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THE INVISIBLE AXION, THE GRAVITINO AND COSMOLOGY

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ABSTRACT

I review the invisible axion and also discuss the gravitino problem in cosmology. Emphasis is placed on the dynamical nature of the $\bar{\theta}$ parameter. It is pointed out that the upper bound of the Peccei-Quinn symmetry breaking scale depends on the particle physics models and can easily go up to 10^{14} GeV. The gravitino with about a 100 GeV mass is not favoured from the observed abundances of light nuclei.

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1. THE STRONG CP PROBLEM

1.1 General Introduction

The invisible axion is applied extensively in this conference. I will review the theoretical background of the invisible axion and its properties. Please, notice that it is a plausible particle which you can apply safely in cosmology.

The strong CP problem belongs to the category of the naturalness problems.¹ For a small parameter ϵ in physics, one needs a natural explanation of it, which is a *naturalness problem*. There are two types of solutions to the naturalness problem, a natural solution in the sense of calculability and a natural solution in the sense of symmetry. The latter type of solution suggested by 't Hooft is very interesting because $\epsilon \rightarrow 0$ increases the symmetry of the Lagrangian. In other words, one can understand $\epsilon = 0$ by imposing such a symmetry.

There are three outstanding naturalness problems: the cosmological constant problem, the gauge hierarchy problem and the strong CP problem. The cosmological constant problem is not understood at present and I have nothing to say about this. The gauge hierarchy problem or the quadratic scalar mass problem is attacked in technicolour theories and in supersymmetric theories. In particular, the supersymmetry solution² to this problem introduces superpartners to the particles we know, so that the quadratic divergence of the scalar mass is tamed. This implies the existence of the gravitino, the superpartner of the graviton. The best-understood solution of the naturalness problems is the Peccei-Quinn solution of the strong CP problem.³ It will not surprise us that we are destined to introduce a new particle (the invisible axion) along this line, as good solutions of naturalness problems usually do.

1.2. Why the $\bar{\theta}$ Parameter?

Due to the existence of the instanton solutions in non-Abelian gauge theories, one has to work in a $\bar{\theta}$ vacuum but not in one of the $|n\rangle$ vacua.⁴ In the $\bar{\theta}$ vacuum, there exists a superselection rule, neglecting quarks,

$$\langle \bar{\theta}' | e^{-\int H dt} | \bar{\theta} \rangle = \delta(\bar{\theta} - \bar{\theta}') \int [dA_\mu] \exp(-S_E) \quad (1)$$

The action also gets a contribution from a formal total divergence term

$$\mathcal{L}_{\text{eff}} = \frac{\bar{\theta}}{32\pi^2} \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \quad (2)$$

which is therefore treated as an effective Lagrangian, and $\bar{\theta} \equiv \theta_{\text{QCD}} + \theta_{\text{QFD}}$ can be interpreted as an effective coupling strength. The magnitude of $\bar{\theta}$ must be very small ($< 10^{-8} \sim 10^{-9}$) from the observed bound on the neutron electric

dipole moment. Therefore, this is a naturalness problem, the so-called *strong CP problem*. Even though the free energy behaves like Fig. 1(a), the $\bar{\theta}$ cannot change due to Eq. (1). Whichever value of $\bar{\theta}$ is chosen, it correctly describes a world having the coupling constant $\bar{\theta}$.

The solution to the strong CP problem must be accompanied by the mechanism of the weak CP violation because one also has to include $\theta_{\text{QFD}} = \text{Arg. det } M_q$. The known strong CP solutions are of three types;

- (a) Natural in the sense of symmetry with hard weak CP violations,
- (b) Natural in the sense of calculability with spontaneous CP violations, and
- (c) Natural in the sense of calculability with soft CP violations.

The solutions (b) and (c) impose the CP symmetry on the hard terms of the effective Lagrangian and on Eq. (2), which implies that the hard weak CP violations are not available.⁵ On the other hand, recent models of Nelson and Barr in the sense of calculability use fine tuning of θ_{QCD} and Yukawa couplings to obtain a Barr texture with a possibility of complex Yukawa couplings.⁶ This type of solution may be useful in GUTS. By far the most attractive solution is of type (a), because it is natural in the sense of 't Hooft: it can accommodate hard weak CP violations, in particular the Kobayashi-Maskawa mechanism, and for the interest of this conference it is applicable to cosmology. Let us now discuss this solution.

1.3. The Dynamical $\bar{\theta}$ Parameter.

If the vacuum sits at a non-zero point $\bar{\theta}$ (see Fig. 1 (a)), it is tempting to make $\bar{\theta}$ roll over to zero. The free energy (Fig. 1 (a)) in a $\bar{\theta}$ vacuum does not change because of a superselection rule. The rolling down can happen only if one makes $\bar{\theta}$ a dynamical variable. For this solution, we need a field degree of freedom. To find this, we examine the massless quark solution of the strong CP problem by Callan, Dashen and Gross.⁴ With a massless quark, the vacuum $|0\rangle$ does not tunnel to any other $|n \neq 0\rangle$ vacuum, and we do not consider Eq. (2) as an effective Lagrangian. The massless quark model has a chiral symmetry: $q \rightarrow e^{i\gamma_5 \alpha} q$, $\theta \rightarrow \theta + 2\alpha$. This solution is not phenomenologically successful, but we can fully use the symmetry they obtained toward a realistic solution. Because $\bar{\theta}$ is a periodic variable, the needed field degree of freedom is a phase field. Namely, one needs a complex scalar field for the necessary field degree of freedom. Thus the dynamical $\bar{\theta}$ parameter or the Peccei-Quinn solution needs both

- (i) A complex scalar field for a dynamical $\bar{\theta}$ parameter, and
- (ii) A global $U(1)_A$ symmetry which is anomalous in the quark sector.

One has to make sure that the dynamical $\bar{\theta}$ parameter of (i) corresponds to the phase field of the global symmetry of (ii). Because it is anomalous in the quark sector, the dynamical field a will feel a *Potential* shown in Fig. 1 (b) and will roll down to zero. Then $\bar{\theta}$ follows a as indicated in Fig. 1. This is possible because the superselection rule does not hold due to $U(1)_A$ symmetry.

Figure 1

Free energy of $\bar{\theta}$ vacuum (a), and invisible axion potentials with the domain wall numbers (b) $N_{DW} = 1$, and (c) $N_{DW} = 2$.

The minimal electroweak model of Weinberg and Salam does not contain an available phase field. To have a dynamical $\bar{\theta}$ parameter ($= a$) in $SU(2) \times U(1)$, Peccei and Quinn had to introduce an additional complex Higgs doublet, so that a dynamical degree of freedom was present. However, if all the possible couplings allowed by $SU(2) \times U(1)$ are present, the needed $U(1)_A$ global symmetry is absent. Namely, by requiring a $SU(2) \times U(1) \times U(1)_A$ symmetry, the Higgs potential does not contain $\mu_{12} H_1^i H_2^j \epsilon_{ij} + c_{12} (H_1^i H_2^j \epsilon_{ij})^2 + \text{h.c.}$, where H_1 and H_2 are the two Higgs doublets, and the Yukawa couplings do not admit $\bar{q}_L u_R H_1^*$ and $\bar{q}_L d_R H_2^*$ terms. The dynamical degree in this model is known to be the Peccei-Quinn-Weinberg-Wilczek (PQWW) axion,^{3,7} whose mass is about 150 KeV and whose interaction with electrons and light quarks is semiweak.

2. THE INVISIBLE AXION AND ITS PROPERTIES

2.1. The Invisible Axion Component

The trouble of the PQW axion is mainly a phenomenological one, that is, it is not observed in decay modes of K and ψ , in reactor experiments and in beam dump experiments. Therefore, a successful solution will be obtained if one decouples the interaction strength of the dynamical degree of freedom $\bar{\theta}$ from the electroweak scale 247 GeV. This can happen only if the needed complex scalar is an $SU(2) \times U(1)$ singlet field,^{8,9} say σ . Indeed, there exist numerous $SU(2) \times U(1)$ singlet scalar fields in spontaneously broken grand unified theories. The vacuum expectation value $\langle \text{VEV} \rangle < \sigma > \equiv v$ can be very large without changing the successful phenomenology of $SU(2) \times U(1)$. Now where is (are) the anomalous quark(s)?

The first solution³ is to introduce heavy quarks $\{Q, \bar{Q}\}$ so that the needed $U(1)_A$ symmetry is present. The singlet σ couples to the heavy quarks by $\bar{Q}Q\sigma + \text{h.c.}$ Then the axion field ($\equiv$ phase of σ) will feel a potential (Fig. 1(b)) due to light quarks interacting with heavy quarks by gluon exchange.

The second solution⁹ is to treat light quarks as the anomalous quarks. But then, one has to introduce an additional Higgs doublet as in the Peccei-Quinn model, so that σ couples to two Higgs doublet systems $\{H_1, H_2\}$ and in turn Higgs doublets couple to the light quark system. In this way the axion field feels the potential of Fig. 1(b).

Being a Goldstone boson, the invisible axion¹⁰⁻¹² a can be properly described as a phase field of a continuous global symmetry $U(1)_I$. The classical Lagrangian is invariant under the transformation

$$\begin{aligned} U(1)_I: \quad a &\rightarrow a + \lambda \\ \phi_1 &\rightarrow e^{i\Gamma_1 \lambda} \phi_1 \\ \phi_2 &\rightarrow e^{i\Gamma_2 \lambda} \phi_2, \text{ etc.} \end{aligned} \quad (3)$$

Since the axion a is neutral, we are interested in neutral components of Higgs fields which are parametrized by

$$\begin{aligned} \phi_1^0 &= \frac{1}{\sqrt{2}} (\rho_1 + v_1) e^{iP_1/v_1} \\ \phi_2^0 &= \frac{1}{\sqrt{2}} (\rho_2 + v_2) e^{iP_2/v_2}, \text{ etc.} \end{aligned} \quad (4)$$

where $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$, $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$, etc. As a simple example, let us take the Peccei-Quinn model of two Higgs doublets, and redefine the two phases by

$$P_1 = a \cos \theta - h \sin \theta; \quad P_2 = a \sin \theta + h \cos \theta \quad (5)$$

Then we obtain $\cos \theta = v_1 \Gamma_1 / (v_1^2 \Gamma_1^2 + v_2^2 \Gamma_2^2)^{1/2}$, $\sin \theta = v_2 \Gamma_2 / (v_1^2 \Gamma_1^2 + v_2^2 \Gamma_2^2)^{1/2}$ and $\lambda' / \lambda = (v_1^2 \Gamma_1^2 + v_2^2 \Gamma_2^2)^{1/2}$. Therefore the axion component is $a_{PQ} = (v_1 \Gamma_1 P_1 + v_2 \Gamma_2 P_2) / (v_1^2 \Gamma_1^2 + v_2^2 \Gamma_2^2)^{1/2}$. The next thing is to find out the charges Γ_1 and Γ_2 . Since the longitudinal component of the Z boson is $h \propto v_1 \Gamma_1^Z P_1 + v_2 \Gamma_2^Z P_2 = v_1 P_1 - v_2 P_2$ (note $\Gamma^Z \equiv 2I_3$, for neutral components), the axion component is $a_{PQ} \propto v_2 P_1 + v_1 P_2$. In general, the axion (or Goldstone) field is obtained from the phase fields of $U(1)_T$ non-trivial Higgs fields by

$$a = \frac{1}{N} \sum_i \Gamma_i v_i P_i, \quad N^2 = \sum_i v_i^2 \Gamma_i^2 \quad (6)$$

where N is the normalization constant. Note that N depends on both the VEV and the magnitude of the $U(1)_T$ charge. The tree-level axion coupling to fermions is

$$-\frac{ia}{N} \sum_i m_i \bar{f}_i \gamma_5 f_i (\Gamma_i^L - \Gamma_i^R); \quad m_i = f_{ij} \frac{v_j}{\sqrt{2}} \quad (7)$$

where f_{ij} is defined by $f_{ij} \bar{\psi}_R^i \psi_L^j \phi_j + \text{h.c.}$ In the heavy quark example,⁸ the tree-level coupling to light quarks vanishes, but they can be present at higher orders.

The current algebra estimate of the axion mass is¹³

$$m_a = \frac{f_\pi m_\pi}{N} \frac{Z^{1/2}}{1+Z} |\xi_p + \xi_Q| \quad (8)$$

where $Z \equiv m_u < \bar{u}u > / m_d < \bar{d}d > \approx 0.56$, and the axion couplings to light quarks via tree-level through mixing and via the two-loop diagram are denoted as ξ_p and ξ_Q respectively;

$$\xi_p = \sum_{i = \text{light } q_L, \bar{q}_R} \Gamma(i) \quad (9)$$

$$\xi_Q = \sqrt{2} \frac{\alpha^2 c}{\pi^2} \sum_{j = \text{heavy } Q_L, \bar{Q}_R} \Gamma(j) \ln \frac{m_{Q_j}^2}{m_u m_d} \quad (10)$$

2.2 Invisible Axion Models

The discussion of the previous subsection can be easily applied to the Dine-Fischler-Srednicki model. It introduces two Higgs doublets and a complex Higgs singlet with the coupling $\epsilon_{ij} H_1^i H_2^j \sigma + \text{h.c.}$ The Yukawa couplings are the same as in the Peccei-Quinn model. The $U(1)_T$ charges are, $H_1: v_2/v_1$, $H_2: v_1/v_2$, $\sigma: -\frac{1}{2}((v_2/v_1) + (v_1/v_2))$, $u_R: -v_1/v_2$, $d_R: -v_2/v_1$, and $q_L: 0$. Note that q_L has zero eigenvalue, which implies that the Kobayashi-Maskawa CP violation is possible. The normalization constant is

$$N = \left\{ \left(\frac{v_2}{v_1} \right)^2 v_1^2 + \left(\frac{v_1}{v_2} \right)^2 v_2^2 + \frac{1}{4} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right)^2 v_0^2 \right\}^{\frac{1}{2}} \quad (11)$$

where $\langle \sigma \rangle = v_0 / \sqrt{2}$. Because N is huge for $v_0 \gg v_1, v_2$, the axion mass and its coupling to quarks are very small. For N_g generations of quarks, we have

$$m_a \cong 3.7 \times 10^{-4} \text{ eV} \left(\frac{10^{11} \text{ GeV}}{v_0} \right) \left(\frac{N_g}{3} \right) \quad (12)$$

In the DFS model, $\xi_p = -3((v_2/v_1) + (v_1/v_2))$ is of order 1.

As mentioned in Sec. 2.1., the heavy quark model also solves the strong CP problem. Since a direct coupling of instantons to heavy quarks is exponentially suppressed, the dominant contribution to the axion potential arises from

Figure 2.

coupling to light quarks at the two-loop level, (see Fig. 2). (A similar diagram for the Z boson has been considered in Ref. 14). The axion coupling to light quarks is⁸

$$- i \epsilon_q \bar{q} \gamma_5 q a, \quad q = u \text{ or } d \quad (13)$$

$$\epsilon_q = \frac{\alpha_c^2}{\pi^2} \frac{m_q}{v_0} \ln \frac{m_Q}{m_q} \quad (14)$$

where $\langle \sigma \rangle = v_0 / \sqrt{2}$. To use the Bardeen-Tye calculation¹³ we derive an effective $U(1)_T$ symmetry including the term (13),

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{1}{2} \partial_\mu a \partial^\mu a + \bar{u} i \not{\partial} u + \bar{d} i \not{\partial} d \\ & - m_u (\bar{u} u \cos \frac{\epsilon_u}{m_u} a + \bar{u} i \gamma_5 u \sin \frac{\epsilon_u}{m_u} a) \\ & - m_d (\bar{d} d \cos \frac{\epsilon_d}{m_d} a + \bar{d} i \gamma_5 d \sin \frac{\epsilon_d}{m_d} a) \end{aligned} \quad (15)$$

$$U(1)_T: \quad a \rightarrow a + \lambda$$

$$\begin{aligned} u & \rightarrow \left(1 - i \frac{\lambda \epsilon_u}{2 m_u} \gamma_5 \right) u \\ d & \rightarrow \left(1 - i \frac{\lambda \epsilon_d}{2 m_d} \gamma_5 \right) d \end{aligned} \quad (16)$$

Now we insert into the formula (8) to find⁸

$$m_a^Q = \frac{f_\pi m_\pi}{N} \frac{\sqrt{Z}}{1+Z} \sqrt{2} \frac{\alpha_c^2}{\pi^2} \ln \frac{m_Q^2}{m_u m_d}. \quad (17)$$

In general, $\xi_Q = \sqrt{2} (\alpha_c^2/\pi^2) \sum_{\text{heavy quarks}} \Gamma(j) \ln(m_Q^2/m_u m_d)$ is of order 10^{-2} . Fig. 3 illustrates how Eq. (8) is derived from light and heavy quark contributions.

Figure 3.

The light quarks (single solid lines) and heavy quarks (double solid lines) contributing to the invisible axion mass.

2.3. Vacuum Structure and Domain Walls

The Peccei-Quinn symmetry is the symmetry of the classical Lagrangian, but is not a symmetry of the full quantum field theory. The anomalous axial current breaks the Peccei-Quinn symmetry, and the axion feels the potential shown in Figs. 1(b) and (c). Note that the gluon anomaly $\theta \tilde{F}\tilde{F}$ term has a period 2π , i.e., we identify θ as $\theta + 2\pi$. The $U(1)_F$ rotation for N_{DW} quarks by $q \rightarrow e^{i\gamma_5 \alpha} q$ changes θ to $\theta + 2N_{DW}\alpha$. If $2N_{DW}\alpha$ is a multiple of 2π , the Lagrangian, $\theta \tilde{F}\tilde{F} + \text{quark terms}$, is left unchanged, which happens for $\alpha = 2\pi l/2N_{DW}$ with $l = 0, 1, 2, \dots, N_{DW}-1$. Under the $U(1)_F$ rotation, the scalar field phases may not rotate by integer multiples of 2π . For example, the DFS model with 3 generations of quarks ($N_{DW} = 6$) has a six-fold vacuum degeneracy, since the phases of Higgs doublets H_1, H_2 , and the Higgs singlet σ change by $-2\pi/6, -2\pi/6$, and $2\pi/6$, respectively. This discrete subgroup $Z(6)$ is unbroken, and the DFS model has the usual six-fold domain wall problem.¹⁵

Here, I only mention the solutions to this domain wall problem:

- (i) Softly broken Peccei-Quinn symmetry,¹⁵
- (ii) One heavy quark invisible axion model,^{8,15,16}
- (iii) Lazarides-Shafi mechanism,¹⁷ and
- (iv) Inflation with the reheating temperature $< v$, where we ordered the solutions according to increasing plausibility.

3. AXION MASS CONSTRAINTS

3.1. From Astrophysics

Since the particle physics bound on the axion mass ($v > 10^4$ GeV from beam dump experiments and $v > 10^5$ GeV from K decay modes) are not so restrictive as the astrophysical and the cosmological bounds, we will discuss the latter two.

Any light and weakly interacting particle has a potential for playing energy loss mechanism in the stellar evolution. From the studies of the evolution of the sun, it is known that the standard model of the sun gives a quite reasonable luminosity. Also, the helium burning red giants constrain the amount of energy loss by the invisible axion. A phenomenological bound on the parameters of the axion is

$$f^2 \exp(-m_a/T_c) < (10^{-11})^2 \quad (18)$$

where T_c is the temperature of the core of the red giant and f is the generic coupling strength of the axion to the u and d quarks and the electron. The factor f comes from the strength of the interaction and the Boltzmann factor is included to take into account the equilibrium number density of the axion in the core of the star. Eq. (18) is not meant to be accurate, but is presented for a qualitative understanding. The right-hand side is obtained from the energy loss rate of a typical red giant $Q_a \leq 100 \text{ erg} \cdot \text{g}^{-1} \cdot \text{sec}^{-1}$. For the PQWW axion, we know the value f , $f \approx 1 \text{ MeV}/250 \text{ GeV}$. Therefore, we obtain $m_a > 20 T_c \approx 292 \text{ KeV}$ which is not a bad estimate compared to the 350 KeV of Dicus et al.^{18c}

The possibility of a small f consistent with astrophysics was first pointed out by Kim in Ref. 8, and in this case one can ignore the Boltzmann factor. Then one obtains $v > 10^{11} m_{e,q}/(\xi_p + \xi_Q)$. More serious calculations¹⁹ were done by Dicus et al in 1980 and by Fukugita et al in 1982. The astrophysical bound is

$$m_a > 350 \text{ KeV or } m_a < (0.1 \sim 0.01) \text{ eV} \quad (19)$$

3.2. From Cosmology

One of the most important advances in cosmology was made by Preskill, Wise and Wilczek, Abbott and Sikivie, and Dine and Fischler.^{20,21} They observed that axions can be created coherently out of a vacuum when the axion field feels the very gentle hills of Fig. 1(b). Now I review the original idea, and use the result to discuss a possibility for raising the Peccei-Quinn symmetry breaking scale.

If we do not consider cosmology, the vacuum is chosen at $\langle a \rangle = 0$ or $\bar{\theta} = 0$. But the cosmological interpretation is quite different for the invisible axion potential of Fig. 1(b), because the valley is so flat that the coherent motion

of invisible axions does not have much chance to settle down to $\langle a \rangle = 0$ if $\langle a \rangle$ was of order v in the beginning. Taking into account the expansion of the universe, the classical motion of the invisible axion is described by

$$\frac{d^2 \langle a \rangle}{dt^2} + 3H \frac{d \langle a \rangle}{dt} + m_a^2(t) \langle a \rangle \cong 0 \quad (20)$$

If the expansion rate is greater than the oscillation rate of the collective motion, the vacuum of the invisible axion has not had a chance to oscillate. On the other hand, if the oscillation rate is greater than the expansion rate, the collective motion is present. This consideration sets the time t_i (or the temperature T_i) when the collective motion of the invisible axion starts to develop. Since T_i is expected to be of order 1 GeV, we have

$$m_a(T_i) \cong 3H = 3 \sqrt{8\pi G \rho / 3} \cong 39.14 \frac{T_i^2}{M_{Pl}} \quad (21)$$

For the temperature-dependent axion mass, we fold to the high temperature estimate³² of the free energy of $\bar{\theta}$ to obtain

$$m_a(T_i) = \frac{0.024}{v} (\Lambda m_u m_d m_s)^{1/2} (9 \ln \frac{\pi T_i^3}{\Lambda}) \left(\frac{\Lambda}{\pi T_i}\right)^4 \xi \quad (22)$$

where Λ is the QCD scale of order 200 MeV and ξ is defined in Eq. (8). From Eqs. (21) and (22), we notice that it does not depend very much on the details of the model unless v is very large.

From Eq. (20), we obtain

$$\frac{(m_a A^2)_f}{(m_a A^2)_i} = \left(\frac{R_i}{R_f}\right)^3 = \left(\frac{T_f}{T_i}\right)^3 \quad (23)$$

where A is defined by $\langle a \rangle = A(t) \cos m_a t$, R is the scale factor, and T_f is the invisible axion cosmic temperature in which we are interested. Therefore, we interpret $\frac{1}{2} m_a A^2$ as the axion number density. The energy density due to axions is

$$\begin{aligned} \rho_a(T_f) &= m_a(T_f) n_a(T_f) = \frac{1}{2} m_a(T_f) m_a(T_f) A_f^2 \\ &= 0.012 \xi^2 \left(\frac{A_i}{v}\right)^2 f_\pi m_\pi (\Lambda m_u m_d m_s)^{1/2} (9 \ln \frac{\pi T_i^3}{\Lambda}) \left(\frac{\Lambda}{\pi}\right)^4 \frac{T_f^3}{T_i} \frac{1}{T_i^6} \end{aligned} \quad (24)$$

where Eqs. (22) and (23) are used to relate A_f to A_i which is known to be of order v . Noting that the critical energy density of our universe is $\rho_c(T_\gamma = 2.7^\circ \text{K}) = 8 \times 10^{-47} h^2 [\text{GeV}^4]$, where h is the Hubble parameter in units of 100 km sec⁻¹ Mpc⁻¹, we obtain in view of Eqs. (21) and (22),

$$\Omega_a = \xi 0.53 \times 10^{-7} h^{-2} \left(\frac{\text{GeV}}{T_i}\right) \left(\frac{T_\gamma}{2.72}\right)^3 \left(\frac{A_i}{v}\right)^2 \frac{v}{M_{Pl}} \quad (25)$$

Because of the huge coefficient, ν must be very small compared to M_{Pl} for the collective invisible axions not to dominate the energy density of our universe. For $\xi = 1$ and $h = 1/2$, we obtain

$$\nu < 2 \times 10^{12} \text{ GeV and } T_i \cong 1 \text{ GeV for } \Omega_a < 10;$$

$$\nu = 2.8 \times 10^{11} \text{ GeV and } T_i \cong 1.5 \text{ GeV for } \Omega_a = 1;$$

$$\nu = 0.75 \times 10^{11} \text{ GeV and } T_i = 2 \text{ GeV for } \Omega_a = 0.2.$$

At present $T_f \cong 0.5 T_\gamma$ and $A_f = (m_a(T_i)/m_a(T_f))^{1/2} (T_f/T_i)^{3/2} A_i \cong 0.5 \times 10^{-21} A_i$ for the case $\Omega_a = 1$. The QCD angle of magnitude 10^{-21} is at present the value to close the universe with $\nu = 2.8 \times 10^{11} \text{ GeV}$, but it is sufficiently small by the particle physics standard.

3.3. The Model-Dependent Upper Bound on ν .

As is evident from Eq. (25), the smaller ξ is, the less energy is contributed by the invisible axion. This seems to be the only reasonable way to raise the Peccei-Quinn symmetry breaking scale ν . We show indeed that this scenario can be achieved depending on invisible axion models.

The temperature-dependent axion mass Eq. (22) can be trusted for $T \gtrsim 200 \text{ MeV}$, because the expression has the maximum point at $e^{3/4} \cdot \Lambda/\pi$. This shape is sketched in Fig. 4. Presumably, if $T \lesssim 100 \text{ MeV}$, we have to use Eq. (8) of the zero temperature axion mass. For $100 \text{ MeV} < T < 200 \text{ MeV}$, we do not have any reliable estimate, and the prediction is weak. For $T_i \lesssim 100 \text{ MeV}$, i.e. $T_i = (\xi M_{Pl}/\nu)^{1/2} \cdot 30 \text{ MeV}$, the Peccei-Quinn symmetry breaking scale is $\nu/\xi > 10^{18} \text{ GeV}$. For $T_i \gtrsim 200 \text{ MeV}$, the valid range of ν is $\nu/\xi < 2.7 \times 10^{15} \text{ GeV}$. The conclusion of the previous subsection falls in this region, and the estimate is reliable.

We proceed to discuss the possibility of a small ξ .

For the heavy quark invisible axion model,⁸ $\xi_p = 0$ and $\xi_Q \cong 0.7 \times 10^{-2}$ for $m_Q = 10^{15} \text{ GeV}$ and $\alpha_c = 1/40$. Following the previous discussion, we obtain $\nu = 0.7 \times 10^{14} \text{ GeV}$ and $T_i = 140 \text{ MeV}$ for $\Omega_a = 1$. Certainly, T_i falls in the

Figure 4.
A schematic view of the axion mass vs. temperature.

question mark region, and the correct value will be a little bit different from this.

In models without heavy quarks, can we make ξ small? Indeed, this can also happen if we make the invisible axion reside in a remote singlet which communicates with the $SU(2) \times U(1)$ Higgs doublets in many steps. For example, with 3 Higgs singlets, we can construct quartic terms

$$\varepsilon_{ij} H_i^{\dagger} H_j^{\dagger} \sigma_1 \sigma_1 + \sigma_1 \sigma_1 \sigma_2 \sigma_2 + \sigma_2 \sigma_2 \sigma_3 \sigma_3 + \text{h.c.} \quad (26)$$

Then, there results from this $\Gamma(\sigma_3) = -3\Gamma(\sigma_2) = 9\Gamma(\sigma_1)$, hence the invisible axion mixing with the phases of H_1 and H_2 is suppressed by a factor 9.5 if $\langle \sigma_1 \rangle \approx \langle \sigma_2 \rangle \approx \langle \sigma_3 \rangle$ (See Eq. (6)). This process can continue at will. Many invisible axion models suggested so far in connection with the domain wall problem reveal this property.

The upper bound on ν depends on the invisible axion models.

For an application of the invisible axion in galaxy formation, see excellent talks by Zee,²³ Turner,²⁴ and Szalay²⁵ at this meeting. For another attempt to understand a small $\bar{\theta}$ at T_i in the new inflationary universe scenario, see Pi's talk²⁶ at this meeting. Other various aspects of the invisible axion are reviewed in Refs. 10-12.

4. THE GRAVITINO PROBLEM

Any stable particle has a forbidden mass region from consideration of the energy density of the universe.^{27,28} For a light gravitino, the allowed mass of the gravitino is $m_{3/2} \lesssim 1 \text{ KeV}$.²⁹ For a stable heavy gravitino, the bound may be 10^{16} GeV . However, a gravitino of this heavy mass is not stable and one has to consider the unstable particle case. Weinberg found a somewhat lower mass for the unstable gravitino by changing the history of the universe even after the original nucleosynthesis era.³⁰ This is possible if the reheating temperature after gravitino decay is sufficiently high ($> 0.4 \text{ MeV}$) so that all synthesized nuclei are burnt and new nucleosynthesis starts from the scratch. This scenario is possible if $m_{3/2} > 10 \text{ TeV}$. The gravitino mass in the region between 1 KeV and 10 TeV is forbidden cosmologically.

However, a 100 GeV gravitino motivated from particle physics falls in this forbidden zone.² Ellis, Linde, and Nanopoulos suggested inflation for dilution of the gravitino number density,³¹ but it became apparent that gravitinos are regenerated after inflation.³²

Recently, we took this problem seriously and calculated the number density of gravitinos using $N=1$ supergravity and supersymmetric gauge Lagrangians.³³ The result is expressed as a function of T_{max} , the reheating temperature after inflation. This estimate is a lower bound of the gravitino number density,

since there may be other sources of gravitino production.³⁴ The gravitino production by two body interactions is most significant at $T \sim T_{\max}$, and hence we have

$$\begin{aligned} \Delta n_{\psi} (at T_f) &= \sum_{ab} \sigma_t(ab \rightarrow c\psi_{\mu}) M \frac{\xi(3)^2}{\pi^4} N_a N_b \sqrt{\frac{90}{\pi^3 N}} T_{\max} T_f^3 \\ &= 3.35 \times 10^{-12} T_f^3 T_9^{\max} (1 - 0.018 \ln T_9^{\max}), \end{aligned} \quad (27)$$

where $M = 2.4 \times 10^{18}$ GeV. The gravitino decays when the temperature of the universe becomes $T_f = (90/32\pi^3 N)^{1/4}$. $(9m_{3/2}^2/8M_{Pl})^{1/2} \approx 10^{-4}$ MeV for the 100 GeV gravitino.

At the time of the nucleosynthesis, the gravitino energy density is not significant unless the reheating temperature is greater than 10^{16} GeV. The nucleosynthesis bound is not restrictive.

The requirement that the entropy in gravitino decay products does not exceed that already present gives a stronger bound $T_{\max} < 2.2 \times 10^{13}$ GeV

The most stringent bound comes from the consideration of D dissociation after 10^8 secs. A gravitino with mass $O(100)$ GeV will dump ultimately 80% of its energy in the form of photons after all hadrons and charged leptons annihilate. The photons lose their energy by producing $e^+ e^-$ until their energy is degraded to 3 GeV. Then they slowly lose energy down to 2.3 MeV.

Below the photon threshold energy, D is not dissociated by the low energy photons. Roughly thirty thousand 2.3 MeV photons are produced from one gravitino decay. The probability for one such photon to destroy D is

$$p(E) = \frac{n_D^{\sigma_D}}{n_e^{\sigma_T}} \quad (28)$$

where E is the photon energy, n_D , n_e , σ_D and σ_T are the D number density, electron number density, D dissociation cross-section and total cross-section of the high energy photons. Integrating this probability function weighted over the photon energy distribution function, we have the condition³⁵

$$(m_{3/2} f) \beta n_{3/2}/n_e E_* \lesssim 1 \quad (29)$$

where we can use $f = 0.8$, $\beta = 0.23$ and $E_* = 100$ MeV for $T \approx 10^{-4}$ MeV. Therefore, we obtain

$$\begin{aligned} T_{\max} &= 2.1 \times 10^9 \text{ GeV for } \delta_B = 10^{-10} \\ T_{\max} &= 2.2 \times 10^{10} \text{ GeV for } \delta_B = 10^{-9} \end{aligned} \quad (30)$$

where $\delta_B = n_B/n_{\gamma}$. This is a very low reheating temperature, and there seem to be conflicts in supergravity models which have 100 GeV gravitino.³⁶ For solutions using flat supergravity potentials, see the talk by Nanopoulos³⁷ at this meeting.

5. CONCLUSION

I emphasized the nature of the dynamical degree of freedom in the Peccei-Quinn solution of the strong CP problem. The invisible axion resides in the phase (the needed dynamical degree) of an $SU(2) \times U(1)$ singlet complex scalar field. Also it is shown that the 10^{14} GeV scale for Peccei-Quinn symmetry breaking is not unreasonable. On the other hand, a 100 GeV gravitino scenario seems to be in trouble from the abundances of light nuclei of our universe.

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