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QED RADIATIVE CORRECTIONS AND RADIATIVE DECAYS OF THE
INTERMEDIATE WEAK BOSONS PRODUCED IN PROTON-ANTIPROTON COLLISIONS

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ABSTRACT

We present here a detailed description of the calculations of QED radiative corrections to the decays of W and Z bosons.

A comparison is made with experimental data. In particular it is shown that it is very unlikely that the three radiative Z^0 events observed by UA1 and UA2 can be explained in terms of standard QED radiative corrections.

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1. INTRODUCTION

The intermediate weak bosons Z^0 and W were discovered in $p\bar{p}$ interactions at CERN [1] by observing $Z^0 \rightarrow l^+l^-$ and $W^\pm \rightarrow l^\pm\nu$, ($l = \mu$ or e) and it is by the study of these purely leptonic decays that the detailed properties -- mass, spin, production cross-section, etc. -- can be measured.

The quantum electrodynamics (QED) radiative corrections must be taken into account as in any other high-energy l^+l^- interaction. Furthermore, the observation of very spectacular events $l^+l^-\gamma$ [2], where the photon energy is of the order of 30 GeV and the invariant mass of the $l^+l^-\gamma$ system is near the Z^0 mass, triggered a stronger interest in the precise knowledge of radiative corrections.

We describe here in detail a Monte Carlo calculation of the QED radiative corrections for $Z^0 \rightarrow l^+l^-$ and $W^\pm \rightarrow l^\pm\nu$, and we compare the predictions with experiment.

2. DESCRIPTION OF THE EVENT GENERATOR

The purpose is to generate simulated events according to first-order radiative corrections. A detector simulation is applied and the events are analysed with the same selection as the data.

The event generation consists of two parts: the generation of a quark-antiquark state according to the structure function of the proton, and the generation of the decays $Z^0 \rightarrow l^+l^-\gamma$, $W^\pm \rightarrow l^\pm\nu\gamma$, where $l = \mu$ or e .

The photon bremsstrahlung in the initial $q\bar{q}$ state has been neglected, since we are interested in the radiative decays of the intermediate weak bosons.

2.1 Quark-antiquark state

A quark-antiquark system is defined with respective fractions X_q and $X_{\bar{q}}$ of the beam energy generated with the known distributions F_u and F_d . The ranges of X_q and $X_{\bar{q}}$ are, however, restricted to values where the production cross-section for the boson is sizeable, i.e. in practice: $M_{\text{boson}} \pm 3\Gamma_{\text{boson}}$. It is obvious that X_q ($X_{\bar{q}}$) has to be above a certain threshold; thus the sea quarks, which are concentrated at low X values, do not contribute effectively. Therefore, only the valence quark contribution has been considered.

The $q\bar{q}$ system is given a transverse momentum with respect to the beam axis corresponding to the measured transverse momenta of Z^0 and W [3] (fig. 1), to avoid using a complicated quantum chromodynamics (QCD) generator.

The centre-of-mass energy of the $q\bar{q}$ system is:

$$\sqrt{s} = (X_q X_{\bar{q}} \cdot 540^2 - p_T^2)^{1/2} .$$

For each event a weight is defined:

$$\text{weight} = P_q \cdot P_{\bar{q}} \cdot \sigma_0$$

where

$$P_q \equiv F_q / X_q$$

$$P_{\bar{q}} \equiv F_{\bar{q}} / X_{\bar{q}} .$$

F_q and $F_{\bar{q}}$ are the fragmentation functions of the corresponding quarks (fig. 2) [4], and σ_0 is the cross-section for

$$q\bar{q} \rightarrow Z \text{ or } W \quad \text{at } \sqrt{s}.$$

The decay of the $q\bar{q}$ state is treated in a separate generator giving $Z^0 \rightarrow l^+ l^- \gamma$ or $W^\pm \rightarrow l^\pm \nu_l \gamma$. The Feynman graphs contributing to the first-order radiative corrections are shown in fig. 3.

2.2 The event generator for bremsstrahlung events

The event generator [5] used to calculate the rate and characteristics of radiative events is similar to generators developed for the calculation of radiative corrections in e^+e^- collisions [6]. The idea is to simulate non-radiative and radiative events at the same time with the appropriate angular distributions and the correct relative probabilities. Firstly, this procedure is described in general terms and then some explicit formulae are given. In the event generator only radiation in the final state is taken into account, both for bremsstrahlung and virtual corrections. The reason is that we are interested in the effect of radiation on the decay process and not on the production process.

The main purpose of the event generator is to describe correctly the hard bremsstrahlung events. In the program a specific photon energy value of $k_0\sqrt{s}/2$ is introduced, below which a photon is called soft and above which it is called hard. For k_0 one can take a value of 0.1% of the maximally allowed photon energy value. The emission of such a soft photon does not affect appreciably the kinematics of the processes:

$$\bar{q}(p_1) + q(p_2) \rightarrow Z^0 \rightarrow l^+(p_+) + l^-(p_-) \quad (1)$$

$$\bar{q}(p_1) + q'(p_2) \rightarrow W^\pm \rightarrow l^\pm + \bar{\nu}_l \quad (2)$$

What is affected by the soft photon emission and by the virtual corrections to the processes (1) and (2) is the size of the cross-section, or in other words the rate of the decays:

$$Z^0 \rightarrow l^+ l^- \quad (3)$$

and

$$W^\pm \rightarrow l^\pm + \bar{\nu}_l \quad (4)$$

Once soft photon emission is considered, virtual corrections should be taken into account in order to avoid infrared divergences. The QED virtual corrections are sufficient in this respect. To these, one can, in principle, add the weak virtual corrections.

The hard bremsstrahlung processes

$$\bar{q}q \rightarrow Z^0 \rightarrow l^+(p_+) + l^-(p_-) + \gamma(k) \quad (5)$$

$$\bar{q}q' \rightarrow W^- \rightarrow l^- + \bar{\nu}_l + \gamma \quad (6)$$

give rise to the radiative decays

$$Z^0 \rightarrow l^+ l^- \gamma \quad (7)$$

$$W^- \rightarrow l^- \bar{\nu}_l \gamma \quad (8)$$

The total decay width of the intermediate vector bosons can be written in the form (for a detailed discussion of the widths, see Consoli et al. [7])

$$\Gamma = \Gamma_0 (1 + \delta_Q + \delta_W) , \quad (9)$$

where δ_Q consists of virtual QED corrections, and soft and hard bremsstrahlung, and δ_W denotes the weak virtual corrections; Γ_0 is the lowest-order width for the decays (3) or (4). We can also write

$$\Gamma = \Gamma_0 (1 + \delta_W)(1 + \delta_Q) , \quad (10)$$

which, expanded to first order in α , gives again expression (9). We prefer [5] to use the form of expression (10) since the weak virtual corrections do not then affect the ratio of soft and hard events. This ratio, under this hypothesis, is uniquely determined by the division of δ_Q^S into δ_Q^S and δ_Q^H , i.e. the soft + virtual and the hard bremsstrahlung part of the correction. The ratio of δ_Q^S and δ_Q^H in turn are determined by the value of k_0 .

Their sum δ_Q is independent of k_0 , in principle, since theoretically k_0 is only needed to add soft and virtual corrections in order to obtain a finite result. In practice, k_0 can also be varied without affecting δ_Q . Decreasing k_0 increases δ_Q^H but decreases δ_Q^S . In order to use $1 + \delta_Q^S$ and δ_Q^H as weights for the generation of either soft or hard photons one should make sure that $1 + \delta_Q^S$ also remains positive for the small k_0 value chosen.

In the event generator a choice is first made between soft and hard bremsstrahlung on the basis of these two weights. If hard bremsstrahlung is chosen, events are generated using the exact multidifferential cross-sections for reactions (5) and (6). For soft emission, the momenta of the two final-state leptons should be generated according to the angular distribution of the reactions (1) and (2). In principle, this is a distribution which differs from the lowest-order angular distribution owing to virtual and soft photon effects.

The event generator considered here uses the lowest-order distribution, since we are not primarily interested in a refined calculation of the angular distribution of the two-body events. After all, the quark axis is not too accurately known. The virtual and soft corrections only matter for the ratio of the number of soft photon events generated relative to the number of hard photon events.

Let us now be more specific as regards the actual formulae used. For the Z^0 case one has

$$\delta_Q = \frac{3}{4} \frac{\alpha}{\pi} \quad (11)$$

and

$$\begin{aligned} \delta_Q^H(k_0) = \frac{2\alpha}{\pi} \left[\left(\ln \frac{S}{m^2} - 1 \right) \left[\ln \frac{1}{k_0} - \frac{1}{4} (1 - k_0)(3 - k_0) \right] - \frac{\pi^2}{6} \right. \\ \left. + \text{Li}_2(k_0) - \frac{1}{4} (1 - k_0)(3 - k_0) \ln(1 - k_0) + \frac{1}{8} (1 - k_0)(5 - k_0) \right], \quad (12) \end{aligned}$$

where $\text{Li}_2(k_0)$ is the dilogarithm

$$\left[\text{Li}_2(k_0) = - \int_0^{k_0} \frac{\ln(1-x)}{x} dx \right] .$$

The lowest-order cross-section for reaction (1) can be written as

$$\frac{d\sigma_0^Z}{d\Omega} = \frac{1}{64\pi^2} \frac{S}{|Z(S)|^2} \left[(V_L^2 + A_L^2)(V_q^2 + A_q^2)(1 + \cos^2 \theta) + 2V_L A_L V_q A_q \cos \theta \right] , \quad (13)$$

where the bremsstrahlung cross-section (5) takes the form

$$\frac{d\sigma}{dp_+^0 dE_\gamma d\Omega d\phi_\gamma} = \frac{1}{16S} \frac{1}{(2\pi)^5} \frac{1}{4} \sum_{\text{spins}} |M_1^Z|^2 , \quad (14)$$

with

$$\begin{aligned} \frac{1}{4} \sum_{\text{spins}} |M_1^Z|^2 &= \frac{4Se^2}{|Z(S)|^2} \left\{ \left[(V_L^2 + A_L^2)(V_q^2 + A_q^2) - 4V_L A_L V_q A_q \right] \right. \\ &\quad \times \left[\frac{(p_1 \cdot p_+)^2 + (p_2 \cdot p_-)^2}{(p_+ \cdot k)(p_- \cdot k)} - \frac{2m^2(p_1 \cdot p_+)^2}{S(p_- \cdot k)^2} - \frac{2m^2(p_2 \cdot p_-)^2}{S(p_+ \cdot k)^2} \right] \\ &\quad + \left[(V_L^2 + A_L^2)(V_q^2 + A_q^2) + 4V_L A_L V_q A_q \right] \\ &\quad \times \left[\frac{(p_1 \cdot p_-)^2 + (p_2 \cdot p_+)^2}{(p_+ \cdot k)(p_- \cdot k)} - \frac{2m^2(p_1 \cdot p_-)^2}{S(p_+ \cdot k)^2} - \frac{2m^2(p_2 \cdot p_+)^2}{S(p_- \cdot k)^2} \right] \left. \right\} , \quad (15) \end{aligned}$$

where we have used the notation

$$Z(S) = S - M_Z^2 + iM_Z \Gamma_Z^{\text{tot}} , \quad (16)$$

where the couplings in the standard model are:

$$V_l = -g(1 - 4 \sin^2 \theta_W) , \quad A_l = -g \quad \text{for leptons ,}$$

$$V_q = g(1 - \frac{8}{3} \sin^2 \theta_W) , \quad A_q = g \quad \text{for u and c quarks} \quad (17)$$

$$V_q = -g(1 - \frac{4}{3} \sin^2 \theta_W) , \quad A_q = -g \quad \text{for d and s quarks}$$

$$g = \frac{e}{2 \sin 2\theta_W} ,$$

and where m is the charged lepton mass and $p_+^0(p_-^0)$ the energy of the positive (negative) lepton.

For the W case one has [7]

$$\delta_Q = \frac{\alpha}{\pi} \left(\frac{77}{24} - \frac{\pi^2}{3} \right) \quad (18)$$

and

$$\delta_Q^H = \frac{\alpha}{\pi} \left[\left(\ln \frac{S}{m^2} - 2 \right) \left[\ln \frac{1}{k_0} - \frac{1}{4} (1 - k_0)(3 - k_0) \right] - \frac{\pi^2}{6} \right. \\ \left. + \text{Li}_2(k_0) - \frac{1}{4} (1 - k_0)(3 - k_0) \ln(1 - k_0) + \frac{1}{24} (1 - k_0)(17 - k_0) \right] . \quad (19)$$

Introducing $W^{-1}(S)$ as the W propagator, similarly to $Z^{-1}(S)$ previously, we write the cross-section for process (2) as

$$\frac{d\sigma_0^W}{d\Omega} = \frac{1}{16S(2\pi)^2} \frac{1}{4} \sum_{\text{spins}} |M_0^W|^2 , \quad (20)$$

with

$$\frac{1}{4} \sum |M_0^W|^2 = \frac{64g_L^2 g_q^2 (p_u \cdot p_e)^2}{|W(S)|^2} = \frac{4g_L^2 g_q^2 S^2 (1 - \cos \theta)^2}{|W(S)|^2}, \quad (21)$$

where the coupling constant g_q is a function of the Cabibbo angle θ_C :

$$g_q = g_L \cos \theta_C$$

$$g_L = \frac{e}{2\sqrt{2} \sin \theta_W}. \quad (22)$$

For the bremsstrahlung cross-section it can be shown [5] that the diagram where a photon is emitted from an internal W line gives contributions to both initial- and final-state radiation. Keeping the latter contribution one obtains

$$\frac{d\sigma_1^W}{dp_L^0 dE_\gamma d\Omega_L d\phi_\gamma} = \frac{1}{16S} \frac{1}{(2\pi)^5} \frac{1}{4} \sum_{\text{spins}} |M_1^W|^2$$

with

$$\begin{aligned} \frac{1}{4} \sum |M_1^W|^2 = & \frac{32e^2 g_L^2 g_q^2}{|W(S)|^2} \left[\left[\frac{q^2}{(p_L \cdot k)(q \cdot k)} - \frac{q^2}{(q \cdot k)^2} \right] [(p_u \cdot p_L)^2 + (p_d \cdot p_v)^2] \right. \\ & \left. - \frac{2m^2 (p_d \cdot p_v)^2}{(p_L \cdot k)^2} \right], \end{aligned} \quad (23)$$

where

$$q = p_L + p_v. \quad (24)$$

3. MAIN FEATURES OF INTERNAL BREMSSTRAHLUNG

First we wish to remark that all the results mentioned below do not depend on the exact value of the parameter k_0 described in the previous section, provided it is small compared to the photon energies we can observe. Therefore, we have fixed that value as 0.005. The hard bremsstrahlung contribution to the total cross-section is then 21% for $W^+ \rightarrow e^+ \nu$ and 43.5% for $Z^0 \rightarrow e^+ e^-$.

In fig. 4, the transverse energy distribution of the photons is shown. In spite of the sharp peaking at low values, the distribution has a long tail and 5% of the $W \rightarrow e \nu$ decays (11% for the Z^0 decay into two electrons) contain a photon of transverse energy greater than 10 GeV. Tables 1a and 1b show the fraction of radiative decays of the W and Z, respectively, as a function of the photon energy. The rate of radiative decays in the electron channel is about twice that in the muon channel. The angle between the photon and the closest charged lepton ($\theta_{\gamma l}$) in the lab. frame is also shown (fig. 5). Again, the distribution has a long tail even for high-energy photons: table 2 gives the scatter plot of $\theta_{\gamma l}$ versus F_γ (the fraction of energy carried out by the photon) in the decay of the W and Z bosons. From this table it is possible to derive the probability of a photon being emitted with $F_\gamma > F_\gamma^0$ and $\theta_{\gamma l} > \theta_0$.

4. EXTERNAL BREMSSTRAHLUNG

4.1 The UA1 experimental set up

The internal bremsstrahlung is not the only phenomenon responsible for topologies with a hard photon. After the decay of the intermediate bosons, the outgoing charged leptons have to go through several layers of material before they are detected: beam pipe, chamber wall, etc.

When going through the beam pipe and surrounding material, a particle travels a number of radiation lengths in matter, depending on the position of the interaction along the beam and the angle of the track. Owing to the corrugated shape of the beam pipe, a track with an angle to the beam smaller than $\sim 75^\circ$ can cross the beam-pipe wall several times, and can even be tangential to that wall, thus having a long range in the iron. Because of this "multiple traversal", and although the mean thickness traversed by electrons from boson decays is only $0.047 X_0$ ($X_0 \simeq 1.6$ cm), the total thickness seen by the track can be as large as $0.3 X_0$, giving a high probability to radiate a sizeable fraction of the lepton energy into a photon (fig. 6).

In the interaction region, the vacuum pipe is well represented by a cylinder of mean radius 69 mm with corrugations of period 22.4 mm. The internal radius varies from 65 mm to 73 mm and the thickness of the wall is 0.17 mm of stainless steel with radiation length $X_0 \approx 1.6$ cm. We have neglected the fact that the beam pipe is made of pieces 60 cm long soldered together. A particle still has to traverse [8] covered tunnels and chamber walls, which have been approximated by a homogeneous cylinder of thickness $0.28\% X_0$ at normal incidence.

The tracks from the decay of a $W^\pm$ or a Z^0 are followed by the program which computes all the intersections with the inner and outer surfaces of the vacuum pipe and then determines the total path length in matter. The position of the vertex is randomised according to the experimental vertex distribution (a Gaussian shape with a σ of 10 cm along the beam direction). Tracks are assumed to be straight: one neglects the curvature over such a small distance (20 cm). The amount of matter penetrated by a track depends on the angle of the track with respect to the beam, and also on the position of the vertex.

4.2 The probability distribution of external bremsstrahlung

Let us first remark that the strong variations of the amount of material traversed make it rather difficult to predict, for one event, the probability of external bremsstrahlung. The only way to improve our knowledge of that probability would be to know the exact position of the corrugations relative to the UA1 reference system.

Many authors have derived formulae for the emission of a photon by a charged particle in an electromagnetic field. In the complete screening limit [9], one finds in the case of UA1:

$$P(E,k)dk = \frac{\Delta r}{X_0} \left[1 + \left(1 - \frac{k}{E}\right)^2 - \left(\frac{2}{3} - \frac{1}{37.5}\right)\left(1 - \frac{k}{E}\right) \right] \frac{dk}{k},$$

where k = energy of the outgoing photon

E = energy of the incident electron.

This formula can be used to generate events if one assumes that the number of radiation lengths $\Delta r/X_0$ is small enough so that all the energy is radiated in one single emission. This is true in our case where $\Delta r/X_0 \approx 0.047$. It becomes only an approximation when the track is exactly tangential to the corrugations.

This formula has also an infrared divergence for $k \rightarrow 0$. It is thus necessary to put a cut-off on the low values of k . This cut-off comes naturally from the resolution of the apparatus. Since the calorimeter resolution is $16\%/\sqrt{E}$, in the energy range of the intermediate boson one has $\Delta E/E \lesssim 3\%$, hence we have used a value of the cut-off which is $k/E = 0.01$. For values below 0.01 the probability distribution has been taken as constant and such that $\int_0^\infty P(E,k) dk = 1$.

Indeed, the physical distributions are not sensitive to the exact value of this cut-off. We also assume that the bremsstrahlung is emitted

along the parent lepton. This is justified by the sharply-peaked angular distribution of the external radiation. Thus, this phenomenon has little effect on the measurement of the lepton energy, since the photon hits, in general, the same cell, but it affects the measurement of the momentum in the central detector. The tail it produces on the $(1/p - 1/E)$ distribution is however at the per cent level, and is hard to dissociate from the effect of the experimental resolution on the momentum p and the energy E , for small photon energies ($E_\gamma < 10$ GeV).

5. UA1 DETECTOR RESOLUTION

The resolution of electromagnetic energies has been measured in test beams and is well known. It amounts to $\Delta E/E = 16\%/\sqrt{E}$ in the central part and $\Delta E_T/E_T = 15\%/\sqrt{E_T}$ in the "bouchons" (electromagnetic end-caps). The error on the momentum of tracks measured in the central detector is more complicated and varies with the momentum and the angle of the track. In order to have a good estimate of this error for the leptons from boson decay we have used the errors computed for each lepton track of our weak boson candidates. It follows that a good parametrization of these errors is given on average by $\Delta(1/p_T) = 0.015 \text{ GeV}^{-1}$.

6. SELECTION CRITERIA

In the experimental sample, strict selection criteria had been applied to the events in order to reduce backgrounds to a negligible amount. These cuts affect sizeably the various distributions and they have also been included in the simulation. In short, they are the following:

$$Z^0 \rightarrow e^+ e^-: \quad p_T^e > 7 \text{ GeV}/c, \quad E_T^e > 15 \text{ GeV}, \quad \theta^e \geq 5^\circ$$

$$Z^0 \rightarrow \mu^+ \mu^-: \quad p_T^\mu > 15 \text{ GeV}/c, \quad \theta^\mu \geq 40^\circ$$

$$W \rightarrow e\nu: \quad p_T^e > 7 \text{ GeV}/c, \quad E_T^e > 15 \text{ GeV}, \quad \theta^e > 5^\circ$$

$$W \rightarrow \mu\nu: \quad p_T^\mu > 15 \text{ GeV}/c, \quad \theta^\mu > 40^\circ$$

where θ is the angle of the charged lepton with respect to the beam axis. The energy of the lepton measured by calorimeters is taken to be equal to the sum of the energies of the cell found and of the two adjacent cells. Thus, the energy of a photon emitted by internal radiation is included in the energy of the lepton if -- and only if -- it hits the same cell as the lepton, or an adjacent cell. A simplified representation of the calorimeter granularity was used.

Figure 7 shows a comparison between the UA1 data on the distribution of $\Delta \equiv [(1/p) - (1/E)]/[\sigma^2(1/p) + \sigma^2(1/E)]^{1/2}$ and our calculation, both for Z and W decays. Figure 7a shows the very good agreement between the data and our calculation. The event at $\Delta = -11$ appears to have an 80 GeV photon, collinear with the charged track in the central chamber. Since that track is very forward, the error on its momentum is much smaller than the average error used here in our calculation. In fact, table 2c shows that the probability of emitting such a photon is of the order of 3×10^{-3} . In figure 7b, one entry at $\Delta \approx -5$ corresponds to the well-known radiative Z^0 decay.

The effect of photon bremsstrahlung, detector resolution, and event selection is shown in fig. 8 for the transverse mass of W events and in fig. 9 for the invariant mass of e^+e^- pairs in Z^0 decays.

7. ENERGY DEPOSITED IN THE ELECTROMAGNETIC CALORIMETER ALONG MUON TRACKS IN THE W OR Z DECAYS INTO MUONS

For each muon we determine which calorimeter cell is hit and we calculate the energy found in that cell and the two adjacent cells.

There are three contributions to the energy measured:

- i) Real photons coming from bremsstrahlung. We used here the photon energy obtained in our generator (fig. 10).
- ii) Ionization energy of the muon in the calorimeter scintillators. We used the distribution obtained in a test beam (fig. 11) parametrized by a Landau function.
- iii) Energy coming from the rest of the event and corresponding to an underlying minimum-bias event (fig. 12).

Figure 13 shows the sums of contributions (i), (ii), and (iii) compared with the UA1 data from $W^+ \rightarrow \mu^+ \nu$ [11]. The agreement is excellent and there is no indication of an extra source of hard photons associated with the W production in the $W^+ \rightarrow \mu^+ \nu$ sample.

8. PROBABILITY OF HARD PHOTON RADIATION IN Z^0 DECAY

UA1 and UA2 have observed in total three Z^0 decays accompanied by very energetic photons²⁻¹¹. Table 3a gives the properties of those events. We can calculate the probability for each event to have $F > F_0$ and $\theta > \theta_{\gamma L}$. In fact, the statistically significant variable is the integral over the configurations with a probability density equal to or smaller than that of the event. It can easily be shown that this number is equal to $\alpha(1 - \ln \alpha)$, where α is the probability that $F > F_0$ and $\theta > \theta_{\gamma L}$.

Two cases were considered: a) Without selection, b) After the selection described in Section 6. Given a total sample of four $Z^0 \rightarrow e^+ e^-$ and five $Z^0 \rightarrow \mu^+ \mu^-$ for UA1 and eight $Z^0 \rightarrow e^+ e^-$ for UA2, one would expect a total of at most 0.08 event (95% C.L.) to be compared to 3 events found.

Using the probabilities obtained without any selection, the number of events expected would be at most 0.1 event (95% C.L.), which is still very far from the observed values.

In conclusion, we can state that the explanation of the radiative z^0 decays observed by UA1 and UA2 in terms of standard QED radiative corrections is very unlikely.

9. CONCLUSION

We have described here in detail the precise calculations of radiative corrections used in the UA1 publications on the observation of the W and Z bosons.

The fact that these radiative corrections do not explain the rate of events with a hard photon in z^0 decays makes the next $p\bar{p}$ run extremely important, since we expect to increase our present statistics by a factor of 5.

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REFERENCES

- [1] G. Arnison et al., Phys. Lett. 122B (1983) 103.
G. Arnison et al., Phys. Lett. 126B (1983) 398.
- [2] G. Arnison et al., Phys. Lett. 135B (1984) 250.
- [3] G. Arnison et al., Phys. Lett. 129B (1983) 273.
- [4] J.F. Owens and E. Reya, Phys. Rev. D 17 (1978) 3003.
M. Gluck, E. Hoffmann and E. Reya, Z. Phys. C 13 (1982) 119.
- [5] F.A. Berends and R. Kleiss, Hard photon effects in W^+ and Z^0 decays,
Leiden University preprint, 1984.
- [6] F.A. Berends and R. Kleiss, Nucl. Phys. B177 (1981) 237.
F.A. Berends, R. Kleiss and S. Jadach, Nucl. Phys. B202 (1982) 63.
- [7] M. Consoli, S. Lo Presti and L. Maiani, Nucl. Phys. B223 (1983) 474.
W. Marciano and A. Sirlin, Phys. Rev. D 8 (1973) 3612.
- [8] B. Sadoulet, UA1-TN/82-34 (1982).
- [9] Yung-su Tsai, Rev. Mod. Phys. 46 (1974) 815; see erratum Rev. Mod.
Phys. 49 (1977) 421.
B. Rossi, High-energy particles (Prentice-Hall, New York, 1952).
- [10] G. Arnison et al., Phys. Lett. 134B (1984) 469.
- [11] P. Bagnaia et al., Evidence for $Z^0 \rightarrow e^+e^-$ at the CERN $\bar{p}p$ collider,
Phys. Lett. 129B (1983) 130.

Table 1

Fraction (in %) of events with $k > k_1$ for electron
and muon decays of $W^\pm$ and Z

a) $W \rightarrow l^\pm \nu \gamma$ ($m = 81$ GeV)

k_1	Electron	Muon
0.01	19.7	10.1
0.05	11.6	5.9
0.1	8.3	4.2
0.2	5.2	2.6
0.3	3.6	1.8
0.4	2.6	1.3
0.5	1.8	0.9
0.6	1.3	0.6
0.7	0.9	0.4
0.8	0.5	0.2
0.9	0.2	0.1

b) $Z^0 \rightarrow l^+ l^- \gamma$ ($m = 90$ GeV)

k_1	Electron	Muon
0.01	41.1	22
0.05	24.2	12.9
0.1	17.3	9.1
0.2	10.9	5.7
0.3	7.5	3.9
0.4	5.4	2.7
0.5	3.8	1.9
0.6	2.7	1.3
0.7	1.8	0.9
0.8	1.1	0.5
0.9	0.5	0.2

Table 2

Considering the photon and the closest charged lepton

this table gives the probability of photon bremsstrahlung as a function of $\theta_{\gamma l}$
(the angle in space between the photon and the charged lepton) and the fraction

of energy carried out by the photon: $F_{\gamma} = E_{\gamma} / (E_{\gamma} + E_l)$.

a) $Z^0 \rightarrow e^+ e^- \gamma$

Angle between photon and closest charged lepton (degrees)

	0.	2.	4.	6.	8.	10.	15.	20.	30.	45.	90.
0.0	0.672E+00	0.330E-02	0.232E-02	0.163E-02	0.141E-02	0.222E-02	0.174E-02	0.247E-02	0.240E-02	0.429E-02	
1.0	0.409E-01	0.363E-02	0.195E-02	0.150E-02	0.130E-02	0.238E-02	0.150E-02	0.245E-02	0.233E-02	0.375E-02	
2.0	0.241E-01	0.184E-02	0.144E-02	0.940E-03	0.521E-03	0.132E-02	0.809E-03	0.132E-02	0.149E-02	0.206E-02	
3.0	0.156E-01	0.131E-02	0.869E-03	0.619E-03	0.415E-03	0.850E-03	0.708E-03	0.120E-02	0.102E-02	0.129E-02	
4.0	0.122E-01	0.111E-02	0.633E-03	0.471E-03	0.399E-03	0.699E-03	0.385E-03	0.988E-03	0.694E-03	0.119E-02	
5.0	0.183E-01	0.170E-02	0.946E-03	0.831E-03	0.503E-03	0.884E-03	0.626E-03	0.952E-03	0.112E-02	0.168E-02	
7.0	0.193E-01	0.162E-02	0.112E-02	0.773E-03	0.650E-03	0.984E-03	0.584E-03	0.107E-02	0.122E-02	0.185E-02	
10.0	0.202E-01	0.191E-02	0.104E-02	0.614E-03	0.607E-03	0.124E-02	0.805E-03	0.117E-02	0.133E-02	0.194E-02	
15.0	0.141E-01	0.127E-02	0.794E-03	0.609E-03	0.463E-03	0.867E-03	0.516E-03	0.752E-03	0.937E-03	0.141E-02	
20.0	0.183E-01	0.196E-02	0.979E-03	0.716E-03	0.663E-03	0.775E-03	0.520E-03	0.113E-02	0.898E-03	0.173E-02	
30.0	0.117E-01	0.108E-02	0.693E-03	0.394E-03	0.245E-03	0.612E-03	0.458E-03	0.694E-03	0.512E-03	0.116E-02	
40.0	0.798E-02	0.713E-03	0.351E-03	0.309E-03	0.198E-03	0.457E-03	0.159E-03	0.402E-03	0.343E-03	0.717E-03	
50.0	0.603E-02	0.730E-03	0.363E-03	0.250E-03	0.220E-03	0.225E-03	0.148E-03	0.298E-03	0.303E-03	0.481E-03	
60.0	0.401E-02	0.287E-03	0.170E-03	0.769E-04	0.784E-04	0.192E-03	0.142E-03	0.807E-04	0.135E-03	0.241E-03	
70.0	0.262E-02	0.307E-03	0.174E-03	0.549E-04	0.124E-03	0.931E-04	0.567E-04	0.471E-04	0.368E-04	0.152E-04	
80.0	0.557E-03	0.623E-04	0.279E-04	0.112E-04	0.558E-05	0.208E-04	0.113E-04	0.237E-04	0.110E-05	0.985E-05	
100.0											

Fraction of photon energy

Table 2

b) $Z^0 \rightarrow \mu^+ \mu^- \gamma$

Angle between photon and closest charged lepton (degrees)

	0.	2.	4.	6.	8.	10.	15.	20.	30.	45.	90.
0.0											
1.0	0.829E+00	0.332E-02	0.223E-02	0.143E-02	0.102E-02	0.237E-02	0.138E-02	0.197E-02	0.214E-02	0.384E-02	
2.0	0.107E-01	0.421E-02	0.188E-02	0.131E-02	0.919E-03	0.208E-02	0.163E-02	0.195E-02	0.214E-02	0.399E-02	
3.0	0.694E-02	0.180E-02	0.133E-02	0.613E-03	0.520E-03	0.109E-02	0.875E-03	0.114E-02	0.124E-02	0.216E-02	
4.0	0.462E-02	0.104E-02	0.875E-03	0.583E-03	0.311E-03	0.856E-03	0.754E-03	0.838E-03	0.928E-03	0.176E-02	
5.0	0.335E-02	0.745E-03	0.792E-03	0.452E-03	0.258E-03	0.567E-03	0.508E-03	0.494E-03	0.105E-02	0.108E-02	
7.0	0.436E-02	0.137E-02	0.108E-02	0.706E-03	0.492E-03	0.925E-03	0.772E-03	0.846E-03	0.128E-02	0.159E-02	
10.0	0.474E-02	0.141E-02	0.933E-03	0.591E-03	0.699E-03	0.824E-03	0.792E-03	0.100E-02	0.121E-02	0.195E-02	
15.0	0.568E-02	0.192E-02	0.905E-03	0.568E-03	0.599E-03	0.109E-02	0.736E-03	0.996E-03	0.138E-02	0.195E-02	
20.0	0.384E-02	0.126E-02	0.518E-03	0.407E-03	0.276E-03	0.631E-03	0.555E-03	0.715E-03	0.101E-02	0.151E-02	
30.0	0.462E-02	0.162E-02	0.864E-03	0.571E-03	0.496E-03	0.109E-02	0.430E-03	0.920E-03	0.107E-02	0.154E-02	
40.0	0.245E-02	0.122E-02	0.501E-03	0.397E-03	0.293E-03	0.544E-03	0.396E-03	0.501E-03	0.775E-03	0.119E-02	
50.0	0.180E-02	0.637E-03	0.261E-03	0.308E-03	0.324E-03	0.463E-03	0.185E-03	0.499E-03	0.354E-03	0.782E-03	
60.0	0.106E-02	0.593E-03	0.274E-03	0.274E-03	0.188E-03	0.337E-03	0.146E-03	0.423E-03	0.201E-03	0.634E-03	
70.0	0.428E-03	0.204E-03	0.100E-03	0.154E-03	0.460E-04	0.138E-03	0.161E-03	0.155E-03	0.136E-03	0.274E-03	
80.0	0.113E-04	0.489E-04	0.380E-06	0.0	0.0	0.428E-04	0.775E-06	0.916E-05	0.341E-04	0.937E-04	
100.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.469E-05	0.0	0.167E-06	

Fraction of photon energy

Table 2

c) $W^- \rightarrow e^- \bar{\nu} \gamma$

Angle between photon and closest charged lepton (degrees)

	0.	2.	4.	6.	8.	10.	15.	20.	30.	45.	90.
0.0											
1.0	0.825E+00	0.211E-02	0.124E-02	0.902E-03	0.663E-03	0.122E-02	0.760E-03	0.108E-02	0.103E-02	0.113E-02	
2.0	0.229E-01	0.207E-02	0.122E-02	0.947E-03	0.611E-03	0.108E-02	0.807E-03	0.108E-02	0.874E-03	0.118E-02	
3.0	0.134E-01	0.121E-02	0.635E-03	0.505E-03	0.451E-03	0.665E-03	0.426E-03	0.666E-03	0.601E-03	0.719E-03	
4.0	0.927E-02	0.892E-03	0.500E-03	0.352E-03	0.273E-03	0.501E-03	0.336E-03	0.496E-03	0.374E-03	0.453E-03	
5.0	0.680E-02	0.694E-03	0.391E-03	0.292E-03	0.250E-03	0.400E-03	0.263E-03	0.364E-03	0.285E-03	0.292E-03	
7.0	0.107E-01	0.935E-03	0.577E-03	0.374E-03	0.282E-03	0.486E-03	0.417E-03	0.568E-03	0.457E-03	0.537E-03	
10.0	0.109E-01	0.107E-02	0.677E-03	0.399E-03	0.333E-03	0.520E-03	0.459E-03	0.504E-03	0.404E-03	0.562E-03	
15.0	0.118E-01	0.100E-02	0.684E-03	0.496E-03	0.330E-03	0.642E-03	0.398E-03	0.533E-03	0.526E-03	0.674E-03	
20.0	0.753E-02	0.756E-03	0.426E-03	0.279E-03	0.226E-03	0.361E-03	0.228E-03	0.394E-03	0.302E-03	0.448E-03	
30.0	0.103E-01	0.840E-03	0.526E-03	0.404E-03	0.360E-03	0.485E-03	0.330E-03	0.527E-03	0.462E-03	0.623E-03	
40.0	0.635E-02	0.539E-03	0.378E-03	0.260E-03	0.159E-03	0.362E-03	0.164E-03	0.288E-03	0.290E-03	0.317E-03	
50.0	0.419E-02	0.436E-03	0.243E-03	0.124E-03	0.134E-03	0.212E-03	0.165E-03	0.210E-03	0.222E-03	0.295E-03	
60.0	0.320E-02	0.296E-03	0.166E-03	0.101E-03	0.573E-04	0.120E-03	0.760E-04	0.919E-04	0.113E-03	0.136E-03	
70.0	0.220E-02	0.219E-03	0.123E-03	0.663E-04	0.503E-04	0.108E-03	0.525E-04	0.522E-04	0.371E-04	0.415E-04	
80.0	0.118E-02	0.959E-04	0.867E-04	0.397E-04	0.424E-04	0.690E-04	0.129E-04	0.243E-04	0.327E-04	0.234E-04	
100.0	0.247E-03	0.414E-04	0.179E-04	0.711E-05	0.123E-05	0.663E-05	0.465E-05	0.523E-05	0.220E-05	0.669E-05	

Fraction of photon energy

Table 2

d) $W^- \rightarrow \mu^- \bar{\nu} \gamma$

Angle between photon and closest charged lepton (degrees)

	0.	2.	4.	6.	8.	10.	15.	20.	30.	45.	90.
0.0	0.911E+00	0.200E-02	0.106E-02	0.863E-03	0.611E-03	0.108E-02	0.802E-03	0.101E-02	0.803E-03	0.115E-02	
1.0	0.602E-02	0.198E-02	0.124E-02	0.929E-03	0.648E-03	0.120E-02	0.833E-03	0.112E-02	0.110E-02	0.123E-02	
2.0	0.324E-02	0.101E-02	0.736E-03	0.465E-03	0.471E-03	0.688E-03	0.422E-03	0.627E-03	0.588E-03	0.775E-03	
3.0	0.253E-02	0.850E-03	0.485E-03	0.391E-03	0.260E-03	0.363E-03	0.347E-03	0.542E-03	0.423E-03	0.625E-03	
4.0	0.172E-02	0.568E-03	0.320E-03	0.277E-03	0.225E-03	0.384E-03	0.242E-03	0.372E-03	0.318E-03	0.358E-03	
5.0	0.283E-02	0.966E-03	0.615E-03	0.389E-03	0.253E-03	0.562E-03	0.432E-03	0.450E-03	0.462E-03	0.683E-03	
7.0	0.276E-02	0.938E-03	0.586E-03	0.431E-03	0.294E-03	0.616E-03	0.342E-03	0.634E-03	0.586E-03	0.643E-03	
10.0	0.300E-02	0.924E-03	0.633E-03	0.538E-03	0.362E-03	0.576E-03	0.451E-03	0.616E-03	0.493E-03	0.754E-03	
15.0	0.181E-02	0.756E-03	0.369E-03	0.339E-03	0.232E-03	0.385E-03	0.275E-03	0.356E-03	0.402E-03	0.482E-03	
20.0	0.228E-02	0.845E-03	0.472E-03	0.353E-03	0.260E-03	0.581E-03	0.380E-03	0.522E-03	0.507E-03	0.698E-03	
30.0	0.132E-02	0.521E-03	0.304E-03	0.217E-03	0.137E-03	0.364E-03	0.210E-03	0.282E-03	0.317E-03	0.500E-03	
40.0	0.755E-03	0.330E-03	0.219E-03	0.152E-03	0.105E-03	0.179E-03	0.111E-03	0.188E-03	0.201E-03	0.263E-03	
50.0	0.440E-03	0.162E-03	0.153E-03	0.982E-04	0.635E-04	0.948E-04	0.110E-03	0.139E-03	0.136E-03	0.196E-03	
60.0	0.186E-03	0.113E-03	0.499E-04	0.245E-04	0.477E-04	0.963E-04	0.468E-04	0.556E-04	0.669E-04	0.108E-03	
70.0	0.284E-04	0.198E-04	0.0	0.104E-04	0.106E-04	0.232E-04	0.113E-04	0.215E-04	0.186E-04	0.672E-04	
80.0	0.0	0.0	0.540E-05	0.0	0.0	0.195E-05	0.0	0.696E-07	0.546E-05	0.490E-05	
100.0											

Fraction of photon energy

Table 3aCharacteristics of radiative Z^0 events

	$e^+e^-\gamma$		$\mu^+\mu^-\gamma$
	UA1	UA2	UA1
E_γ	38.8 ± 1.5	24.4 ± 1.4	28.3 ± 3
E_{L_1}	9 ± 1	11.4 ± 0.9	$50.6^{+5.8}_{-4.7}$
E_{L_2}	61 ± 1.2	68.5 ± 1.6	42.2^{+44}_{-14}
$F = E_\gamma / (E_\gamma + E_{L_1})$	0.81	0.68	0.35
$\theta_{\gamma L_1}$	14.4	31.8	7.9

Table 3bProbabilities of hard photon bremsstrahlung for the radiative Z^0 events

	$e^+e^-\gamma$		$\mu^+\mu^-\gamma$
	UA1	UA2	UA1
a) No cut	1.1×10^{-3}	1.4×10^{-3}	1.2×10^{-2}
Upper limit at 95% C.L.	2×10^{-3}	1.7×10^{-3}	1.8×10^{-2}
b) After selection	1.4×10^{-5}	2.0×10^{-4}	7.8×10^{-3}

Figure captions

Fig. 1 : W and Z transverse-momentum distributions

Fig. 2 : Fragmentation functions F_u and F_d .

Fig. 3 : Feynman diagrams contributing to the first-order radiative corrections.

a) W decay

b) Z decay

Fig. 4 : Fraction of energy carried by the photon: $E_\gamma/(M/2)$:

a) $Z^0 \rightarrow e^+ e^- \gamma$

b) $Z^0 \rightarrow \mu^+ \mu^- \gamma$

c) $W^\pm \rightarrow e^\pm \nu \gamma$

d) $W^\pm \rightarrow \mu^\pm \nu \gamma$

Fig. 5 : Angle between the photon and the closest charged lepton.

a) $Z^0 \rightarrow e^+ e^- \gamma$

b) $Z^0 \rightarrow \mu^+ \mu^- \gamma$

c) $W^\pm \rightarrow e^\pm \nu \gamma$

d) $W^\pm \rightarrow \mu^\pm \nu \gamma$

Fig. 6 : Amount of matter along a track as a function of the angle with respect to the beam axis (UA1).

Fig. 7 : $(1/p - 1/E)/\sqrt{[\sigma^2(1/p) + \sigma^2(1/E)]}$ (from UA1 data).

a) $W \rightarrow e \nu \gamma$

b) $Z^0 \rightarrow e^+ e^- \gamma$

Fig. 8 : Transverse mass for $W \rightarrow e \nu \gamma$. The dotted curve corresponds to one selection criteria described in Section 6. The other curve corresponds to events for which $E_T^\nu > 30$ GeV and $E_T^e > 30$ GeV.

Fig. 9 : e^+e^- invariant mass for $Z^0 \rightarrow e^+e^-\gamma$. The dotted line includes photon bremsstrahlung only. The other curve is obtained after smearing the electron energies according to the calorimeter resolution.

Fig. 10 : Photon energy spectrum along the muon direction.

a) $Z^0 \rightarrow \mu^+\mu^-\gamma$

b) $W^\pm \rightarrow \mu^\pm\nu\gamma$.

Fig. 11 : $\Delta E/\Delta X$ measured in a test beam along the muon tracks, in the electromagnetic calorimeter of UA1.

Fig. 12 : Energy obtained along the muon tracks owing to the underlying minimum-bias event (in the UA1 electromagnetic calorimeter).

Fig. 13 : Calculation of the total energy along a muon track compared with the experimental measurement (UA1).

a) $Z^0 \rightarrow \mu^+\mu^-\gamma$

b) $W \rightarrow \mu\nu\gamma$

c) Comparison with experiment for 14 $W \rightarrow \mu\nu$ events

Fig. 1

Fig. 2

a) W DECAY:

b) Z DECAY:

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13