned} \quad (3.6)$$

in agreement with explicit instanton calculations.

We now add to $L_{\text{eff}}^{\text{inv}}$ the soft breaking terms:

$$L_{\text{eff}}^{\text{break}} = m_\lambda (\pi_\lambda + \pi_\lambda^*) - \mu^2 (\pi^* \pi)^{1/2} - \hat{\mu}^2 (\pi + \pi^*) \quad (3.7)$$

which have the same transformation properties as the three possible^{1*} soft breaking terms of Eqs. (3.3).

The scalar potential originating from $L_{\text{eff}} = L_{\text{eff}}^{\text{inv}} + L_{\text{eff}}^{\text{break}}$ is:

$$\begin{aligned} V &= \alpha \left(\pi_\lambda \pi_\lambda^* \right)^{2/3} |L|^2 + \beta \left(\pi^* \pi \right)^{1/2} \left| \frac{\pi_\lambda}{\pi} - m \right|^2 - \\ &\quad - m_\lambda (\pi_\lambda + \pi_\lambda^*) + \hat{\mu}^2 (\pi + \pi^*) + \mu^2 (\pi^* \pi)^{1/2}; \\ L &= \log[\pi \pi_\lambda / \Lambda^5]. \end{aligned} \quad (3.8)$$

The boundedness of the potential requires (in the effective as well as in the underlying theory) a relation between m , μ and $\hat{\mu}$, i.e.,

$$\beta |m|^2 + \mu^2 - 2 |\hat{\mu}^2| > 0, \quad (3.9)$$

which we shall assume to be satisfied.

Treating m , m_λ , μ and $\hat{\mu}$ as being all of the same order and much smaller than Λ , it is easy to find the minimum of the potential (3.8). Defining

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For simplicity, however, we shall ignore, for SQCD, the possibility of a non-vanishing vacuum angle (i.e., of non-real mass parameters).

$$\langle \pi_\lambda \rangle \cdot \langle \pi \rangle = \Lambda^5 (1 + \varepsilon) \quad (3.10)$$

and

$$\langle \pi_\lambda \rangle / \langle \pi \rangle = \hat{m} ,$$

one finds

$$\begin{aligned} \hat{m} &= \frac{1}{3\beta} \left[\beta m - m_\lambda + \left(m_\lambda^2 + 4\beta^2 m^2 - 2\beta m m_\lambda + 3\beta \mu^2 + 6\beta \hat{\mu}^2 \right)^{1/2} \right], \\ \varepsilon \cdot \left(\hat{m} \Lambda^5 \right)^{1/6} &= \frac{1}{3\alpha} \left[2\beta m - 2m_\lambda - \left(m_\lambda^2 + 4\beta^2 m^2 - 2\beta m m_\lambda + 3\beta \mu^2 + 6\beta \hat{\mu}^2 \right)^{1/2} \right]. \end{aligned} \quad (3.11)$$

Clearly, in the SUSY limit one gets $\hat{m} \rightarrow m$ and $\varepsilon \rightarrow 0$, thus recovering one of the SUSY vacua of Eqs. (3.6).

One also finds that, thanks to the constraint (3.9), this minimum always exists and that it is indeed the absolute minimum, with

$$V_{\min} = -2m_\lambda \langle \pi_\lambda \rangle < 0 \quad (3.12)$$

At this point, we can compute the scalar, pseudoscalar and fermion mass matrices. A combination of S and T (predominantly S) gives a heavy [i.e., mass $O(\Lambda)$] split supermultiplet, whilst a different combination (predominantly T) gives a light split supermultiplet. Obviously, we are interested in this latter. Straightforward but lengthy calculations yield the following mass spectrum in the light supermultiplet

$$\begin{aligned} M_F^2 / m^2 &= \frac{1}{4} \left[1 + 9 \left(\frac{\hat{m}}{m} \right)^2 + 6 \frac{\hat{m}}{m} \right], \\ M_P^2 / m^2 &= \frac{1}{2} \left[1 - 3 \left(\frac{\hat{m}}{m} \right)^2 + 10 \frac{\hat{m}}{m} \right] + 2 \frac{m_\lambda \hat{m}}{m^2} + \frac{1}{2} \frac{\mu^2}{m^2} \quad (3.13) \\ M_S^2 / m^2 &= 6 \left(\frac{\hat{m}}{m} \right)^2 - 2 \left(\frac{\hat{m}}{m} \right) - 2 \frac{m_\lambda \hat{m}}{m^2}, \end{aligned}$$

with $\hat{m}$ given by Eq. (3.11). One immediately notices that

$$\text{STr } M^2 \Big|_{\text{light sector}} = \frac{1}{2} \mu^2. \quad (3.14)$$

The validity of such a simple supertrace mass formula is proven in the Appendix.

A simple numerical study of Eqs. (3.13) shows that, in a range of the SUSY breaking parameters m_λ , μ^2 and $\hat{\mu}^2$, the lightest particle in the spectrum is the spin $\frac{1}{2}$ fermion in spite of the fact that no unbroken chiral symmetry protects it. The most favourable situation for a light fermion appears to be the one in which $M_P^2 \approx M_S^2$ and, from Eq. (3.14), $M_F^2 = M_P^2 - 1/4 \mu^2$. It can be shown, however, that irrespective of the values of the breaking parameters, it is not possible to get M_F^2 less than $\sim \frac{1}{2} \min(M_P^2, M_S^2)$. In other words, one cannot get rid of the scalar partners in this case. The opposite result will be achieved in the next model we are going to examine.

3.3 SQCD with $N = M = 3$: a model of composite leptons?

The model we shall investigate now is the natural reduction of the one-family model of Ref. 10) after colour degrees of freedom (as well as hypercolour ones) have been eliminated. It can thus be seen as a prototype model for one family of leptons (say e, ν_e).

The fundamental chiral preon superfields transform under the gauge group $SU(3)_{HC} \times SU(2)_L \times U(1)_Y$ as^{1*}

$$\begin{aligned} \Phi_{\alpha}^{i=1,2} &\equiv (3, 2, -1/3) & ; & & \Phi_{\alpha}^3 &\equiv (3, 1, 2/3) \\ \tilde{\Phi}_2^{\alpha} &\equiv (\bar{3}, 1, 4/3) & ; & & \tilde{\Phi}_{1,3}^{\alpha} &\equiv (\bar{3}, 1, -2/3) \times 2 \end{aligned} \quad (3.15)$$

As in Ref. 10), we shall first discuss the model in the absence of the $SU(2)_L \times U(1)_Y$ gauging, when it reduces itself to SQCD with $N = M = 3$. However, unlike what was done there, we shall introduce from the start soft SUSY breakings and determine (rather than guess) the resulting vacuum order parameters. We shall then compute the light spectrum before and after the $SU(2)_L \times U(1)_Y$ gauging.

In the absence of the weak gauging, the effective Lagrangian of this theory can be written as

$$L_{eff} = L_{eff}^{inv.} + L_{eff}^{break} \quad (3.16)$$

where $L_{eff}^{inv.}$ can be taken from Ref. 7) of the form

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In order to make this model anomaly-free, two hypercolour singlet superfields (spectators) transforming as $(1, 2, 1)$ and $(1, 1, -2)$ must be added. This also rids the model of a Witten $SU(2)$ anomaly. We thank N. Seiberg for mentioning this last point to us.

$$L_{\text{eff}}^{\text{inv}} = \frac{1}{\alpha \Lambda^4} (S^* S)_D + \frac{1}{\beta \Lambda^2} (T_{ij}^* T_{ij})_D + \left[S \log \det \left(\frac{T}{\Lambda^2} \right) - m_{ij} T_{ij} \right]_F + \text{h.c.} \quad (3.17)$$

with

$$S = \pi_\lambda + \theta \chi_\lambda + \dots$$

$$T_{ij} \equiv \Phi_\alpha^i \tilde{\Phi}_j^\alpha = \pi_{ij} + \theta \chi_{ij} + \dots \quad (3.18)$$

The sum over repeated indices in (3.17) is understood.

Before turning to $L_{\text{eff}}^{\text{break}}$, two remarks are in order. The first one is that, for the sake of simplicity, we have taken the kinetic D-terms for S and T in (3.17) to be canonical up to a trivial rescaling, rather than scale-invariant. The formulae would be somewhat more complicated with scale-invariant kinetic terms, such as those given in Ref. 7), but the physics would remain the same. The second observation is that, in the case $N = M$, the $U(1)_V$ symmetry is known³⁾ to be spontaneously broken by the condensates

$$\langle \det_{\alpha i} \phi_\alpha^i \rangle, \langle \det_{\beta j} \tilde{\phi}_j^\beta \rangle = O(\Lambda^{2N}). \quad (3.19)$$

This fact requires the introduction of at least two more superfields^{1*)}:

$$X = \det_{\alpha i} \Phi_\alpha^i, \quad \tilde{X} = \det_{\beta j} \tilde{\Phi}_j^\beta \quad (3.20)$$

in $L_{\text{eff}}^{\text{inv}}$. When this is done, one recovers¹⁷⁾, in the chiral SUSY limit $m_{ij} = 0$, enough massless fermions to saturate 't Hooft's anomaly matching conditions and to agree with the counting of massless fermions given in Ref. 10). On the other hand, the superfields X and $\tilde{X}$ are neutral with respect to the $SU(2)_L \times U(1)_Y$ interactions. That is why we shall neglect them in the following discussion.

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A useful discussion with N. Seiberg on this point is acknowledged.

The SUSY mass terms $m_{ij} T_{ij}|_F$ must be compatible with the $SU(2)_L \times U(1)_Y$ gauging. This restricts the non-vanishing entries in m_{ij} to m_{31} and m_{33} . For the moment, we shall take $m_{ij} = 0$. Similar restrictions hold for a SUSY breaking bilinear mass term $\hat{\mu}_{ij} \phi_i \tilde{\phi}_j$ which we shall also neglect for the time being, together with a gaugino mass term. Some effects due to the switching-on of these mass parameters will be discussed later on.

In conclusion, in this first analysis we shall limit ourselves to soft breaking terms given by bosonic masses of the type:^{1*}

$$L_{break} = - \sum_i \left(\mu_i^2 \phi_i^* \phi_i + \tilde{\mu}_i^2 \tilde{\phi}_i^* \tilde{\phi}_i \right) \quad (3.21)$$

Using the symmetry properties of each term in (3.21), one can immediately see that they are reproduced, at the effective Lagrangian level, by:

$$L_{eff}^{break} = - \frac{1}{\Lambda^2} \left(\mu_i^2 + \tilde{\mu}_i^2 \right) \pi_{ij} \pi_{ij}^* \equiv - \frac{1}{\Lambda^2} \bar{\mu}_{ij}^2 \pi_{ij} \pi_{ij}^* ; \mu_i^2, \tilde{\mu}_i^2 > 0 \quad (3.22)$$

with the $SU(2)_L$ invariance forcing the constraint $\mu_1 = \mu_2$. For simplicity, we shall also take $\tilde{\mu}_1 = \tilde{\mu}_2$, so that

$$\bar{\mu}_{11}^2 = \bar{\mu}_{22}^2 = \bar{\mu}_{12}^2 = \bar{\mu}_{21}^2 \equiv \bar{\mu}^2 \quad (3.23)$$

The scalar potential resulting from L_{eff} [Eqs.(3.17), (3.22)] is:

$$V = \alpha \Lambda^4 \left| \log \frac{\det \pi}{\Lambda^6} \right|^2 + \beta \Lambda^2 \pi_\lambda \pi_\lambda^* \sum_{i,j} \left| (\pi^{-1})_{ij} \right|^2 + \frac{1}{\Lambda^2} \bar{\mu}_{ij}^2 \left| \pi_{ij} \right|^2 \quad (3.24)$$

The stationarity conditions on V can be satisfied with

$$\langle \pi_\lambda \rangle = \langle \pi_{ij} \rangle = 0 \quad , \quad i \neq j \quad (3.25)$$

(1*)

One can always arrive at such diagonal bosonic masses through an $SU(M) \times SU(M)$ redefinition of Φ_i and $\tilde{\Phi}_i$.

and

$$\alpha \Lambda^6 L = - \bar{\mu}_{ii}^2 \left| \langle \pi_{ii} \rangle \right|^2, \quad \forall i, \quad i = 1, 2, 3. \quad (3.26)$$

where $L \equiv \log((\langle \pi_{11} \rangle \langle \pi_{22} \rangle \langle \pi_{33} \rangle) / \Lambda^6)$. For small SUSY breaking, i.e., for

$$\bar{\mu}_{ij}^2 / \Lambda^2 \ll 1 \quad (3.27)$$

they yield

$$\begin{aligned} \langle \pi_{11} \rangle &= \langle \pi_{22} \rangle \equiv \langle \pi \rangle = \left(\bar{\mu}_{33} / \bar{\mu} \right)^{1/3} \cdot \Lambda^2, \\ \langle \pi_{33} \rangle &= \left(\bar{\mu} / \bar{\mu}_{33} \right)^{2/3} \cdot \Lambda^2. \end{aligned} \quad (3.28)$$

One can also argue that (3.25) and (3.28) provide the true minimum of V , at least in a large range of SUSY breaking parameters.

We are now in a position to compute the various mass matrices. Let us start from the fermions χ_λ and χ_{ij} . The off-diagonal fermions χ_{ij} ($i \neq j$) are immediately seen to be massless, because of the vanishing of $\langle \pi_\lambda \rangle$. The remaining mass matrix takes the simple form:

$$M_F = \sqrt{\alpha\beta} \Lambda^3 \begin{pmatrix} \chi_\lambda & \chi_{11} & \chi_{22} & \chi_{33} \\ 0 & \pi^{-1} & \pi^{-1} & \pi_{33}^{-1} \\ \pi^{-1} & & & \\ \pi^{-1} & & 0 & \\ \pi_{33}^{-1} & & & \end{pmatrix} \quad (3.29)$$

which implies two massless states:

$$\begin{aligned} \chi_3 &\equiv \chi_{11} - \chi_{22}, \\ \chi_8 &\equiv \langle \pi \rangle (\chi_{11} + \chi_{22}) - 2 \langle \pi_{33} \rangle \chi_{33} \end{aligned} \quad (3.30)$$

and a Dirac particle of mass

$$M_{\chi_{\text{heavy}}} = \sqrt{\alpha\beta} \Lambda^3 \left(2 \langle \pi \rangle^{-2} + \langle \pi_{33} \rangle^{-2} \right)^{1/2}. \quad (3.31)$$

Thus we get eight massless fermions (χ_{ij} , $i \neq j$, χ_3 and χ_8). If we add to them the two fermions in X and $\tilde{X}$ [Eq. (3.20)], we end up with a total of ten massless fermions. This is precisely the number needed to saturate the 't Hooft conditions on the anomaly-free $U(1)_X$ symmetry. In general, for $N = M$, this symmetry does not transform^{7),8)} the scalars ϕ and $\tilde{\phi}$ and is therefore unbroken by the condensates (3.25) and (3.26). One can actually show¹⁰⁾ that the number of massless fermions is N^2+1 in the absence of weak gauging.

We now turn to the bosons. The off-diagonal sectors (i.e., $i \neq j$) give two-by-two mass matrices of the form:

$$\begin{pmatrix} \pi_{ij}^* & \pi_{ji} \end{pmatrix} \begin{pmatrix} \beta \bar{\mu}_{ij}^2 & -\beta \bar{\mu}_{ii} \bar{\mu}_{jj} \\ -\beta \bar{\mu}_{ii} \bar{\mu}_{jj} & \beta \bar{\mu}_{jj}^2 \end{pmatrix} = M_{ij}^2, \quad (3.32)$$

where use has been made of the stationarity condition (3.26) in writing the off-diagonal elements. Hence:

$$\det M_{ij}^2 = \beta^2 \left[\bar{\mu}_{ij}^2 \bar{\mu}_{ji}^2 - \bar{\mu}_{ii}^2 \bar{\mu}_{jj}^2 \right] = \beta^2 \left(\mu_i^2 - \mu_j^2 \right) \left(\tilde{\mu}_i^2 - \tilde{\mu}_j^2 \right). \quad (3.33)$$

We see from (3.33) that, for generic values of μ_i^2 and $\tilde{\mu}_i^2$, we have two massive eigenstates, whilst for $\mu_i = \mu_j$ (or $\tilde{\mu}_i = \tilde{\mu}_j$) we have one massless and one massive eigenstate^{1*}. Because of the $SU(2)_L$ invariance, this latter possibility holds in the 1,2 sector ($\mu_1 = \mu_2$) for which we find the massless eigenstates:

$$\pi_{12} - \pi_{21}^* \equiv \pi^{(+)}; \quad \pi_{21} - \pi_{12}^* \equiv \pi^{(-)}, \quad (3.34)$$

while the orthogonal combinations are massive, with $M^2 = 2\beta\mu_{11}^2$. The 1,3 and 2,3 sectors will be taken to have masses $o(\beta\mu^2)$.

(1*)=====

Eq. (3.33) clearly shows that for certain ranges of the μ^2 and $\tilde{\mu}^2$ parameters, one eigenvalue becomes negative, signalling that the diagonal vacuum (3.28) has to be replaced by an off-diagonal one.

In the diagonal sector ($i = j$) it is more convenient to work with real (scalar) and imaginary (pseudoscalar) components. One then finds that the fields

$$\begin{aligned} \text{Im} (\pi_{11} - \pi_{22}) &\equiv \pi^{(0)}, \\ \text{Im} \left[(\pi_{11} + \pi_{22}) \langle \pi \rangle - 2 \pi_{33} \langle \pi_{33} \rangle \right] &\equiv \pi_8 \end{aligned} \quad (3.35)$$

are massless, while all the remaining pseudoscalars and all the scalars take masses $O(\beta\mu^2)$.

It is easy to see that, indeed, four exact (massless) Goldstone bosons are expected with the soft breaking terms that have been chosen. One can also verify that

$$STr M^2 = 2\beta \sum_{i,j} (\mu_i^2 + \tilde{\mu}_j^2), \quad (3.36)$$

in agreement with the supertrace mass formula which is derived in the Appendix.

Before discussing the effects of gauging $SU(2)_L \times U(1)_Y$, we comment on those resulting from a non-vanishing gluino mass and/or SUSY mass $m_{ij} = \delta_{i3} \delta_{j3} m$. In the presence of m_λ and m , the scalar potential (3.24) is modified into:

$$\begin{aligned} V \rightarrow V' &= \alpha \Lambda^4 \left| b g \frac{\det \pi}{\Lambda^6} \right|^2 + \beta \Lambda^2 \left| \pi_\lambda \pi_{ij}^{-1} - m \delta_{i3} \delta_{j3} \right|^2 \\ &+ \frac{1}{\Lambda^2} (\mu_i^2 + \tilde{\mu}_j^2) |\pi_{ij}|^2 - m_\lambda (\pi_\lambda + \pi_\lambda^*). \end{aligned} \quad (3.37)$$

As long as $\mu_i^2 + \tilde{\mu}_j^2 \gg m^2 + m_\lambda^2$, the values of $\langle \pi_{ij} \rangle$ given in (3.25) and (3.28) are left essentially unchanged, while π_λ develops an expectation value of the form:

$$\Lambda^{-2} \langle \pi_\lambda \rangle = m_\lambda f_1 + m f_2 \quad (3.38)$$

with f_1 and f_2 known functions of ratios of μ_i^2 and $\tilde{\mu}_j^2$. As a result of (3.38), the previously massless fermions pick up a mass of $O(m_\lambda + m)$ from the F-terms in (3.17), in agreement with the fact that the R-symmetry $[U(1)_X]$ protecting them has been explicitly broken.

We finally turn to the effects due to the $SU(2)_L \times U(1)_Y$ gauging, taking $m_\lambda = m = 0$ for simplicity. Considering the $SU(2)_L \times U(1)_Y$ transformations of the

composite superfields S and T_{ij} , it is straightforward to gauge $SU(2)_L \times U(1)_Y$. One simply makes the replacements

$$\begin{aligned} (S^* S)_D &\rightarrow (S^* S)_D \\ (T_{ij}^* T_{ij})_D &\rightarrow (T^+)_{ji} \exp \left[\left(\frac{\vec{t}}{2} \cdot \vec{V}_2 \right)_{ii'} + \left(\frac{Y_i + Y_j}{2} \right) \delta_{ii'} B \right] T_{i'j} \end{aligned} \quad (3.39)$$

where $\vec{V}_2$ and B are the vector superfields containing the $SU(2)_L$ gauge bosons $\vec{V}_\mu$ and the $U(1)_Y$ gauge boson B_μ respectively (they include the coupling constant factors g_2 and g_Y). One further adds kinetic terms for these gauge superfields and, possibly, SUSY breaking masses for their gauginos. The change in the scalar potential (3.24) is easily found to be:

$$\begin{aligned} \delta V|_{\text{gauge}} &= \frac{g_Y^2}{8} \left[-|\pi_{11}|^2 - |\pi_{21}|^2 + 2|\pi_{31}|^2 + |\pi_{12}|^2 + \right. \\ &\quad \left. + |\pi_{22}|^2 - |\pi_{13}|^2 - |\pi_{23}|^2 \right]^2 + \frac{g_Y^2}{8} \left[(|\pi_{1j}|^2 - |\pi_{2j}|^2)^2 + 4|\pi_{1j}|^2 |\pi_{2j}|^2 \right] \end{aligned} \quad (3.40)$$

One can check that, as long as the minimum is flavour diagonal, the new term (3.40) does not affect the position of the minimum. The effects on the spectrum, however, are much less trivial. A technicolour-type Higgs mechanism occurs with the three Goldstones $\pi^{(+)}$, $\pi^{(-)}$ and $\pi^{(0)}$ of Eqs. (3.34) and (3.35) providing the longitudinal components to the massive $W^\pm$ and Z^0 . These have the standard model expressions and masses:

$$\begin{aligned} M(W^\pm)^2 &= \beta g_z^2 \frac{\langle \pi \rangle^2}{\Lambda^2} = \beta g_z^2 \Lambda^2 \left(\frac{\mu_{33}}{\mu} \right)^{2/3}, \\ M(Z^0)^2 &= M(W^\pm)^2 / \cos^2 \theta_W ; \quad \sin^2 \theta_W = \frac{g_Y^2}{g_Y^2 + g_z^2}. \end{aligned} \quad (3.41)$$

There is also, of course, a massless photon, with the electric charge given by $Q = T_{3L} + Y/2$.

The spin zero SUSY partners of $\pi^\pm$, π^0 also get a mass² term of $O(\beta g^2 (\langle \pi \rangle^2 / \Lambda^2)) = O(M(W)^2, M(Z^0)^2)$ on top of their previous masses $O(\beta \mu^2)$. On the other hand, the pseudoscalar π_8 remains massless at this stage.

In the fermionic sector, $\chi_{12} \equiv \chi^{(+)}$, $\chi_{21} \equiv \chi^{(-)}$ and χ_3 combine with the fermionic partners of $W^\pm$ and Z^0 , $\tilde{W}^\pm$ and $\tilde{Z}^0$, to give three Dirac fermions whose tree level masses (if $m_\lambda = m = 0$) are the same as the $W^\pm$ and Z^0 masses. When m_λ and/or m are non-vanishing, there is an extra $\chi^+ \chi^-$ and $\chi^0 \chi^0$ mass term which is induced by $\langle \pi_\lambda \rangle$ and the three Dirac fermions become six Weyl particles with the mass splittings $\Delta M = 0(m_\lambda, m)$.

Of the remaining five composite massless fermions (at $m_\lambda = m = 0$), four represent a family of leptons, including an $SU(2)_L \times U(1)_Y$ singlet, the left-handed antineutrino. The identification would be:

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv \begin{pmatrix} \chi_{13} \\ \chi_{23} \end{pmatrix}_L ; \quad e_L^c \equiv \chi_{32} ; \quad \nu_L^c \equiv \chi_{31} . \quad (3.42)$$

The fifth massless fermion is χ_8 . Together with the fermions in X and $\tilde{X}$ of Eq. (3.20), we have a total of seven massless fermions. This is the correct number to match the $U(1)_X^3$ anomaly, since the four massless weak gauginos now also contribute in the underlying theory, but three of them have acquired a mass from the dynamical Higgs phenomenon.

The extra light states that we have obtained besides the wanted lepton family may turn out to be phenomenologically harmless, since they are either decoupled from the $SU(2)_L \times U(1)_Y$ currents ($X, \tilde{X}$) or coupled through them only to heavy systems. In any case, it seems to us premature to take the model seriously and try to defend it from phenomenological dangers. We plan to come back to these questions within a more complete scheme incorporating also colour (quarks) and, hopefully, families.

Our last point concerns a mechanism for generating small masses for the composite leptons in the limit in which the $U(1)_X$ symmetry is only broken at the weak gauging level (i.e., $m_\lambda = m_{ij} = 0$). A photino mass is capable of generating a radiative mass to the charged lepton from the diagram of Fig. 2. This diagram uses, besides a photino mass, the $SU(2)_L \times U(1)_Y$ breaking $\langle \pi_{23} \pi_{32} \rangle$ propagator which can be obtained from Eq. (3.32). The result of the (convergent) calculation is particularly simple for $m_Y^2 \ll \mu_1^2, \tilde{\mu}_1^2$:

$$m_{e^+} = \frac{e^2}{12\pi^2} m_{\tilde{\gamma}} \frac{\bar{\mu} \tilde{\mu}_{33}}{(m_+^2 - m_-^2)} \log \left(\frac{m_+^2}{m_-^2} \right) ; \quad m_\nu = 0 , \quad (3.43)$$

where $\beta m_{\pm}^2$ are the eigenvalues of the mass matrix M_{ij}^2 of Eq. (3.32) with $i, j=2,3$. They are assumed to be both non-vanishing.

Equation (3.43) shows an interesting way of generating masses for fermions which enjoy the kind of double protection mechanism that was proposed in Ref. 10). Indeed, the m_Y factor signals the SUSY protection, whilst the $\bar{\mu}_{33}$ factor is connected to the $SU(2)_L \times U(1)_Y$ protection [$SU(2)_L$ is unbroken for $\mu_{33} \rightarrow 0$, as can be seen from Eq. (3.28)]. A similar effect would generate quark masses through massive gluino exchange in a model incorporating the $SU(3)_C$ gauging.

In conclusion, our results can be viewed as an explicit dynamical realization of the mechanisms envisaged in Ref. 10) for generating a hierarchy of light mass scales. The crucial ingredients needed to achieve this goal have been the non-perturbative effects leading to the breakdown of the non-renormalization theorems and the inclusion of soft supersymmetry breaking terms in order to determine the vacuum expectation values.

We hope that the results we have shown will provide useful hints as to which directions should be taken in the challenging search for a model of composite quarks and leptons.

NOTE ADDED

We have already checked that in the extension of the $N = M = 3$ case to the more realistic $N = M = 6$ case, where also quarks are present [cf. Ref. 10)], the anomalies which had forced us to introduce spectator fields are automatically absent.

APPENDIX

A supertrace mass formula

We shall derive here a general mass formula holding for any effective Lagrangian of the type:

$$L = d(S_i, S_i^*)|_D + f(S_i)|_F + h.c. + \varepsilon g(\phi_i, \phi_i^*), \quad (A.1)$$

where S_i are chiral superfields, $\phi_i(\phi_i^*)$ is the $\theta = 0$ component of $S_i(S_i^*)$ and εg denotes SUSY breaking terms which are characterized by a small parameter ε . Using the following formulae¹⁸⁾ for the spin zero and spin $\frac{1}{2}$ mass matrices:

$$M_{\phi_i \phi_j}^2 = J_{ie}^{-1} V_{ej+} = J_{ie}^{-1} \left[(J_{nm}^{-1})_{ej+} f'_n f'_m{}^* + \right. \\ \left. + (J_{nm}^{-1})_e f'_n f''_{mj}{}^* + (J_{nm}^{-1})_{j+} f''_{ne} f'_m{}^* + (J_{nm}^{-1}) f''_{ne} f''_{mj}{}^* \right] \quad (A.2)$$

$$M_{\psi_i \psi_j} = J_{ie}^{-1} (M'_F)_{ej} = J_{ie}^{-1} \left[f''_{ej} - J_{hk}^{-1} f'_h \frac{\partial^3 d}{\partial \phi_k^* \partial \phi_e \partial \phi_j} \right] = \\ = J_{ie}^{-1} \left[f''_{ej} - J_{hk}^{-1} f'_h (J_{ke})_j \right], \quad (A.3)$$

where

$$J_{nm} = \frac{\partial^2 d}{\partial \phi_n \partial \phi_m^*}; \quad (J_{nm})_{j+} = \frac{\partial}{\partial \phi_j^*} J_{nm}; \\ f'_i = \frac{\partial f}{\partial \phi_i}, \quad f''_i = \frac{\partial f^*}{\partial \phi_i^*}. \quad (A.4)$$

(sum over repeated indices is understood),

===== (1*)

In the presence of SUSY breaking, there are also non-trivial mass matrices of the $M_{\phi_i \phi_j}^2$ and $M_{\phi_i^* \phi_j^*}^2$ type. While relevant for the spectrum, such matrices do not contribute to the supertrace.

one finds:

$$\begin{aligned} \text{STr } |M|^2 &= \sum_J (-)^{2J} (2J+1) T_L M_J^2 = \\ &= \epsilon J_{il}^{-1} \frac{\partial^2 g}{\partial \phi_l^* \partial \phi_i} + J_{il}^{-1} f'_n f'_m T_{il, nm} , \end{aligned} \quad (\text{A.5})$$

where

$$T_{il, nm} = (J_{nm}^{-1})_{li} - J_{nu}^{-1} (J_{uv})_{it} J_{vz}^{-1} (J_{zt})_{el} J_{tm}^{-1} \quad (\text{A.6})$$

In our case, J is diagonal, i.e., $J_{nm} = \delta_{nm} J_{nn}$ and so one has

$$T_{ii, ii} = (J_{ii}^{-3}) | (J_{ii})_i |^2 - J_{ii}^{-2} (J_{ii})_{ii} , \quad (\text{A.7})$$

while all the other entries are zero. In particular, if $d(S_i, S_i^*) = \sum_1 (S_i S_i^*)^P_i$, as we took in the examples, one gets

$$T_{ii, ii} = 0 ; \quad \text{STr } M^2 = \epsilon J_{il}^{-1} \frac{\partial^2 g}{\partial \phi_l^* \partial \phi_i} . \quad (\text{A.8})$$

In any case, even if the additional term in Eq. (A.5) is present, it is of higher order in ϵ , since, from the minimization conditions, one finds

$$f' = O(\epsilon) . \quad (\text{A.9})$$

Notice that $\text{STr } M^2$ does not get contributions from SUSY breaking terms which are linear in the fields, such as gaugino mass terms, for instance.

The final point concerns the validity of the supertrace formula for the light sector alone at $O(\varepsilon)$:

$$|M_F|_{ij}^2 = (M_B^2)_{ij} + \varepsilon g_{ij}, \quad (A.10)$$

where now we went over to the basis in which kinetic terms are canonical. Diagonalizing the SUSY mass matrices $|M_F|^2$ and M_B^2 , we find that to $O(\varepsilon)$ the shift of each eigenvalue is given by:

$$(M_B^2)_n - |M_F|_n^2 = \varepsilon g_{nn} \quad (A.11)$$

If we take the trace of this relation over the light sector, we get:

$$\text{STr} |M|^2 \Big|_{\text{light}} = \varepsilon \sum_{n=\text{light}} g_{nn} \quad (A.12)$$

In the case of SQCD with $N = 2$, $M = 1$ (Section 3.2), the soft breaking terms are all in the direction of the light eigenstates, so that one obtains $\text{STr} M^2 \Big|_{\text{all}} = \text{STr} M^2 \Big|_{\text{light}}$, explaining the validity of Eq. (3.14).

In the case of $N = M = 3$, part of the SUSY breaking lies in the direction of the component of π_{ij} which, together with π_λ , acquires a heavy mass. In this case, the $\text{STr} M^2 \Big|_{\text{light}}$ is slightly more complicated than the right-hand side of Eq. (3.36).

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FIGURE CAPTIONS

Fig. 1 : Vacuum energy as a function of θ_v for SU(3), SYM theory. Solid lines denote the true ground state at each θ_v while dashed lines represent the other two false vacua. The value of K for the true vacuum changes at $\theta = \pi(\text{mod } 2\pi)$.

Fig. 2 : Diagram inducing a radiative mass to the charged lepton.

Fig. 1

Fig. 2