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MULTIPLE DIFFRACTION THEORY OF $p\bar{p}$ SCATTERING AT 546 GeV

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A B S T R A C T

The proton and antiproton are pictured as clusters of partons and diffractive multiple scattering theory is used to describe their elastic collisions. The theory developed is related to the geometrical model of Chou and Yang and generalizes it in several ways. The calculated cross-sections show good agreement with the experimental ones when the average parton-parton scattering amplitude is given a suitable momentum-transfer dependence. The proton form factors used in the analysis are derived directly from experiment and explain the slope changes seen in the $p\bar{p}$ differential cross-section near the forward direction.

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It has been clear for some years that hadrons have a composite structure. The evident impossibility of separating their constituents as such, however, has prevented us from learning much about the forces which act between those constituents. We ask therefore whether it is not possible to learn something of the behaviour of those elementary interactions, at least at small momentum transfers through careful analysis of hadron-hadron elastic scattering? We propose to investigate that possibility by using a diffractive form of multiple scattering theory to discuss proton-antiproton scattering at a centre-of-mass energy of 546 GeV. The theory¹⁾ we shall apply is one which has had many successful applications to the collisions of hadrons with nuclei.

We assume that the proton and the antiproton each consist of a cluster of particles which we shall call partons (although these are not necessarily the partons seen by the electroweak interactions). As these clusters pass through each other, the partons interact in pairs i, j , scattering one another with invariant scattering amplitudes $f_{ij}(t)$. We expect these amplitudes to be predominantly imaginary, or absorptive in character, a property which represents schematically the effect of the grossly inelastic processes such as quark annihilation or gluon emission, which change the parton population. The scattering described by the amplitudes f_{ij} will be diffractive in character and much of it shadow scattering. To find the elastic scattering of two such clusters, we require the amplitude that both clusters remain intact and unexcited following the collision.

We write the invariant amplitude for $p\bar{p}$ scattering as an impact parameter integral

$$F(t) = i \int J_0(b\sqrt{-t}) \left\{ 1 - e^{-\Omega(b)} \right\} b db \quad (1)$$

in which $-t$ is the squared momentum transfer in the centre-of-mass system, and the integration is carried out over the impact plane (perpendicular to the incident momentum). The function $\Omega(b)$ represents principally an opacity effective for clusters passing with a relative impact parameter b , but we shall take it to be complex and thereby include refractive as well as absorptive effects.

Let us consider two colliding clusters containing different numbers of partons, say A in one and B in the other, and assume that all of the partons in both clusters are distributed independently with the same characteristic densities, $\rho_a(\vec{r})$ in one cluster and $\rho_b(\vec{r})$ in the other ($\int \rho_a(\vec{r}) d\vec{r} = \int \rho_b(\vec{r}) d\vec{r} = 1$).

Then, according to the diffractive multiple scattering theory, the complex opacity is given approximately by

$$\Omega(\vec{b}) = AB \int \rho_a(\vec{r}) \langle \Gamma_{ij}(\vec{b} + \vec{s} - \vec{s}') \rangle \rho_b(\vec{r}') d\vec{r} d\vec{r}' \quad (2)$$

in which $\vec{s}$ and $\vec{s}'$ are the components of $\vec{r}$ and $\vec{r}'$ in the impact plane, Γ_{ij} is the profile function characteristic of the parton pair ij ,

$$\Gamma_{ij}(\vec{b}) = \frac{1}{2\pi i} \int e^{-i\vec{q} \cdot \vec{b}} f_{ij}(t) d^2q \quad (3)$$

and in the latter integrand $t = -q^2$. The average $\langle \rangle$ indicated in the expression (2) is to be taken over all partons i in cluster a and all partons j in cluster b . There is no requirement that the partons all be the same as long as they have similar and uncorrelated spatial distributions in each cluster.

If we let $f(t)$ be the parton-parton scattering amplitude, averaged in this sense, and substitute the expression (3) for the profile function into Eq. (2) we find

$$\Omega(\vec{b}) = \frac{AB}{2\pi i} \int e^{-i\vec{q} \cdot \vec{b}} G_a(t) G_b(t) f(t) d^2q \quad (4)$$

in which G_a and G_b are the form factors of the densities ρ_a and ρ_b respectively.

The complex opacity given by Eqs. (2) and (4) is the leading term of an expansion which assumes that simple binary collisions involving different partons play a rôle which dominates that of collisions in which individual partons collide multiply. That will be the case as long as neither cluster is too opaque to an average parton in the other (although it may be quite opaque to the other cluster taken as a whole). We shall make this assumption for the present, in the interest of simplicity. Correction terms can be added to the opacity should the parameters eventually warrant that.

For the case of $p\bar{p}$ scattering charge symmetry requires $A = B$ and $G_a = G_b = G_p$, where G_p is the form factor for the parton density of the proton. It is convenient in that case to define

$$K(i+\alpha) = A^2 f(0) \quad (5)$$

and then to rewrite Eq. (4) as

$$\Omega(b) = K(1-i\alpha) \int_0^\infty J_0(qb) G_p^2(t) \frac{f(t)}{f(0)} q dq \quad (6)$$

An opacity function having several of the features of Eq. (6) has been used as the basis of the geometrical model by Chou and Yang²⁾. Their model is a particularly simple one which has made several successful quantitative predictions and seems to embody a good deal of semi-quantitative wisdom as well. It corresponds in the present context to assuming in effect that parton-parton scattering is isotropic so that $f(t)/f(0)$ is set equal to one and eliminated from Eq. (5). It assumes furthermore that the interactions are purely absorptive, so that $\alpha = 0$. The zeros in the differential cross-section introduced by the latter assumption have been a mixed blessing. One of them has correctly pointed to a dip in the observed cross-section; no experimental trace has been found of another.

It is worth noting that the de Broglie wavelength for the $p\bar{p}$ system at 546 GeV in the centre-of-mass system is $\lambda/2\pi = 3.6 \times 10^{-17}$ cm, a remarkably small distance in relation to the size of the proton. While the partons have smaller momenta than the hadrons, the wavelength for most parton pairs is only likely to be a modest multiple of the $p\bar{p}$ wavelength noted. We would therefore expect parton-parton scattering to be strongly forward-peaked, in the usual diffraction-theoretical sense, unless the interaction range is smaller, say than 10^{-3} times the radius of the proton. We prefer for these reasons to rely on the more general form of multiple scattering theory, while acknowledging that it contains a significantly greater number of quantities which remain to be distributed.

A feature the multiple scattering model shares with the geometrical model is that the size and shape of the hadrons as expressed by the form factors $G(t)$ play an essential fixed rôle in determining the scattering amplitudes. Changes which take place with increasing energy such as increasing and redistributed opacity are ascribed to energy dependence of the averaged amplitude $f(s,t)$.

The differential cross-sections which are derived from the model are inevitably quite sensitive in shape and magnitude to the precise representation used for the form factor $G(t)$. We shall assume that $G_p(t)$ can be identified with the electric form factor of the proton $G_E(t)$, as measured in electron scattering experiments. Moreover, rather than use a simplified but inaccurate parametrization for G_E when the results depend so sensitively upon it, we shall make direct use of two parametrizations which seem best to summarize the

available electron scattering results. One is a parametrization suggested by Felst³⁾

$$G_e(t) = (1 - 3.04 t + 1.54 t^2 - .068 t^3)^{-1} \quad (7)$$

with t in $(\text{GeV}/c)^2$, which is based on data available from all save the lower energy accelerators in 1973. The second is a more complicated four-pole parametrization due to Borkowski, Simon, Walther and Wendling (BSWW)⁴⁾, which pays more specific attention to the small momentum-transfer data accumulated at the lower energy accelerators.

Comparing the theory with experiment at the present stage is a matter of demonstrating that Eqs. (1) and (6) can fit the observed data and then seeing what that fit can tell us about the function $A^2 f(t)$. We have done that by computing for several simple parametrizations of this function the total cross-section $\sigma_t = 4\pi \text{Im } F(0)$ and the differential cross-section $d\sigma_{el}/d|t| = \pi |F(t)|^2$. The constant K in Eq. (6) tends, within narrow limits, to be determined by the total cross-section and α by the shape of the differential cross-section near the diffraction minimum. However, with those constants optimized and $f(t)/f(0)$ still taken equal to one, no accurate fit of the theory to the data⁵⁾ of the CERN UA4 experiment can be found.

To proceed further requires that we make explicit guesses about the form of the function $f(t)/f(0)$. It is clear that we can only learn by that means about the behaviour of the function within a limited range of t . That is essentially the range within which the squared form factor $G_p^2(t)$ which multiplies $f(t)/f(0)$ in Eq. (6) has not decreased too appreciably. We cannot therefore expect to learn about the behaviour of $f(t)/f(0)$ at large momentum transfers. The analysis of elastic scattering, in other words, only permits us to examine the behaviour of $f(t)/f(0)$ within a cone defined by the proton form factor. But by the same token we need only represent the function parametrically over the same limited range. Many different parametric representations could no doubt serve that purpose. Of several single-parameter representations which we have chosen to try somewhat arbitrarily, the best fit was given by $f(t)/f(0) = (1 - at)^{-\frac{1}{2}}$. The differential cross-section which results from that assumption, with the parameters $K = 83 \text{ mb}$, $\alpha = 0.18$, $a = 2.0 (\text{GeV}/c)^{-2}$ and the use of the Felst form factor, Eq. (7), is shown together with the experimental data in Fig. 1. The theory can be seen to follow fairly well the general trend of the data [which are still incomplete and preliminary for $-t > 0.5 (\text{GeV}/c)^2$]. The total cross-section for this case has been adjusted to the experimental⁵⁾ value of 61.9 mb. The ratio of the elastic and total cross-sections is then found to be 0.203 where the experimental ratio

is 0.215. The ratio of the real to the imaginary part of the forward scattering amplitude is calculated to be $\rho = 0.133$.

Reassuringly equivalent results, with some improvements of the fit at small momentum transfers were obtained by using the BSWW form factor in a similar way. This differential cross-section, shown in Fig. 2, was based on the parameters $K = 81.3$ mb, $\alpha = 0.190$ and $a = 1.80$ (GeV/c)⁻². The parameters calculated from these are then $\sigma_t = 61.9$ mb, $\sigma_{el}/\sigma_t = 0.193$ and $\rho = 0.143$.

Another parametrization of the ratio $f(t)/f(0)$ which does nearly as well is $(1-a't)^{-1}$ with $a' \sim \frac{1}{2}a$. The function $f(t)/f(0)$, of course, should be expected to contain a t -dependent complex phase factor as well, but we have found that for plausible phases its influence would only be felt appreciably at the larger momentum transfers where the present rough and preliminary character of the experimental data would offer little means of discriminating between different choices. We have therefore omitted the variable phase factor entirely. A similar simplification has been made in many nuclear applications of the theory.

The most accurate measurements of scattering are naturally made where the intensity is highest, near the forward direction. In this region, the UA4 collaboration has observed a rapid change in the logarithmic slope of the differential cross-section⁶⁾ which has been the subject of considerable interest. We find that both the Felst and BSWW form factors contain changes of slope in this region which tend to be magnified in the theoretical scattered intensities. Plots of the logarithmic slopes versus $-t$ are given in Fig. 3 for the differential cross-sections shown in Figs. 1 and 2. The UA4 group reports, for example, a slope averaged from $-t = 0.03$ to 0.10 (GeV/c)² of $b = 15.3 \pm 0.3$ (GeV/c)⁻², and averaged from $-t = 0.21$ to 0.50 (GeV/c)² a slope of 13.4 ± 0.3 (GeV/c)⁻². Both of these measurements, as can be seen from Fig. 3, are quite consistent with the slopes furnished by the BSWW form factor, while the slope furnished by the Felst form factor remains a bit high in the second interval. The change of slope, in other words, is better represented by the BSWW form factor, which embodies more experimental data in this low momentum transfer region, and is presumably more accurate there. If that is so, then to the degree that the present theory is reliable, the changing slopes would appear to have a natural explanation in the detailed shape of the proton charge distribution.

The most direct interpretation of the large anisotropy we find in the ratio $f(t)/f(0)$ is that it is diffractive in nature and indicates a parton-parton interaction range considerably larger than the relevant de Broglie wavelength. If that is the case then the mean squared interaction radius may be estimated,

though perhaps only roughly, by $\langle r^2 \rangle = 3a$. With the value $a = 1.9 \text{ (GeV/c)}^{-2}$ typical of the calculations noted earlier we find a root mean square radius $\langle r^2 \rangle^{\frac{1}{2}} = 0.47 \text{ fm}$. We believe this range to be an energy-dependent parameter and find indications of significantly smaller values in the studies we have undertaken of proton-proton scattering at ISR energies.

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FIGURE CAPTIONS

- Fig. 1 Calculated differential cross-section based on Felst form factor, Eq. (7), shown together with UA4 results.
- Fig. 2 Calculated differential cross-section based on BSWW form factor, shown together with UA4 results.
- Fig. 3 Variation of logarithmic slope of the differential cross-section with $-t$ for the Felst and BSWW form factors.

Fig. 1

Fig. 2

Fig. 3