THE UNRUH EFFECT IN EXTENDED THERMOMETERS

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A B S T R A C T

The Unruh effect (heating by acceleration in vacuum) is discussed for the case of extended thermometers. An ambiguity in the temperature, corresponding to the well-known ambiguity in temperature of an extended body in thermal equilibrium in a gravitational field, is noted. The generality of the effect is related, following Sewell, to PCT symmetry. Application is made to the case of a spin-half particle in a confining and accelerating electromagnetic field.

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1. INTRODUCTION

The Unruh effect [1] is the registration of a nonzero temperature in vacuum by a uniformly accelerated thermometer. For acceleration a the Unruh temperature is

$$T_U = a \hbar / (2\pi c k)$$

where $2\pi\hbar$ is Planck's constant, k is Boltzmann's constant, and c the velocity of light. In what follows we take $\hbar = c = 1$, so that

$$\beta_U = (kT_U)^{-1} = 2\pi/a \quad (1)$$

The effect arises in relativistic quantum field theory. It is related to Hawking radiation from black holes [2]. For reviews see [3] and [4].

Two of us [5] have considered the theoretical possibility of using the spin of an electron in a magnetic field as a thermometer in this connection. We found that a serious treatment would involve taking account of the fluctuation of the electron motion about its mean and so dealing with an extended quantum system. In this paper we treat the Unruh effect in such extended thermometers in a general way. Other authors [1, 6] have considered particular model thermometers with spatial extension, but did not follow up the consequences of such extension in the case of uniform linear acceleration. They did not notice therefore an ambiguity in the temperature which comes from the extension.

Our general argument will be from aspects of relativistic quantum field theory related to PCT symmetry. In this we are inspired by the work of Sewell [7]. Moreover, we must refer to that work for mathematical rigour, to which the present paper makes no pretention. Note, by the way, that one of us [8] has traced out explicitly the role of PCT in the case of free fields.

At the end of the paper we return briefly to the particular problem of an electron in an accelerating and confining electromagnetic field.

2. KMS CONDITION

We begin by recalling the Kubo-Martin-Schwinger characterization of a state of temperature T [9, 10]. With Hamiltonian H the density matrix is

$$\left. \begin{aligned} \rho &= Z^{-1} e^{-H/(kT)} = Z^{-1} e^{-\beta H} \\ Z &= \text{Tr} e^{-H/(kT)} \end{aligned} \right\} \quad (2)$$

The expectation value of a product of Heisenberg operators $A(t)$ and $B(t')$ is

$$\langle A(t) B(t') \rangle = \text{Tr} (A(t) B(t') \rho) \quad (3)$$

Using

$$A(t-i\beta) = e^{\beta H} A(t) e^{-\beta H} \quad (4)$$

and the cyclic invariance of the trace we come to the KMS condition

$$\langle A(t-i\beta) B(t') \rangle = \langle B(t') A(t) \rangle \quad (5)$$

or alternatively

$$\langle A(t-i\beta/2) B(t'+i\beta/2) \rangle = \langle B(t') A(t) \rangle \quad (6)$$

Conversely, if the KMS condition, (5) or (6), holds for all A and B , then the density matrix is (1).

Let now a second system (the 'thermometer') be weakly coupled to the first (the 'system'). Let the interaction Hamiltonian in the interaction representation be

$$\sum_m a_m(t) A_m(t) \quad (7)$$

where operators A_m belong to the system and operators a_m to the thermometer. For initial and final states i and f of the thermometer and I and F of the system the time-integrated transition probability is in lowest order perturbation theory, denoting thermometer energy eigenvalues by E ,

$$\left| \int dt e^{i(E_f - E_i)t} \sum_m \langle f | a_m(0) | i \rangle \langle F | A_m(t) | I \rangle \right|^2 \quad (8)$$

$$= \left. \iint dt dt' e^{i(E_f - E_i)(t - t')} \sum_{mn} \langle I | A_n^\dagger(t') | F \rangle \langle F | A_m(t) | I \rangle \langle i | a_n^\dagger(0) | f \rangle \langle f | a_m(0) | i \rangle \right\} \quad (9)$$

After the disappearance of transients we have then a transition rate per unit time from i to f , summing over final states F and with thermal averaging over initial states I ,

$$\left. T_{fi} = \int_{-\infty}^{\infty} dt e^{i(E_f - E_i)t} \sum_{mn} \langle A_n^\dagger(0) A_m(t) \rangle \langle i | a_n^\dagger(0) | f \rangle \langle f | a_m(0) | i \rangle \right\} \quad (10)$$

where we have taken

$$t' = 0$$

Because of the Hermiticity condition

$$\sum a_m A_m = \sum A_m^\dagger a_m^\dagger \quad (11)$$

we could also have obtained (10) with a and A replaced by their Hermitian conjugates. If we at the same time interchange i and f , and the summation indices m and n , and change the sign of the integration variable t , we have

$$T_{if} = \left. \begin{aligned} & \int_{-\infty}^{\infty} dt e^{i(E_f - E_i)t} \\ & \sum_{mn} \langle A_m(t) A_n^\dagger(0) \rangle \langle i | a_n^\dagger(0) | f \rangle \langle f | a_m(0) | i \rangle \end{aligned} \right\} \quad (12)$$

In this last expression make the change of variable

$$t \rightarrow t - i\beta$$

and deform the integration contour back to real t . Then

$$T_{if} = \left. \begin{aligned} & \int_{-\infty}^{\infty} dt e^{i(E_f - E_i)(t - i\beta)} \\ & \sum_{mn} \langle A_m(t - i\beta) A_n^\dagger(0) \rangle \langle i | a_n^\dagger(0) | f \rangle \langle f | a_m(0) | i \rangle \end{aligned} \right\}$$

Finally using the KMS condition (5) in the form

$$\langle A_m(t - i\beta) A_n^\dagger(0) \rangle = \langle A_n^\dagger(0) A_m(t) \rangle$$

and looking again at (10) we see that

$$T_{if} = e^{\beta(E_f - E_i)} T_{fi} \quad (13)$$

This means that (in this weak coupling approximation) an incoherent mixture of the energy eigenstates of the thermometer will be in equilibrium with the system when the occupation probabilities ρ_i and ρ_f of any two states are such that

$$\rho_f / \rho_i = e^{-\beta E_f} / e^{-\beta E_i} \quad (14)$$

i.e. when the thermometer also is at temperature $(k\beta)^{-1}$.

We will see that a relation like (13) again holds, with the Unruh β , (1), for a thermometer uniformly accelerated in vacuum.

3. UNIFORM ACCELERATION

Consider a family of inertial frames $F(\tau)$, where τ runs between $\pm\infty$. Let neighbouring members be obtained from one another by a combination of infinitesimal translation in time and infinitesimal boost in the z direction. Then the coordinates attached to a given event in two neighbouring frames are related by

$$\left. \begin{aligned} X_{\tau+\delta\tau} &= X_{\tau} \\ Y_{\tau+\delta\tau} &= Y_{\tau} \\ Z_{\tau+\delta\tau} &= Z_{\tau} - (a\delta\tau) t_{\tau} \\ t_{\tau+\delta\tau} &= t_{\tau} - (a\delta\tau) z_{\tau} - \delta\tau \end{aligned} \right\} \quad (15)$$

where a is the acceleration parameter specifying the rate of boosting. Since x and y enter trivially here and subsequently, we will usually not mention them again explicitly. Instead of the other two coordinates the combinations

$$u = t + z + a^{-1} \quad v = t - z - a^{-1} \quad (16)$$

are convenient. Then (15) becomes

$$\begin{aligned} \delta u_{\tau} &= - (a\delta\tau) u_{\tau} \\ \delta v_{\tau} &= + (a\delta\tau) v_{\tau} \end{aligned} \quad (17)$$

which integrate to

$$\left. \begin{aligned} u_{\tau} &= u_0 e^{-a\tau} \\ v_{\tau} &= v_0 e^{+a\tau} \end{aligned} \right\} \quad (18)$$

or

$$\left. \begin{aligned} u_0 &= u_{\tau} e^{a\tau} \\ v_0 &= v_{\tau} e^{-a\tau} \end{aligned} \right\} \quad (19)$$

Now the whole family of reference frames $F(\tau)$ give a highly redundant parametrization of space time. To reduce this redundancy we specify that each such frame be used only for the space-like surface

$$t_{\tau} = \frac{1}{2} (u_{\tau} + v_{\tau}) = 0 \quad (20)$$

With this restriction we define ξ by

$$z_{\tau} + a^{-1} = u_{\tau} = -v_{\tau} = \xi + a^{-1} \quad (21)$$

From (19), dropping the suffix 0 on quantities referred to the standard inertial frame $F(0)$,

$$\left. \begin{aligned} u &= (\xi + a^{-1}) e^{a\tau} \\ v &= -(\xi + a^{-1}) e^{-a\tau} \end{aligned} \right\} \quad (22)$$

or

$$\left. \begin{aligned} z + a^{-1} &= (\xi + a^{-1}) \cosh a\tau \\ t &= (\xi + a^{-1}) \sinh a\tau \end{aligned} \right\} \quad (23)$$

The pair $(\xi + a^{-1}, \tau)$ are usually called 'Rindler coordinates' [11, 12]. For real values they do not cover the whole of space time, but do parametrize uniquely the 'right Rindler wedge'

$$z + a^{-1} > |t| \quad (24)$$

and the 'left Rindler wedge'

$$z + a^{-1} < |t| \quad (25)$$

We have recalled the introduction of the Rindler coordinates at some length to stress that they can be regarded as describing events in a succession of inertial frames, so that in what follows we need ordinary quantum field theory referred to inertial frames only.

Let $\Psi(\tau)$ be the Heisenberg-picture state vector of a quantum system in reference frame $F(\tau)$. Then $\Psi(\tau + \delta\tau)$ is given by the appropriate combination of time translation operator (Hamiltonian) H_τ and boost operator B_τ :

$$\Psi(\tau + \delta\tau) = -i\delta\tau (H_\tau - a B_\tau) \Psi(\tau) \quad (26)$$

When $a = 0$, $\Psi(\tau)$ is just the ordinary Schrödinger-picture state vector, and when $a \neq 0$ we have a generalization of that picture which is well adapted to uniformly accelerated systems.

To accelerate the 'thermometer' we suppose appropriate external fields to be applied. To produce uniform acceleration these must look the same in all frames $F(\tau)$ at the relevant instants $t_\tau = 0$. Then

$$K = H_\tau - a B_\tau \quad (27)$$

will be independent of τ . This is the appropriate generalization of ordinary translation invariance in time. Let

$$K = W + U + V \quad (28)$$

where W belongs to the thermometer, U to the quantum field in which it is immersed, and V is the interaction term. In the absence of V , the eigenstates of W ,

$$W \Phi_n = W_n \Phi_n \quad (29)$$

would be stationary. We are interested in the transitions between these states induced by V . Let V be a sum of products of operators a belonging to the thermometer and A belonging to the field. In the interaction representation

$$V(t) = \int d\xi \sum_m a_m(\xi, \tau) A_m(\xi, \tau) \quad (30)$$

with

$$\left. \begin{aligned} a_m(\xi, \tau) &= e^{iW\tau} a_m(\xi, 0) e^{-iW\tau} \\ A_m(\xi, \tau) &= e^{iU\tau} A_m(\xi, 0) e^{-iU\tau} \end{aligned} \right\} \quad (31)$$

Then in analogy with (10)

$$T_{fi} = \int_{-\infty}^{\infty} d\tau e^{i(W_f - W_i)\tau} \iint d\xi d\eta \sum_{mn} \langle A_n^\dagger(\eta, 0) A_m(\xi, \tau) \rangle \langle i | a_n^\dagger(\eta, 0) | f \rangle \langle f | a_m(\xi, 0) | i \rangle \quad (32)$$

where $\langle \rangle_0$ denotes an expectation value in the ordinary (Minkowski) vacuum of the quantum field, which we suppose to be the initial state of the field. Likewise in analogy with (12)

$$T_{if} = \int_{-\infty}^{\infty} d\tau e^{i(W_f - W_i)\tau} \iint d\xi d\eta \sum_{mn} \langle A_m(\xi, \tau) A_n^\dagger(\eta, 0) \rangle \langle i | a_n^\dagger(\eta, 0) | f \rangle \langle f | a_m(\xi, 0) | i \rangle \quad (33)$$

We would come then to the analogue of (13)

$$T_{if} = e^{(W_f - W_i)\beta} T_{fi} \quad (34)$$

and to the conclusion that there is an equilibrium thermometer temperature $1/\beta$, if we had for some β the analogue of the KMS condition (6)

$$\langle A(\xi, \tau - i\beta) B(\xi', 0) \rangle_0 = \langle B(\xi', 0) A(\xi, \tau) \rangle_0. \quad (35)$$

So this we now consider.

4. ACCELERATION KMS CONDITION

The operators A and B in (35) are, by construction (31), just Lorentz transforms of ordinary Heisenberg field operators -- for the quantum field without the thermometer. In general Lorentz transformation of operators not only changes space-time arguments, but also mixes tensor and spinor indices. However we can choose the decomposition (30) to be in terms of components A_n which do not mix under boosts in the z direction (e.g. $E_x \pm iB_y$ rather than E_x and B_y). Then

$$A(\xi, \tau) = e^{n(A)a\tau} A_L(u, v) \quad (36)$$

where A_L is a Heisenberg operator in a fixed laboratory frame, and from (22)

$$\left. \begin{aligned} u &= t + z + a^{-1} = (a^{-1} + \xi) e^{a\tau} \\ v &= t - z - a^{-1} = -(a^{-1} + \xi) e^{-a\tau} \end{aligned} \right\} \quad (37)$$

and where $n(A)$ is the integer (or half-integer) for the Bosonic (or Fermionic) operator which characterizes its behaviour under boosting.

Taking units such that $a = 1$, the left-hand side of (35) is then

$$\langle A(\xi, \tau - i\beta) B(\xi', 0) \rangle_0 = e^{-i\beta n(A)} \langle A_L(u e^{-i\beta}, v e^{i\beta}) B_L(u', v') \rangle_0. \quad (38)$$

Considering this as a function of $(\tau - i\beta)$ we argue that an analytic continuation from the real axis into the lower half plane exists, and at a certain point equals the RHS of (35).

In terms of a complete set of intermediate states $|n\rangle$ we have for real arguments the representation

$$\left. \begin{aligned} \langle A_L(u, v) B_L(u', v') \rangle_0 = \\ \iint dE dP \sigma(E, P) \exp \left[-iE(u+v-u'-v')/2 + iP(u-v-u'+v')/2 \right] \end{aligned} \right\} \quad (39)$$

where

$$\sigma(E, P) = \sum_n \langle 0 | A_L(0, 0) | n \rangle \langle n | B_L(0, 0) | 0 \rangle \delta(E_n - E) \delta(P_n - P) \quad (40)$$

The substitutions

$$u \rightarrow u e^{-i\beta} \quad v \rightarrow v e^{i\beta} \quad (41)$$

change the modulus of the exponential factor in (39) from unity to

$$\exp \left[-\frac{1}{2} (E-P) u \sin \beta + \frac{1}{2} (E+P) v \sin \beta \right] \quad (42)$$

Remembering that

$$E - P \geq 0 \quad E + P \geq 0 \quad (43)$$

and that for a point in the right Rindler wedge

$$u > 0 \quad v < 0$$

we expect (39) to converge for

$$\sin \beta \geq 0 \quad (44)$$

Moreover on the boundary of this region, at $\beta = \pi$, we recognize in (39) the representation of

$$\langle A_L(-u, -v) B_L(u', v') \rangle_0 \quad (45)$$

And so we have for real τ

$$\left. \begin{aligned} \langle A(\xi, \tau - i\pi) B(\xi', 0) \rangle_0 &= \\ e^{-i\pi n(A)} \langle A_L(-u, -v) B_L(u', v') \rangle_0 & \end{aligned} \right\} \quad (46)$$

Now the points (u, v) and (u', v') being in the same Rindler wedge, $(-u, -v)$ and (u', v') are in opposite Rindler wedges -- and so have space-like separation. The corresponding operators then commute (in the Bosonic case) or anticommute (in the Fermionic case). Eq. (46) can be replaced by

$$\left. \begin{aligned} \langle A(\xi, \tau - i\pi) B(\xi', 0) \rangle_0 &= \\ e^{+i\pi n(A)} \langle B_L(u', v') A_L(-u, -v) \rangle_0 & \end{aligned} \right\} \quad (47)$$

This is true in the first instance for real ξ, τ, u, v . But the expression on the right can now be continued as before, so that for $\alpha \leq \pi$ we can define

$$\left. \begin{aligned} \langle A(\xi, \tau - i\pi - i\alpha) B(\xi', 0) \rangle_0 &= \\ e^{(i\pi - i\alpha)n(A)} \langle B_L(u', v') A_L(-u e^{-i\alpha}, -v e^{i\alpha}) \rangle_0 & \end{aligned} \right\} \quad (48)$$

On the boundary, $\alpha = \pi$, using, from (36) at $\tau = 0$,

$$B(\xi', 0) = B_L(u', v')$$

we come to (35).

We are well aware that our assertions about the existence of spectral representations, and their convergence in the absence of the obvious threat, are

naïve and devoid of rigour. We expect that they would indeed hold in a regularized theory, and that constructive rigour could be pursued from that direction. We refer to Sewell [7] for axiomatic rigour.

5. AMBIGUITY

Consider in equations (23) the change of variables

$$\left. \begin{aligned} \bar{\xi} &= \xi - \xi_0 \\ \bar{\tau} &= (1 + a \xi_0) \tau \\ \bar{a} &= (1 + a \xi_0)^{-1} a \end{aligned} \right\} \quad (49)$$

Then

$$\begin{aligned} z + a^{-1} &= (\bar{\xi} + \bar{a}^{-1}) \cosh \bar{a} \bar{\tau} \\ t &= (\bar{\xi} + \bar{a}^{-1}) \sinh \bar{a} \bar{\tau} \end{aligned} \quad (50)$$

With a trivial shift in the z variable

$$\bar{z} + \bar{a}^{-1} = z + a^{-1} \quad (51)$$

the equations would be exactly the same as before. This means that a given object performing a given motion can equally well be attributed a different "uniform" acceleration $\bar{a}$ instead of a , just by taking $\xi = \xi_0$ rather than $\xi = 0$ as the "centre". And we would come, by the preceding arguments, to a different Unruh temperature:

$$\bar{T}_u / T_u = \bar{a} / a \quad (52)$$

This ambiguity is not important for a small thermometer so long as we require the "central" point, $\xi = \xi_0$, to be inside and not outside the object in question. But the existence of such an ambiguity is an interesting point of principle. This ambiguity in temperature is related to the corresponding

ambiguity in the unit of time and therefore of energy. Standard clocks attached to different points in the "uniformly" accelerated object do not keep time with one another. We could in fact define a local temperature $T_u(\xi)$, referred at each point to the local unit as defined by the local clock, which would not be constant over the object. The same point comes up for an object in thermal equilibrium in a static gravitational field. The temperature as usually defined in thermodynamics is of course uniform. But because of the gravitational red shift the unit natural at one height is not that natural at another. If we insist on the local unit at every point we come to a "temperature" which is non-uniform, and which is not that of ordinary thermodynamics. See for example Tolman [13].

6. ELECTRON THERMOMETER

The 'thermometer' of the above argument is supposed to be a Lorentz invariant quantum system of finite spatial extent. One can imagine [1, 6] for example a relativistic quantum field confined in a box whose walls are, for it, impenetrable, but which are freely penetrated by the original quantum field whose temperature is to be taken. One can imagine such a box held together by some unspecified physical force and uniformly accelerated by some other.

In an attempt to be a little more realistic it has been suggested [5] to consider an electron confined and accelerated by electromagnetic fields. Let these fields be so shaped in space and time that they are constant in the series of inertial frames by which we define uniform acceleration. The depolarization of the electron spin in the direction of a magnetic field is in suitable conditions a temperature indicator. Note that any increase of realism here is only relative. For we do not enquire into how such electromagnetic fields could be generated, nor into whether the necessary material support, including perhaps space-charge, should be taken into account.

In what follows we look a little more closely at the behaviour of an electron in these circumstances.

The Hamiltonian and boost operators of QED are

$$H = \int d^3x \quad T_{00} \quad (53)$$

$$B = - \int d^3x \quad z \quad T_{00} \quad (54)$$

with

$$\begin{aligned} T_{00} = & \psi^\dagger (m\beta - e\vec{\alpha} \cdot \vec{A} + e\phi) \psi \\ & + \frac{1}{2} i (\vec{\nabla} \psi^\dagger \cdot \vec{\alpha} \psi - \psi^\dagger \vec{\alpha} \cdot \vec{\nabla} \psi) \\ & + \dots \end{aligned} \quad \left. \vphantom{\begin{aligned} T_{00} = & \psi^\dagger (m\beta - e\vec{\alpha} \cdot \vec{A} + e\phi) \psi \\ & + \frac{1}{2} i (\vec{\nabla} \psi^\dagger \cdot \vec{\alpha} \psi - \psi^\dagger \vec{\alpha} \cdot \vec{\nabla} \psi) \\ & + \dots \end{aligned}} \right\} \quad (55)$$

where the terms ... not given explicitly do not contain the fermion field. Then the thermometer Hamiltonian W of (28) is

$$W = \int d^3x (1+az) \left\{ \psi^\dagger (m\beta - e\vec{\alpha} \cdot \vec{A} + e\phi) \psi + \frac{1}{2} i (\vec{\nabla} \psi^\dagger \cdot \vec{\alpha} \psi - \psi^\dagger \vec{\alpha} \cdot \vec{\nabla} \psi) \right\} \quad (56)$$

where A and ϕ are the applied external fields only. The corresponding terms in the radiation field go into V in (28). Because of the zero in $(1+az)$ at $z = -a^{-1}$, W falls into two parts (as also do U and V) which can be considered separately. In what follows we retain only the right Rindler wedge

$$(1 + az) > 0 \quad (57)$$

and denote the corresponding part of W by W_R .

To diagonalize W_R we have to solve in the region (57) the c-number Dirac equation

$$\left\{ (1 + az)(m\beta \vec{\alpha} \cdot (i\vec{\nabla} + e\vec{A}) + e\phi) - \frac{1}{2}ia\alpha_z \right\} u = E u \quad (58)$$

The ground state of W_R would be a Dirac sea with all states u of negative E occupied. This would reduce in the absence of A and ϕ to the Rindler Fermion vacuum -- not the Minkowski Fermion vacuum. If we start with a state nearer to the latter, then there will be some excitation from negative to positive E (pair creation). But this effect will presumably be negligible near $z = 0$ if we assume

$$a \ll m \quad (59)$$

Then we are concerned only with the excitation of the extra electron trapped in the potential well around $z = 0$.

If the fields are neither too strong nor too strongly varying, a non-relativistic approximation should be adequate. So we treat m as large in (58) and perform a Foldy-Wouthuysen transformation. In doing this we treat acceleration a as of order m^{-1} (not doing so got us into trouble in [5]). Then the equivalent 1-particle 2-spinor nonrelativistic Hamiltonian is

$$H = m + maz + e\phi + \frac{1}{2m}(i\vec{\nabla} + e\vec{A})^2 - \frac{e}{2m}\vec{\sigma} \cdot \vec{B} \quad (60)$$

neglecting terms of order m^{-2} and higher.

In this approximation spin and orbital motions decouple. The thermal distribution over the energy levels then implies a spin density matrix

$$\exp\left(\frac{e}{2m} \frac{\vec{\sigma} \cdot \vec{B}}{kT_0}\right) = \exp\left(\frac{\pi e}{ma} \vec{\sigma} \cdot \vec{B}\right) \quad (61)$$

So the fact that the electron is not localized on a classical path does not influence the thermal distribution over spin states -- in this approximation. The quantum treatment confirms, in this order, the semiclassical treatment of [5].

In higher orders in m^{-1} spin orbit interaction will appear (including the terms which were omitted in [5] only because of their $(g-2)$ dependence). The general argument of the present paper will still imply a thermal distribution over the coupled levels. But the situation as regards spin considered separately will be more complicated.

We neglected above that a particle can escape from the potential well by thermal excitation over, or quantum tunnelling through, the confining barrier. However, we expect these effects to be exponentially small for large m .

7. CONCLUSION

The main novelty in this paper is the extension of the Unruh analysis to extended thermometers, and the discussion of the associated temperature ambiguity. In insisting on an ambiguity, rather than an inhomogeneity, in temperature, we insist on the ordinary usage in thermodynamics of a temperature uniform over a body in thermal equilibrium. We are not here tempted by the alternative convention, which makes equilibrium temperature vary with height in a gravitational field, in a way related to the gravitational red shift. For we deal with an extended quantum system, whose quantum states are not localized, but belong to the extended system as a whole. It is a distribution over these extended states which is characterized by the temperature.

We think that our demonstration of the generality of the Unruh effect, in terms of PCT and related ideas, may also have some value to readers who find difficult the rigorous mathematical treatment of Sewell [7]. For quite different treatments of interacting fields see [14] and [15].

We did not in fact use the PCT theorem, but did use the central idea [16] that time and one space component can be reversed by analytically continued Lorentz boost. And we used, in a primitive way, analyticity notions like those involved in the axiomatic proof of PCT [17].

Having insisted on the generality of the discussion, let us insist finally on some limitations.

In quantum mechanics only an arbitrarily massive body can be well localized for an arbitrary time. We have avoided this problem by invoking external fields to accelerate our electron in §6, without discussing the massive material support for these fields. One would have to pay attention to this question in any practical proposal, but the very discouraging orders of magnitude of [5] inhibit us from investing effort in this direction.

Finally our treatment, like all others that we have seen, requires weak coupling between thermometer and system. Indeed some such weak coupling idea (although not necessarily represented by the smallness of a coupling constant) seems essential to the concept 'thermometer'. Without such a restriction final thermal equilibrium would seem to require merging together of the 'system' and what was to be the 'instrument'.

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