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QUARKONIUM SPECTROSCOPY

W. Buchmüller
CERN - Geneva

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1. INTRODUCTION

Over the past ten years heavy quarkonia have been extensively studied^{1),2)}, both experimentally and theoretically. They occur as families of narrow resonances and are believed to be bound states of heavy quark-antiquark pairs. So far, two sets of resonances have been discovered: the ϕ - and T - spectroscopies, which are related to the c - and b -quark species.

Heavy quarkonia provide direct evidence for the quark structure of hadrons, and the interest in the ϕ - and T - families has always been stimulated by the hope that these systems may be the "hydrogen atoms" of strong interactions. It is the theme of these lectures to discuss to what extent this hope has been fulfilled and to point out where the quarkonium spectroscopy is still incomplete.

The bound state structure of quarkonia is very simple. As the constituents are heavy, a non-relativistic treatment based on the Schrödinger equation and a static potential is sufficient to a good approximation. The phenomenological success of potential

models, as well as theoretical limitations of this approach, are discussed in Section 2. Similar to atomic physics the non-relativistic potential model can be extended to account for electromagnetic and hadronic transitions and spin-dependent forces, as we will see in Sections 3 and 4. Because of their simplicity, quarkonia are promising systems with respect to quantitative QCD tests; the present status as well as some theoretical expectations are discussed in Section 5. Due to the gluonic degrees of freedom bound states in addition to those predicted by potential models are expected. Such "extra" states are the subject of Section 6. Section 7 contains a brief summary.

Before we proceed to a discussion of potential models, let us briefly recall a few basic experimental facts³⁾. Most of our knowledge about the ϕ - and T -spectroscopies has been obtained at electron-positron storage rings where quarkonium resonances are seen in the reaction (cf. Fig. 1)

$$e^+e^- \rightarrow \text{hadrons} \quad .$$

At centre-of-mass energies between 3.0 GeV and 4.5 GeV, the ϕ -family (J/ψ , ψ' , ...) was discovered and the T -resonances (T , T' , ...) were found in the range from 9.4 GeV to 11 GeV (cf. Fig. 2). Further states have been discovered by studying the decays of the resonances produced in e^+e^- collisions. The analysis of the cascade decays

$$\begin{array}{c} \psi' \rightarrow \gamma \gamma \psi \\ \quad \quad \quad \hookrightarrow e^+e^- \end{array}$$

and the inclusive photon spectrum

$$\psi' \rightarrow \gamma + \text{anything}$$

led to the discovery of the charmonium χ states as well as the η_c and η_c' . More recently, analogous states in the T -spectroscopy, χ_b and χ_b' , have been found. In addition to photon transitions, hadronic transitions such as

$$\psi' \rightarrow \psi \pi \pi, \quad T' \rightarrow T \pi \pi, \quad ,$$

have been observed.

Fig. 1 Quarkonium resonances in the reaction $e^+e^- \rightarrow \text{hadrons}$.
 $p_-(p_+)$ is the electron (positron) four-momentum;
 $s = (p_- - p_+)^2$ is the total centre-of-mass energy squared.

Fig. 2 Cross-section for $e^+e^- \rightarrow \text{hadrons}$, showing the T resonances [CUSB data, from Ref. 3)].

An impressive amount of experimental information about the ϕ and T systems has been accumulated as a few numbers may illustrate: the Mark III group⁴⁾ has analyzed 2.7×10^6 ϕ decays, the latest Crystal Ball analysis⁵⁾ of the inclusive γ spectrum from ϕ' involves 1.8×10^6 hadronic ϕ' decays, and the T mass is known up to $\Delta M/M \sim 10^{-6}$ 2),

$$M_T = 9460.0 \pm 0.1 \text{ MeV} ,$$

which corresponds to QED accuracy in hadron spectroscopy! As we will see, however, there are still important aspects in the theory of quarkonia which have not yet been tested and further experimental work at electron-positron as well as hadron colliders is needed to complete our understanding of heavy quark-antiquark bound states.

2. HEAVY QUARKS AND THE $(Q\bar{Q})$ POTENTIAL

The theory of hadron structure is quantum chromodynamics (QCD)⁶⁾. The fundamental building blocks are the colour triplet spin- $\frac{1}{2}$ u-, d-, s-, c-, b-, (t-) quarks and a colour octet of spin-1 gluons. The resulting quark-antiquark attraction is described by a Coulomb-like potential with a "running coupling" whose strength depends on the quark-antiquark separation r ,

$$\alpha_s(r) \equiv \frac{g_s^2(r)}{4\pi} = \frac{12\pi}{33-2n_f} \frac{1}{\ln \frac{1}{r\Lambda}}, \quad (2.1)$$

where n_f is the number of "effective" flavours and Λ the QCD scale parameter, i.e., $\Lambda \sim 300$ MeV. Due to the increase of $\alpha_s(r)$ with r , a quark-antiquark pair cannot be separated. For the "confinement" radius, i.e., the typical hadronic size, one obtains from (2.1):

$$r_c \sim \frac{1}{\Lambda} \sim 0.7 \text{ fm}. \quad (2.2)$$

Light quarks are quarks whose intrinsic ("current") mass is small compared to the QCD scale, i.e., $m_Q \ll \Lambda$. Examples are the u- and d-quark masses: $m_u \sim 5$ MeV, $m_d \sim 9$ MeV. The approximate chiral symmetry associated with light quarks is spontaneously broken through the formation of vacuum condensates,

$$\langle 0 | \bar{u}u | 0 \rangle \sim \langle 0 | \bar{d}d | 0 \rangle \sim \Lambda^3. \quad (2.3)$$

This generates an effective ("constituent") mass $\tilde{m}_Q$ for light quarks,

$$\tilde{m}_Q \sim \Lambda \sim \frac{1}{2} m_p \sim \frac{1}{3} m_N, \quad (2.4)$$

and leads to the additive quark model as an approximate description of ordinary hadrons.

Heavy quarks are quarks whose current mass is large compared to the scale parameter Λ . So far we know two heavy quark species: the c-quark with $m_c \sim 1.5$ GeV and the b-quark with $m_b \sim 4.8$ GeV. Bound states of heavy quarks can be treated as non-relativistic two-body systems to good approximation. Thus with respect to their J^{PC} quantum numbers, these $(Q\bar{Q})$ bound states form a positronium-like spectrum. The total spin $\vec{S} = \vec{S}_Q + \vec{S}_{\bar{Q}}$ can take the values 0 and 1. The angular momentum $\vec{L}$ and $\vec{S}$ form the total angular

momentum $\vec{J} = \vec{L} + \vec{S}$. One distinguishes

singlet states: $S = 0, J = L$,
and
triplet states: $S = 1, J = \begin{cases} 1 \\ L-1, L, L+1 \end{cases}, \begin{matrix} L=0 \\ L \neq 0 \end{matrix}$

with parity and C-parity

$$P = (-)^{L+1}, \quad C = (-)^{L+S}. \quad (2.5)$$

Of particular interest are the triplet states with $L = 0$. They carry the quantum numbers of the photon, $J^{PC} = 1^{--}$, and are therefore dominantly produced in e^+e^- collisions (cf. Fig. 3). Figure 4 shows the narrow ($b\bar{b}$) resonances of the T spectroscopy as well as the various observed hadronic and photon transitions.

Fig. 3 Mixing between γ and $(Q\bar{Q})$ resonances, the dominant production mechanism for hadrons in the final state.

The mass spectrum of a quarkonium family is computed by means of the Schrödinger equation:

$$M_n(Q\bar{Q}) = 2m_Q + E_n(m_Q, V), \quad (2.6)$$

$$\left[-\frac{1}{m_Q} \Delta + V(r) \right] \psi_n(r) = E_n \psi_n(r).$$

Obviously, the main problem of quantitative spectroscopy is the choice of the correct $(Q\bar{Q})$ potential $V(r)$. So far the potential has not yet been computed from first principles, although the progress in lattice-QCD⁷⁾ appears promising. At present, one therefore has to rely on simple models.

Fig. 4 T-spectroscopy. The quantum numbers of the various states are given by $n^{2S+1}L_J$, $L = S, P, D, \dots$ (see text). Full (dashed) lines correspond to observed (predicted) states.

(i) QCD-like models

In this class of models, the known asymptotic behaviour of the static $(Q\bar{Q})$ potential in QCD is incorporated. One has

$$V(r) \underset{r \ll \frac{1}{\Lambda}}{\sim} -\frac{4}{3} \frac{d_S(r)}{r}, \quad (2.7a)$$

$$V(r) \underset{r \gg \frac{1}{\Lambda}}{\sim} k r, \quad (2.7b)$$

where k is the "string tension". Equation (2.7a) corresponds to a static Coulomb field at short distances whereas (2.7b) reflects the linear increase of the binding energy with distance for a colour-electric flux tube (cf. Fig. 5). In QCD-like models the potential at intermediate distances is chosen ad hoc.

Fig. 5 Field configurations which determine the $(Q\bar{Q})$ potential at small and large distances.

The simplest choice is the Cornell model⁸⁾ for which the first extensive study of quarkonia was carried out:

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + kr, \quad \alpha_s = \text{Const.} \quad (2.8)$$

Already for $(b\bar{b})$ bound states, the vacuum polarization corrections to α_s are important quantitatively. Models with "running coupling" $\alpha_s(r)$ have been studied by many authors. A particularly simple and successful example was suggested by Richardson⁹⁾. The Fourier transform

$$\tilde{V}(q) = \int d^3x e^{-i\vec{q}\vec{x}} V(r), \quad r = |\vec{x}|, \quad q = |\vec{q}|, \quad (2.9)$$

has the asymptotic behaviour

$$\tilde{V}(q) \underset{q \gg \Lambda}{\sim} -\frac{4}{3} \frac{48\pi^2}{33-2n_f} \frac{1}{q^2 \ln \frac{q^2}{\Lambda^2}}, \quad (2.10a)$$

$$\tilde{V}(q) \underset{q \ll \Lambda}{\sim} -8\pi k \frac{1}{q^2}, \quad (2.10b)$$

where k is the string tension introduced in (2.7b). Richardson's interpolating formula for (2.10a) and (2.10b) reads

$$\tilde{V}(q) = -\frac{4}{3} \frac{48\pi^2}{33-2n_f} \frac{1}{q^2 \ln \left(\frac{q^2}{\Lambda^2} + 1 \right)}. \quad (2.11)$$

Equation (2.11) defines a potential which depends only on a single scale parameter Λ as it is the case for the true QCD potential. Consequently, the string tension k and Λ are related. Richardson's fit uses $\Lambda = 398$ MeV which yields $k = 0.147$ GeV².

The potential (2.11) describes the ψ and T spectroscopies remarkably well. Yet, in order to relate Λ to a well-defined QCD scale parameter, e.g. $\Lambda_{\overline{MS}}$, one has to proceed to a next level of sophistication and incorporate next-to-leading order perturbative QCD corrections for the $(Q\bar{Q})$ potential. This has been done¹⁰⁾ and the running coupling of the potential, $\alpha_p(q) = (-3/16\pi)q^2\tilde{V}(q)$, has been studied in terms of its β -function. For T spectroscopy, however, this development is phenomenologically rather unimportant and we will therefore not discuss it further in these lectures.

(ii) Empirical models

Theoretically unbiased one may also attempt to describe quarkonium spectroscopy using simple empirical potentials. Examples are the logarithmic¹¹⁾ and small power potentials. Such models check the QCD-content of QCD-like models. A successful example has been suggested by Martin¹²⁾

$$V(r) = A + B r^{0.1} \quad (2.12)$$

How well do potential models work? Two QCD-like models and Martin's model are compared with experiment in Table 1. All of them work very well, in fact better than one should expect. Within the theoretical uncertainties, the three models cannot be distinguished. Figure 6 shows various potentials with the mean square radii of the two quarkonium families. The ϕ and T spectroscopies probe distances between 0.1 fm and 1.0 fm. In this range the phenomenological potential appears to be essentially logarithmic and there is evidence neither for a Coulombic nor a linearly rising part.

Fig. 6 Different $(Q\bar{Q})$ potentials [from Ref. 13]: (1) Ref. 12), (2) Ref. 10), (3) Ref. 14), (4) Ref. 8).

	State	Experiment Refs. 2), 3)	Cornell Ref. 8)	"QCD" Ref. 10)	Martin Ref. 12)
(c $\bar{c}$)	2S	0.589 (0.46 $\pm$ 0.06)	0.589* (0.44)	0.60 (0.46)	0.592* (0.40)
	1P	0.425	0.427*	0.42	0.407
(b $\bar{b}$)	2S	0.561 (0.44 $\pm$ 0.03)	0.560* (0.39)	0.56* (0.44)	0.560* (0.41)
	3S	0.890 (0.32 $\pm$ 0.03)	0.898 (0.27)	0.89 (0.32)	0.900 (0.35)
	1P	0.440 ($\pm$ 1)	0.463	0.43	0.401
	2P	0.796 ($\pm$ 5)	0.811	0.79	0.782

Table 1 Predictions of three potential models compared with experiment. Excitation energies for different (c $\bar{c}$) and (b $\bar{b}$) states are given in GeV. The ratios of leptonic widths with respect to the ground state are listed in brackets. The asterisk denotes input to fix the parameters of the potential.

There are, however, two hints that the potential at distances below 0.1 fm may be as predicted by perturbative QCD. The leptonic width of the T resonance, $\Gamma_{ee}(T) \equiv \Gamma(T \rightarrow e^+e^-)$, is proportional to the square of the wave function at the origin,

$$\Gamma_{ee}^{(0)}(T) = \frac{16\pi e_b^2 d_{EH}^2}{M_T^2} |\psi_T(0)|^2, \quad (2.13)$$

and thereby particularly sensitive to the short distance part of the potential. Contrary to other models QCD-like models with running coupling and QCD corrections to $\Gamma_{ee}^{(0)}$ taken into account yield the correct value for the T leptonic width¹³⁾. Theory and experiment are compared in Table 2. The second hint comes from the recently observed χ_b states in the T spectroscopy (cf. Fig. 4). Next to T, the χ_b resonances are the quarkonia of smallest size¹³⁾

$$\begin{array}{ccccccc} \langle r^2 \rangle_T^{\frac{1}{2}} & < & \langle r^2 \rangle_{\chi_b}^{\frac{1}{2}} & < & \langle r^2 \rangle_{3/4}^{\frac{1}{2}} & < & \dots \\ 0.23 f_m & & 0.39 f_m & & 0.42 f_m & & \end{array} \quad (2.14)$$

The sensitivity of their centre-of-gravity (c.o.g.) position with respect to the short distance behaviour of the $(Q\bar{Q})$ potential is shown in Table 3: the more singular the potential, the larger the mass difference $M(\chi_b) - M(T)$. In the pure Coulombic case, one has $M(\chi_b) - M(T) = M(T') - M(T)$.

	"QCD"- model	Experiment
$\Gamma_{ee}(T)$ (keV)	1.07 ± 0.24	1.24 ± 0.05

Table 2 T leptonic width of a "QCD"- model¹³⁾ compared with experiment²⁾.

	Martin Ref. 12)	Richardson Ref. 9)	Cornell Ref. 8)	Experiment Ref. 2)
$M_{c.o.g.}(\chi_b) - M(T)$ [MeV]	401	436	463	440 ± 1

Table 3 χ_b - T mass differences for different potentials compared with experiment.

Given the empirical fact that potential models describe quarkonium energy levels with an accuracy up to a few MeV, one may conclude from the success of Richardson's potential that the regime below 0.1 fm is governed by perturbative QCD. Obviously, for the final evidence we have to wait for the ζ -spectroscopy of the t -quark.

Voloshin has made a prediction for the χ_b - T mass difference based on QCD sum rules¹⁵⁾

$$M(\chi_b) - M(T) = 0.37 \pm 0.03 \text{ GeV} \quad , \quad (2.15)$$

and it has been emphasized¹⁶⁾, even after¹⁷⁾ the experimental

discovery of the χ_b states, that the $\chi_b - T$ mass difference should distinguish between potential models and QCD sum rules. It is conceivable¹⁸⁾, however, that the discrepancy between (2.15) and the experimental result (cf. Table 3) is a problem of the specific calculation in Ref. 15) and not of the QCD sum rule approach in general.

What is the theoretical significance of the phenomenological success of potential models? Consistency of the non-relativistic potential model demands that corrections are not too large. A complete treatment of relativistic corrections¹⁹⁾ has not been achieved and appears impossible without a deeper understanding of the confinement problem. Yet the order of magnitude of relativistic corrections to mass differences is easily estimated,

$$\Delta_{\text{REL}}(M_{T'} - M_T) \sim \langle \frac{v^2}{c^2} \rangle_T (M_{T'} - M_T) \sim 45 \text{ MeV} . \quad (2.16)$$

Fig. 7 Contribution to the T mass arising in second order perturbation theory due to light quarks.

Furthermore "coupled channel" corrections, first evaluated by the Cornell group⁸⁾, are important. They appear in second order perturbation theory as a consequence of heavy quark - light quark bound state intermediate states (cf. Fig. 7) and reflect vacuum polarization effects of light quarks. A recent calculation of Heikkilä, Törnqvist and Ono²⁰⁾ yields

$$\Delta_{\text{CC}}(M_{T'} - M_T) \sim -20 \text{ MeV} . \quad (2.17)$$

In view of (2.16) and (2.17) the accuracy of potential models up to a few MeV appears almost suspicious! It may be due, however, to cancellations between various large contributions. The empirical fact of the flavour independence of the $(Q\bar{Q})$ potential, which is evident from Fig. 8, suggests that the total effect of relativistic and coupled channel corrections is small.

Fig. 8 Evidence for flavour independence of the $(Q\bar{Q})$ potential. Comparison of the potentials V_ψ and V_T , constructed by means of the inverse scattering method from ψ and T data, from Ref. 21).

Even if the non-relativistic potential model is theoretically self-consistent, one may still argue that the empirical $(Q\bar{Q})$ potential is only an effective potential and has nothing to do with the static energy of an infinitely heavy quark-antiquark pair in QCD. This point of view has been advocated by the ITEP group [c.f., e.g., Ref. 16)], based on the existence of the gluon condensate,

$$\eta \equiv \langle 0 | : \frac{ds}{\pi} G_{\mu\nu}^a(0) G^{a\mu\nu}(0) : | 0 \rangle \sim (330 \text{ MeV})^4, \quad (2.18)$$

which is said to be due to "vacuum fluctuations" of spacial and temporal size

$$R_G \sim T_G \sim \eta^{-1/4} \sim 0.6 \text{ fm}. \quad (2.19)$$

The validity of the potential picture requires that the characteristic quark oscillation time,

$$T_Q \sim \frac{1}{M_{\psi'} - M_\psi} \sim \frac{1}{M_{T'} - M_T} \sim 0.3 \text{ fm}, \quad (2.20)$$

is much larger than the gluon oscillation time, i.e., $T_Q \gg T_G$. This is not the case for (2.19) and (2.20).

Obviously there is no exact two-body problem in any quantum field theory. Yet a pure dimensional analysis does not seem sufficient to argue away the possibility of the approximate validity of potential models, which may well be a consequence of some factors of 4π . Bell and Bertlmann²²⁾ have studied the connection between potential models and the QCD sum rule approach but a simple link is still missing. It seems that we have to wait for the next test of flavour independence in $(t\bar{t})$ spectroscopy and for progress in lattice calculations for a final answer to the question of the theoretical significance of the empirical $(Q\bar{Q})$ potential.

3. ELECTROMAGNETIC AND HADRONIC TRANSITIONS

Until recently, it seemed that potential models had one serious problem: all models yield branching ratios for the electromagnetic transitions $\psi' \rightarrow \gamma \chi_J$ (cf. Table 4, Fig. 9) which are a factor 2-3 larger than the experimentally measured values²³⁾. Electromagnetic E1-transitions between S- and P-states are given by

$$\Gamma_J = \frac{4}{27} e_Q^2 \alpha_{EM} k_\gamma^3 |\langle 1P | r | 2S \rangle|^2 (2J+1) \quad , \quad (3.1)$$

where e_Q , α_{EM} and k_γ are the quark charge, the electromagnetic fine structure constant and the photon momentum respectively. The partial width Γ_J is proportional to the square of an overlap integral involving the 1P and 2S wave functions. Since the early days of charmonium spectroscopy, it has been emphasized²⁴⁾ that due to the sensitivity to details of the wave functions, the discrepancy between theory and experiment should not be taken too serious, and that even in atomic physics Hartree-Fock calculations of E1 rates are frequently off by 50%. Nevertheless, a discrepancy of almost 300% is rather disturbing and the puzzle has only recently been resolved by McClary and Byers²⁵⁾. They pointed out that, due to the node in the 2S wave function, the matrix element $\langle 1P | r | 2S \rangle$ has a positive and a negative contribution which partly cancel each other. Thus a small change in the wave functions, due for instance to relativistic corrections, can have a large effect on the overlap integral. A quantitative analysis²⁵⁾ which incorporates relativistic and coupled channel corrections removes indeed the discrepancy with experiment, as can be seen from Table 4. For comparison, we have also listed the remarkable results which Henriques, Kellett and Moorhouse²⁶⁾ obtained already in 1976 in a relativistic treatment based on the Bethe-Salpeter equation.

State	Experiment	Theory, non-relativistic	Theory, relativistic	
			McClary+Byers ²⁵⁾	Henriques et al. ²⁶⁾
J=2	7.7±1.7	18±3	10±2	6.5±1.2
J=1	8.8±1.9	23±4	11±2	9.8±1.8
J=0	9.7±2.2	27±5	7.5±1.5	10±2

Table 4 Branching ratios for the radiative decay $\psi' \rightarrow \chi_J$. For the first two columns, see Ref. 23).

Fig. 9 Radiative transitions $\psi' \rightarrow \gamma \chi_J$ ($J = 2, 1, 0$).

In the T system relativistic corrections are expected to be much smaller than in the ψ system ($\langle v^2/c^2 \rangle_{J/\psi} \sim 0.23$, $\langle v^2/c^2 \rangle_T \sim 0.08$). Thus, it is comforting that recent measurements of the electromagnetic transitions $T' \rightarrow \gamma \chi_{1,2}^b$ are in agreement with potential model predictions as shown in Table 5.

State	Experiment	Theory, non-relativistic
J=1	6.1±1.4	7.1±2.5
J=2	5.9±1.4	7.1±2.5

Table 5 Branching ratios for the transition $T' \rightarrow \gamma \chi_J^b$. The dipole matrix element is taken from Ref. 13).

In addition to electromagnetic transitions hadronic transitions²⁷⁾⁻³¹⁾ between different quarkonium states, such as $\psi' \rightarrow \phi\pi\pi$, $\psi' \rightarrow \phi\eta$, $T' \rightarrow T\pi\pi$, etc. have been extensively studied experimentally as well as theoretically. Figure 10 shows a QCD graph which contributes to the transition $T' \rightarrow T\pi^+\pi^-$. Between the successive emission of the two gluons the $(Q\bar{Q})$ system propagates as a colour octet state. As the quark motion is slow, a multipole expansion²⁷⁾ can be performed which leads to the scaling law⁸⁾

$$\Gamma(T' \rightarrow T\pi^+\pi^-) = \left(\frac{\bar{r}_T}{\bar{r}_{\psi}} \right)^4 \Gamma(\psi' \rightarrow \psi\pi^+\pi^-), \quad (3.2)$$

where $\bar{r}_{T(\psi)}$ are the mean square radii of T and J/ψ . The relation (3.2) led to the prediction¹⁾

$$\Gamma(T' \rightarrow T\pi^+\pi^-)|_{TH} = (5 \pm 1) \text{ keV},$$

in remarkable agreement with experiment^{1),2)}

$$\Gamma(T' \rightarrow T\pi^+\pi^-)|_{Exp} = (5.2 \pm 1.6) \text{ keV}.$$

Predictions for η transitions²⁹⁾ are also in agreement with experiment, whereas the theoretical expectation for the dipion mass spectrum is at variance with the experimentally measured distribution^{1),2)}.

Fig. 10 Contribution to the hadronic decay $T' \rightarrow T\pi^+\pi^-$.

4. SPIN-DEPENDENT FORCES

There exists an extensive literature³²⁾ on spin-dependent forces in heavy quark-antiquark systems. The most popular approach has been to start from the Bethe-Salpeter equation and an instantaneous kernel with some suitably chosen Lorentz structure, and to deduce the corresponding Breit-Fermi Hamiltonian³³⁾. Since the work of Henriques, Kellett and Moorhouse²⁶⁾, it is known that kernels which correspond to "scalar exchange" at large distances and "vector exchange" at small distances describe well the fine and hyperfine splittings of quarkonia. At a more fundamental level, spin-dependent forces have been considered in the bag model³⁴⁾⁻³⁶⁾. The results obtained are similar to those of the "scalar+vector exchange" ansatz. Most closely related to QCD is the approach of Eichten and Feinberg³⁷⁾ and Gromes^{38),39)}. These authors start from the Wilson loop and relate the various pieces of the spin Hamiltonian to correlation functions of electric and magnetic field strengths; these correlation functions are then either evaluated perturbatively or related to the static ($Q\bar{Q}$) potential.

The basic structure of the spin Hamiltonian can also be obtained in a very elementary way⁴⁰⁾ by making use of the intuitive flux tube picture of electric confinement and standard techniques⁴¹⁾ used to derive the spin Hamiltonian for the hydrogen atom or positronium from classical electrodynamics. For simplicity, let us first consider a ($Q\bar{q}$) system with $m_Q \gg m_{\bar{q}} \gg \Lambda$ in the limit $m_Q \rightarrow \infty$. At small distances, $r \ll 1/\Lambda$, the antiquark $\bar{q}$ moves in the static colour field generated by the quark Q . The total energy of the system contains a piece ΔE which depends on the spin $\vec{s}$ of $\bar{q}$ ⁴¹⁾,

Fig. 11 a) Static colour electric field generated by the quark Q ($r \ll 1/\Lambda$);
b) rotating electric flux tube ($r \gg 1/\Lambda$).

$$\Delta E = -\mu \vec{s} \cdot \vec{B}' + \vec{\omega}_T \cdot \vec{s} \quad , \quad (4.1)$$

where μ , $\vec{B}'$ and $\vec{\omega}_T$ are the magnetic moment, the magnetic field in the instantaneous co-moving rest frame of $\bar{q}$ and the Thomas frequency, which can be expressed in terms of the angular momentum $\vec{L}$ and the static ($Q\bar{q}$) energy $V_s(r)$,

$$\vec{\omega}_T = -\frac{1}{2m_{\bar{q}}^2} \vec{L} \cdot \frac{1}{r} \frac{dV_s}{dr} \quad . \quad (4.2)$$

In the rest frame of Q , one has a static electric field $\vec{E}$ and a vanishing magnetic field, $\vec{B} = 0$. In the co-moving frame of $\bar{q}$ this leads to

$$\vec{E}' = \vec{E} + O(v^2) \quad , \quad \vec{B}' = -\vec{v} \times \vec{E} + O(v^2) \quad , \quad (4.3)$$

where $\vec{v}$ is the velocity of $\bar{q}$. Using $\mu = g/m_{\bar{q}}$ and $\vec{E} = -(1/g)\vec{\nabla}V_s$, one obtains from (4.1)-(4.3) the familiar spin-orbit interaction

$$\Delta E = \frac{1}{2m_{\bar{q}}^2} \vec{L} \cdot \vec{s} \frac{1}{r} \frac{dV_s}{dr} \quad . \quad (4.4)$$

In order to obtain this result, we have used the standard coupling of a spin to a magnetic field, i.e., Eq. (4.1), and our knowledge about the field configuration generated by a static quark. The spin dependent interaction energy for large $Q\bar{q}$ separation can be obtained in a similar manner once the field configuration at large distances is known. This information is provided by the flux tube picture of electric confinement (cf. Fig. 11b), according to which a static tube contains only colour electric flux, i.e., $\vec{E} \neq 0$, $\vec{B} = 0$. If we assume that the fields in a flux tube segment moving with velocity $\vec{v}$ can be obtained by Lorentz transformation from the static case, we have in the rest frame of Q $\vec{B} = \vec{v} \times \vec{E} + O(v^2)$. Yet in the co-moving rest frame of $\bar{q}$, we obtain $\vec{B}' = 0$! From (4.1) and (4.2) we thus conclude

$$\Delta E = -\frac{1}{2m_{\bar{q}}^2} \vec{L} \cdot \vec{s} \frac{1}{r} \frac{dV_L}{dr} \quad , \quad (4.5)$$

where V_L is the static ($Q\bar{q}$) energy for distances $r \gg 1/\Lambda$. Equations (4.4) and (4.5) are identical except for the sign.

The generalization to a $(Q\bar{Q})$ system, i.e., $m_Q = m_{\bar{Q}} = m$, is straightforward and magnetostatic interactions are easily taken into account. Assuming, in the spirit of the "Coulomb+linear" potential model that the sum of the asymptotic expressions for $r \gg 1/\Lambda$ and $r \ll 1/\Lambda$ is a reasonable approximation also at intermediate distances, one obtains with $V_S = -(4/3)(\alpha_s/r)$, $V_L = kr$ ⁴⁰⁾:

$$\begin{aligned}
 H_{\text{SPIN}} = & -\frac{1}{2m^2} \vec{L} \cdot \vec{S} \frac{k}{r} + \frac{2}{m^2} \vec{L} \cdot \vec{S} \frac{d_s}{r^3} \\
 & + \frac{4}{3} \frac{d_s}{m^2} \frac{3(\vec{S}_1 \cdot \hat{\vec{r}})(\vec{S}_2 \cdot \hat{\vec{r}}) - \vec{S}_1 \cdot \vec{S}_2}{r^3} \\
 & + \frac{32\pi}{9} \frac{d_s}{m^2} \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{r}) , \\
 \vec{S} = & \vec{S}_1 + \vec{S}_2 , \quad \vec{L} = \vec{r} \times \vec{p} , \quad \hat{\vec{r}} = \frac{\vec{r}}{r} .
 \end{aligned} \tag{4.6}$$

The Hamiltonian (4.6) is identical with the expression obtained from a "scalar+vector exchange" Bethe-Salpeter kernel ³²⁾. Except for the sign of the first term, it also agrees with the result of Eichten and Feinberg ³⁷⁾. According to a recent paper by Gromes ³⁹⁾, however, this sign difference results from an unjustified assumption about the correlation function between electric and magnetic fields and should be reversed. It thus appears that spin dependent forces of heavy quarks are essentially understood, and that the phenomenologically successful "scalar + vector exchange" ansatz, which corresponds to the naive flux tube picture of electric confinement, can be derived ^{37), 39)} within QCD.

	ΔM (MeV)	Flux tube model ^{a)} Refs. 40), 42)	Eichten + Feinberg ^{b)} Ref. 37)	Experiment Refs. 23), 43)
(c $\bar{c}$)	$\Delta M(1P)$	134	131	140 ± 5
	$r(1P)$	0.61	1.0	0.49 ± 0.02
(b $\bar{b}$)	$\Delta M(1P)$	38	50	41 ± 3
	$r(1P)$	0.73	1.0	0.93 ± 0.3
	$\Delta M(2P)$	31	36	33 ± 5
	$r(2P)$	0.72	1.0	0.85 ± 0.4

Table 6 ψ and T fine structure, total and relative P-state splittings [cf. Eq. (4.7)]. a) $\alpha_s(\psi) = 0.53$, $\alpha_s(T) = 0.28$; b) $\alpha_s(\psi) = 0.341$, $\alpha_s(T) = 0.227$.

According to the Hamiltonian (4.6), spin-orbit forces are long ranged whereas tensor and spin-spin forces are short ranged. Contrary to the fine structure, the hyperfine splittings may therefore be suitable quantities for quantitative QCD tests. Predictions obtained from (4.6) for P-state splittings are compared with results of Eichten and Feinberg and experimental data in Table 6 where

$$\Delta M(nP) = M(n^3P_2) - M(n^3P_0) ,$$

$$r(nP) = \frac{M(n^3P_2) - M(n^3P_1)}{M(n^3P_1) - M(n^3P_0)} , \quad (4.7)$$

denote the total mass difference and the ratio of upper to lower mass difference. The CUSB results⁴³⁾ for the χ_b states favour the predictions of Eichten and Feinberg yet the errors are large and preliminary Crystal Ball data⁴⁴⁾ read

$$\Delta M(\chi_b) = 52 \pm 7 \text{ MeV} , \quad r(\chi_b) = 0.58 \pm 0.3 .$$

Beyond any doubt, fine structure splittings are a relativistic effect. From Table 6, one reads off

$$\frac{\Delta M(\chi_b)}{\Delta M(\chi_c)} \Big|_{\text{EXP}} \sim 0.29 ,$$

whereas the velocities of b- and c- quarks, calculated in potential models, yield

$$\frac{\langle \frac{v^2}{c^2} \rangle_{\chi_b}}{\langle \frac{v^2}{c^2} \rangle_{\chi_c}} \Big|_{\text{TH}} \sim 0.28 .$$

One important aspect of spin dependent forces remains to be tested experimentally. So far no singlet P-states (1P_1) have been observed. The spin-spin forces given by the Hamiltonian (4.6) are short ranged, which one also expects from a comparison⁴⁵⁾ of the $\psi' - \eta'_c$ and $\psi - \eta_c$ mass differences. The 1P_1 masses should be equal to the centre of gravity of the 3P_J states, i.e.,

$$M(^1P_1) = \frac{5}{9} M(^3P_2) + \frac{3}{9} M(^3P_1) + \frac{1}{9} M(^3P_0) . \quad (4.8)$$

A discovery of the ψ and Υ 1P_1 -states at hadron colliders would complete our understanding of the spin forces of heavy quarks.

5. QUANTITATIVE QCD TESTS

In spite of considerable experimental and theoretical efforts, quantitative QCD tests^{46), 47)} have turned out to be very difficult. Scaling violations in deep inelastic scattering are obscured by "higher twist" contributions and α_s determinations from jet analyses seem to depend strongly on fragmentation models. It is conceivable that heavy quarkonia will lead to an accurate determination of the strong coupling α_s and the related QCD scale parameter $\Lambda_{\overline{\text{MS}}}$ ⁴⁶⁾.

The domain of QCD perturbation theory are distances

$$r \ll \frac{1}{\Lambda} \sim 0.5 \text{ fm} . \quad (5.1)$$

As we saw in Section 2, quarkonia populate the range of intermediate distances,

$$\begin{aligned} \langle r^2 \rangle_{\frac{3}{2}\psi}^{\frac{1}{2}} &\sim 0.4 \text{ fm}, \dots, \langle r^2 \rangle_{\psi'}^{\frac{1}{2}} \sim 0.9 \text{ fm}, \\ \langle r^2 \rangle_{\chi}^{\frac{1}{2}} &\sim 0.2 \text{ fm}, \dots, \langle r^2 \rangle_{\chi'''}^{\frac{1}{2}} \sim 0.9 \text{ fm}. \end{aligned} \quad (5.2)$$

For some physical quantities, however, a factorization into a non-perturbative and a perturbative part can be achieved and reliable predictions are possible. The hadronic width of the η_b ⁴⁸⁾, for instance, is given by

$$\Gamma_{\text{HAD}}(\eta_b) \sim |\psi(0)|^2 |M(b\bar{b} \rightarrow 2G)|^2, \quad (5.3)$$

where $\psi(0)$ and $M(b\bar{b} \rightarrow 2G)$ are the $(b\bar{b})$ wave function at the origin and the on-shell scattering matrix element shown in Fig. 12. In general, hadronic and electromagnetic decay widths take the form (5.3). Their ratios are therefore determined by perturbation theory alone (at least up to a certain accuracy), whereas the absolute widths involve the non-perturbative quantity $\psi(0)$ which can be calculated in potential models. A complete discussion of hadronic and electromagnetic decays has been given by Remiddi⁴⁹⁾.

At present the only determination of α_s from quarkonia is based on the hadronic width of the T resonance. The calculation of Mackenzie and Lepage⁵⁰⁾ yields

$$\alpha_s^3(0.48 M_T) = \frac{\Gamma_{\text{HAD}}(T)}{\Gamma_{\mu\mu}(T)} \alpha_{\text{EM}}^2 \frac{81\pi e_b^2}{10(\pi^2-9)}, \quad (5.4)$$

Fig. 12 Perturbative amplitude $M(b\bar{b} \rightarrow 2G)$ for the decay $\eta_b \rightarrow$ hadrons.

from which one obtains²⁾

$$\alpha_s(0.48 M_T) = 0.165 \pm 0.005 , \quad (5.5)$$

$$\Lambda_{\overline{MS}} = 118^{+16}_{-15} \text{ MeV} .$$

Additional, new information comes from the χ_b states where the branching ratios for the radiative transitions $1^3P_{2,1} \rightarrow \gamma 1^3S_1$ have been measured. Together with the theoretical E1 widths (which are believed to be reliable, cf. Section 3), one obtains for the hadronic widths of the χ_b states²⁾

$$\Gamma_{\text{HAD}}(1^3P_2) = 152^{+63}_{-50} \text{ keV} , \quad (5.6)$$

$$\Gamma_{\text{HAD}}(1^3P_1) = 41^{+40}_{-20} \text{ keV} .$$

This yields the ratio

$$\frac{\Gamma_{\text{HAD}}(1^3P_1)}{\Gamma_{\text{HAD}}(1^3P_2)} \sim 0.27 , \quad (5.7)$$

which is compatible with the result of lowest order QCD,

$$\frac{\Gamma_{\text{HAD}}(1^3P_1)}{\Gamma_{\text{HAD}}(1^3P_2)} = \frac{5}{3\pi} \alpha_s \ln(m_b R_c) \sim 0.22 , \quad (5.8)$$

where we have used α_s of (5.5), $m_b = 4.9 \text{ GeV}$, $R_c^{-1} = 0.4 \text{ GeV}$.

More information on $\Lambda_{\overline{MS}}$ is expected from a measurement of the $T - \eta_b$ hyperfine splitting⁵¹⁾,

$$\Delta M(T-\eta_b) = \frac{8}{9} \frac{\Gamma_{ee}(T)}{e_b^2 d_{EM}^2} \alpha_{\overline{MS}}(m_b) \left[1 + 5.6 \frac{d_{\overline{MS}}(m_b)}{\pi} \right] \quad (5.9)$$

The pseudoscalar state η_b may be seen in the cascade shown in Fig. 13 for which Kuang and Yan have estimated a branching ratio of 0.5%³⁰⁾. A theoretically particularly clean quantity is the ratio of hadronic widths⁵²⁾

$$\frac{\Gamma_{HAD}(\chi_{J=0}^b)}{\Gamma_{HAD}(\chi_{J=2}^b)} = \frac{15}{4} \left(1 + 9.5 \frac{d_s}{\pi} \right), \quad (5.10)$$

which, together with the $T-\eta_b$ hyperfine splitting, is given in Table 7 for different values of $\Lambda_{\overline{MS}}$.

$\Lambda_{\overline{MS}}$ (MeV)	100	200	300
$\Delta M(T-\eta_b)$ (MeV)	27	35	42
$\Gamma_{HAD}(\chi_0^b)/\Gamma_{HAD}(\chi_2^b)$	5.5	5.8	6.1

Table 7 $T - \eta_b$ hyperfine splitting and χ_b hadronic widths ratio for different values of $\Lambda_{\overline{MS}}$.

A lower bound on the QCD scale parameter can be obtained from the empirical $(Q\bar{Q})$ potential. As down to 0.1 fm no sign of a Coulombic behaviour has become visible, $\Lambda_{\overline{MS}}$ cannot be arbitrarily small. In Fig. 14 the perturbative QCD potential for different values of $\Lambda_{\overline{MS}}$ is compared with the empirical $(Q\bar{Q})$ potential at small distances ($r < r_c$, $1/r_c^2 \Lambda_{\overline{MS}}^2 = 100$). This leads to the estimate¹³⁾ $\Lambda_{\overline{MS}} > 100$ MeV.

Fig. 13 Cascade decay $\Upsilon'' \rightarrow \chi_b^{S=0} \pi\pi \rightarrow \gamma \eta_b \pi\pi$.

Fig. 14 Comparison of the perturbative QCD potential for different values of $\Lambda_{\overline{MS}}$ with the phenomenological ($Q\bar{Q}$) potential for distances $r < r_c = 1/10\Lambda_{\overline{MS}}$, from Ref. 13).

Heavy quarkonia are a promising testing ground for perturbative QCD. As they are non-relativistic systems, the bound state dynamics is very simple, and it may be possible to control and separate non-perturbative from genuine perturbative effects. Potential dangers for this programme are, however, already apparent: they include the dependence on the renormalization scheme^{53),54)}, relativistic⁵⁵⁾ and coupled channel²⁰⁾ corrections, and possible deviations from factorization. Nevertheless, agreement between various independent determinations of $\Lambda_{\overline{MS}}$ from heavy quarkonia would provide a non-trivial quantitative confirmation of quantum chromodynamics.

6. "EXTRA" STATES

Even more difficult than quantitative QCD tests appears the verification of a qualitative QCD prediction: the existence of states due to gluonic degrees of freedom which are not predicted by the quark model. So far, there exists no unequivocal evidence for glueballs¹⁾.

Quarkonium families should also contain "extra" states. In the flux tube picture there are $(Q\bar{Q})$ states with an excited flux tube configuration. In the string model, which corresponds to the zero-width limit of the flux tube, such "vibrational" states (cf. Fig. 15) have first been discussed by Giles and Tye⁵⁶⁾. A specific prediction⁵⁷⁾ for the first excited state in the T family with $J^{PC} = 1^{--}$,

$$\begin{aligned} M(T_{V18}) - M(T) &= 990 \pm 90 \text{ MeV}, \\ \Gamma_{ee}(T_{V18}) &= 0.20 \pm 0.15 \text{ keV}, \end{aligned} \quad (6.1)$$

turned out not to be correct. The experimental upper bound³⁾ for the leptonic width of a 1^{--} resonance between T'' and T''' reads $\Gamma_{ee} < 20 \text{ eV}$. Thus the approximations made in Ref. 57) to estimate the properties of the "extra" states were not adequate.

Hasenfratz, Horgan, Kuti and Richard³⁴⁾ have used the bag model to calculate properties of quark-antiquark-gluon bound states. They have predicted an axial vector ($J^{PC} = 1^{--}$) with an excitation energy

Fig. 15 Ground state and excited state of a string (zero-width flux tube).

$$M(b\bar{b}G) - M(\Upsilon) = 1040 \pm 200 \text{ MeV}, \quad (6.2)$$

which, unfortunately, is difficult to test experimentally. More recent calculations give somewhat larger excitation energies. Ono⁵⁸⁾ expects $(Q\bar{Q}G)$ states with $\Delta M \sim 1.3 \text{ GeV}$ and a lattice calculation⁵⁹⁾ also yields states with $\Delta M = (1.3 \pm 0.2) \text{ GeV}$ for the T system.

The discovery of "extra" states in the ϕ or T families would be of great importance. It could provide direct evidence for the gluonic degrees of freedom and more insight into the structure of bound states in QCD.

7. SUMMARY

We have discussed various aspects of quarkonium physics. The ϕ and T spectroscopies are well described by potential models. The known part of the $(Q\bar{Q})$ potential between 0.1 fm and 1.0 fm is essentially logarithmic, yet the success of QCD-like potentials with respect to the T leptonic width and the centre of gravity of the recently discovered χ_b states suggests that the Coulombic region starts below 0.1 fm. The ζ -spectroscopy of the t -quark should therefore provide direct evidence for asymptotic freedom.

The long-standing puzzle of the charmonium E1 transitions has been resolved. The discrepancy between theory and experiment in the ϕ family has been removed by incorporating relativistic corrections. For the T system, where the bound quarks move much slower than in the ϕ system, non-relativistic models describe the data well.

What is the relation between the phenomenological $(Q\bar{Q})$ potential and the static energy of an infinitely heavy $(Q\bar{Q})$ pair in QCD? The phenomenological success of potential models, in particular the evidence for flavour independence, suggests that the difference between the two quantities is small, and that deviations due to relativistic and coupled channel corrections, as well as the gluon condensate are not very large. The final answer will be provided by lattice calculations and the $(\bar{t}t)$ spectroscopy.

Spin dependent forces appear to be essentially understood within QCD: starting from the Wilson loop, a Hamiltonian has been derived which agrees with the successful "scalar+vector exchange" ansatz and the intuitive flux tube picture. The measured ϕ and T

fine structure splittings are in agreement with theoretical predictions.

There are reasons to believe that the T spectroscopy may lead to quantitative QCD tests. Valuable information will come from the $T - \eta_b$ hyperfine splitting and the hadronic χ_b widths. Furthermore, quarkonium spectroscopies should contain "extra" states, due to gluonic degrees of freedom, which may be detectable at hadron colliders.

If our hopes with respect to quantitative QCD tests should be fulfilled, the T system would indeed be the "hydrogen atom" of QCD: so far the basic binding force and the spin forces have been studied; a test of radiative corrections would probe, like the Lamb shift, the underlying field theory at the quantum level.

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REFERENCES

- 1) K. Gottfried, Proc. HEP-83, Brighton (UK), eds. J. Guy and C. Costain (1983).
- 2) P.M. Tuts, Proc. Lepton Photon Symp., Cornell University, eds. D.G. Cassel and D.L. Kreinick (1983)
- 3) See, for instance,
K. Berkelmann, Phys. Rep. 98C:145 (1983);
J. Lee Franzini and P. Franzini, Ann. Rev. Nucl. Part. Sci 33:1 (1983);
E.D. Bloom and C.W. Peck, *ibid.* 33:143 (1983).
- 4) K.F. Einsweiler, Proc. HEP-83, Brighton (UK), eds. J. Guy and C. Costain, (1983).
- 5) E.D. Bloom and C.W. Peck, Ref. 3).
- 6) T. Barnes, these proceedings.
- 7) J. Kuti, Proc. Lepton Photon Symp., Cornell University, eds. D.G. Cassel and D.L. Kreinick (1983).
- 8) E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D17:3090 (1978); 21:203 (1980).
- 9) J.L. Richardson, Phys. Lett. 82B:272 (1979).
- 10) W. Buchmüller, G. Grunberg and S.-H.H. Tye, Phys. Rev. Lett. 45:103 (1980); 45:587(E) (1980).
- 11) C. Quigg and J.L. Rosner, Phys. Lett. 71B:153 (1977);
M. Machacek and Y. Tomozawa, Prog. Theor. Phys. 58:1890 (1977).

- 12) A. Martin, Phys. Lett. 100B:511 (1981).
- 13) W. Buchmüller and S.-H.H. Tye, Phys. Rev. D24 (1981) 132.
- 14) G. Bhanot and S. Rudaz, Phys. Lett. 78B:119 (1978).
- 15) M.B. Voloshin, preprint ITEP-21 (1980).
- 16) M.A. Shifman, Proc. Lepton Photon Symp., Bonn, ed. W. Pfeil (1981).
- 17) M.A. Shifman, Ann. Rev. Nucl. Part. Sci. 33:199 (1983).
- 18) H.R. Rubinstein, private communication.
- 19) See, for instance,
E. Eichten, preprint Fermilab-Conf.-83/101 THY (1983);
Ref. 13), Appendix B.
- 20) K. Heikkilä, N.A. Törnqvist and S. Ono, Phys. Rev. D29:110 (1984).
- 21) C. Quigg and J.L. Rosner, Phys. Rev. D23:2625 (1981).
- 22) J.S. Bell and R.A. Bertlmann, CERN preprint TH.3769 (1983).
- 23) J.E. Gaiser, Proc. 2nd Moriond Workshop on New Flavours, Les Arcs, eds. J. Tran Thanh Van and L. Montanet, (1982).
- 24) K. Gottfried, Proc. Lepton Photon Symp., Hamburg, ed. F. Gutbrod (1977).
- 25) R. McClary and N. Byers, Phys. Rev. D28:1692 (1983).
- 26) A.B. Henriques, B.H. Kellett and R.G. Moorhouse, Phys. Lett. 64B:85 (1976).
- 27) K. Gottfried, Phys. Rev. Lett. 40:598 (1978).
- 28) A. Billoire, R. Lacaze, A. Morel and H. Navelet, Nucl. Phys. B155:493 (1979).
- 29) T.M. Yan, Phys. Rev. D22:1652 (1980).
- 30) Y.P. Kuang and T.M. Yan, Phys. Rev. D24:2874 (1981).
- 31) For a recent review, see
M.E. Peskin, preprint SLAC-PUB-3273 (1983).
- 32) For a recent review, see
J.L. Rosner, Univ. of Chicago preprint EFI 83/17 (1983).
- 33) For recent work, see Ref. 25);
P. Moxhay and J.L. Rosner, Phys. Rev. D28:1132 (1983);
H.J. Schnitzer, Phys. Lett. 134B:253 (1984).
- 34) P. Hasenfratz, R.R. Horgan, J. Kuti and J.-M. Richard, Phys. Lett. 95B:299 (1980).
- 35) K. Johnson, unpublished.
- 36) J. Baacke, Y. Igarashi and G. Kasperidus, Z. Phys. C13:131 (1982).
- 37) E. Eichten and F. Feinberg, Phys. Rev. D23:2724 (1981).
- 38) D. Gromes, Z. Phys. C22:265 (1984).
- 39) D. Gromes, Heidelberg preprint HD-THEP-84-5 (1984).
- 40) W. Buchmüller, Phys. Lett. 112B:479 (1982).
- 41) J.D. Jackson, Classical Electrodynamics, 2nd edition, J.Wiley and Sons, New York (1975) p. 541.
- 42) W. Buchmüller, Proc. 2nd Moriond Workshop on New Flavours, Les Arcs, eds. J. Tran Thanh Van and L. Montanet (1982).
- 43) C. Klopfenstein et al., CUSB collaboration, Phys. Rev. Lett. 51:160 (1983).

- 44) J.K. Bienlein, private communication.
- 45) A. Martin and J.-M. Richard, Phys. Lett. 115B:323 (1982).
- 46) A.J. Buras, Proc. Lepton Photon Symp., Bonn, ed. W. Pfeil (1981).
- 47) G.P. Lepage, Proc. Lepton Photon Symp., Cornell University, eds. D.G. Cassel and D.L. Kreinick (1983).
- 48) R. Barbieri, G. Curci, E. D'Emilio and E. Remiddi, Nucl. Phys. B154:535 (1979).
- 49) E. Remiddi, Proc. Workshop on Physics at Lear, Erice, eds. U. Gastaldi and R. Klapisch (1982).
- 50) P.B. Mackenzie and G.P. Lepage, Phys. Rev. Lett. 47:1244 (1981).
- 51) W. Buchmüller, Y.J. Ng and S.-H.H. Tye, Phys. Rev. D24:3003 (1981);
R. Barbieri, R. Gatto and E. Remiddi, Phys. Lett. 106B:497 (1981);
S.N. Gupta, S.F. Radford and W.W. Repko, Phys. Rev. D26:3305 (1982).
- 52) R. Barbieri, M. Caffo, R. Gatto and E. Remiddi, Phys. Lett. 95B:93 (1980).
- 53) S. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D28:228 (1983).
- 54) G. Grunberg, Phys. Lett. 135B:455 (1984).
- 55) A.T. Aerts and L. Heller, Phys. Rev. D29:513 (1984).
- 56) R.C. Giles and S.-H.H. Tye, Phys. Rev. Lett. 37:1175 (1976).
- 57) W. Buchmüller and S.-H.H. Tye, Phys. Rev. Lett. 44:850 (1980).
- 58) S. Ono, Orsay preprint LPTHE 84/13 (1984).
- 59) N.A. Campbell, L.A. Griffiths, C. Michael and P.E.L. Rakow, Liverpool preprint LTH 113 (1984).